The analysis of Lobo and Visser applied to both Natario and Casimir warp drives. Physical reactions of gravitational repulsive behavior between the positive mass of the spaceship and the negative mass of the warp bubble

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Abstract

Warp Drives are solutions of the Einstein Field Equations that allows superluminal travel within the framework of General Relativity. There are at the present moment two known solutions: The Alcubierre warp drive discovered in 1994 and the Natario warp drive discovered in 2001. The major drawback concerning warp drives is the huge amount of negative energy able to sustain the warp bubble.In order to perform an interstellar space travel to a "nearby" star at 20 light-years away in a reasonable amount of time a ship must attain a speed of about 200 times faster than light.However the negative energy density at such a speed is directly proportional to the factor 10^{48} which is 1.000.000.000.000.000.000.000.000 times bigger in magnitude than the mass of the planet Earth!! Although the energy conditions of General Relativity forbids the existence of negative energy the Casimir Effect first predicted by Casimir in 1948 and verified experimentally by Lamoreaux in 1997 allows sub-microscopical amounts of it.We introduce in this work a shape function that will low the negative energy density requirements in the Natario warp drive from $10^{48}\frac{Joules}{Meter^3}$ to $10^{-7}\frac{Joules}{Meter^3}$ a low and affordable level. However reducing the negative energy density requirements of the warp drive to arbitrary low levels works only for empty bubbles not for bubbles with real spaceships inside because the positive mass of the spaceship exerts over the negative mass of the bubble a gravitational repulsive force and a spaceship with a large positive mass inside a bubble of small negative mass destroys the bubble.According to Lobo and Visser we can reduce the negative energy density of the warp bubble only to the limit when the negative energy becomes a reasonable fraction of the positive mass/energy of the spaceship.and no less otherwise the bubble is destroyed.The analysis of Lobo and Visser must be taken in account when considering bubbles with real spaceships inside otherwise the warp drive may not work.We reproduce in this work the analysis of Lobo and Visser for the Natario and Casimir warp drives.The work of Lobo and Visser is the third most important work in warp drive science immediately after the works of Alcubierre and Natario and the Lobo-Visser paper must also be considered a seminal paper like the ones of both Alcubierre and Natario.

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1 Introduction

The Warp Drive as a solution of the Einstein Field Equations of General Relativity that allows superluminal travel appeared first in 1994 due to the work of Alcubierre.([1]) The warp drive as conceived by Alcubierre worked with an expansion of the spacetime behind an object and contraction of the spacetime in front.The departure point is being moved away from the object and the destination point is being moved closer to the object.The object do not moves at all[1].It remains at the rest inside the so called warp bubble but an external observer would see the object passing by him at superluminal speeds(pg 8 in [1])(pg 1 in [2])(pg 34 in [3]).

Later on in 2001 another warp drive appeared due to the work of Natario.([2]).This do not expands or contracts spacetime but deals with the spacetime as a "strain" tensor of Fluid Mechanics(pg 5 in [2]). Imagine the object being a fish inside an aquarium and the aquarium is floating in the surface of a river but carried out by the river stream.The warp bubble in this case is the aquarium whose walls do not expand or contract. An observer in the margin of the river would see the aquarium passing by him at a large speed but inside the aquarium the fish is at the rest with respect to his local neighborhoods.

However there are 3 major drawbacks that compromises the warp drive physical integrity as a viable tool for superluminal interstellar travel.

The first drawback is the quest of large negative energy requirements enough to sustain the warp bubble. In order to travel to a "nearby" star at 20 light-years at superluminal speeds in a reasonable amount of time a ship must attain a speed of about 200 times faster than light.However the negative energy density at such a speed is directly proportional to the factor 10^{48} which is 1.000.000.000.000.000.000.000.000 times bigger in magnitude than the mass of the planet Earth!!!

Another drawback that affects the warp drive is the quest of the interstellar navigation:Interstellar space is not empty and from a real point of view a ship at superluminal speeds would impact asteroids,comets,interstellar space dust and photons.

The last drawback raised against the warp drive is the fact that inside the warp bubble an astronaut cannot send signals with the speed of the light to control the front of the bubble because an Horizon(causally disconnected portion of spacetime)is established between the astronaut and the warp bubble.

We can demonstrate that the Natario warp drive can "easily" overcome these obstacles as a valid candidate for superluminal interstellar travel(see [4] and [5]).

The quest for negative energy densities in warp drive spacetimes is very important:The energy conditions of General Relativity allows the existence of positive energy densities only and the warp drive which requires negative energy densities is sustained by the equations of General Relativity.

So according to General Relativity the warp drive is impossible because it violates all the knows energy conditions.This was also stated by both Alcubierre and Natario (pg 8 in [1] pg 1 in [2]).

[1] do not violates Relativity

Although Classical Physics forbids the existence of negative energy densities there exists an effect in Quantum Physics that allows its existence however in very small sub-microscopical amounts.

The Casimir Effect first predicted theoretically by Casimir in 1948 [6] and experimentally demonstrated by Lamoreaux in 1997 [7] allows the existence of negative energy densities.Lamoreaux obtained experimentally negative energy densities of $10^{-4}\frac{Joules}{Meter^3}$.This is an extremely small value:In order to get an idea of how small this value is consider the fact that a body of 1 kilogram in a cubic meter of space possesses an energy density of $9 \times 10^{16}\frac{Joules}{Meter^3}$. So the values obtained by Lamoreaux are 10^{20} times lighter than the ones of a body of 1 kilogram in a cubic meter of space or better:100.000.000.000.000.000.000 times lighter than the ones of a body of 1 kilogram in a cubic meter of space.

Alcubierre was the first to propose the Casimir Effect as a source of negative energy to sustain a warp drive(pg 9 in [1]) however the Casimir Effect can generate sub-microscopical amounts of it and warp drives needs astronomical quantities.

In this work we propose a modification of the geometry of the Natario warp drive in order to get negative energy density requirements compatible with the ones obtained by Lamoreaux for the Casimir Effect.We introduce a new Natario shape function that will low the negative energy density requirements from $10^{48}\frac{Joules}{meters^3}$ to $10^{-7}\frac{Joules}{meters^3}$ even for a spaceship moving at 200 times light speed.

This is a result 1000 times lighter than the ones obtained by Lamoreaux in 1997 and it proofs that the Casimir Effect can undoubtely generate and sustain a Natario warp drive spacetime.

Our result is only a demonstration of how far can the mathematics go in order to ameliorate the negative energy density requirements needed to sustain a warp drive.A real warp drive in interstellar space would need large and macroscopical amounts of negative energy density in order to deflect hazardous interstellar matter protecting the ship from impacts with asteroids comets or photons from gamma radiation.[2]

Some years ago Lobo and Visser presented a very interesting work (see abs of [10]).They noted that in both Alcubierre and Natario warp drive spacetimes the center of the bubble where a spaceship would reside as an Eulerian observer in the position $rs = 0$ is always described as empty space.Since we need the gravitational repulsive force between positive and negative masses to deflect incoming Doppler blueshifted photons and asteroids in order to protect the spaceship and the crew members from the hazardous interstellar matter and asteroids or Doppler blueshifted photons are objects of positive mass/energy we cannot forget that the spaceship inside the bubble is also a positive mass object and if we place a spaceship inside the bubble then gravitational repulsive forces between the positive mass of the spaceship and the negative mass of the walls of the bubble would arise. Neither Alcubierre nor Natario addressed the problem of spaceships inside the warp bubble.

In our work we can low the negative energy density needed to sustain a Natario warp drive bubble moving at 200 times light speed from 10^{48} to 10^{-7} but this can be done only for empty bubbles not for bubbles carrying spaceships because the large positive mass of the spaceship would repel the low negative mass of the bubble destroying the bubble and we would have no warp drive after all.

[2]See Appendices H and M in [4] and [5]

Lobo and Visser outlined the fact that in order for a warp bubble to be stable a certain relation between the positive mass of the spaceship coupled with the negative mass of the warp bubble and the bubble radius must exists otherwise taking arbitrary values for these measures the bubble is destroyed.According to Lobo and Visser the amount of negative energy in the warp bubble able to sustain a large spaceship must be a reasonable fraction of the positive mass of the spaceship.(see summary pg 13 in [10]). Besides low levels of negative energy cannot deflect the incoming hazardous interstellar matter but macroscopical amounts of it can do.

In this work we demonstrate the validity of the arguments of Lobo and Visser using two spaceships: The NASA Space Shuttle with 100 metric tons (100.000 kilograms) and the Star Trek Enterprise with 3.250.000 metric tons (3.250.000.000 kilograms) with certain configurations of the warp bubble negative energy density and radius and for low amounts of negative energy both the Shuttle and the Enterprise destroys the bubble due to gravitational repulsive forces.For intermediate amounts of negative energy density the warp bubble can sustain the Shuttle but cannot sustain the Enterprise because the Enterprise is 32.500 heavier than the Shuttle and the negative energy that sustains the Shuttle cannot cope with the gravitational repulsive force of the Enterprise.

Only with high amounts of negative energy density that although far from being a reasonable fraction of the Enterprise mass a warp bubble can sustain the Enterprise proving the argument of Lobo and Visser as valid.

Any future study of warp drive geometry concerning bubbles with real flesh-and-bone spaceships inside must take in account the analysis of Lobo and Visser or the warp drive will simply not work

As a matter of fact we can affirm that the Lobo and Visser study is so relevant that must be regarded as the third most important work on warp drive science immediately after the works of Alcubierre and Natario and the Lobo-Visser paper must be considered also a seminal paper just like the ones of both Alcubierre and Natario.

In this work we cover only the Natario warp drive and we avoid comparisons between the differences of the models proposed by Alcubierre and Natario since these differences were already deeply covered by the existing available literature.However we use the Alcubierre shape function to define its Natario counterpart.

We adopt here the Geometrized system of units in which $c = G = 1$ for geometric purposes and the International System of units for energetic purposes

2 The Natario warp drive continuous shape function

Introducing here $f(rs)$ as the Alcubierre shape function that defines the Alcubierre warp drive spacetime we can construct the Natario shape function $n(rs)$ that defines the Natario warp drive spacetime using its Alcubierre counterpart.Below is presented the equation of the Alcubierre shape function.[3].

$$f(rs) = \frac{1}{2}[1 - tanh[@(rs - R)] \tag{1}$$

$$rs = \sqrt{(x - xs)^2 + y^2 + z^2} \tag{2}$$

According with Alcubierre any function $f(rs)$ that gives 1 inside the bubble and 0 outside the bubble while being $1 > f(rs) > 0$ in the Alcubierre warped region is a valid shape function for the Alcubierre warp drive.(see eqs 6 and 7 pg 4 in [1] or top of pg 4 in [2]).

