On The Confinement Of Quarks Without Applying the Bag Pressure

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Abstract

We explain the fatal error in quantum chromodynamics. By applying this correction to the dynamics of quarks, we can confine quarks in hadrons. We will show why quarks do not obey the Pauli exclusion principle and why we cannot observe free quarks. In addition, we obtain the correct size of hadrons.

1 Introduction

Two electrons with identical quantum numbers cannot exist in the hydrogen atom, because each electron is subluminal and its phase velocity is superluminal. When there are 2 electrons with identical quantum numbers in a hydrogen atom or with identical energy levels in a cubic box, the second electron exists at every location (space-time coordinates) with exactly identical wave function characteristics to those of the first electron. In other words, the two electrons simultaneously exist at an exact point in equal time. This phenomenon is the consequence of the probabilistic characteristic of a wave function and quantum mechanics. In other words, the wave equation does not provide us with more information about the exact location of each electron. The energy and absolute value of the momentum of each electron are exactly determined, but they do not have a specific location. At a specific time, they are ubiquitous at every location where the wave function does not have a zero value. However, we can have 3 identical quarks with identical spin states in baryons. To explain this phenomenon, we propose a strange theorem.

Theorem 1. *Quarks are Superluminal particles.*

Any specific change of each state of the wave function in its associated Hilbert space will propagate in the space-time coordinate with the phase velocity of the wave function in space-time. In other words, the particles communicate with one another via their phase velocities [6]. We postulate that quarks are superluminal. Because each quark is superluminal, its phase velocity should be subluminal; thus, quarks with identical spins can occupy the same energy level in hadrons. In other words, the first quark is unaware of the spin and the characteristics of the second quark, because their phase velocities are subluminal. If we change the wave function of the second quark, this change will propagate with a speed less

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than c to other locations of space-time in the bag. The phase velocity is not in space-like regions. In other words, two quarks with identical energies and momenta are located at different points in the bag. Quantum mechanics postulate that at a specific time, a subluminal particle with a specific energy momentum does not have a specific location. In other words, it is ubiquitous in the bag. However, because the phase velocity of a superluminal particle is subluminal, superluminal particle is no longer ubiquitous. Thus, two superluminal particles that are confined in a cubic box no longer exist at the exact space-time point. Thus, it is not necessary for them to obey Pauli Exclusion Principle. Note that the exclusion principle is applicable for two identical particles with identical wave function characteristics.

Theoretically, as we mentioned previously, the wave function of a single superluminal particle cannot collapse, because its phase velocity of collapse is subluminal and obeys causality [6].

Before the wave function collapses, the particle does not have a specific location. We create its location by doing an experiment and measuring its location. However, after we determined the location of a particle, the particle should not be detected in other locations even in notably far space-like locations that have no causal relationship with the location of the collapsed particle. When ψ_{space} of a subluminal particle collapses, it communicates via its phase velocity (at infinite velocity in the reference frame of the collapsed wave function) to other locations in space-time that the wave function should not collapse at other locations of the universe. Thus, a particle cannot be detected in two space-like locations, although two locations do not have causal relations with each other. However, if the particle is superluminal, its phase velocity is subluminal and cannot perform this communication in space-like regions of space-time. The phase velocity must be superluminal to allow the collapse of the wave function. Because quarks are superluminal, we never observe a single free quark.

2 Wave equation of the hydrogen atom with superluminal electron

There is a big difference between the ordinary hydrogen atom and a model with superluminal electron. In the subluminal model, we have negative potential energy. When we increase the energy of the electron in the subluminal model, the momentum of the electron decreases; thus, the wavelength of the electron increases, and the electron increases its distance from the proton. In the subluminal model, although the energy cannot be less than the mass of the particle, the minimum momentum can be zero.

$$E^2 = c^2P^2 + m^2c^4 \tag{1}$$

Thus, the wave length has no maximum, and it can approach infinity, which results in the escape of an electron from the hydrogen atom according to the Wilson-Sommerfeld rule. The minimum principal quantum number for the minimum radius of hydrogen atom is $n = 1$.

