On the Confinement of Quarks without Applying the Bag Pressure

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Abstract

We explain a fatal error in quantum chromodynamics. By applying a correction to the dynamics of quarks, we can confine quarks in hadrons. We will show why quarks do not obey the Pauli exclusion principle and why we cannot observe free quarks. In addition, we obtain correct hadron sizes.

1 Introduction

Two electrons with identical quantum numbers cannot exist in a hydrogen atom, because each electron is subluminal and its phase velocity is superluminal. When there are 2 electrons with identical quantum numbers in a hydrogen atom or with identical energy levels in a cubic box, the second electron exists at every location (space-time coordinates) with exactly identical wave function characteristics to those of the first electron. In other words, the two electrons simultaneously exist at an exact point at the same time. This phenomenon is a consequence of the probabilistic characteristics of wave functions and quantum mechanics. Specifically, the wave equation does not provide us with more information about the exact location of each electron. The energy and absolute value of the momentum of each electron are exactly determined, but they do not have specific locations. At a given time, they are ubiquitous at every location where the wave function does not vanish. However, we can have 3 identical quarks with identical spin states in baryons. To explain this phenomenon, we propose a strange theorem:

Theorem 1. *Quarks are superluminal particles.*

Any specific change in the state of a wave function in its associated Hilbert space will propagate in space-time coordinates with the phase velocity of the wave function in space-time. Specifically, particles communicate with each other at their phase velocities [7]. We postulate that quarks are superluminal. Because each quark is superluminal, its phase velocity should be subluminal; thus, quarks with identical spins can occupy the same energy level in hadrons. In other words, the first quark is unaware of the spin and characteristics of the second quark, because their phase velocities are subluminal. If we change the wave function of the second quark, this change will propagate at less than c to other space-time

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locations in the bag. The phase velocity is not measured in a space-like region. Specifically, two quarks with identical energies and momenta are located at different points in the bag. Quantum mechanics postulates that, at a specific time, a subluminal particle with a specific energy-momentum does not have a specific location. In other words, it is ubiquitous in the bag. However, because the phase velocity of a superluminal particle is subluminal, a superluminal particle is no longer ubiquitous. Thus, two superluminal particles that are confined in a cubic box no longer exist at the exact space-time point. Thus, it is not necessary for them to obey the Pauli exclusion principle. The exclusion principle is applicable to two identical particles with identical wave function characteristics.

Theoretically, as we mentioned previously, the wave function of a single superluminal particle cannot collapse, because the phase velocity of collapse is subluminal and obeys causality [7].

Before the wave function collapses, the particle does not have a specific location. We create its location by performing an experiment and measuring its location. However, after we determine the location of a particle, the particle should not be detected in other locations even in notably far space-like locations that have no causal relation with the location of the collapsed particle. When ψ_{space} of a subluminal particle collapses, it communicates at its phase velocity (at infinite velocity in the reference frame of the collapsed wave function) to other locations in space-time that the wave function should not collapse at other locations of the universe. Thus, a particle cannot be detected in two space-like locations, although two locations do not have a causal relation with each other. However, if the particle is superluminal, its phase velocity is subluminal, and it cannot perform this communication in space-like regions of space-time. The phase velocity must be superluminal to allow for the collapse of the wave function. Because quarks are superluminal, we never observe free quarks. Note that, although we can identify quarks in hadrons using deep inelastic scattering, before scattering, the wave functions of quarks are confined in hadrons, and it is not necessary for the wave functions to communicate with the entire universe to be able to collapse. The above argument is applicable to free quarks.

2 Wave equation of a hydrogen atom with a superluminal electron

There is a significant difference between an ordinary hydrogen atom and a model with a superluminal electron. In the subluminal model, we have negative potential energy. When we increase the energy of the electron in the subluminal model, the momentum of the electron decreases; thus, the wavelength of the electron increases, and the electron increases its distance from the proton. In the subluminal model, although the energy cannot be less than the mass of the particle, the minimum momentum can be zero.

$$E^2 = c^2P^2 + m^2c^4 \tag{1}$$

Thus, the wavelength has no maximum, i.e., it can approach infinity, which results in the escape of an electron from the hydrogen atom according to the Wilson-Sommerfeld rule. The minimum principal quantum number for the minimum radius of the hydrogen atom is $n = 1$.

However, in the superluminal model, although the minimum amount of relativistic energy is zero, the momentum has a non-zero minimum: It cannot be less than the mass of the electron, namely, $m_s c$.

$$c^2P^2 = E^2 + m_s^2c^4 \tag{2}$$

$$E = \frac{m_sc^2}{\sqrt{\beta^2 - 1}} \tag{3}$$

$$P = \frac{m_sv}{\sqrt{\beta^2 - 1}} \tag{4}$$

We see that the electron has a maximum wavelength $\lambda = \hbar/cm_s$. Thus, by the Wilson-Sommerfeld rule, the electron cannot have an infinite wavelength and thus cannot escape the hydrogen atom. This fact sets a limit on the maximum radius of the bag. Thus, the electron in the superluminal model is confined. For the superluminal model, the principal quantum number of the maximum radius of the bag is $n = 1$.

$$\frac{(m_\circ^2c^4 + E^2)^{1/2}}{\hbar c}2\pi r = 1 \tag{5}$$

When the electron energy increases, its momentum increases, but its wavelength decreases; thus, it becomes increasingly confined. The electron falls deeper into the hydrogen atom or bag, which is in contrast to our observation in the subluminal model.