In the Alcubierre shape function xs is the center of the warp bubble where the ship resides. R is the radius of the warp bubble and @ is the Alcubierre parameter related to the thickness.According to Alcubierre these can have arbitrary values.We outline here the fact that according to pg 4 in [1] the parameter @ can have arbitrary values.rs is the path of the so-called Eulerian observer that starts at the center of the bubble $xs = R = rs = 0$ and ends up outside the warp bubble $rs > R$.

According to Natario(pg 5 in [2]) any function that gives 0 inside the bubble and $\frac{1}{2}$ outside the bubble while being $0 < n(rs) < \frac{1}{2}$ in the Natario warped region is a valid shape function for the Natario warp drive.

The Natario warp drive continuous shape function can be defined by:

$$n(rs) = \frac{1}{2}[1 - f(rs)] \tag{3}$$

$$n(rs) = \frac{1}{2}[1 - [\frac{1}{2}[1 - tanh[@(rs - R)]]]] \tag{4}$$

This shape function gives the result of $n(rs) = 0$ inside the warp bubble and $n(rs) = \frac{1}{2}$ outside the warp bubble while being $0 < n(rs) < \frac{1}{2}$ in the Natario warped region.

Note that the Alcubierre shape function is being used to define its Natario shape function counterpart.

For the Natario shape function introduced above it is easy to figure out when $f(rs) = 1$(interior of the Alcubierre bubble) then $n(rs) = 0$(interior of the Natario bubble) and when $f(rs) = 0$(exterior of the Alcubierre bubble)then $n(rs) = \frac{1}{2}$(exterior of the Natario bubble).

[3] $tanh[@(rs + R)] = 1$,$tanh(@R) = 1$ for very high values of the Alcubierre thickness parameter $@ >> |R|$

Another Natario warp drive valid shape function can be given by:

$$n(rs) = [\frac{1}{2}][1 - f(rs)^{WF}]^{WF} \tag{5}$$

Its derivative square is :

$$n'(rs)^2 = [\frac{1}{4}]WF^4[1 - f(rs)^{WF}]^{2(WF-1)}[f(rs)^{2(WF-1)}]f'(rs)^2 \tag{6}$$

The shape function above also gives the result of $n(rs) = 0$ inside the warp bubble and $n(rs) = \frac{1}{2}$ outside the warp bubble while being $0 < n(rs) < \frac{1}{2}$ in the Natario warped region(see pg 5 in [2]).

Note that like in the previous case the Alcubierre shape function is being used to define its Natario shape function counterpart. The term WF in the Natario shape function is dimensionless too:it is the warp factor.For a while it is important to outline only that the warp factor $WF >> |R|$ is much greater then the modulus of the bubble radius.

For the second Natario shape function introduced above it is easy to figure out when $f(rs) = 1$(interior of the Alcubierre bubble) then $n(rs) = 0$(interior of the Natario bubble) and when $f(rs) = 0$(exterior of the Alcubierre bubble)then $n(rs) = \frac{1}{2}$(exterior of the Natario bubble).

- Numerical plot for the second shape function with @ = 50000 and warp factor with a value $WF = 200$

rs	$f(rs)$	$n(rs)$	$f'(rs)^2$	$n'(rs)^2$
$9,99970000000E+001$	1	0	$2,650396620740E-251$	0
$9,99980000000E+001$	1	0	$1,915169647489E-164$	0
$9,99990000000E+001$	1	0	$1,383896564748E-077$	0
$1,00000000000E+002$	$0,5$	$0,5$	$6,250000000000E+008$	$3,872591914849E-103$
$1,00001000000E+002$	0	$0,5$	$1,383896486082E-077$	0
$1,00002000000E+002$	0	$0,5$	$1,915169538624E-164$	0
$1,00003000000E+002$	0	$0,5$	$2,650396470082E-251$	0

- Numerical plot for the second shape function with @ = 75000 and warp factor with a value $WF = 200$

rs	$f(rs)$	$n(rs)$	$f'(rs)^2$	$n'(rs)^2$
$9,99980000000E+001$	1	0	$5,963392481410E-251$	0
$9,99990000000E+001$	1	0	$1,158345097767E-120$	0
$1,00000000000E+002$	$0,5$	$0,5$	$1,406250000000E+009$	$8,713331808411E-103$
$1,00001000000E+002$	0	$0,5$	$1,158344999000E-120$	0
$1,00002000000E+002$	0	$0,5$	$5,963391972940E-251$	0

- Numerical plot for the second shape function with @ = 100000 and warp factor with a value $WF = 200$

rs	$f(rs)$	$n(rs)$	$f'(rs)^2$	$n'(rs)^2$
$9,99990000000E+001$	1	0	$7,660678807684E-164$	0
$1,00000000000E+002$	$0,5$	$0,5$	$2,500000000000E+009$	$1,549036765940E-102$
$1,00001000000E+002$	0	$0,5$	$7,660677936765E-164$	0

The plots in the previous page demonstrate the important role of the thickness parameter @ in the warp bubble geometry wether in both Alcubierre or Natario warp drive spacetimes.For a bubble of 100 meters radius $R = 100$ the regions where $1 > f(rs) > 0$(Alcubierre warped region) and $0 < n(rs) < \frac{1}{2}$(Natario warped region) becomes thicker or thinner as @ becomes higher.

Then the geometric position where both Alcubierre and Natario warped regions begins with respect to R the bubble radius is $rs = R - \epsilon < R$ and the geometric position where both Alcubierre and Natario warped regions ends with respect to R the bubble radius is $rs = R + \epsilon > R$

As large as @ becomes as smaller ϵ becomes too.We will address this issue later in this work.

Since we will work in the equatorial plane of the Natario warp drive[4] the area of the warped region is the circular crown delimited by the difference between the area of the circle of the end of the warped region minus the area of the circle of the beginning of the warped region.Then for a warp bubble of radius R we have according to the plots of the previous page:

- 1)-Radius R_{end} of the circle of the end of the warped region:

$$R_{end} = R + \epsilon \tag{7}$$

- 2)-Radius R_{beg} of the circle of the beginning of the warped region:

$$R_{beg} = R - \epsilon \tag{8}$$

- 1)-Area S_{end} of the circle of the end of the warped region:

$$S_{end} = \pi R_{end}^2 = \pi (R + \epsilon)^2 \tag{9}$$

- 2)-Area S_{beg} of the circle of the beginning of the warped region:

$$S_{beg} = \pi R_{beg}^2 = \pi (R - \epsilon)^2 \tag{10}$$

The area of the warped region is then given by:

$$S_{warp} = S_{end} - S_{beg} = \pi R_{end}^2 - \pi R_{beg}^2 = \pi (R + \epsilon)^2 - \pi (R - \epsilon)^2 = 4\pi\epsilon R \tag{11}$$

[4]see Appendix A

3 The Weak Energy Condition(WEC) in General Relativity and the Casimir Effect

The equation of the weak energy density condition (WEC) in General Relativity is given by:(see eq 4.72 pg 124 in [9],see eq 2.1.6 pg 41 in [8])

$$\rho = T_{\mu\nu} u^{\mu} u^{\nu} \geq 0 \tag{12}$$

From above according to the WEC the term $T_{\mu\nu}$ represents a stress-emergy-momentum-tensor(SEMT) in a given local inertial frame that must be always positive-no negative masses are allowed in General Relativity(pg 124 in [9]) and the terms $u^{\mu}u^{\nu}$ are timelike vectors.

The WEC states that the energy density of any distribution of matter obeying a SEMT given by $T_{\mu\nu}$ as seen by an observer in a given spacetime moving with a four-velocity vector given by u^{μ} or u^{ν} must be non-negative for any future-directed timelike vectors u^{μ} or u^{ν}.(pg 41 in [8]).

However as stated by both Alcubierre and Natario the warp drive requires negative energy density and violates the WEC(and all the other energy conditions).(see pg 8 in [1],pg 1 in [2])

If negative masses are not allowed by General Relativity and the warp drive needs negative masses this means to say that the warp drive is impossible.

This is true from the point of view of the Classical Physics domain where General Relativity belongs:However the Quantum Physics domain allows sub-microscopical amounts of negative energy densities in a clear violation of the WEC.

The only known process to generate experimentally these sub-microscopical amounts of negative energy densities is the Casimir Effect.[5][6]

The Casimir Effect states that the vacuum energy density between two parallel conduction plates separated by a distance d is given by:(pg 42 in [8])

$$\rho = -\frac{\pi^2}{720}\frac{\hbar}{d^4} \tag{13}$$

The equation above was written in the Geometrized System of Units $c = G = 1$.In the International System of Units the same equation would be:

$$\rho = -\frac{\pi^2}{720}\frac{\hbar c}{d^4} \tag{14}$$

In the equation above $\pi = 3,1415926536$,$\hbar = \frac{h}{2\pi}$,h is the Planck Constant given by $h = 6,626 \times 10^{-34} Joules \times Seconds$ and $c = 3 \times 10^8 \frac{Meters}{Seconds}$ is the light speed.Except for the distance between the plates d all the terms above are constants.Then the equation of the Casimir Effect is better rewritten as:

[5]see Wikipedia the Free Encyclopedia on Energy Condition and Casimir Effect

[6]see also Appendix D

$$\rho = -\frac{C}{d^4} \tag{15}$$

With C being the Casimir Factor defined as:

$$C = \frac{\pi^2}{720}\hbar c \tag{16}$$

Computing the values of C we have:

$$C = \frac{\pi^2}{720}\hbar c = \frac{9,8696044012}{720} \times 1,0545606529 \times 10^{-34} \times 3 \times 10^8 = 4,3367068589 \times 10^{-28} \tag{17}$$

This is an extremely small value and the total negative energy density we can extract from the Casimir Effect is directly proportional to C and inversely proportional to the fourth power of the distance d.But since C is always a constant then the total negative energy density we can extract from the Casimir Effect depends only of the distance d of the separation of the plates.