However, in the superluminal model, although the minimum amount of relativistic energy is zero, the momentum has a specific minimum. It cannot be less than the mass of the electron, which is m_sc.

$$c^2P^2 = E^2 + m_s^2c^4 \tag{2}$$

$$E = \frac{m_s c^2}{\sqrt{\beta^2 - 1}} \tag{3}$$

$$P = \frac{m_s v}{\sqrt{\beta^2 - 1}} \tag{4}$$

We see that the electron has a maximum wavelength $\lambda = \hbar/cm$. Thus, by the Wilson-Sommerfeld rule, the electron cannot gain infinite wavelength and cannot escape the hydrogen atom. This fact sets a limit on the maximum radius of the bag. Thus, the electron in the superluminal model is confined. For the superluminal model, the principal quantum number of the maximum radius of the bag is $n = 1$.

$$\frac{(m_\circ^2 c^4 + E^2)^{1/2}}{\hbar c} 2\pi r = 1 \tag{5}$$

When the electron energy increases, its momentum increases, but its wavelength decreases; thus, it becomes increasingly confined. The electron falls deeper in the hydrogen atom or bag, which is contrary to our observation in the subluminal model.

At this point, we seek to derive and solve the wave function of confined superluminal electron in the hydrogen bag. First, we study the radial Dirac equation. The Dirac equation for a subluminal particle with real mass leads to the equations shown below[3]

$$\hbar c \frac{dg(r)}{dr} + (1+\kappa)\hbar c \frac{g(r)}{r} - [E + m_\circ c^2 + \frac{Z\alpha}{r}] f(r) = 0 \tag{6}$$

$$\hbar c \frac{df(r)}{dr} + (1-\kappa)\hbar c f(r) r + [E - m_\circ c^2 + \frac{Z\alpha}{r}] g(r) = 0 \tag{7}$$

The normalized solutions are proportional to

$$\begin{aligned} &f(r) \approx -\frac{1}{\Gamma(2\gamma+1)} (2\lambda r)^{\gamma-1} e^{-\lambda r} \times \\ &\left\{ (\frac{(n'+\gamma) m_\circ c^2}{E} - \kappa) F(-n', 2\gamma+1; 2\lambda r) + n' F(1-n', 2\gamma+1; 2\lambda r) \right\} \end{aligned} \tag{8}$$

$$\begin{aligned} &g(r) \approx \frac{1}{\Gamma(2\gamma+1)} (2\lambda r)^{\gamma-1} e^{-\lambda r} \times \\ &\left\{ (\frac{(n'+\gamma) m_\circ c^2}{E} - \kappa) F(-n', 2\gamma+1; 2\lambda r) - n' F(1-n', 2\gamma+1; 2\lambda r) \right\} \end{aligned} \tag{9}$$

For normalizable wave functions, γ should be positive. κ is the Dirac quantum number, and

$$\lambda = \frac{(m_\circ^2 c^4 - E^2)^{1/2}}{\hbar c} \tag{10}$$

$$q = 2\lambda r \tag{11}$$

$$\gamma = +\sqrt{\kappa^2 - (Z\alpha)^2} = +\sqrt{(j + \frac{1}{2})^2 - (Z\alpha)^2} \tag{12}$$

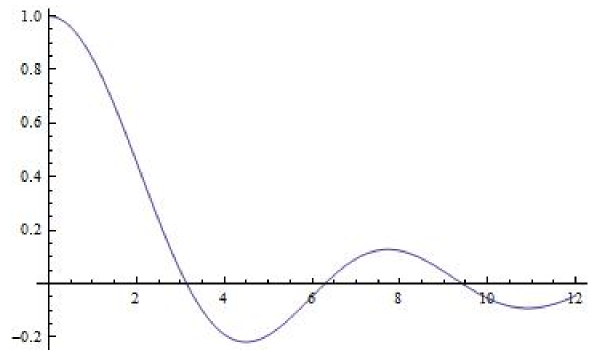

Figure 1: real part of $e^{-ix}F(1,3,2ix)$

To terminate hyper geometric series, we should discard the negative values of n'

$$n = n' + |\kappa| = n' + j + \frac{1}{2} \quad n = 1, 2, 3 \tag{13}$$

The solution for the hydrogen atom provides a hyper geometric function, which is an associated Laguerre polynomial and is characteristic of the wave function for Coulomb potential.

$$L_n^m(x) = \frac{(n+m)!}{n!m!} F(-n, m+1, x) \tag{14}$$

where $L_n^m(x)$ is the associated Laguerre function. look at (8) and (9).