At this point, we seek to derive and solve the wave function of a confined superluminal electron in the hydrogen bag. First, we study the radial Dirac equation. The Dirac equation for a subluminal particle with real mass leads to the following [3]

$$\hbar c\frac{dg(r)}{dr} + (1+\kappa)\hbar c\frac{g(r)}{r} - [E + m_\circ c^2 + \frac{Z\alpha}{r}]f(r) = 0 \tag{6}$$

$$\hbar c\frac{df(r)}{dr} + (1-\kappa)\hbar c f(r) r + [E - m_\circ c^2 + \frac{Z\alpha}{r}]g(r) = 0 \tag{7}$$

The normalized solutions are proportional to

$$\begin{aligned} f(r) \approx & -\frac{1}{\Gamma(2\gamma+1)}(2\lambda r)^{\gamma-1}e^{-\lambda r} \times \\ & \left\{ (\frac{(n'+\gamma)m_\circ c^2}{E} - \kappa)F(-n', 2\gamma+1; 2\lambda r) + n'F(1-n', 2\gamma+1; 2\lambda r) \right\} \end{aligned} \tag{8}$$

$$\begin{aligned} g(r) \approx & \frac{1}{\Gamma(2\gamma+1)}(2\lambda r)^{\gamma-1}e^{-\lambda r} \times \\ & \left\{ (\frac{(n'+\gamma)m_\circ c^2}{E} - \kappa)F(-n', 2\gamma+1; 2\lambda r) - n'F(1-n', 2\gamma+1; 2\lambda r) \right\} \end{aligned} \tag{9}$$

For normalizable wave functions, γ should be positive. κ is the Dirac quantum number, and

$$\lambda = \frac{(m_\circ^2c^4 - E^2)^{1/2}}{\hbar c} \tag{10}$$

$$q = 2\lambda r \tag{11}$$

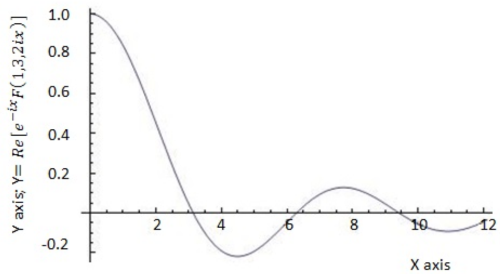

Figure 1: real part of the radial wave function $e^{-ix}F(1,3,2ix)$

$$\gamma = +\sqrt{\kappa^2 - (Z\alpha)^2} = +\sqrt{(j+\frac{1}{2})^2 - (Z\alpha)^2} \tag{12}$$

To terminate the hypergeometric series, we should discard the negative values of n':

$$n = n' + |\kappa| = n' + j + \frac{1}{2} \quad n = 1,2,3 \tag{13}$$

The solution for the hydrogen atom is a hypergeometric function, which is an associated Laguerre polynomial and is characteristic of a wave function in the Coulomb potential.

$$L_n^m(x) = \frac{(n+m)!}{n!m!}F(-n, m+1, x) \tag{14}$$

where $L_n^m(x)$ is the associated Laguerre function [see (8) and (9)].

We mimic the above procedure for the superluminal model with imaginary mass and obtain

$$\hbar c\frac{dg(r)}{dr} + (1+\kappa)\hbar c\frac{g(r)}{r} - [E + im_\circ c^2 + \frac{Z\alpha}{r}]f(r) = 0 \tag{15}$$

$$\hbar c\frac{df(r)}{dr} + (1-\kappa)\hbar c f(r)r + [E - im_\circ c^2 + \frac{Z\alpha}{r}]g(r) = 0 \tag{16}$$

We define λ as

$$\lambda = \frac{(m_\circ^2 c^4 + E^2)^{1/2}}{\hbar c} \tag{17}$$

We solve the above equation and exactly mimic the provided method in the reference for the solution of the Coulomb potential [3]. Finally, we obtain

$$\begin{gathered} g(r) \approx (2\lambda r)^{\gamma-1}e^{-i\lambda r} \times \\ \left\{(\frac{(n'+\gamma)m_\circ c^2}{E} - \kappa)F(-n', 2\gamma+1; 2i\lambda r) - n'F(1-n', 2\gamma+1; 2i\lambda r)\right\} \end{gathered} \tag{18}$$

$$\begin{gathered} f(r) \approx -(2\lambda r)^{\gamma-1}e^{-i\lambda r} \times \\ \left\{(\frac{(n'+\gamma)m_\circ c^2}{E} - \kappa)F(-n', 2\gamma+1; 2i\lambda r) + n'F(1-n', 2\gamma+1; 2i\lambda r)\right\} \end{gathered} \tag{19}$$

In the above equations, $F(-n', 2\gamma+1; 2i\lambda r)$ is normalized for only negative values of n' if

$$-n' < 2\gamma + 1 \tag{20}$$

For example, for $j = \frac{1}{2}$(which gives $\gamma = 1$), and $n' = -1$ we have a well-behaved wave function (figure 1). For $-n' = 2\gamma+1$, the behavior of the wave function $F(-n', 2\gamma+1; 2i\lambda r)$ is similar to $cos(r)$. For negative n', the above hypergeometric equations are similar to the spherical Bessel function of the first type. From (18) and (19), the relation between the hypergeometric series and the Bessel functions is

$$J_\nu(x) = \frac{e^{-ix}}{\nu!}(\frac{x}{2})^\nu F(\nu + \frac{1}{2}, 2\nu + 1, 2ix) \tag{21}$$

The spherical Bessel function of the first type is defined as

$$j_\nu(x) = \sqrt{\frac{\pi}{2x}} J_{\nu+1/2}(x) \tag{22}$$

We observed that the solution for the subluminal hydrogen atom is a Laguerre polynomial. However, we see that $f(r)$ and $g(r)$ for a superluminal electron in the Coulomb potential is similar to the spherical Bessel function of the first type. The spherical Bessel functions appear in only two similar cases. The first case is a particle trapped in an infinite three-dimensional radial well potential. The solutions to this problem are spherical Bessel functions of the first type. Similarly, the solutions to the MIT bag model, which postulated the existence of an unknown pressure and the vanishing of the Dirac current outside the bag, are also spherical Bessel functions of the first type [2, 4].

To create a superluminal Dirac equation for quarks, we can use imaginary mass or substitute the following matrix $\beta_s = i\beta$to calculate $f(r)$ and $g(r)$. However, when we want to construct the Dirac current, we will encounter a problem. The correct method is to consider the following non-Hermitian matrices, where $\beta_s = \beta\gamma_5$ [1, 5]

$$\alpha = \begin{pmatrix} 0 & \sigma \\ \sigma & 0 \end{pmatrix} \qquad \beta_s = \begin{pmatrix} 0 & I \\ -I & 0 \end{pmatrix} \tag{23}$$

This method satisfies all of the required properties of the superluminal Dirac equation. It appears that we should reformulate the QCD Lagrangian and the gluon-gluon interaction term. It can also be shown that the new superluminal (tachyonic) Dirac equation is CP invariant [5]. These facts may provide a solution to the strong CP problem [6]. Note that we did not postulate that the strong force is the electromagnetic force among superluminal particles. However, even if the force among the particles was repulsive in the above equation or its strength with respect to distance did not follow a $\frac{1}{r^2}$ law, the factor that determines whether the system is stable and whether the superluminal positron can escape the proton is the energy of the system and not the attractive or repulsive forces among the particles.