The total negative energy density we can extract from the Casimir Effect is better written by the following equation:

$$\rho = -\frac{4,3367068589 \times 10^{-28}}{d^4} \tag{18}$$

The mass of the proton is $1.672621777 \times 10^{-27}$ kilograms[7] and C is a fixed value 10 times lighter than the magnitude of a proton mass.

So in order to obtain reasonable amounts of negative energy densities by the Casimir Effect the distance d between the plates must be sub-microscopically small or the effect will not be noticed.

For two plates separated by a distance $d = 1$ meter the total negative energy density would be:

$$\rho = -\frac{4,3367068589 \times 10^{-28}}{1^4} = -4,3367068589 \times 10^{-28} Joules/Meter^3 \tag{19}$$

For a distance $d = 10$ meters the total negative energy density would be:

$$\rho = -\frac{4,3367068589 \times 10^{-28}}{(10)^4} = -4,3367068589 \times 10^{-32} Joules/Meter^3 \tag{20}$$

These values are impossible to be measured experimentally at such distances.

The Casimir Effect was theorized in 1948 by Casimir (pg 3 in [6]) but in 1948 Casimir did not possessed the needed technology to demonstrate it experimentally and the Casimir Effect remained a mathematical conjecture until 1997.

Lamoreaux in 1997 demonstrated experimentally the Casimir Effect for a distance between plates of about $d = 0,6\mu m$ to $d = 6\mu m$ (pg 1 in [7]).$1\mu m = 10^{-6} meters$.

[7] see Wikipedia the Free Encyclopedia

Computing the Casimir Effect for a distance $d = 1\mu m$ or $d = 10^{-6} meters$ we should expect for:

$$\rho = -\frac{4,3367068589 \times 10^{-28}}{(10^{-6})^4} = -\frac{4,3367068589 \times 10^{-28}}{(10^{-24})} = -4,3367068589 \times 10^{-4} Joules/Meter^3 \quad (21)$$

The value above is within the range of values obtained by Lamoreaux for the Casimir Effect and although microscopically small it can be measured.

The work of Lamoreaux is very important:He obtained microscopically small amounts of negative energy density and unfortunately the warp drive needs astronomical amounts of it but at least Lamoreaux demonstrated that the Casimir Effect is real and the negative energy exists.

Alcubierre was the first to propose the use of the Casimir Effect to generate the warp drive spacetime distortion.(pg 9 in [1]).However warp drives needs astronomical amounts of negative energy density and the Casimir Effect unfortunately can only provide sub-microscopical small amounts of it.

In this work we propose a modification in the geometry of the warp drive spacetime in order to be satisfied by the small amounts of negative energy generated by the Casimir Effect.

A modification in the equations of the warp drive can reduce dramatically the needs of negative energy needed to sustain it making the requirements of the warp drive negative energy density as small as the experimental results obtained by Lamoreaux for the Casimir Effect.

In this work we demonstrate that this is perfectly possible for the Natario warp drive spacetime:As a matter of fact we are about to propose the so-called Casimir warp drive. Remember that the Einstein field equation of General Relativity allows curved or distorted spacetime geometries if we can generate the needed mass-energy configurations that will distort the spacetime geometry according to our needs.If we want a certain spacetime geometric configuration we need to generate a distribution of mass and energy specially tailored for the needed configuration. So warp drives can be classified by two conditions:geometry(the left side of the Einstein field equation) and energy(the right side of the Einstein field equation).

- 1)-From the point of view of geometry the warp drive can be classified as an Alcubierre or a Natario warp drive since there are no other known solutions.
- 2)-From the point of view of the energy that will distort the spacetime geometry generating the warp drive effect the only known solution is the Casimir warp drive.

4 The problem of the negative energy in the Natario warp drive spacetime:The unphysical nature of warp drive

The negative energy density for the Natario warp drive is given by(see pg 5 in [2])

$$\rho = T_{\mu\nu}u^{\mu}u^{\nu} = -\frac{1}{16\pi}K_{ij}K^{ij} = -\frac{v_s^2}{8\pi}\left[3(n'(rs))^2\cos^2\theta + \left(n'(rs) + \frac{r}{2}n''(rs)\right)^2\sin^2\theta\right] \tag{22}$$

Converting from the Geometrized System of Units to the International System we should expect for the following expression:

$$\rho = -\frac{c^2}{G}\frac{vs^2}{8\pi}\left[3(n'(rs))^2\cos^2\theta + \left(n'(rs) + \frac{rs}{2}n''(rs)\right)^2\sin^2\theta\right]. \tag{23}$$

Rewriting the Natario negative energy density in cartezian coordinates we should expect for[8]:

$$\rho = T_{\mu\nu}u^{\mu}u^{\nu} = -\frac{c^2}{G}\frac{v_s^2}{8\pi}\left[3(n'(rs))^2(\frac{x}{rs})^2 + \left(n'(rs) + \frac{r}{2}n''(rs)\right)^2(\frac{y}{rs})^2\right] \tag{24}$$

In the equatorial plane($1+1$ dimensional spacetime with $rs = x - xs$,$y = 0$ and center of the bubble $xs = 0$):

$$\rho = T_{\mu\nu}u^{\mu}u^{\nu} = -\frac{c^2}{G}\frac{v_s^2}{8\pi}\left[3(n'(rs))^2\right] \tag{25}$$

Note that in the above expressions the warp drive speed vs appears raised to a power of 2. Considering our Natario warp drive moving with $vs = 200$ which means to say 200 times light speed in order to make a round trip from Earth to a nearby star at 20 light-years away in a reasonable amount of time(in months not in years) we would get in the expression of the negative energy the factor $c^2 = (3 \times 10^8)^2 = 9 \times 10^{16}$ being divided by $6,67 \times 10^{-11}$ giving $1,35 \times 10^{27}$ and this is multiplied by $(6 \times 10^{10})^2 = 36 \times 10^{20}$ coming from the term $vs = 200$ giving $1,35 \times 10^{27} \times 36 \times 10^{20} = 1,35 \times 10^{27} \times 3,6 \times 10^{21} = 4,86 \times 10^{48}$!!!

A number with 48 zeros!!!The planet Earth have a mass[9] of about $6 \times 10^{24}kg$

This term is 1.000.000.000.000.000.000.000.000 times bigger in magnitude than the mass of the planet Earth!!!or better:The amount of negative energy density needed to sustain a warp bubble at a speed of 200 times faster than light requires the magnitude of the masses of 1.000.000.000.000.000.000.000.000 planet Earths for both Alcubierre and Natario cases!!!!

And multiplying the mass of Earth by c^2 in order to get the total positive energy "stored" in the Earth according to the Einstein equation $E = mc^2$ we would find the value of $54 \times 10^{40} = 5,4 \times 10^{41} Joules$.

Earth have a positive energy of $10^{41} Joules$ and dividing this by the volume of the Earth(radius $R_{Earth} =$ 6300 km approximately) we would find the positive energy density of the Earth.Taking the cube of the Earth radius $(6300000m = 6,3 \times 10^6)^3 = 2,5 \times 10^{20}$ and dividing $5,4 \times 10^{41}$ by $(4/3)\pi R_{Earth}^3$ we would find the value of $4,77 \times 10^{20}\frac{Joules}{m^3}$. So Earth have a positive energy density of $4,77 \times 10^{20}\frac{Joules}{m^3}$ and we are talking about negative energy densities with a factor of 10^{48} for the warp drive while the quantum theory allows only microscopical amounts of negative energy density.

[8] see Appendix A

[9] see Wikipedia:The free Encyclopedia

So we would need to generate in order to maintain a warp drive with 200 times light speed the negative energy density equivalent to the positive energy density of 10^{28} Earths!!!!

A number with 28 zeros!!!.Unfortunately we must agree with the major part of the scientific community that says:"Warp Drive is impossible and unphysical!!".

Remember that the Casimir Effect is the only physical known source of negative energy density and Lamoreaux obtained experimentally negative energy densities of about $10^{-4}\frac{Joules}{Meter^3}$ while a body of density of 1 kilogram per cubic meter possesses a density of about $9 \times 10^{16}\frac{Joules}{Meter^3}$

However looking better to the expression of the negative energy density in the equatorial plane of the Natario warp drive:

$$\rho = T_{\mu\nu}u^{\mu}u^{\nu} = -\frac{c^2}{G}\frac{v_s^2}{8\pi}\left[3(n'(rs))^2\right] \tag{26}$$

We can see that a very low derivative and hence its square can perhaps obliterate the huge factor of 10^{48} ameliorating the negative energy requirements to sustain the warp drive.

In section 5 we will explain how the second Natario shape function introduced in section 2 allows the reduction of the negative energy density requirements to arbitrary low values completely obliterating the factor 10^{48} which is 1.000.000.000.000.000.000.000.000 times bigger in magnitude than the mass of the planet Earth!!!...

The values obtained with our new Natario shape function will make the Natario warp drive negative energy density requirements falls well within the Lamoreaux experimental values obtained for the Casimir Effect.

Why we cannot use the first Natario shape function $n(rs)$ defined in the section 2 as:

$$n(rs) = \frac{1}{2}[1 - f(rs)] \tag{27}$$

With $f(rs)$ the Alcubierre shape function being:[10].

$$f(rs) = \frac{1}{2}[1 - tanh[@(rs - R)] \tag{28}$$

In order to get the total energy requirements needed to sustain the Natario warp drive??

The square of the derivative of the Alcubierre shape function is given by:

$$f'(rs)^2 = \frac{1}{4}[\frac{@^2}{cosh^4[@(rs - R)]}] \tag{29}$$

In the equatorial plane $y = 0$ and we can neglect the second order derivative of the Natario shape function and consequently its square.The square of the first order derivative is then given by:

$$n'(rs)^2 = \frac{1}{4}f'(rs)^2 \tag{30}$$

[10] $tanh[@(rs + R)] = 1$,$tanh(@R) = 1$ for very high values of the Alcubierre thickness parameter $@ >> |R|$

$$n'(rs)^2 = \frac{1}{4}(\frac{1}{4}[\frac{@^2}{cosh^4[@(rs-R)]}]) \tag{31}$$

$$n'(rs)^2 = \frac{1}{16}[\frac{@^2}{cosh^4[@(rs-R)]}] \tag{32}$$

An interesting feature is the fact that the square of the derivative of the Natario shape function in the equatorial plane is 4 times lower than its Alcubierre counterpart.