We mimic the above procedure for the superluminal model with imaginary mass and obtain

$$\hbar c \frac{dg(r)}{dr} + (1+\kappa)\hbar c \frac{g(r)}{r} - [E + im_\circ c^2 + \frac{Z\alpha}{r}]f(r) = 0 \tag{15}$$

$$\hbar c \frac{df(r)}{dr} + (1-\kappa)\hbar c f(r) r + [E - im_\circ c^2 + \frac{Z\alpha}{r}]g(r) = 0 \tag{16}$$

we define λ as

$$\lambda = \frac{(m_\circ^2 c^4 + E^2)^{1/2}}{\hbar c} \tag{17}$$

We solve the above equation and exactly mimic the provided method in the reference for the solution of Coulomb potential [3]. Finally, we obtain

$$g(r) \approx (2\lambda r)^{\gamma-1} e^{-i\lambda r} \times$$
$$\left\{ (\frac{(n'+\gamma)m_\circ c^2}{E} - \kappa) F(-n', 2\gamma+1; 2i\lambda r) - n' F(1-n', 2\gamma+1; 2i\lambda r) \right\} \tag{18}$$

$$f(r) \approx -(2\lambda r)^{\gamma-1} e^{-i\lambda r} \times$$
$$\left\{ (\frac{(n'+\gamma)m_\circ c^2}{E} - \kappa) F(-n', 2\gamma+1; 2i\lambda r) + n' F(1-n', 2\gamma+1; 2i\lambda r) \right\} \tag{19}$$

In the above equations, $F(-n', 2\gamma+1; 2i\lambda r)$ is normalized for only negative values of n' if

$$-n' < 2\gamma + 1 \tag{20}$$

For example, for $j = \frac{1}{2}$(which gives $\gamma = 1$), and $n' = -1$ we have a well behaved wave function (figure 1). For $-n' = 2\gamma+1$, the behavior of the wave function $F(-n', 2\gamma+1; 2i\lambda r)$ is similar to $cos(r)$. for negative n', the above hyper geometric equations are similar to the spherical Bessel function of the first type. From (18) and (19) the relation between the hyper geometric series and the Bessel functions is

$$J_\nu(x) = \frac{e^{-ix}}{\nu!}(\frac{x}{2})^\nu F(\nu + \frac{1}{2}, 2\nu + 1, 2ix) \tag{21}$$

The spherical Bessel function of the first type is defined as

$$j_\nu(x) = \sqrt{\frac{\pi}{2x}} J_{\nu+1/2}(x) \tag{22}$$

We saw that the solution for subluminal hydrogen atom is Laguerre polynomial. However, we see that $f(r)$ and $g(r)$ for a superluminal electron with Coulomb potential is similar to the spherical Bessel function of the first type. The spherical Bessel functions appear in only two similar cases. The first case is a particle trapped in an infinite three-dimensional radial well potential. The solution to this problem is the spherical Bessel function of the first type. Similarly, the solution to the MIT bag model, which postulated the existence of an unknown pressure and the vanishing of the Dirac current outside the bag, is also spherical Bessel functions of first type[2, 4].

To create a superluminal Dirac equation for quarks, we can use imaginary mass or substitute the following matrix $\beta_s = i\beta$ to calculate $f(r)$ and $g(r)$. However, when we want to construct the Dirac current, we will face a problem. The correct method is to consider the following non-Hermitian matrices $\beta_s = \beta\gamma_5$ [1, 5]

$$\alpha = \begin{bmatrix} 0 & \sigma \\ \sigma & 0 \end{bmatrix} \qquad \beta_s = \begin{bmatrix} 0 & I \\ -I & 0 \end{bmatrix} \tag{23}$$

This method satisfies all required properties of a superluminal Dirac equation. Note that we did not postulate that the strong force is actually the electromagnetic force among superluminal particles. However, even if the force among the particles was repulsive in the above equation or its strength with respect to distance was not $\frac{1}{r}$, the factor that determines whether the system is stable and whether the superluminal positron can escape the proton is the energy of the system and not the attractive or repulsive force among the particles.