Note that the universe for a superluminal positron in a hydrogen atom is the bag. Its beginning is the boundary of the bag, and its infinity is the center of the bag. The same law that prevents the electron from falling into the proton in the subluminal model prohibits the superluminal positron or electron from escaping from the hydrogen bag. When studying the inter-quark potential, we consider the following conjecture:

Conjecture. The strong force is simply the superluminal effect of the electromagnetic force among superluminal particles.

Without applying any pressure or infinite potential, we have confined the superluminal electron with the appropriate bag radius in the hydrogen atom. In other words, we solved a modified Dirac equation for superluminal particles and substituted the attractive Coulomb potential in the absence of any infinite potential. The solutions were spherical Bessel functions of the first type.

The confinement of quarks in hadrons has a similar mechanism to the above example. It appears that we no longer require the non-Abelian $SU(3)$ symmetry of the strong force to confine quarks in hadrons. This method indicates that we should consider another symmetry group for QCD. Unfortunately, it is not clear why the net electric charge of the bag must be an integer value. In the next section, we provide some elementary examples of computing cross sections using our new insights into quantum chromodynamics.

3 Quantum Electrodynamics of Superluminal Particles

The quantum field theory of superluminal particles is a problematic field theory due to the tachyon condensation effect. In addition, quarks in hadrons do not obey the Pauli exclusion principle, which is a fact that is not considered in tachyonic field theory.

In this section, we use a heuristic approach for the calculation of cross sections in strong interactions. In the superluminal Klein-Gordon equation, the mass term is imaginary, but all other parameters, including the Klein-Gordon current $[j^\mu = (\rho, j)]$, are similar to the subluminal ones. To compute cross sections in the subluminal Dirac and Klein-Gordon equations, we use the flux relation:

$$F = |v_A - v_B|.2E_A.2E_B = 4(|p_A|E_B + |p_B|E_A = 4((P_A.P_B)^2 - m_A^2 m_B^2)) \tag{24}$$

It can be shown that, if we use the superluminal energy-momentum relation (2) instead of (1), the above flux relation remains valid. Thus, we conclude that the cross section formulas for superluminal and subluminal particles have similar expressions.

In the center-of-mass frame, the process $AB \to CD$ for spinless particles, has a differential cross section of [8]

$$\frac{d\sigma}{d\Omega}|_{cm} = \frac{1}{64\pi^2(E_A + E_B)^2}\frac{p_f}{p_i}|\mathcal{M}|^2 \tag{25}$$

where for the amplitude,

$$\mathcal{M} = (ie(p_A + p_C)^\mu)(\frac{g_{\mu\nu}}{q^2})(ie(p_B + p_D)^\nu) \tag{26}$$

$d\Omega$ is the element of the solid angle about P_C, $|P_A| = |P_B| = p_i$, and $|P_C| = |P_D| = p_f$. In the superluminal quark model, if quarks exist at the boundary of the bag, then their speeds will approach infinity, their energies will approach zero, and their momenta will reach the minimum value $m_s c$ (non-relativistic region). In contrast, at the center of the bag, their speeds will approach the speed of light, and their energies and momenta will approach infinity (relativistic region).

In the subluminal model, the energy of the system in the denominator of (25) can never be less than the mass of the interacting particles; thus, the cross section for the minimum initial energy of the interacting particles cannot increase dramatically, but in the superluminal model, if quarks exist at the boundary of the bag (non-relativistic limit and infinite velocity, which in QCD is called a large distance), their cross sections can diverge because the energy in the denominator of the above equation (25) can approach zero. Thus, the cross section diverges at the boundary, and a quark cannot escape from the bag. Therefore, instead of equation (25), we use lattice QCD.

From equation (25), for the very-high-energy subluminal spinless interaction, we have [8]

$$\frac{d\sigma}{d\Omega}|_{cm} = \frac{\alpha^2}{4(E_A+E_B)^2}(\frac{3+cos\theta}{1-cos\theta})^2 \tag{27}$$

where θ is the scattering angle. To obtain this formula, we neglect the mass and equate the energy and momentum in (26). For the superluminal model, the technique is similar and produces a similar result. Thus, equation (27) is applicable to superluminal spinless particles at very high energies. In this limit, all interactions between quarks in hadrons, including QCD and QED interactions, are calculated using one superluminal equation (27), which is also related to the subluminal QED formula. Thus, we falsely conclude that, at small distances, the QCD running coupling constant, which is a function of the energy-momentum of the virtual gluons exchanged between quarks $(p_A - p_C)^2$, disappears. Moreover, the QCD interactions between subluminal particles are negligible, and as a result, we have only the subluminal QED result and not QCD. However, there is no change in the running coupling constant, which can be concluded based on our conjecture.

If we mimic the above procedure for a spinless quark that has approximately zero energy and moves at the boundary of the hadron $(E_i, P_i) = (0, m_s c)$ and $(E_f, P_f) = (0, m_s c)$, we obtain

$$\frac{d\sigma}{d\Omega}|_{cm} = \frac{\alpha^2}{4(E_A+E_B)^2}(\frac{1+cos\theta}{1-cos\theta})^2 \tag{28}$$

Note that we have assumed that the energy of each quark before and after the interaction remains constant $(E_i, P_i) = (0, m_s c)$ and $(E_f, P_f) = (0, m_s c)$.

From equation (5) and considering the experimental values in a typical bag, such as a proton, the ratio of the energies of the quarks to their masses is extremely large. Thus, it is probable that one quark losses all of its energy and moves toward the boundary of the bag and the others gain all of the energy in the bag and move toward the center of the bag. This behavior is completely evident in the parton distribution function $F(x)$, and this low energy region of quarks, namely, $E < m_s c$, is interpreted as a sea of quarks and gluons as $x \to 0$ [8]. If quarks were subluminal, then the energy of a quark could never be less than its mass, and as a result, the shape of the parton distribution function would be different.