Back again to the negative energy density in the Natario warp drive:

$$\rho = T_{\mu\nu}u^\mu u^\nu = -\frac{c^2}{G}\frac{v_s^2}{8\pi}\left[3(n'(rs))^2\right] \tag{33}$$

Inserting the results of the squares of the derivatives of the Natario shape function we get:

$$\rho = T_{\mu\nu}u^\mu u^\nu = -\frac{c^2}{G}\frac{v_s^2}{8\pi}\left[\frac{3}{16}[\frac{@^2}{cosh^4[@(rs-R)]}]\right] \tag{34}$$

Now we must discuss a little bit of warp drive basics:

- 1)-According to Natario(pg 5 in [2]) any function that gives 0 inside the bubble and $\frac{1}{2}$ outside the bubble while being $0 < n(rs) < \frac{1}{2}$ in the Natario warped region is a valid shape function for the Natario warp drive.

Then inside the bubble and outside the bubble the Natario shape function have always constant or fixed values(0 inside the bubble and $\frac{1}{2}$ outside the bubble) and its derivative is zero.Hence the region where the Natario shape function vary its values resulting in non-null derivatives is the Natario warped region($0 < n(rs) < \frac{1}{2}$) which means to say the walls of the warp bubble.

- 2)-Since the negative energy density depends on non-null derivatives of the Natario shape function this means to say that the negative energy density resides in the Natario warped region(warp bubble walls

Then the region where

$$\rho = T_{\mu\nu}u^\mu u^\nu = -\frac{c^2}{G}\frac{v_s^2}{8\pi}\left[3(n'(rs))^2\right] \tag{35}$$

is the Natario warped region($0 < n(rs) < \frac{1}{2}$) with $n'(rs))^2 \neq 0$

Lets define the beginning of the Natario warped region where $n(rs)$ ceases to be zero as the point a and the end of the Natario warped region where $n(rs)$ is about to reach the value of $\frac{1}{2}$ as the point b.Remember that rs is the path of the so-called Eulerian observer that starts at the center of the bubble $xs = R = rs = 0$ and ends up outside the warp bubble $rs > R$.So we have a certain value for rs in the beginning of the Natario warped region which is a and another value for rs in the end of the Natario warped region which is b.

The difference $b - a$ is the width of the Natario warped region.

From section 2 we know that @ is the Alcubierre parameter related to the thickness of the bubble which can possesses arbitrary values and as large @ is as thicker or thinner the bubble becomes.Then for a very small thickness or width we must have a thickness parameter @ $>> |R|$ which means to say a very large value for @.

Back again to the negative energy density in the Natario warp drive:

$$\rho = T_{\mu\nu} u^{\mu} u^{\nu} = -\frac{c^2}{G} \frac{v_s^2}{8\pi} \left[\frac{3}{16} [\frac{@^2}{cosh^4[@(rs - R)]}] \right] \tag{36}$$

Note that a large @ multiplied by $\frac{c^2}{G}\frac{v_s^2}{8\pi}$(which is 10^{48} for 200 times faster than light) will make the negative energy requirements even worst.The first shape function introduced in section 2 is not suitable for a real Natario warp drive spacetime.In section 5 we will explain how the second shape function introduced in section 2 will satisfy the Natario warp drive requirements well within the range of the experimental values obtained by Lamoreaux for the Casimir Effect.

5 Reducing the negative energy density requirements in the Natario warp drive in a $1+1$ dimensional spacetime to the experimental limits obtained by Lamoreaux for the Casimir Effect:The Casimir warp drive

From section 3 we know that Lamoreaux demonstrated experimentally the existence of exotic matter with negative energy densities however at sub-microscopical limits of about $10^{-4}\frac{Joules}{Meter^3}$.

This an incredible small amount of negative energy density:a body of mass 1 kilogram in a volume of one cubic meter possesses an energy density of $9 \times 10^{16}\frac{Joules}{Meter^3}$

So the negative energy density obtained by Lamoreaux is 10^{20} times lighter than the one of a 1 kilogram body in a cubic meter of space or better: 100.000.000.000.000.000.000 times lighter than the one of a 1 kilogram body in a cubic meter of space !!

From section 4 we know that the negative energy needed to sustain a warp drive wether in the Alcubierre or Natario geometries with a speed of 200 times faster than light in order to visit stars at 20 light-years away in a reasonable amount of time demands an amount of negative energy density of about $10^{48}\frac{Joules}{Meter^3}$.

This is an astronomical amount of negative energy density impossible to be generated by any laboratory with any present or future technology:Earth mass is nearly 10^{24} kilograms and this number is 10^{24} times bigger in magnitude than the mass of the Earth or better:1.000.000.000.000.000.000.000.000 times bigger in magnitude than the mass of the Earth!!

Remember also that warp drives can be classified according to their geometries as Alcubierre or Natario cases since there are no other known topologies.But the most important thing here is not the geometry:

The most important thing considering warp drives is the source of negative energy density that will generate the spacetime geometrical distortion.The Casimir Effect is until now the only known source available.

Now we are ready to demonstrate how the negative energy density requirements can be greatly reduced for the Natario warp drive in a $1+1$ dimensional spacetime in order to fill the Lamoreaux experimental limits.We will introduce here the so-called Casimir warp drive.

According to Natario(pg 5 in [2]) any function that gives 0 inside the bubble and $\frac{1}{2}$ outside the bubble while being $0 < n(rs) < \frac{1}{2}$ in the Natario warped region is a valid shape function for the Natario warp drive.

A Natario warp drive valid shape function can be given by:

$$n(rs) = [\frac{1}{2}][1 - f(rs)^{WF}]^{WF} \tag{37}$$

Its derivative square is :

$$n'(rs)^2 = [\frac{1}{4}]WF^4[1 - f(rs)^{WF}]^{2(WF-1)}[f(rs)^{2(WF-1)}]f'(rs)^2 \tag{38}$$

The shape function given in the previous page gives the result of $n(rs) = 0$ inside the warp bubble and $n(rs) = \frac{1}{2}$ outside the warp bubble while being $0 < n(rs) < \frac{1}{2}$ in the Natario warped region(see pg 5 in [2]).

Note that the Alcubierre shape function $f(rs)$ is being used to define its Natario shape function counterpart. The term WF in the Natario shape function is dimensionless too:it is the warp factor that will squeeze the region where the derivatives of the Natario shape function are different than 0.[11]The warp factor is always a fixed integer number directly proportional to the modulus of the bubble radius.$WF > |R|$.

For the Natario shape function introduced above it is easy to figure out when $f(rs) = 1$(interior of the Alcubierre bubble) then $n(rs) = 0$(interior of the Natario bubble) and when $f(rs) = 0$(exterior of the Alcubierre bubble)then $n(rs) = \frac{1}{2}$(exterior of the Natario bubble).

We must analyze the differences between this new Natario shape function with warp factors compared to the original Natario shape function presented in Section 2 and mainly the differences between their derivative squares essential to low the negative energy density requirements in the $1+1$ Natario warp drive spacetime.In order to do so we need to use the Alcubierre shape function.

- 1)-Alcubierre shape function and its derivative square:[12].

$$f(rs) = \frac{1}{2}[1 - tanh[@(rs - R)] \tag{39}$$

$$f'(rs)^2 = \frac{1}{4}[\frac{@^2}{cosh^4[@(rs - R)]}] \tag{40}$$

- 2)-original Natario shape function and its derivative square:

$$n(rs) = \frac{1}{2}[1 - f(rs)] \tag{41}$$

$$n'(rs)^2 = \frac{1}{16}[\frac{@^2}{cosh^4[@(rs - R)]}] \tag{42}$$

- 3)-Natario shape function with warp factors and its derivative square:

$$n(rs) = [\frac{1}{2}][1 - f(rs)^{WF}]^{WF} \tag{43}$$

$$n'(rs)^2 = [\frac{1}{4}]WF^4[1 - f(rs)^{WF}]^{2(WF-1)}[f(rs)^{2(WF-1)}]f'(rs)^2 \tag{44}$$

$$n'(rs)^2 = [\frac{1}{4}]WF^4[1 - f(rs)^{WF}]^{2(WF-1)}[f(rs)^{2(WF-1)}][\frac{1}{4}[\frac{@^2}{cosh^4[@(rs - R)]}]] \tag{45}$$

[11]See the tables plotted in section 2

[12]$tanh[@(rs + R)] = 1$,$tanh(@R) = 1$ for very high values of the Alcubierre thickness parameter $@ >> |R|$

$$n'(rs)^2 = [\frac{1}{16}]WF^4[1 - f(rs)^{WF}]^{2(WF-1)}[f(rs)^{2(WF-1)}][\frac{@^2}{cosh^4[@(rs - R)]}] \tag{46}$$

- 4)-negative energy density in the $1 + 1$ Natario warp drive spacetime:

$$\rho = T_{\mu\nu}u^{\mu}u^{\nu} = -\frac{c^2}{G}\frac{v_s^2}{8\pi}\left[3(n'(rs))^2\right] \tag{47}$$

We already know that the region where the negative energy density is concentrated is the warped region in both Alcubierre ($1 > f(rs) > 0$) and Natario ($0 < n(rs) < \frac{1}{2}$) cases.

And we also know that for a speed of 200 times light speed the negative energy density is directly proportional to 10^{48} resulting from the term $\frac{c^2}{G}\frac{v_s^2}{8\pi}$.

So in order to get a physically feasible Natario warp drive the derivative of the Natario shape function must obliterate the factor 10^{48}.

Examining first the negative energy density from the original Natario shape function:

$$\rho = T_{\mu\nu}u^{\mu}u^{\nu} = -\frac{c^2}{G}\frac{v_s^2}{8\pi}\left[3(n'(rs))^2\right] \tag{48}$$

$$n'(rs)^2 = \frac{1}{16}[\frac{@^2}{cosh^4[@(rs - R)]}] \tag{49}$$

$$\rho = T_{\mu\nu}u^{\mu}u^{\nu} = -\frac{c^2}{G}\frac{v_s^2}{8\pi}\left[\frac{3}{16}[\frac{@^2}{cosh^4[@(rs - R)]}]\right] \tag{50}$$

We already know from section 4 that @ is the Alcubierre parameter related to the thickness of the bubble and a large $@ > |R|$ means a bubble of very small thickness.On the other hand a small value of $@ < |R|$ means a bubble of large thickness.But @ cannot be zero and cannot be $@ << |R|$ so independently of the value of @ the factor $\frac{c^2}{G}\frac{v_s^2}{8\pi}$ still remains with the factor 10^{48} from 200 times light speed which is being multiplied by $@^2$ making the negative energy density requirements even worst!!