Note that the universe for a superluminal positron in the hydrogen atom is the bag. Its beginning is the boundary of the bag, and its infinity is the center of the bag. The same law that does not permit the electron to fall on the proton in the subluminal model prohibits the superluminal positron or electron to escape from the hydrogen bag. By studying the inter quarks potential, we consider the following conjecture

Conjecture. The strong force is only the superluminal effect of the electromagnetic force among superluminal particles.

Without applying any pressure or infinite potential, we have confined the superluminal electron with the appropriate bag radius in the hydrogen atom. In other words, we solved a modified Dirac equation for superluminal particles and substituted the attractive Coulomb potential in the absence of any infinite potential. The solutions were spherical Bessel functions of the first type.

The confinement of quarks in hadrons has a similar mechanism to the above example. It appears that we no longer require $SU(3)$ symmetry of the strong force to confine quarks in hadrons. This method indicates that we should consider another symmetry group for QCD. Although it is not clear why the net electric charge of the bag must be an integer value, The author is completely confident that if we consider the superluminal correction for quarks, we can solve QCD at all energy values.

3 Appendix

In the appendix, we solve the Dirac equation for the superluminal hydrogen atom. We mimic the method from reference [3]. The electric potential is

$$V = -\frac{Ze^2}{r} \tag{24}$$

The radial Dirac equations are

$$\frac{dG}{dr} = -\frac{k}{r}G + [\frac{E + imc^2}{\hbar c} + \frac{Z\alpha}{r}]F(r) \tag{25}$$

$$\frac{dF}{dr} = \frac{k}{r}F - [\frac{E - imc^2}{\hbar c} + \frac{Z\alpha}{r}]G(r) \tag{26}$$

where we use $G = rg$ and $F = rf$, and

$$\alpha = \frac{e^2}{\hbar c} = \frac{1}{137} \tag{27}$$

for small r near the origin, $E \pm imc^2$ is omitted. thus, we have

$$\frac{dG}{dr} + \frac{k}{r}G + -\frac{Z\alpha}{r}F(r) = 0 \tag{28}$$

$$\frac{dF}{dr} - \frac{k}{r}F + \frac{Z\alpha}{r}G(r) = 0 \tag{29}$$

We attempt the ansatz $G = ar^\gamma$ $F = br^\gamma$

$$a\gamma r^{\gamma-1} + \kappa ar^{\gamma-1} - Z\alpha br^{\gamma-1} = 0 \tag{30}$$

$$b\gamma r^{\gamma-1} - \kappa br^{\gamma-1} + Z\alpha ar^{\gamma-1} = 0 \tag{31}$$

which indicates that

$$a(\gamma + \kappa) - bZ\alpha = 0 \tag{32}$$

$$aZ\alpha + b(\gamma - \kappa) = 0 \tag{33}$$

The determinant of coefficients must vanish, which yields

$$\gamma^2 = \kappa^2 - (Z\alpha)^2 \tag{34}$$

$$\gamma = \pm\sqrt{\kappa^2 - (Z\alpha)^2} = \pm\sqrt{(j + \frac{1}{2})^2 - Z^2\alpha^2} \tag{35}$$

We choose

$$q = 2\lambda r \tag{36}$$

and

$$\lambda = \frac{\sqrt{E^2 + m_\circ^2 c^4}}{\hbar c} \tag{37}$$

which results in

$$\frac{dG}{dq} = \frac{-kG}{q} + [\frac{E + imc^2}{2\lambda\hbar c} + \frac{Z\alpha}{q}]F(q) \tag{38}$$

$$\frac{dF}{dq} = -[\frac{E - imc^2}{2\lambda\hbar c} + \frac{Z\alpha}{q}]G + \frac{k}{q}F(q) \tag{39}$$

For $q \to \infty$, we have

$$\frac{dG}{dq} = \frac{E + imc^2}{2\lambda\hbar c}F \tag{40}$$

$$\frac{dF}{dq} = -\frac{E - imc^2}{2\lambda\hbar c}G \tag{41}$$

using (36) and (37), we obtain

$$\frac{d^2G}{d^2q} = -\frac{E^2 + m^2c^2}{4\lambda^2\hbar^2c^2}G = -\frac{1}{4}G \tag{42}$$

$$\frac{d^2F}{d^2q} = -\frac{E^2 + m^2c^2}{4\lambda^2\hbar^2c^2}F = -\frac{1}{4}F \tag{43}$$