At this stage, we study the general form of the cross sections of tachyonic spin half particles. The superluminal Dirac equation can be written as

$$H\psi = c(\alpha.p)\psi + \beta_s m_s c^2\psi = c(\alpha.p)\psi + \beta\gamma_5 m_s c^2\psi \tag{29}$$

or in its abbreviated form as

$$(i\gamma^\mu\partial_\mu - \gamma^5 m)\psi(x) = 0 \tag{30}$$

The tachyonic Lagrangian is [5]

$$\pounds = \frac{i}{2}(\overline{\psi}\gamma^5\gamma^\mu(\partial_\mu\psi) - (\partial_\mu\overline{\psi})\gamma^5\gamma^\mu\psi) - m\overline{\psi}\psi \tag{31}$$

The Dirac current is [1]

$$\rho = \psi^\dagger\gamma_5\psi, \qquad j = c(\psi^\dagger\Sigma\psi) \tag{32}$$

where

$$\gamma_5 = \begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix}, \qquad \Sigma = \begin{pmatrix} \sigma & 0 \\ 0 & \sigma \end{pmatrix} \tag{33}$$

Thus, the Dirac current can also be written as

$$J^\mu = c(\overline{\psi}\gamma^\mu\gamma^5\psi) \tag{34}$$

and the tachyonic Hamiltonian is [5, 11] ,

$$H = H_5 + H_I \tag{35}$$

$$H_5 = \alpha.p + \beta\gamma^5 m \tag{36}$$

Its interaction Hamiltonian will be

$$H_I = J^\mu A_\mu \tag{37}$$

Because (34) is different from the subluminal current, the cross section will be different. It is not clear whether we should use Fermi-Dirac or Bose-Einstein statistics for superluminal quarks in the first quantization because they do not obey the Pauli exclusion principle. In tachyonic field theory, the tachyonic propagator is written as [9, 11]

$$S_T(p) = \frac{1}{\not{p} - \gamma^5(m + i\epsilon)} = \frac{\not{p} - \gamma^5 m}{p^2 + m^2 + i\epsilon} \tag{38}$$

$$< 0|T\psi_\xi(x)\overline{\psi}_{\xi'}(y)\gamma^5|0 >= iS_T(x-y)_{\xi\xi'} \tag{39}$$

$$S_T(x-y) = \int \frac{d^4k_\nu}{(2\pi)^4} e^{-ik_\nu.(x-y)} \frac{\not{k}_\nu - \gamma^5 m_\nu}{k_\nu^2 + m_\nu^2 + i\epsilon} \tag{40}$$

In addition, we have

$$\sum_\sigma (-\sigma) u_\sigma(p) \bigotimes \overline{u}_\sigma(p)\gamma^5 = \frac{\not{k}_\nu - \gamma^5 m}{2m} \qquad \sum_\sigma (-\sigma)\nu_\sigma(p) \bigotimes \overline{\nu}_\sigma(p)\gamma^5 = \frac{\not{k}_\nu + \gamma^5 m}{2m} \tag{41}$$

Therefore, for quark pair production in (e^+e^-) collisions, we have

$$e^+(P_1, r_1) + e^-(P_2, r_2) \to q^+(P_1', s_1) + q^-(P_2', s_2) \tag{42}$$

Its amplitude will be

$$\mathcal{M}_(r_1, r_2, s_1, s_2) = iqe[\overline{u}_{s2}(P_2')\gamma_\alpha\gamma_5 v_{s1}(P_1')]_{(q)} \frac{1}{(p_1+p_2)^2} [\overline{v}_{r1}(P_1)\gamma^\alpha u_{r2}(P_2)]_{(e)} \tag{43}$$

We compute the cross section according to the method in reference [8]

The following gamma relations are useful:

$$(\gamma^5)^2 = 1 \qquad \gamma^{5\dagger} = \gamma^5 \qquad \gamma^5\gamma^\mu = -\gamma^\mu\gamma^5 \tag{44}$$

We have

$$L_e^{uv} = \frac{1}{2}\sum_{s'} [\overline{u}(k')\gamma^\mu u(k)][\overline{u}(k')\gamma^\nu u(k)]^* \tag{45}$$

$$L^q_{uv} = \frac{1}{2}\sum_{s'}[\overline{u}(p')\gamma_\mu\gamma_5 u(p)][\overline{u}(p')\gamma_\nu\gamma_5 u(p)]^* \tag{46}$$

$$L_e^{uv} = \frac{1}{2}Tr((\not{k}' + m_e)\gamma^\mu(\not{k}' + m_e)\gamma^\nu) \tag{47}$$

$$L^q_{uv} = \frac{1}{2}Tr((\not{p}' - \gamma_5 m_q)\gamma_\mu(\not{p}' - \gamma_5 m_q)\gamma_\nu) \tag{48}$$

$$L_e^{uv} = \frac{1}{2}Tr(\not{k}'\gamma^\mu\not{k}\gamma^\nu) + \frac{1}{2}m_e^2 Tr(\gamma^\mu\gamma^\nu) \tag{49}$$

$$L^q_{uv} = \frac{1}{2}Tr(\not{p}'\gamma_\mu\not{p}\gamma_\nu) + \frac{1}{2}m_q^2 Tr(\gamma_\mu\gamma_5\gamma_\nu\gamma_5) \tag{50}$$

$$L_e^{uv} = 2(k'^\mu k^\nu + k'^\nu k^\mu - (k'.k - m_e^2)g^{\mu\nu}) \tag{51}$$

$$L^q_{uv} = 2(p'^\mu p^\nu + p'^\nu p^\mu - (k'.k + m_q^2)g_{\mu\nu}) \tag{52}$$

Therefore, the amplitude will be

$$\overline{\mathcal{M}}^2 = \frac{8e^2q^2}{(k-k')^2}[(k'.p')(k.p) + (k'.p)(k.p') - m_e^2 p.p' + m_q^2 k'.k - 2m_e^2 m_q^2] \tag{53}$$

where q is the quark electric charge. This result can be compared with subluminal electron muon scattering [8, 12]:

$$\overline{\mathcal{M}}^2 = \frac{8e^4}{(k-k')^2}[(k'.p')(k.p) + (k'.p)(k.p') - m_e^2 p.p' - m_\mu^2 k'.k + 2m_e^2 m_\mu^2] \tag{54}$$

In the extreme relativistic limit, we ignore the masses of electrons and quarks, and the cross section will be similar to the electron muon scattering cross section.

$$\frac{d\sigma}{d\Omega}|_{cm} = \frac{e^2 e_q^2}{64(E_A + E_B)^2}(1 + \cos^2\theta) \tag{55}$$

Unfortunately, the total cross section is one third of the value that we obtain from traditional QCD calculations of electron quark scattering, which considers the color factor.