Examining now the negative energy density from the Natario shape function with warp factors:

$$n'(rs)^2 = [\frac{1}{16}]WF^4[1 - f(rs)^{WF}]^{2(WF-1)}[f(rs)^{2(WF-1)}][\frac{@^2}{cosh^4[@(rs - R)]}] \tag{51}$$

$$\rho = T_{\mu\nu}u^{\mu}u^{\nu} = -\frac{c^2}{G}\frac{v_s^2}{8\pi}\left[[\frac{3}{16}]WF^4[1 - f(rs)^{WF}]^{2(WF-1)}[f(rs)^{2(WF-1)}][\frac{@^2}{cosh^4[@(rs - R)]}]\right] \tag{52}$$

Comparing both negative energy densities we can clearly see that the differences between the equations is the term resulting from the warp factor which is:

$$WF^4[1 - f(rs)^{WF}]^{2(WF-1)}[f(rs)^{2(WF-1)}] \tag{53}$$

Inside the bubble $f(rs) = 1$ and $[1 - f(rs)^{WF}]^{2(WF-1)} = 0$ resulting in a $n'(rs)^2 = 0$.This is the reason why the Natario shape function with warp factors do not have derivatives inside the bubble.

Outside the bubble $f(rs) = 0$ and $[f(rs)^{2(WF-1)}] = 0$ resulting also in a $n'(rs)^2 = 0$.This is the reason why the Natario shape function with warp factors do not have derivatives outside the bubble.

Using the Alcubierre warped region we have:

In the Alcubierre warped region $1 > f(rs) > 0$.In this region the derivatives of the Natario shape function do not vanish because if $f(rs) < 1$ then $f(rs)^{WF} << 1$ resulting in an $[1 - f(rs)^{WF}]^{2(WF-1)} << 1$ .Also if $f(rs) < 1$ then $[f(rs)^{2(WF-1)}] << 1$ too if we have a warp factor $WF > |R|$.

Note that if $[1 - f(rs)^{WF}]^{2(WF-1)} << 1$ and $[f(rs)^{2(WF-1)}] << 1$ their product $[1 - f(rs)^{WF}]^{2(WF-1)}[f(rs)^{2(WF-1)}] <<<< 1$

Note that inside the Alcubierre warped region $1 > f(rs) > 0$ when $f(rs)$ approaches 1 $n'(rs)^2$ approaches 0 due to the factor $[1 - f(rs)^{WF}]^{2(WF-1)}$ and when $f(rs)$ approaches 0 $n'(rs)^2$ approaches 0 again due to the factor $[f(rs)^{2(WF-1)}]$.

Back again to the negative energy density using the Natario shape function with warp factors:

$$\rho = T_{\mu\nu}u^{\mu}u^{\nu} = -\frac{c^2}{G}\frac{v_s^2}{8\pi}\left[[\frac{3}{16}]WF^4[1 - f(rs)^{WF}]^{2(WF-1)}[f(rs)^{2(WF-1)}][\frac{@^2}{cosh^4[@(rs - R)]}]\right] \tag{54}$$

Independently of the thickness parameter @ or the bubble radius R for a warp factor $WF = 600$ we have the following situations considering the Alcubierre warped region $1 > f(rs) > 0$:

- 1)-in the beginning of the Alcubierre warped region when $f(rs) = 0,9$ then $[f(rs)^{2(WF-1)}] = [(0,9)^{2(600-1)}] = (0,9)^{2(599)} = (0,9)^{1198} = 1,5223913522 \times 10^{-55}$
- 2)-in an intermediate region of the Alcubierre warped region when $f(rs) = 0,8$ then $[f(rs)^{2(WF-1)}] = [(0,8)^{2(600-1)}] = (0,8)^{2(599)} = (0,8)^{1198} = 7,9763539287 \times 10^{-117}$
- 3)-in the middle of the Alcubierre warped region when $f(rs) = 0,5$ then $[f(rs)^{2(WF-1)}] = [(0,5)^{2(600-1)}] = (0,5)^{2(599)} = (0,5)^{1198} = 0$
- 4)-in the end of the Alcubierre warped region when $f(rs) = 0,1$ then $[f(rs)^{2(WF-1)}] = [(0,1)^{2(600-1)}] = (0,1)^{2(599)} = (0,1)^{1198} = 0$

Note that when $f(rs) = 9,9999990000 \times 10^{-1}$ the term $[f(rs)^{2(WF-1)}] = 9,9988020717 \times 10^{-1}$ and this does not ameliorate the factor 10^{48} but the term $[1 - f(rs)^{WF}]^{2(WF-1)} = 0$ because as stated before when $f(rs)$ approaches 1 $n'(rs)^2$ approaches 0 due to the factor $[1 - f(rs)^{WF}]^{2(WF-1)}$

Note also that the Natario shape function with warp factors completely obliterated the term $\frac{c^2}{G}\frac{v_s^2}{8\pi}$ with the factor 10^{48} from 200 times light speed making the negative energy density requirements physically feasible!!

And remember that 10^{48} is 1.000.000.000.000.000.000.000.000 times bigger in magnitude than the mass of the planet Earth which is 10^{24} kilograms !!!

Lamoreaux in 1997 obtained for the negative energy density from the Casimir Effect the experimental value of $10^{-4}\frac{Joules}{Meter^3}$.

Remember again that this is an incredible small amount of negative energy density:a body of mass 1 kilogram in a volume of one cubic meter possesses an energy density of $9 \times 10^{16}\frac{Joules}{Meter^3}$

So the negative energy density obtained by Lamoreaux is 10^{20} times lighter than the one of a 1 kilogram body in a cubic meter of space or better: 100.000.000.000.000.000.000 times lighter than the one of a 1 kilogram body in a cubic meter of space !!

The peak of energy for our modified Natario shape function occurs in the neighborhoods of the region where $f(rs) = 0,9$ and multiplying 10^{48} by 10^{-55} we get $10^{-7}\frac{Joules}{Meter^3}$

So our modified shape function can sustain a Natario warp bubble moving at a speed of 200 times faster than light with negative energy density requirements 1000 times lighter than the ones obtained by Lamoreaux.

The Casimir Effect can without shadows of doubt generate and sustain a Natario warp drive spacetime.

Writing together the equations of the Natario negative energy density with our modified shape function and the Casimir negative energy density

$$\rho = T_{\mu\nu}u^\mu u^\nu = -\frac{c^2}{G}\frac{v_s^2}{8\pi}\left[[\frac{3}{16}]WF^4[1-f(rs)^{WF}]^{2(WF-1)}[f(rs)^{2(WF-1)}][\frac{@^2}{cosh^4[@(rs-R)]}]\right] \tag{55}$$

$$\rho = -\frac{\pi^2}{720}\frac{\hbar c}{d^4} \tag{56}$$

We get finally:

$$\rho = -\frac{c^2}{G}\frac{v_s^2}{8\pi}\left[[\frac{3}{16}]WF^4[1-f(rs)^{WF}]^{2(WF-1)}[f(rs)^{2(WF-1)}][\frac{@^2}{cosh^4[@(rs-R)]}]\right] = -\frac{\pi^2}{720}\frac{\hbar c}{d^4} \tag{57}$$

Remember that the most important thing concerning warp drives is the negative energy density needed to generate and sustain the warp bubble.The formula given above is the equation of the so-called Casimir warp drive.

6 The analysis of Lobo and Visser applied to both Natario and Casimir warp drives. Physical reactions of gravitational repulsive behavior between the positive mass of the spaceship and the negative mass of the warp bubble

When both Alcubierre and Natario developed their theories about the warp drive they considered the center of the warp bubble as the position where $rs = 0$ from the point of view of the Eulerian observer where the spaceship resides but the center of the bubble was described as being empty space in their works.They did not mentioned masses in the center of the bubble.[13](See abs of [10])

But we know that a warp bubble is designed to carry a spaceship at superluminal speeds and spaceships are objects of positive masses.Also we know that positive and negative masses repels each other.

Adapted from the negative mass in Wikipedia:The free Encyclopedia:

"if we have a small object with equal inertial and passive gravitational masses falling in the gravitational field of an object with negative active gravitational mass (a small mass dropped above a negative-mass planet, say), then the acceleration of the small object is proportional to the negative active gravitational mass creating a negative gravitational field and the small object would actually accelerate away from the negative-mass object rather than towards it."

We count on the negative mass in the front of the Natario warp bubble to deflect asteroids comets Doppler blueshifted photons or space dust and debris that otherwise would pose a very serious threat to the crew members.And asteroids comets or space dust and debris are objects of positive masses.[14]

Also we know that we need large amounts of negative energy density in order to generate the large repulsive gravitational fields able to deflect these hazardous objects.And the negative energy density calculated for the Casimir warp drive in the previous section although able to mathematically ameliorate the negative energy density requirements for a warp bubble moving at 200 times light speed may be not enough to protect the ship and the crew members from impacts with the hazardous interstellar matter.

Since positive and negative masses reacts with each other in a repulsive way would be interesting to analyze what happens with the warp bubble if we place a spaceship of large positive mass inside the bubble exactly in the center of the bubble $rs = 0$.We consider the spaceship a point-like particle however very massive with all its mass in the center of the bubble $rs = 0$.We are about to reproduce in this work the analysis of Lobo and Visser (abs of [10]) using sample mathematical arguments accessible to beginners or introductory students.

[13]We consider in this work the center of the bubble as the center of mass CM frame for any spaceship

[14]See Appendices H and M in [4] and [5]

When we calculated the negative energy density for a Natario warp drive moving with 200 times light speed in the previous section we arrived at a result 1000 times lighter than the one found by Lamoreaux for the Casimir Effect.However the center of the bubble in our calculations was empty like in the same situations described by Alcubierre and Natario.

According to Lobo and Visser a finite mass spaceship placed in the center of the warp bubble would change the whole picture affecting out results making the Casimir Effect a not valid option after all.