We have $G \approx e^{\pm\frac{iq}{2}}$, but we choose the negative sign

$$G = \sqrt{imc^2 + E}e^{\frac{-iq}{2}}(\phi_1 + \phi_2) \tag{44}$$

$$F = \sqrt{imc^2 - E}e^{\frac{-iq}{2}}(\phi_1 - \phi_2) \tag{45}$$

by substituting into equation (38) and (39), we obtain

$$\begin{aligned} &\sqrt{imc^2 + E} \times \frac{-i}{2}e^{\frac{-iq}{2}}(\phi_1 + \phi_2) + \sqrt{imc^2 + E}e^{\frac{-iq}{2}}(\phi_1' + \phi_2') \\ &= \frac{-k}{q}\sqrt{imc^2 + E}e^{\frac{-iq}{2}}(\phi_1 + \phi_2) + [\frac{E + imc^2}{2\lambda\hbar c} + \frac{Z\alpha}{q}][\sqrt{imc^2 - E}]e^{\frac{-iq}{2}}(\phi_1 - \phi_2) \end{aligned} \tag{46}$$

$$\begin{aligned} &\sqrt{imc^2 - E} \times \frac{-i}{2}e^{\frac{-iq}{2}}(\phi_1 - \phi_2) + \sqrt{imc^2 - E}e^{\frac{-iq}{2}}(\phi_1' - \phi_2') \\ &= -[\frac{E - imc^2}{2\lambda\hbar c} + \frac{Z\alpha}{q}][\sqrt{imc^2 + E}]e^{\frac{-iq}{2}}(\phi_1 + \phi_2) + \frac{k}{q}\sqrt{imc^2 - E}e^{\frac{-iq}{2}}(\phi_1 - \phi_2) \end{aligned} \tag{47}$$

or

$$\sqrt{imc^2+E}\times e^{\frac{-iq}{2}}[\frac{-i}{2}(\phi_1+\phi_2)+(\phi_1'+\phi_2')]$$
$$=\frac{-k}{q}\sqrt{imc^2+E}e^{\frac{-iq}{2}}(\phi_1+\phi_2)+[\frac{E+imc^2}{2\lambda\hbar c}+\frac{Z\alpha}{q}][\sqrt{imc^2-E}]e^{\frac{-iq}{2}}(\phi_1-\phi_2) \quad (48)$$

$$\sqrt{imc^2-E}\times e^{\frac{-iq}{2}}[\frac{-i}{2}(\phi_1-\phi_2)+(\phi_1'-\phi_2')]$$
$$=-[\frac{E-imc^2}{2\lambda\hbar c}+\frac{Z\alpha}{q}][\sqrt{imc^2+E}]e^{\frac{-iq}{2}}(\phi_1+\phi_2)+\frac{k}{q}\sqrt{imc^2-E}e^{\frac{-iq}{2}}(\phi_1-\phi_2) \quad (49)$$

dividing by $e^{-\frac{iq}{2}}$ and further dividing the first equation by $(imc^2+E)^{\frac{1}{2}}$ and the second equation by $(imc^2-E)^{\frac{1}{2}}$, we obtain

$$[\frac{-i}{2}(\phi_1+\phi_2)+(\phi_1'+\phi_2')]$$
$$=\frac{-k}{q}(\phi_1+\phi_2)+[\frac{E+imc^2}{2\lambda\hbar c}+\frac{Z\alpha}{q}]\frac{\sqrt{imc^2-E}}{\sqrt{imc^2+E}}(\phi_1-\phi_2) \quad (50)$$

$$[\frac{-i}{2}(\phi_1-\phi_2)+(\phi_1'-\phi_2')]$$
$$=-[\frac{E-imc^2}{2\lambda\hbar c}+\frac{Z\alpha}{q}]\frac{\sqrt{imc^2+E}}{\sqrt{imc^2-E}}(\phi_1+\phi_2)+\frac{k}{q}(\phi_1-\phi_2) \quad (51)$$