From our previous findings about spin half particles, equations (53) and (54), we can deduce one interesting fact when calculating the cross section that is always valid: only the second power of the mass appears in the cross section. Thus, if we use the superluminal Lagrangian for quantum chromodynamic calculations to find the net results, we can simply change the sign of the superluminal particle mass $m_q^2 \to -m_q^2$ that appears in the cross section and use traditional QED calculations to save time and omit the γ^5 terms for the mass in the Dirac tachyonic equations (29) to (31). In other words, because there is no imaginary term in the cross section, we can easily use QED calculations to obtain QCD results.

The running coupling constant of QCD and other renormalizations of the electric charge and mass are similar to QED but with one important difference: the mass term in QCD is imaginary; therefore, the second power of the mass is negative, and the virtual gluon in

QCD is time-like, not space-like. Additionally, $q^2 > 0$ which makes the sign of the term in the logarithm positive.

$$\alpha(Q^2) = \frac{\alpha(\mu^2)}{1 - \frac{\alpha(\mu^2)}{3\pi} log(\frac{Q^2}{\mu^2})} \qquad Q^2 < 0, \qquad \mu^2 < 0, \qquad q^2 = -Q^2 \tag{56}$$

Another interesting fact about the superluminal Lagrangian is that, in QED currents, which are the interactions among subluminal particles, we have no γ^5 term, but in charged weak currents, which are the interactions among superluminal quarks and subluminal leptons, and neutrinos with undetermined speeds according to the academic community, we have the term $(1 - \gamma^5)$, which creates a left-hand term. Finally, in QCD, which describes the interactions among superluminal particles, we have a γ^5 term.

In other words, QED uses polar vectors, the charged weak currents break parity and use both polar and axial vectors, and QCD uses axial vectors. In contrast, the weak neutral current, includes a combination of γ^5 and the unit tensor I and, thus, has mixed parity. It seems that γ^5 is related to both superluminalities and the experimental value of the Weinberg angle in the weak hypercharge isospin relation. Why is the subtraction of the QED and QCD current equal to the charged weak current? All of these points may hint at further unification among fields.

$$c_V^f = T_f^3 - 2\sin^2\theta_W Q_f, \qquad c_A^f = T_f^3 \tag{57}$$

$$J_\mu^{NC}(\nu) = (\overline{u}_\nu \gamma_\mu \frac{1}{2}(1 - \gamma^5) u_\nu) \tag{58}$$

$$J_\mu^{NC}(e) = (\overline{u}_e \gamma_\mu \frac{1}{2}(c_V^e - c_A^e \gamma^5) u_e) \tag{59}$$

$$J_\mu^{NC}(q) = (\overline{u}_q \gamma_\mu \frac{1}{2}(c_V^q - c_A^q \gamma^5) u_q) \tag{60}$$

$$J_\mu^{CC}(e - \nu) = (\overline{u}_\nu \gamma_\mu \frac{1}{2}(1 - \gamma^5) u_e) \tag{61}$$

$$J_\mu^{CC}(q - q') = (\overline{u}_q \gamma_\mu \frac{1}{2}(\gamma^5 - 1) u_{q'}) \tag{62}$$

$$J_\mu(q) = (\overline{u}_q \gamma_\mu \gamma^5 u_q) \tag{63}$$

$$J_\mu(e) = (\overline{u}_e \gamma_\mu u_e) \tag{64}$$

For quark-antiquark scattering or, in other words, for Bhabha scattering of quarks, we can write

$$q^+(P_1, r_1) + q^-(P_2, r_2) \to q^+(P_1', s_1) + q^-(P_2', s_2) \tag{65}$$

$$\mathcal{M} = \mathcal{M}_a + \mathcal{M}_b \tag{66}$$

$$\mathcal{M}_a = -iq^2[\overline{u}(P_2')\gamma_\alpha\gamma_5 u(P_2)]\frac{1}{(p_1 - p_1')^2}[\overline{v}(P_1)\gamma^\alpha\gamma^5 v(P_1')] \tag{67}$$

$$\mathcal{M}_b = iq^2[\overline{u}(P_2')\gamma_\alpha\gamma_5 v(P_1')]\frac{1}{(p_1+p_2)^2}[\overline{v}(P_1)\gamma^\alpha\gamma^5 u(P_2)] \tag{68}$$

The final shape of the cross section can be found from the QED result [12] and by changing the sign of the superluminal particle mass term in the cross section.

As we discussed earlier, it can be easily shown that, for a specific amplitude in QCD, the superluminal Dirac Lagrangian is CP invariant (see [8])

$$\mathcal{M}_{cp} = \mathcal{M}^\dagger \tag{69}$$

In other words, there is no CP violation in QCD and, thus, no need for the introduction of particles, such as axions. In addition, we should review our past results about grand unified theories and proton mass decays.

4 Appendix

In the appendix, we solve the Dirac equation for the superluminal hydrogen atom. We mimic the method from reference [3]. The electric potential is

$$V = -\frac{Ze^2}{r} \tag{70}$$

The radial Dirac equations are

$$\frac{dG}{dr} = -\frac{k}{r}G + [\frac{E+imc^2}{\hbar c} + \frac{Z\alpha}{r}]F(r) \tag{71}$$

$$\frac{dF}{dr} = \frac{k}{r}F - [\frac{E-imc^2}{\hbar c} + \frac{Z\alpha}{r}]G(r) \tag{72}$$

where we use $G = rg$ and $F = rf$, and

$$\alpha = \frac{e^2}{\hbar c} = \frac{1}{137} \tag{73}$$

for small r near the origin; $E \pm imc^2$ is omitted. Thus, we have

$$\frac{dG}{dr} + \frac{k}{r}G + -\frac{Z\alpha}{r}F(r) = 0 \tag{74}$$

$$\frac{dF}{dr} - \frac{k}{r}F + \frac{Z\alpha}{r}G(r) = 0 \tag{75}$$

We attempt the ansatz $G = ar^\gamma$ $F = br^\gamma$

$$a\gamma r^{\gamma-1} + \kappa a r^{\gamma-1} - Z\alpha b r^{\gamma-1} = 0 \tag{76}$$

$$b\gamma r^{\gamma-1} - \kappa b r^{\gamma-1} + Z\alpha a r^{\gamma-1} = 0 \tag{77}$$

which indicate that

$$a(\gamma+\kappa) - bZ\alpha = 0 \tag{78}$$

$$aZ\alpha + b(\gamma-\kappa) = 0 \tag{79}$$

The determinant of the coefficients must vanish, which yields

$$\gamma^2 = \kappa^2 - (Z\alpha)^2 \tag{80}$$

$$\gamma = \pm\sqrt{\kappa^2 - (Z\alpha)^2} = \pm\sqrt{(j+\frac{1}{2})^2 - Z^2\alpha^2} \tag{81}$$