We decided to choose two spaceships with very different masses when compared to each other in order to reproduce the Lobo and Visser analysis in details.Our spaceships are:

- 1)-The NASA Space Shuttle with 100 metric tons.(100.000) kilograms[15]
- 2)-The Star Trek Enterprise with 3.250.000 metric tons.(3.250.000.000) kilograms[16]

We will demonstrate according to Lobo and Visser that there exists a relation between the following elements:

- 1)-The negative mass of the warp bubble
- 2)-The positive mass of the spaceship
- 3)-The warp bubble radius

If we want to keep the integrity of the warp bubble stable the values of the measures shown above cannot be taken arbitrarily or the warp bubble will be destroyed.

Lobo and Visser arrived at the conclusion that the negative mass of the warp field must be an appreciable fraction of the positive mass of the ship in order to keep the bubble stable.(see summary page 13 in [10]).In this work we complement the analysis of Lobo and Visser demonstrating also that the radius of the warp bubble must be taken into account if we want to keep the bubble stable and the value of the radius cannot be taken arbitrarily

We will now examine their results using a different approach and at the end we will arrive at the same conclusions proving that their point of view is entirely correct.

When we computed the negative energy density needed to sustain a Natario warp drive bubble we used the Natario equation with the terms $\frac{c^2}{G} \times \frac{vs^2}{8\pi}$ and the squares derivatives of the shape function $n'(rs)^2$.We also used the warp factor WF.The term $\frac{c^2}{G} \times \frac{vs^2}{8\pi}$ for s speed of 200 times faster than light gives a number of a magnitude of 10^{48} $\frac{Joules}{Meters^3}$ for the negative energy density needed to sustain a Natario warp drive but we reduced the requirements to 10^{-7} $\frac{Joules}{Meters^3}$ using derivatives squares of adapted shape functions and warp factors.

[15]See Appendix B
[16]See Appendix C

But what we did was to use mathematical artifacts to reduce the amount of negative energy stored in the walls of the warp bubble manipulating functions and constants. Our calculations did not took in consideration if the bubble was empty or filled with a spaceship of a large weight.

The most important point of view in the analysis of Lobo and Visser is the fact that empty bubbles can have arbitrary levels of energy density while charged bubbles[17] cannot because the positive mass of the spaceship inside the bubble reacts with the negative mass of the warp bubble(see abs of [10]) and a spaceship of large positive mass inside a bubble of low negative mass will destroy the bubble due to the gravitational repulsive forces between positive and negative masses and we will have no warp drive after all.

Besides a bubble of low negative mass will not be able to deflect the hazardous interstellar matter [18] that otherwise would harm the crew members.

Considering now the Shuttle with 100.000 kilograms and the Enterprise with 3.250.000.000 kilograms as point-like particles with all their masses in their center of mass CM frames exactly in the center of their respective bubbles $rs = 0$ one bubble for the Shuttle and the other for the Enterprise and each warp bubble have a radius R of 100 meters.We also consider all the differential elements of negative energy density or negative mass density as the points of the circumference line with length $L = 2\pi R$ .The beginning of the warped region $R_{beg} = R-\epsilon$ and the end of the warped region $R_{end} = R+\epsilon$ coincides when $\epsilon = 0$.

So we place the mass $M > 0$ of each spaceship in the center C of each bubble and we took two differential elements of negative mass $n1 < 0$ and $n2 < 0$ and $n1 = n2$ in each bubble each differential element almost close together to each other in the positions $P1$ and $P2$ over the line of the circumference length of each bubble.The distance d between $P1$ and $P2$ is $d = 1\mu$ in each bubble while the distance between $P1$,$P2$ and each center C of each bubble is R.[19]

The force $F2$ between $P1$ and $P2$ is weak because although d have a small value and the gravitational force is inversely proportional to d the product of the masses $-n1 \times -n2 = n1n2$ is a very small number resulting in a weak attractive gravitational force because $n1$ and $n2$ possesses low values. This attractive force $F2$ keeps the points $P1$ and $P2$ together hence keeps the integrity of the warp bubble

Since $R >> d$ and the gravitational force is inversely proportional to R we would expect a repulsive gravitational force $F1$ between C and $P1$ or C and $P2$ weaker in modulus when compared to the attractive force $F2$ or better: $|F2| >> |F1|$.However since $|M| >> |n1|$ or $|M| >> |n2|$ the product $M \times n1 = -Mn1$ or $M \times n2 = -Mn2$ can give big numbers in modulus due to the large value of M.Besides the repulsive gravitational force is directly proportional to M so a large M can make the repulsive force $|F1|$ between the center C of each bubble and the points $|P1P2|$ stronger than the attractive force $|F2|$ between the points $|P1P2|$ that keeps the integrity of the bubble or better:$|F1| >> |F2|$.

If this happens the bubble is destroyed and we have no warp drive after all!!

[17]Bubbles with spaceships inside

[18]See Appendices H and M in [4] and [5]

[19]See Appendices B and C for details

In order to keep $|F2| >> |F1|$ when $M >> (|n1|+|n2|)$ the radius of the bubble R must be enormous in order to keep the force $F1$ always weaker than $F2$.However a large R means a bubble of large circumference length $L = 2\pi R$.

Considering the bubbles placed over a bidimensional plane B the length of the part of the circumference in front of the spaceships (front hemisphere) is given by $\frac{1}{2}L = \pi R$.Consider also a large set of points $A1$ to An scattered across the plane outside the circumference but directly in front of the circumference line and directly in the course of the spaceships.As far the bubble moves forward as far many of these points $A1$ to An will be intersected by by the front hemisphere line. Now consider the points $A1$ to An as asteroids and each intersection between the front hemisphere line and each one of these points is really an impact!!!!

A bubble of small radius R will have a small lenght L and a small front hemisphere line and will intersect a small number of points when compared to the front hemisphere line of a larger circumference hence it will suffer less impacts when compared to a bubble of large radius.

We will now evaluate the behavior of the Shuttle and Enterprise bubbles each bubble with 100 meters radius with 3 different amounts of negative mass in the differential elements.[20].

- 1)-10^{-21} kilograms.Exactly the negative mass obtained by Lamoreaux for the Casimir Effect.[21]
- 2)- 10^{-7} kilograms
- 3)-10 kilograms

The case of 10^{-21} kilograms is very interesting:by a manipulation of the warp factor WF from the previous section we can arrive at the results obtained by Lamoreaux for the negative energy density ameliorating the factor 10^{48} but unfortunately this is valid only for empty bubbles.

Placing the masses of the Shuttle and the Enterprise in the center of their respective bubbles we can see that the bubbles are completely destroyed by the huge masses of the spaceships when compared to the Casimir mass.

A warp bubble able to sustain the Shuttle would need a radius of 10^7 meters.At this distance the repulsive force $|F1|$ have a magnitude of 10^{-40} Newtons and the attractive force $|F2|$ have a magnitude of 10^{-39} Newtons.Since $10^{-39} > 10^{-40}$ this bubble is stable however $10^7 = 10.000.000$ a bubble of 10.000 kilometers;Not practical at all!!!!!!And think about the asteroids in front of the bubble.The Casimir Effect can only sustain empty bubbles not "charged" ones.

A warp bubble able to sustain the Enterprise would need a radius of 10^{10} meters.At this distance the repulsive force $|F1|$ have a magnitude of 10^{-41} Newtons and the attractive force $|F2|$ have a magnitude of 10^{-39} Newtons. Since $10^{-39} > 10^{-41}$ this bubble is stable however $10^{10} = 10.000.000.000$ a bubble of 10.000.000 kilometers:Not practical at all!!!!!.And this bubble would collide with a larger number of asteroids when compared to the Shuttle bubble.Without shadows of doubt the Casimir Effect must be ruled out from "charged" warp bubbles.

[20]See Appendices B and C for details

[21]Considering each differential element of negative energy density equal to the amount of negative energy density obtaimed in the experience and divided by c^2

The case of 10^{-7} kilograms is a better scenario:a warp bubble of 100 meters radius with differential elements of 10^{-7} kilograms can sustain the Shuttle but cannot sustain the Enterprise because the Enterprise is 32.500 times heavier than the Shuttle so this bubble do not have strength to transport the Enterprise.And 10^{-7} kilograms of negative mass can deflect incoming Doppler blueshifted photons from interstellar space but cannot protect the ship against asteroids.

The case of 10 kilograms is an excellent scenario!!!!:a warp bubble of 100 meters radius with differential elements of 10 kilograms can sustain the Shuttle and can sustain the Enterprise.And 10 kilograms of negative mass can deflect incoming Doppler blueshifted photons from interstellar space and can also deflect micrometeoroids.

Now we can understand the importance of the analysis of Lobo and Visser:

While in the previous section we used mathematical techniques to low the energy density requirements from 10^{48} to 10^{-7} these results works only for empty bubbles not for charged bubbles.

So any future serious study on warp drive geometry concerning real "flesh-and-bone" spaceships wether in Alcubierre Natario or any other unknown warp drive solution of the Einstein field equations of General Relativity still waiting to be discovered must take the analysis of Lobo and Visser in account or the warp drive simply will not work

Although 10 kilograms are far from being considered a reasonable fraction of the spaceship mass concerning the Shuttle or the Enterprise this amount of negative mass is a better result when compared to 10^{-7} kilograms. and without shadows of doubt a better result when compared to 10^{-21} kilograms.This proves the validity of the argument of Lobo and Visser(see abs and summary pg 13 of [10])

We just finished to demonstrate here the analysis of Lobo and Visser and our examples shows clearly the relation between the positive mass of the spaceship coupled with the negative mass of the bubble and the bubble radius proving in fact that the analysis of Lobo and Visser is entirely correct.

The work of Lobo and Visser is the third most important work on warp drive science immediately after the works of Alcubierre and Natario and the Lobo-Visser paper must also be considered a seminal paper just like the papers of Alcubierre and Natario.

7 Conclusion

In this work we demonstrated the analysis of Lobo and Visser proving that their arguments about the relations between the positive mass of the spaceship coupled to the negative mass of the warp bubble and the bubble radius in which the negative mass of the bubble must be a significant fraction of the positive mass of the ship is entirely correct.