However, we had

$$\frac{\sqrt{imc^2-E}}{\sqrt{imc^2+E}}=\frac{imc^2-E}{\sqrt{-m^2c^4-E^2}}=\frac{imc^2-E}{i\hbar c\lambda} \quad (52)$$

and

$$\frac{\sqrt{imc^2+E}}{\sqrt{imc^2-E}}=\frac{imc^2+E}{\sqrt{-m^2c^4-E^2}}=\frac{imc^2+E}{i\hbar c\lambda} \quad (53)$$

Thus

$$[\frac{-i}{2}(\phi_1+\phi_2)+(\phi_1'+\phi_2')]$$
$$=\frac{-k}{q}(\phi_1+\phi_2)+[\frac{E+imc^2}{2\lambda\hbar c}+\frac{Z\alpha}{q}]\frac{imc^2-E}{i\hbar c\lambda}(\phi_1-\phi_2) \quad (54)$$

$$[\frac{-i}{2}(\phi_1-\phi_2)+(\phi_1'-\phi_2')]$$
$$=-[\frac{E-imc^2}{2\lambda\hbar c}+\frac{Z\alpha}{q}]\frac{imc^2+E}{i\hbar c\lambda}(\phi_1+\phi_2)+\frac{k}{q}(\phi_1-\phi_2) \quad (55)$$

By adding the two above equations

$$-i\phi_1 + 2\phi_1' = -2\frac{k}{q}\phi_2 + (\frac{E+imc^2}{2\lambda\hbar c})(\frac{imc^2-E}{i\hbar c\lambda})[\phi_1 - \phi_2]$$
$$+\frac{Z\alpha}{q}(\frac{imc^2-E}{i\hbar c\lambda})[\phi_1-\phi_2] - [\frac{E-imc^2}{2\lambda\hbar c}][\frac{imc^2+E}{i\hbar c\lambda}[\phi_1+\phi_2]$$
$$-\frac{Z\alpha}{q}\times\frac{imc^2+E}{i\hbar c\lambda}(\phi_1+\phi_2)$$
$$= -2\frac{k}{q}\phi_2 + (\frac{-m^2c^4-E^2}{2i\lambda^2\hbar^2c^2})(\phi_1-\phi_2) + \frac{Z\alpha}{q}(\frac{imc^2-E}{i\hbar c\lambda})(\phi_1-\phi_2)$$
$$-[\frac{E^2+m^2c^4}{2i\lambda^2\hbar^2c^2}](\phi_1+\phi_2) - \frac{Z\alpha}{q}\frac{imc^2+E}{i\hbar c\lambda}(\phi_1+\phi_2)$$
$$= \frac{-2k\phi_2}{q} - \frac{1}{2i}(\phi_1-\phi_2) - \frac{1}{2i}(\phi_1+\phi_2)$$
$$+\frac{Z\alpha}{q}(\frac{imc^2-E}{i\hbar mc\lambda})(\phi_1-\phi_2) - \frac{Z\alpha}{q}(\frac{imc^2+E}{i\hbar c\lambda})(\phi_1+\phi_2) \tag{56}$$

or

$$-i\phi_1 + 2\phi_1' = -\frac{2k}{q}\phi_2 - \frac{1}{i}\phi_1 +$$
$$\frac{Z\alpha}{q}(\frac{imc^2-E}{h\hbar\lambda c})(\phi_1-\phi_2) - \frac{Z\alpha}{q}(\frac{imc^2+E}{i\hbar c\lambda})(\phi_1+\phi_2) \tag{57}$$

By subtracting two equations, we have

$$-i\phi_2 + 2\phi_2' = -\frac{2k}{q}\phi_1 - \frac{E^2+m^2c^4}{2i\hbar^2c^2\lambda^2}(\phi_1-\phi_2)$$
$$+\frac{Z\alpha}{q}\frac{(imc^2-E)}{i\hbar c\lambda}(\phi_1-\phi_2) + \frac{(E^2+m^2c^4)}{2i\hbar^2c^2\lambda^2}(\phi_1+\phi_2)$$
$$+\frac{Z\alpha}{q}\frac{imc^2+E}{i\hbar c\lambda}(\phi_1+\phi_2) \tag{58}$$

or

$$-i\phi_2 + 2\phi_2' = -\frac{2k}{q}\phi_1 + \frac{\phi_2}{i} + \frac{Z\alpha}{q}\frac{(imc^2-E)}{i\hbar c\lambda}(\phi_1-\phi_2)$$
$$+\frac{Z\alpha}{q}\frac{imc^2+E}{i\hbar c\lambda}(\phi_1+\phi_2) \tag{59}$$