We choose

$$q = 2\lambda r \tag{82}$$

and

$$\lambda = \frac{\sqrt{E^2 + m_\circ^2 c^4}}{\hbar c} \tag{83}$$

which results in

$$\frac{dG}{dq} = \frac{-kG}{q} + [\frac{E + imc^2}{2\lambda\hbar c} + \frac{Z\alpha}{q}]F(q) \tag{84}$$

$$\frac{dF}{dq} = -[\frac{E - imc^2}{2\lambda\hbar c} + \frac{Z\alpha}{q}]G + \frac{k}{q}F(q) \tag{85}$$

For $q \to \infty$, we have

$$\frac{dG}{dq} = \frac{E + imc^2}{2\lambda\hbar c}F \tag{86}$$

$$\frac{dF}{dq} = -\frac{E - imc^2}{2\lambda\hbar c}G \tag{87}$$

Using (82) and (83), we obtain

$$\frac{d^2G}{d^2q} = -\frac{E^2 + m^2c^2}{4\lambda^2\hbar^2c^2}G = -\frac{1}{4}G \tag{88}$$

$$\frac{d^2F}{d^2q} = -\frac{E^2 + m^2c^2}{4\lambda^2\hbar^2c^2}F = -\frac{1}{4}F \tag{89}$$

We have $G \approx e^{\pm\frac{iq}{2}}$, but we choose the negative sign

$$G = \sqrt{imc^2 + E}e^{\frac{-iq}{2}}(\phi_1 + \phi_2) \tag{90}$$

$$F = \sqrt{imc^2 - E}e^{\frac{-iq}{2}}(\phi_1 - \phi_2) \tag{91}$$

by substituting into equations (84) and (85), we obtain

$$\begin{aligned}&\sqrt{imc^2 + E} \times \frac{-i}{2}e^{\frac{-iq}{2}}(\phi_1 + \phi_2) + \sqrt{imc^2 + E}e^{\frac{-iq}{2}}(\phi_1' + \phi_2')\\ &= \frac{-k}{q}\sqrt{imc^2 + E}e^{\frac{-iq}{2}}(\phi_1 + \phi_2) + [\frac{E + imc^2}{2\lambda\hbar c} + \frac{Z\alpha}{q}][\sqrt{imc^2 - E}]e^{\frac{-iq}{2}}(\phi_1 - \phi_2)\end{aligned} \tag{92}$$

$$\begin{aligned}&\sqrt{imc^2 - E} \times \frac{-i}{2}e^{\frac{-iq}{2}}(\phi_1 - \phi_2) + \sqrt{imc^2 - E}e^{\frac{-iq}{2}}(\phi_1' - \phi_2')\\ &= -[\frac{E - imc^2}{2\lambda\hbar c} + \frac{Z\alpha}{q}][\sqrt{imc^2 + E}]e^{\frac{-iq}{2}}(\phi_1 + \phi_2) + \frac{k}{q}\sqrt{imc^2 - E}e^{\frac{-iq}{2}}(\phi_1 - \phi_2)\end{aligned} \tag{93}$$

or

$$\sqrt{imc^2+E}\times e^{\frac{-iq}{2}}[\frac{-i}{2}(\phi_1+\phi_2)+(\phi_1'+\phi_2')]$$
$$=\frac{-k}{q}\sqrt{imc^2+E}e^{\frac{-iq}{2}}(\phi_1+\phi_2)+[\frac{E+imc^2}{2\lambda\hbar c}+\frac{Z\alpha}{q}][\sqrt{imc^2-E}]e^{\frac{-iq}{2}}(\phi_1-\phi_2) \quad (94)$$

$$\sqrt{imc^2-E}\times e^{\frac{-iq}{2}}[\frac{-i}{2}(\phi_1-\phi_2)+(\phi_1'-\phi_2')]$$
$$=-[\frac{E-imc^2}{2\lambda\hbar c}+\frac{Z\alpha}{q}][\sqrt{imc^2+E}]e^{\frac{-iq}{2}}(\phi_1+\phi_2)+\frac{k}{q}\sqrt{imc^2-E}e^{\frac{-iq}{2}}(\phi_1-\phi_2) \quad (95)$$

Dividing by $e^{-\frac{iq}{2}}$ and further dividing the first equation by $(imc^2+E)^{\frac{1}{2}}$ and the second equation by $(imc^2-E)^{\frac{1}{2}}$, we obtain

$$[\frac{-i}{2}(\phi_1+\phi_2)+(\phi_1'+\phi_2')]$$
$$=\frac{-k}{q}(\phi_1+\phi_2)+[\frac{E+imc^2}{2\lambda\hbar c}+\frac{Z\alpha}{q}]\frac{\sqrt{imc^2-E}}{\sqrt{imc^2+E}}(\phi_1-\phi_2) \quad (96)$$

$$[\frac{-i}{2}(\phi_1-\phi_2)+(\phi_1'-\phi_2')]$$
$$=-[\frac{E-imc^2}{2\lambda\hbar c}+\frac{Z\alpha}{q}]\frac{\sqrt{imc^2+E}}{\sqrt{imc^2-E}}(\phi_1+\phi_2)+\frac{k}{q}(\phi_1-\phi_2) \quad (97)$$

However, we had

$$\frac{\sqrt{imc^2-E}}{\sqrt{imc^2+E}}=\frac{imc^2-E}{\sqrt{-m^2c^4-E^2}}=\frac{imc^2-E}{i\hbar c\lambda} \quad (98)$$

and

$$\frac{\sqrt{imc^2+E}}{\sqrt{imc^2-E}}=\frac{imc^2+E}{\sqrt{-m^2c^4-E^2}}=\frac{imc^2+E}{i\hbar c\lambda} \quad (99)$$