We started with a brief discussion of the problem of the energy conditions in General Relativity that requires always positive energy densities while the energy density for the warp drive is always negative.The Casimir Effect predicted theoretically in 1948 by Casimir [6] and experimentally demonstrated in 1997 by Lamoreaux [7] is the only known experimental source of negative energy density.Lamoreaux obtained $10^{-4}\frac{Joules}{Meters^3}$ of negative energy density.This is a sub-microscopical quantity 10^{20} times lighter than the one of a 1 kilogram body in a cubic meter of space or better: 100.000.000.000.000.000.000 times lighter than the one of a 1 kilogram body in a cubic meter of space but at least we know that negative energy densities exists in Nature and are more than a theoretical prediction.

For the negative energy density needed to travel at 200 times light speed we lowered the total amount from $10^{48}\frac{Joules}{Meters^3}$ which have a magnitude of 1.000.000.000.000.000.000.000.000 the magnitude of the mass of the Earth to $10^{-7}\frac{Joules}{Meters^3}$ using a Natario shape function with warp factors derived from the modulus of the bubble radius.Our result is 1000 times lighter than the negative energy density of $10^{-4}\frac{Joules}{Meters^3}$ obtained experimentally by Lamoreaux in 1997 for the Casimir Effect. Combined the equation for the negative energy density in the Natario warp drive with the equation of the negative energy density for the Casimir Effect we obtained the equation of the Casimir warp drive.

But unfortunately although the mathematical capability to lower the negative energy density from 10^{48} to 10^{-7} is a great achievement[22] it is not enough for warp bubbles containing real "flesh-and-bone" spaceships.Alcubierre,Natario and ourselves all of us worked with empty bubbles.Mathematical techniques to lower the negative energy density to extremely and arbitrary low levels works only for empty bubbles and not for bubbles with spaceships.

For bubbles with spaceships inside we can low the negative energy requirements only to the limit in which the negative energy of the bubble is a reasonable fraction of the positive mass of the spaceship.This is the most important point of view in the analysis of Lobo and Visser.

In order to verify the validity of the Lobo sand Visser arguments we used two spaceships:the NASA Space Shuttle with 100 metric tons(100.000 kilograms) and the Star Trek Enterprise with 3.250.000 metric tons(3.250.000.000 kilograms).We took two bubbles(one for the Shuttle and the other for the Enterprise) each one of the bubbles with 100 meters of radius and we placed both the Shuttle and the Enterprise in the center of their own respective bubbles because we considered both the Shuttle and the Enterprise as massive point-like particles with the mass of each spaceship concentrated in each center-of-mass CM frame.

[22]Otherwise with the factor 10^{48} always present all the warp drive discussions would be useless

In order to study the repulsive gravitational forces between the positive masses of the spaceships and the negative masses of the bubbles we took 2 differential elements of negative mass equals between each other as 2 points from the line of circumference length of each bubble and we used 3 different values for the differential elements of negative mass:

- 1)-10^{-21} kilograms.Exacrly the Casimir mass obtained by Lamoreaux.
- 2)-10^{-7} kilograms
- 3)-10 kilograms

A warp bubble of 100 meters radius with a differential element of negative mass of 10^{-21} kilograms cannot support the weight neither the Shuttle nor the Enterprise and the bubble is completely destroyed by the repulsive gravitational forces from the Shuttle or the Enterprise either.

A warp bubble of 100 meters radius with a differential element of negative mass of 10^{-7} kilograms can support the weight of the Shuttle but cannot support the weight of the Enterprise because the Enterprise weights 32.500 times more than the Shuttle and the bubble is completely destroyed by the repulsive gravitational forces from the Enterprise.

A warp bubble of 100 meters radius with a differential element of negative mass of 10 kilograms can support the weight of the Shuttle or the weight of the Enterprise either.

We lowered the negative energy density to travel at 200 times light speed in the Natario warp drive from 10^{48} to 10^{-7}. Now due to Lobo and Visser we know that we must stop with a negative energy density able to give a differential element of negative mass of at least 10 kilograms and no less!!

The point above outlines the importance of the work of Lobo and Visser as the third most relevant work on warp drive science immediately after the works of Alcubierre and Natario and the Lobo-Visser paper must also be considered a seminal paper just like the papers of both Alcubierre and Natario.

However 10 kilograms of a differential element of negative mass represents an amount of negative energy density far beyond the Casimir Effect capability but Lobo and Visser mentions almost in the bottom of pg 1 in [10] the fact that the Casimir Effect is not the only option and we can have negative energy densities at classical and macroscopical levels [23].This would be the ideal solution

Lastly and in order to terminate this work we are confident to affirm that the Natario warp drive will survive the passage of the Century XXI and will arrive to the Future.The Natario warp drive as a valid candidate for faster than light interstellar space travel will arrive to the the Century $XXIV$ on-board the future starships up there in the middle of the stars helping the human race to give his first steps in the exploration of our Galaxy

Live Long And Prosper

[23]Non-Minimally coupled scalar fields

8 Appendix A:The Natario warp drive negative energy density in Cartezian coordinates

The negative energy density according to Natario is given by(see pg 5 in [2])[24]:

$$\rho = T_{\mu\nu}u^{\mu}u^{\nu} = -\frac{1}{16\pi}K_{ij}K^{ij} = -\frac{v_s^2}{8\pi}\left[3(n'(rs))^2\cos^2\theta + \left(n'(rs) + \frac{r}{2}n''(rs)\right)^2\sin^2\theta\right] \tag{58}$$

In the bottom of pg 4 in [2] Natario defined the x-axis as the polar axis.In the top of page 5 we can see that $x = rs\cos(\theta)$ implying in $\cos(\theta) = \frac{x}{rs}$ and in $\sin(\theta) = \frac{y}{rs}$

Rewriting the Natario negative energy density in cartezian coordinates we should expect for:

$$\rho = T_{\mu\nu}u^{\mu}u^{\nu} = -\frac{1}{16\pi}K_{ij}K^{ij} = -\frac{v_s^2}{8\pi}\left[3(n'(rs))^2(\frac{x}{rs})^2 + \left(n'(rs) + \frac{r}{2}n''(rs)\right)^2(\frac{y}{rs})^2\right] \tag{59}$$

Considering motion in the equatorial plane of the Natario warp bubble (x-axis only) then $[y^2 + z^2] = 0$ and $rs^2 = [(x - xs)^2]$ and making $xs = 0$ the center of the bubble as the origin of the coordinate frame for the motion of the Eulerian observer then $rs^2 = x^2$ because in the equatorial plane $y = z = 0$.

Rewriting the Natario negative energy density in cartezian coordinates in the equatorial plane we should expect for:

$$\rho = T_{\mu\nu}u^{\mu}u^{\nu} = -\frac{1}{16\pi}K_{ij}K^{ij} = -\frac{v_s^2}{8\pi}\left[3(n'(rs))^2\right] \tag{60}$$

[24] $n(rs)$ is the Natario shape function.Equation written in the Geometrized System of Units $c = G = 1$

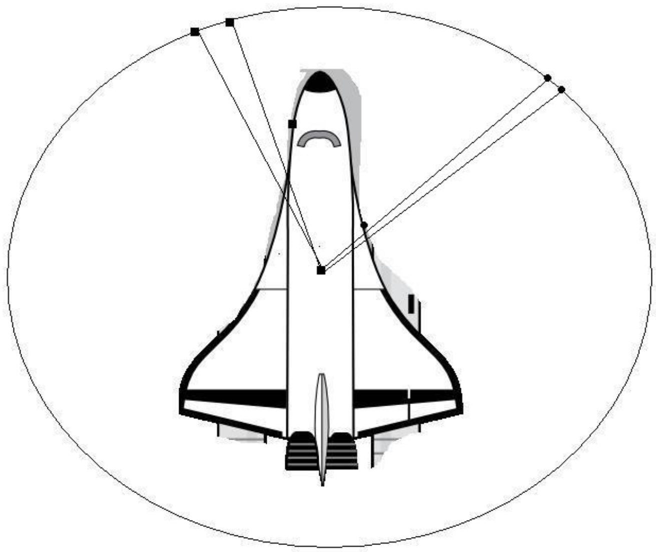

Figure 1: Artistic representation of the NASA Space Shuttle inside a Natario warp drive bubble).(Source:Internet)

9 Appendix B:Artistic presentation of the NASA Space Shuttle inside a Natario warp drive bubble

Above is being presented a Natario warp drive carrying the NASA Space Shuttle inside as a point-like particle with all its mass centered in the center of mass CM frame placed in the center of the warp bubble $rs = 0$.The point in the center of the bubble is the point C where the positive Shuttle mass M resides and the points $P1$,$P2$ in the left side of the Shuttle or the points $P3$,$P4$ in the right side of the Shuttle are the locations of the differential elements of negative mass $-n1$,$-n2$,$-n3$ and $-n4$ respectively and placed exactly over the bubble radius R.[25] [26]

All the differential elements possesses the same negative mass $-n1 = -n2 = -n3 = -n4$.

[25]The beginning and the end of the warped region coincides so according to section 2 $R_{end} = R + \epsilon$ and $R_{beg} = R - \epsilon$ with $\epsilon = 0$ giving the result $R_{end} = R_{beg}$

[26]The total negative mass mass can be obtained by a contour integral of all the differential elements of negative mass over the circumference length $L = 2\pi R$

Note that $P1$ is close to $P2$ and $P3$ is close to $P4$.The distance d between $P1$ to $P2$ or $P3$ to $P4$ is $d = 1\mu m$. [27] The point C applies over each set of points $P1P2$ or $P3P4$ a repulsive[28] gravitational force $F1 < 0$ while the force $F2 > 0$ between $P1$ and $P2$ or $P3$ and $P4$ is always attractive.[29]

Since $d << R$ we would expect an attractive gravitational force $F2$ between $P1P2$ or $P3P4$ stronger than the repulsive gravitational force $F1$ exerted by C over each set of points.Or better:$|F2| >> |F1|$.

However $M >> (|n1| + |n2| + |n3| + |n4|)$ so in certain circumstances $|F1| >> |F2|$ even when $d << R$

If the force $|F1| << |F2|$ then the force $F2$ that keeps the set of points $P1P2$ and $P3P4$ together is stronger than the repulsive gravitational force $F1$ generated by the point C over each set of points and the warp bubble keeps its integrity.

