Five conjectures on primes based on the observation of Poulet and Carmichael numbers

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Abstract. In this paper I enunciate five conjectures on primes, based on the study of Fermat pseudoprimes and on the author’s believe in the importance of multiples of 30 in the study of primes.

Conjecture 1:

For any $p, q$ distinct primes, $p > 30$, there exist $n$ positive integer such that $p - 30*n$ and $q + 30*n$ are both primes.

Note:
This conjecture is based on the observation of 2-Poulet numbers (see my paper “A conjecture about 2-Poulet numbers and a question about primes”).

Conjecture 2:

For any $p, q, r$ distinct primes there exist $n$ positive integer such that the numbers $30*n - p$, $30*n - q$ and $30*n - r$ are all three primes.

Note:
This enunciation is obviously equivalent to the enunciation that there exist $m$ such that $p + 30*m$, $q + 30*m$ and $r + 30*m$ are all three primes (take $x = 30*n - p$, $y = 30*n - q$ and $z = 30*n - r$. Then there exist $k$ such that $30*k - 30*n + p$, $30*k - 30*n + q$ and $30*k - 30*n + r$ are all three primes).

Note:
This conjecture implies of course that for any pair of twin primes $(p, q)$ there exist a pair of primes $(30*n - p, 30*n - q)$ so that there are infinitely many pairs of twin primes.

Note:
This conjecture is based on the observation of 3-Carmichael numbers (see my paper "A conjecture about primes based on heuristic arguments involving Carmichael numbers").

**Conjecture 3:**

There exist an infinity of pairs of distinct primes \((p, q)\), where \(p < q\), both of the same form from the following eight ones: \(30k + 1\), \(30k + 7\), \(30k + 11\), \(30k + 13\), \(30k + 17\), \(30k + 19\), \(30k + 23\) and \(30k + 29\) such that the number \(pq + (q - p)\) is prime.

**Note:**

This conjecture is based on the observation of Carmichael numbers.

**Examples:**

: \(31 \times 151 + (151 - 31) = 4801\) prime;
: \(37 \times 127 + (127 - 37) = 4789\) prime;
: \(41 \times 101 + (101 - 41) = 4201\) prime;
: \(13 \times 103 + (103 - 13) = 1429\) prime;
: \(17 \times 47 + (47 - 17) = 829\) prime;
: \(19 \times 109 + (109 - 19) = 2161\) prime;
: \(23 \times 53 + (53 - 23) = 1249\) prime.

**Conjecture 4:**

There exist an infinity of pairs of distinct primes \((p, q)\), where \(p < q\), both of the same form from the following eight ones: \(30k + 1\), \(30k + 7\), \(30k + 11\), \(30k + 13\), \(30k + 17\), \(30k + 19\), \(30k + 23\) and \(30k + 29\) such that the number \(pq - (q - p)\) is prime.

**Note:**

This conjecture is based on the observation of Carmichael numbers.

**Examples:**

: \(31 \times 61 - (61 - 31) = 1861\) prime;
: \(7 \times 37 - (37 - 7) = 229\) prime;
: \(11 \times 41 - (41 - 11) = 421\) prime;
: \(13 \times 73 - (73 - 13) = 919\) prime;
: \(17 \times 47 - (47 - 17) = 769\) prime;
: \(19 \times 139 - (139 - 19) = 2521\) prime;
: \(23 \times 293 - (293 - 23) = 6469\) prime.
Conjecture 5:

For any p prime there exist an infinity of primes q, q > p, where p and q are both of the same form from the following eight ones: 30*k + 1, 30*k + 7, 30*k + 11, 30*k + 13, 30*k + 17, 30*k + 19, 30*k + 23 and 30*k + 29 such that the number p*q - (q - p) is prime.