Thus,

$$[\frac{-i}{2}(\phi_1+\phi_2)+(\phi_1'+\phi_2')]$$
$$=\frac{-k}{q}(\phi_1+\phi_2)+[\frac{E+imc^2}{2\lambda\hbar c}+\frac{Z\alpha}{q}]\frac{imc^2-E}{i\hbar c\lambda}(\phi_1-\phi_2) \quad (100)$$

$$[\frac{-i}{2}(\phi_1-\phi_2)+(\phi_1'-\phi_2')]$$
$$=-[\frac{E-imc^2}{2\lambda\hbar c}+\frac{Z\alpha}{q}]\frac{imc^2+E}{i\hbar c\lambda}(\phi_1+\phi_2)+\frac{k}{q}(\phi_1-\phi_2) \quad (101)$$

By adding the above two equations,

$$
\begin{aligned}
&-i\phi_1+2\phi_1'=-2\frac{k}{q}\phi_2+(\frac{E+imc^2}{2\lambda\hbar c})(\frac{imc^2-E}{i\hbar c\lambda})[\phi_1-\phi_2]\\
&+\frac{Z\alpha}{q}(\frac{imc^2-E}{i\hbar c\lambda})[\phi_1-\phi_2]-[\frac{E-imc^2}{2\lambda\hbar c}][\frac{imc^2+E}{i\hbar c\lambda}[\phi_1+\phi_2]\\
&-\frac{Z\alpha}{q}\times\frac{imc^2+E}{i\hbar c\lambda}(\phi_1+\phi_2)\\
&=-2\frac{k}{q}\phi_2+(\frac{-m^2c^4-E^2}{2i\lambda^2\hbar^2c^2})(\phi_1-\phi_2)+\frac{Z\alpha}{q}(\frac{imc^2-E}{i\hbar c\lambda})(\phi_1-\phi_2)\\
&-[\frac{E^2+m^2c^4}{2i\lambda^2\hbar^2c^2}](\phi_1+\phi_2)-\frac{Z\alpha}{q}\frac{imc^2+E}{i\hbar c\lambda}(\phi_1+\phi_2)\\
&=\frac{-2k\phi_2}{q}-\frac{1}{2i}(\phi_1-\phi_2)-\frac{1}{2i}(\phi_1+\phi_2)\\
&+\frac{Z\alpha}{q}(\frac{imc^2-E}{i\hbar mc\lambda})(\phi_1-\phi_2)-\frac{Z\alpha}{q}(\frac{imc^2+E}{i\hbar c\lambda})(\phi_1+\phi_2)
\end{aligned}
\tag{102}
$$

or

$$
\begin{aligned}
&-i\phi_1+2\phi_1'=-\frac{2k}{q}\phi_2-\frac{1}{i}\phi_1+\\
&\frac{Z\alpha}{q}(\frac{imc^2-E}{h\hbar\lambda c})(\phi_1-\phi_2)-\frac{Z\alpha}{q}(\frac{imc^2+E}{i\hbar c\lambda})(\phi_1+\phi_2)
\end{aligned}
\tag{103}
$$

By subtracting two equations, we have

$$
\begin{aligned}
&-i\phi_2+2\phi_2'=-\frac{2k}{q}\phi_1-\frac{E^2+m^2c^4}{2i\hbar^2c^2\lambda^2}(\phi_1-\phi_2)\\
&+\frac{Z\alpha}{q}\frac{(imc^2-E)}{i\hbar c\lambda}(\phi_1-\phi_2)+\frac{(E^2+m^2c^4)}{2i\hbar^2c^2\lambda^2}(\phi_1+\phi_2)\\
&+\frac{Z\alpha}{q}\frac{imc^2+E}{i\hbar c\lambda}(\phi_1+\phi_2)
\end{aligned}
\tag{104}
$$

or

$$
\begin{aligned}
&-i\phi_2+2\phi_2'=-\frac{2k}{q}\phi_1+\frac{\phi_2}{i}+\frac{Z\alpha}{q}\frac{(imc^2-E)}{i\hbar c\lambda}(\phi_1-\phi_2)\\
&+\frac{Z\alpha}{q}\frac{imc^2+E}{i\hbar c\lambda}(\phi_1+\phi_2)
\end{aligned}
\tag{105}
$$

Summarizing, we obtain

$$
\phi_1'=(i-\frac{Z\alpha E}{qi\hbar c\lambda})\phi_1-(\frac{k}{q}+\frac{Z\alpha}{q}\frac{mc^2}{\hbar c\lambda})\phi_2 \tag{106}
$$

$$\phi_2' = (-\frac{k}{q} + Z\alpha\frac{mc^2}{\hbar c\lambda q})\phi_1 + \frac{Z\alpha}{q}\frac{E}{i\hbar c\lambda}\phi_2 \tag{107}$$

We use a power series. We separate a factor q^γ, which describes the behavior of the solution for $q \to 0$

$$\phi_1 = q^\gamma \sum \alpha_m q^m \tag{108}$$

$$\phi_2 = q^\gamma \sum \beta_m q^m \tag{109}$$

Inserting this equation into equations (106) and (107), we obtain

$$\begin{aligned}&\sum(m+\gamma)\alpha_m q^{m+\gamma-1} = i\sum \alpha_m q^{m+\gamma} - \frac{Z\alpha E}{i\hbar c\lambda}\sum \alpha_m q^{m+\gamma-1}\\ &-(k + \frac{Z\alpha mc^2}{\hbar c\lambda})\sum \beta_m q^{m+\gamma-1}\end{aligned} \tag{110}$$

and

$$\begin{aligned}&\sum \beta_m(m+\gamma)q^{m+\gamma-1} = (-k + \frac{Z\alpha mc^2}{\hbar c\lambda})\sum \alpha_m q^{m+\gamma-1}\\ &+\frac{Z\alpha E}{i\hbar c\lambda}\sum \beta_m q^{m+\gamma-1}\end{aligned} \tag{111}$$

By comparing the coefficients, we obtain

$$\alpha_m(m+\gamma) = i\alpha_m - 1 - \frac{Z\alpha E\alpha_m}{h\hbar c\lambda} - (k + \frac{Z\alpha mc^2}{\hbar\lambda c})\beta_m \tag{112}$$

$$\beta_m(m+\gamma) = (-k + \frac{Z\alpha mc^2}{\hbar c\lambda})\alpha_m + \frac{Z\alpha E}{i\hbar c\lambda}\beta_m \tag{113}$$