Summarizing, we obtain

$$\phi_1' = (i - \frac{Z\alpha E}{qi\hbar c\lambda})\phi_1 - (\frac{k}{q} + \frac{Z\alpha}{q}\frac{mc^2}{\hbar c\lambda})\phi_2 \tag{60}$$

$$\phi_2' = (-\frac{k}{q} + Z\alpha\frac{mc^2}{\hbar c\lambda q})\phi_1 + \frac{Z\alpha}{q}\frac{E}{i\hbar c\lambda}\phi_2 \tag{61}$$

We use the power series. We separate a factor q^γ, which describes the behavior of the solution for $q \to 0$

$$\phi_1 = q^\gamma \sum \alpha_m q^m \tag{62}$$

$$\phi_2 = q^\gamma \sum \beta_m q^m \tag{63}$$

Inserting this equation into equations (60) and (61), we obtain

$$\sum(m+\gamma)\alpha_m q^{m+\gamma-1} = i\sum \alpha_m q^{m+\gamma} - \frac{Z\alpha E}{i\hbar c\lambda}\sum \alpha_m q^{m+\gamma-1}$$
$$-(k + \frac{Z\alpha mc^2}{\hbar c\lambda})\sum \beta_m q^{m+\gamma-1} \tag{64}$$

and

$$\sum \beta_m (m+\gamma) q^{m+\gamma-1} = (-k + \frac{Z\alpha mc^2}{\hbar c\lambda})\sum \alpha_m q^{m+\gamma-1}$$
$$+\frac{Z\alpha E}{i\hbar c\lambda}\sum \beta_m q^{m+\gamma-1} \tag{65}$$

By comparing the coefficient, we obtain

$$\alpha_m(m+\gamma) = i\alpha_m - 1 - \frac{Z\alpha E\alpha_m}{h\hbar c\lambda} - (k + \frac{Z\alpha mc^2}{\hbar\lambda c})\beta_m \tag{66}$$

$$\beta_m(m+\gamma) = (-k + \frac{Z\alpha mc^2}{\hbar c\lambda})\alpha_m + \frac{Z\alpha E}{i\hbar c\lambda}\beta_m \tag{67}$$

From the above equation, we obtain

$$\frac{\beta_m}{\alpha_m} = \frac{(-k + \frac{Z\alpha mc^2}{\hbar c\lambda})}{m + \gamma - \frac{Z\alpha E}{i\hbar c\lambda}} = \frac{(k - \frac{Z\alpha mc^2}{\hbar c\lambda})}{n' - m} \tag{68}$$

$$n' = \frac{Z\alpha E}{i\hbar c\lambda} - \gamma \tag{69}$$

For $m = 0$, we obtain

$$\frac{\beta_\circ}{\alpha_\circ} = \frac{k - \frac{Z\alpha mc^2}{\hbar c\lambda}}{n'} = \frac{k - (n'+\gamma)\frac{mc^2}{E}}{n'} \tag{70}$$

inserting (68) into (66) and (67), we obtain

$$\alpha_m(m+\gamma) = i\alpha_{m-1} - \frac{Z\alpha E\alpha_m}{i\hbar c\lambda} - (k + \frac{z\alpha mc^2}{\hbar c\lambda})\frac{(k - \frac{Z\alpha mc^2}{\hbar c\lambda})}{(m+\gamma - \frac{Z\alpha E}{i\hbar c\lambda})}\alpha_m \tag{71}$$

$$\alpha_m[(m+\gamma) + \frac{Z\alpha E}{i\hbar c\lambda} - \frac{(k + \frac{z\alpha mc^2}{\hbar c\lambda})(k - \frac{Z\alpha mc^2}{\hbar c\lambda})}{(m-n')} = i\alpha_{m-1} \tag{72}$$

$$\alpha_m[m+\gamma + \frac{Z\alpha E}{i\hbar c\lambda} + \frac{(k + \frac{z\alpha mc^2}{\hbar c\lambda})(k - \frac{Z\alpha mc^2}{\hbar c\lambda})}{(n'-m)} = i\alpha_{m-1} \tag{73}$$

$$\alpha_m[(m+\gamma + \frac{Z\alpha E}{i\hbar c\lambda})(n'-m) + k^2 - \frac{Z^2\alpha^2 m^2 c^4}{\hbar^2 c^2\lambda^2}] = i\alpha_{m-1}(n'-m) \tag{74}$$