From the above equation, we obtain

$$\frac{\beta_m}{\alpha_m} = \frac{(-k + \frac{Z\alpha mc^2}{\hbar c\lambda})}{m+\gamma-\frac{Z\alpha E}{i\hbar c\lambda}} = \frac{(k - \frac{Z\alpha mc^2}{\hbar c\lambda})}{n'-m} \tag{114}$$

$$n' = \frac{Z\alpha E}{i\hbar c\lambda} - \gamma \tag{115}$$

For $m = 0$, we obtain

$$\frac{\beta_\circ}{\alpha_\circ} = \frac{k - \frac{Z\alpha mc^2}{\hbar c\lambda}}{n'} = \frac{k - (n'+\gamma)\frac{mc^2}{E}}{n'} \tag{116}$$

Inserting (114) into (112) and (113), we obtain

$$\alpha_m(m+\gamma) = i\alpha_{m-1} - \frac{Z\alpha E\alpha_m}{i\hbar c\lambda} - (k+\frac{z\alpha mc^2}{\hbar c\lambda})\frac{(k-\frac{Z\alpha mc^2}{\hbar c\lambda})}{(m+\gamma-\frac{Z\alpha E}{i\hbar c\lambda})}\alpha_m \tag{117}$$

$$\alpha_m[(m+\gamma) + \frac{Z\alpha E}{i\hbar c\lambda} - \frac{(k+\frac{z\alpha mc^2}{\hbar c\lambda})(k-\frac{Z\alpha mc^2}{\hbar c\lambda})}{(m-n')} = i\alpha_{m-1} \tag{118}$$

$$\alpha_m[m+\gamma + \frac{Z\alpha E}{i\hbar c\lambda} + \frac{(k+\frac{z\alpha mc^2}{\hbar c\lambda})(k-\frac{Z\alpha mc^2}{\hbar c\lambda})}{(n'-m)} = i\alpha_{m-1} \tag{119}$$

$$\alpha_m[(m+\gamma+\frac{Z\alpha E}{i\hbar c\lambda})(n'-m) + k^2 - \frac{Z^2\alpha^2 m^2 c^4}{\hbar^2 c^2\lambda^2}] = i\alpha_{m-1}(n'-m) \tag{120}$$

If we expand the bracket on the left-hand side of the above equation and use equation (115), we obtain

$$(m+\gamma+\frac{Z\alpha E}{i\hbar c\lambda})(\frac{Z\alpha E}{i\hbar c\lambda}-\gamma-m) = -2m\gamma - m^2 - \gamma^2 - (\frac{Z\alpha E}{\hbar c\lambda})^2 \tag{121}$$

Thus, we have

$$\alpha_m[-2m\gamma - m^2 - \gamma^2 - (\frac{Z\alpha E}{\hbar c\lambda})^2 + k^2 - \frac{Z^2\alpha^2 m^2 c^4}{\hbar^2 c^2\lambda^2}] = i\alpha_{m-1}(n'-m) \tag{122}$$

$$\alpha_m[-m(2\gamma+m) + (Z\alpha)^2 - (\frac{Z\alpha E}{\hbar c\lambda})^2 - \frac{Z^2\alpha^2 m^2 c^4}{\hbar^2 c^2\lambda^2}] = i\alpha_{m-1}(n'-m) \tag{123}$$

with

$$\gamma^2 = k^2 - (Z\alpha)^2 \tag{124}$$

We conclude that

$$\alpha_m[-m(2\gamma+m) + (Z\alpha)^2(1-\frac{E^2+m^2c^4}{\hbar^2 c^2\lambda^2})] = i\alpha_{m-1}(n'-m) \tag{125}$$

which can be written as

$$\begin{aligned}\alpha_m &= \frac{-(n'-m)}{m(2\gamma+m)} i\alpha_{m-1} \\ &= \frac{(-1)^m(n'-1)...(n'-m)\alpha_\circ i^m}{m!(2\gamma+1)...(2\gamma+m)} = \frac{(1-n')(2-n')...(m-n')(i)^m}{m!(2\gamma+1)...(2\gamma+m)}\alpha_\circ\end{aligned} \tag{126}$$

$$\beta_m = \frac{(+k - \frac{Z\alpha mc^2}{\hbar c\lambda})}{n' - m} \frac{(-1)^m (n'-1)...(n'-m)\alpha_\circ i^m}{m!(2\gamma+1)...(2\gamma+m)} \tag{127}$$

$$\beta_m = \frac{(+k - \frac{Z\alpha mc^2}{\hbar c\lambda})(-1)^m (n'-1)...(n'-m+1)\alpha_\circ i^m}{m!(2\gamma+1)...(2\gamma+m)} \tag{128}$$

Using (116), we conclude that

$$\beta_m = \frac{(+k - \frac{Z\alpha mc^2}{\hbar c\lambda})(-1)^m (n'-1)...(n'-m+1) i^m}{m!(2\gamma+1)...(2\gamma+m)} \frac{n'\beta_\circ}{(+k - \frac{Z\alpha mc^2}{\hbar c\lambda})} \tag{129}$$

$$\beta_m = \frac{n'(n'-1)...(n'-m+1)(-1)^m i^m}{m!(2\gamma+1)...(2\gamma+m)} \beta_\circ \tag{130}$$

The above equation is the confluent hyper geometric function

$$F(a, c; x) = 1 + \frac{a}{c} x + \frac{a(a+1)}{c(c+1)} \frac{x^2}{2!} + ... \tag{131}$$

$$\phi_1 = \alpha_\circ q^\gamma F(1-n', 2\gamma+1; iq) \tag{132}$$

$$\begin{aligned} \phi_2 &= \beta_\circ q^\gamma F(-n', 2\gamma+1; iq) \\ &= (\frac{\kappa - Z\alpha mc^2/\hbar c\lambda}{n'})\alpha_\circ q^\gamma F(-n', 2\gamma+1; iq) \end{aligned} \tag{133}$$

For negative values of n' the above series is normalized if we choose the appropriate γ. Using (90), (91), $G = rg$ and $F = rf$, we can construct the normalized wave functions f and g.

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