If we expand the bracket on the left hand side of the above equation and use equation (69), we obtain

$$(m+\gamma + \frac{Z\alpha E}{i\hbar c\lambda})(\frac{Z\alpha E}{i\hbar c\lambda} - \gamma - m) = -2m\gamma - m^2 - \gamma^2 - (\frac{Z\alpha E}{\hbar c\lambda})^2 \tag{75}$$

Thus, we have

$$\alpha_m[-2m\gamma - m^2 - \gamma^2 - (\frac{Z\alpha E}{\hbar c\lambda})^2 + k^2 - \frac{Z^2\alpha^2 m^2 c^4}{\hbar^2 c^2\lambda^2}] = i\alpha_{m-1}(n'-m) \tag{76}$$

$$\alpha_m[-m(2\gamma+m) + (Z\alpha)^2 - (\frac{Z\alpha E}{\hbar c\lambda})^2 - \frac{Z^2\alpha^2 m^2 c^4}{\hbar^2 c^2\lambda^2}] = i\alpha_{m-1}(n'-m) \tag{77}$$

with

$$\gamma^2 = k^2 - (Z\alpha)^2 \tag{78}$$

We conclude that

$$\alpha_m[-m(2\gamma+m) + (Z\alpha)^2(1 - \frac{E^2 + m^2c^4}{\hbar^2 c^2\lambda^2})] = i\alpha_{m-1}(n'-m) \tag{79}$$

which can be writen as

$$\begin{aligned}\alpha_m &= \frac{-(n'-m)}{m(2\gamma+m)} i\alpha_{m-1} \\ &= \frac{(-1)^m(n'-1)...(n'-m)\alpha_\circ i^m}{m!(2\gamma+1)...(2\gamma+m)} = \frac{(1-n')(2-n')...(m-n')(i)^m}{m!(2\gamma+1)...(2\gamma+m)}\alpha_\circ\end{aligned} \tag{80}$$

$$\beta_m = \frac{(+k - \frac{Z\alpha mc^2}{\hbar c\lambda})}{n' - m}\frac{(-1)^m(n'-1)...(n'-m)\alpha_\circ i^m}{m!(2\gamma+1)...(2\gamma+m)} \tag{81}$$

$$\beta_m = \frac{(+k - \frac{Z\alpha mc^2}{\hbar c\lambda})(-1)^m(n'-1)...(n'-m+1)\alpha_\circ i^m}{m!(2\gamma+1)...(2\gamma+m)} \tag{82}$$

Using (70), we conclude that

$$\beta_m = \frac{(+k - \frac{Z\alpha mc^2}{\hbar c\lambda})(-1)^m(n'-1)...(n'-m+1)i^m}{m!(2\gamma+1)...(2\gamma+m)}\frac{n'\beta_\circ}{(+k - \frac{Z\alpha mc^2}{\hbar c\lambda})} \tag{83}$$

$$\beta_m = \frac{n'(n'-1)...(n'-m+1)(-1)^m i^m}{m!(2\gamma+1)...(2\gamma+m)}\beta_\circ \tag{84}$$

The above equation is the confluent hyper geometric function

$$F(a,c;x) = 1 + \frac{a}{c}x + \frac{a(a+1)}{c(c+1)}\frac{x^2}{2!} + ... \tag{85}$$

$$\phi_1 = \alpha_\circ q^\gamma F(1-n', 2\gamma+1; iq) \tag{86}$$

$$\begin{aligned}\phi_2 &= \beta_\circ q^\gamma F(-n', 2\gamma+1; iq)\\ &= (\frac{\kappa - Z\alpha mc^2/\hbar c\lambda}{n'})\alpha_\circ q^\gamma F(-n', 2\gamma+1; iq)\end{aligned} \tag{87}$$

For negative value of n' the above series is normalized if we choose the appropriate γ. using (44) , (45) and the fact that $G = rg$ and $F = rf$, we can construct the normalized wave functions f and g.

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