But if the force $|F1| >> |F2|$ then the force $F2$ that keeps the set of points $P1P2$ and $P3P4$ together is weaker than the repulsive gravitational force $F1$ generated by the point C over each set of points and the warp bubble cannot keep its integrity.Each set of points will be disrupted because the force $F1$ between C and $P1$ or between C and $P2$ will be stronger than the force $F2$ that keeps $P1$ and $P2$ together and due to the angle between $P1$ and $P2$ the warp bubble will be destroyed.

In order to keep $|F2| >> |F1|$ when $M >> (|n1| + |n2| + |n3| + |n4|)$ the radius of the bubble R must be enormous in order to keep the force $F1$ always weaker than $F2$.

But an enormous radius have disadvantages:it would demands a large bubble circumference length

Consider the picture of the Shuttle inside a Natario warp bubble in a bidimensional plane B

For a warp bubble of radius R the circumference length is $L = 2\pi R$ and the length of the part of the circumference in the front of the Shuttle (front hemisphere) would be $\frac{L}{2} = \pi R$

Now consider many points $A1$ $A2$ $A3.....An$ defined in the plane B outside the bubble front hemisphere but right in front of the bubble front hemisphere

As far as the bubble moves forward the line of the circumference of the front hemisphere of the bubble will intersect these points As larger the front hemisphere circumference line is as many of these points will be intersected.

A bubble of short radius will have a smaller circumference length hence a smaller front hemisphere with a smaller line and will intersect a smaller number of points An when compared to a bubble of larger radius and a larger circumference line.

[27] The same distance of Lamoreaux for the Casimir Effect

[28] The multiplication of a positive mass M by a negative mass $-n1$ gives a negative product $-Mn1$.The minus sign characterizes the repulsive gravitational behavior

[29] The multiplication of two negative masses $-n1$ and $-n2$ gives a positive product $n1n2$ because $-1 \times -1 = 1$ so the minus sign is cancelled and the gravitational force is positive and attractive

Now imagine that these $A1$ to An points represents positions of asteroids in interstellar space directly in front of the Shuttle course.Each intersection between the line of the circumference of the front hemisphere and a point An defined in the plane B is really an impact.A bubble of small radius will suffer less impacts than a bubble of larger radius.

And remember that we need large outputs of negative energy density in order to generate the deflective gravitational fields that will protect the ship and crew members against collisions with the interstellar matter.[30]

- The plot below represents the mass of the NASA Space shuttle in the center of the Natario warp bubble and differential elements of negative mass placed over the bubble warped region.All masses are given in kilograms.

M	$n1$	$n2$	$n3$	$n4$
$1,0000000000E+05$	$1,0000000000E-07$	$1,0000000000E-07$	$1,0000000000E-07$	$1,0000000000E-07$

- Below is being presented a numerical plot for the integrity of the Natario warp drive bubble with the NASA Space Shuttle mass in the center of the bubble. $G = 6,6700000000E-11$ in SI units.All the forces are given in Newtons and the distances in meters.$F1$ is always negative while $F2$ is always positive.

R	d	$F1$	$F2$
$1,0000000000E+02$	$1,0000000000E-06$	$6,6700000000E-17$	$6,6700000000E-13$
$9,0000000000E+01$	$1,0000000000E-06$	$8,2345679012E-17$	$6,6700000000E-13$
$8,0000000000E+01$	$1,0000000000E-06$	$1,0421875000E-16$	$6,6700000000E-13$
$7,0000000000E+01$	$1,0000000000E-06$	$1,3612244898E-16$	$6,6700000000E-13$
$6,0000000000E+01$	$1,0000000000E-06$	$1,8527777778E-16$	$6,6700000000E-13$
$5,0000000000E+01$	$1,0000000000E-06$	$2,6680000000E-16$	$6,6700000000E-13$
$4,0000000000E+01$	$1,0000000000E-06$	$4,1687500000E-16$	$6,6700000000E-13$
$3,0000000000E+01$	$1,0000000000E-06$	$7,4111111111E-16$	$6,6700000000E-13$
$2,0000000000E+01$	$1,0000000000E-06$	$1,6675000000E-15$	$6,6700000000E-13$
$1,0000000000E+01$	$1,0000000000E-06$	$6,6700000000E-15$	$6,6700000000E-13$
$5,0000000000E+00$	$1,0000000000E-06$	$2,6680000000E-14$	$6,6700000000E-13$
$3,0000000000E+00$	$1,0000000000E-06$	$7,4111111111E-14$	$6,6700000000E-13$
$1,0000000000E+00$	$1,0000000000E-06$	$6,6700000000E-13$	$6,6700000000E-13$
$5,0000000000E-01$	$1,0000000000E-06$	$2,6680000000E-12$	$6,6700000000E-13$
$1,0000000000E-01$	$1,0000000000E-06$	$6,6700000000E-11$	$6,6700000000E-13$
$1,0000000000E-02$	$1,0000000000E-06$	$6,6700000000E-09$	$6,6700000000E-13$

Above are the plots for a Natario warp bubble with the mass of the Shuttle in the CM frame in the center of the bubble $rs = 0$ reacting with differential elements of negative mass each element with $|10^{-7}|$ kilograms.[31]

[30] See Appendices H and M in [4] and [5]

[31] A remarkable amount of negative mass since Lamoreaux obtained only 10^{-21} kilograms of negative mass considering the negative energy density equal to a differential element of negative energy and being divided by c^2

Note that for a warp bubble of 100 meters radius the repulsive force $|F1|$ have a magnitude of 10^{-17} Newtons and the attractive force $|F2|$ have a magnitude of 10^{-13} Newtons Since $10^{-13} >> 10^{-17}$ this bubble is stable however $|10^{-7}|$ kilograms may deflect incoming blueshifted photons from interstellar space but cannot cope with large asteroids.

A warp bubble of 1 meter radius have the repulsive force $|F1|$ equal in magnitude to the attractive force $|F2|$ and a bubble with 50 centimeters [32] will collapse because the repulsive force $|F1|$ have a magnitude of 10^{-12} and the attractive force $|F2|$ have a magnitude of 10^{-13} and $10^{-12} > 10^{-13}$. Bubbles with radius shorter than 50 centimeters will always collapse.

- Below is being presented a plot of the mass of the NASA Space shuttle in the center of the Natario warp bubble and differential elements of negative mass placed over the bubble warped region.All masses are given in kilograms.

M	$n1$	$n2$	$n3$	$n4$
$1,0000000000E+05$	$4,8185631766E-21$	$4,8185631766E-21$	$4,8185631766E-21$	$4,8185631766E-21$

- Below is being presdented a numerical plot for the integrity of the Natario warp drive bubble with the NASA Space Shuttle mass in the center of the bubble. $G = 6,6700000000E-11$ in SI units.All the forces are given in Newtons and the distances in meters.$F1$ is always negative while $F2$ is always positive.

R	d	$F1$	$F2$
$1,0000000000E+07$	$1,0000000000E-06$	$3,2139816388E-40$	$1,5486773575E-39$
$1,0000000000E+06$	$1,0000000000E-06$	$3,2139816388E-38$	$1,5486773575E-39$
$1,0000000000E+04$	$1,0000000000E-06$	$3,2139816388E-34$	$1,5486773575E-39$
$1,0000000000E+03$	$1,0000000000E-06$	$3,2139816388E-32$	$1,5486773575E-39$
$1,0000000000E+02$	$1,0000000000E-06$	$3,2139816388E-30$	$1,5486773575E-39$

Above are the plots for a Natario warp bubble with the mass of the Shuttle in the CM frame in the center of the bubble $rs = 0$ reacting with differential elements of negative mass each element with $|10^{-21}|$ kilograms exactly the mass of Lamoreaux for the Casimir Effect. [33]

Note that in this case for a warp bubble of 100 meters radius the repulsive force $|F1|$ have a magnitude of 10^{-30} Newtons and the attractive force $|F2|$ have a magnitude of 10^{-39} Newtons. Since $10^{-30} >> 10^{-39}$ this bubble not stable and would collapse

So a warp bubble with 100 meters of radius is stable or not depending on the amount of the negative mass in the borders of the bubble.

Note that for a warp bubble of 10^7 meters radius the repulsive force $|F1|$ have a magnitude of 10^{-40} Newtons and the attractive force $|F2|$ have a magnitude of 10^{-39} Newtons.Since $10^{-39} > 10^{-40}$ this bubble is stable however $10^7 = 10.000.000$ a bubble of 10.000 kilometers;Not practical at all!!!!!!

[32] Remember that we consider the Shuttle a point-like particle with all its mass concentrated in the CM frame in the center of the bubble $rs = 0$

[33] In section 5 we arrived at a result 1000 times lighter than the ones obtained by Lamoreaux for the Casimir Effect. Manipulating the warp factor WF we can get the same results of Lamoreaux

Like we said before:

-)-In order to keep $|F2| >> |F1|$ the radius of the bubble must be enormous in order to weak the force$|F1|$ when $|M|| >> (|n1| + |n2| + |n3| + |n4|)$.An enormous radius will keep $|F1|$ always weaker than $|F2|$.

- Below is being presented a plot of the mass of the NASA Space shuttle in the center of the Natario warp bubble and differential elements of negative mass placed over the bubble warped region.All masses are given in kilograms.

M	$n1$	$n2$	$n3$	$n4$
$1,0000000000E+05$	$1,0000000000E+01$	$1,0000000000E+01$	$1,0000000000E+01$	$1,0000000000E+01$

- Below is being presdented a numerical plot for the integrity of the Natario warp drive bubble with the NASA Space Shuttle mass in the center of the bubble. $G = 6,6700000000E-11$ in SI units.All the forces are given in Newtons and the distances in meters.$F1$ is always negative while $F2$ is always positive.

R	d	$F1$	$F2$
$1,0000000000E+02$	$1,0000000000E-06$	$6,6700000000E-09$	$6,6700000000E+03$

Above are the plots for a Natario warp bubble with the mass of the Shuttle in the CM frame in the center of the bubble $rs = 0$ reacting with differential elements of negative mass each element with $|10|$ kilograms.

Note that for a warp bubble of 100 meters radius the repulsive force $|F1|$ have a magnitude of 10^{-9} Newtons and the attractive force $|F2|$ have a magnitude of 10^3 Newtons. Since $10^3 >> 10^{-9}$ this bubble is stable and $|10|$ kilograms of negative mass can generate repulsive gravitational fields able to deflect not only Doppler blueshifted photons but also can deflect interstellar dust and micrometeorites.[34]

[34] See Appendices H and M in [4] and [5]

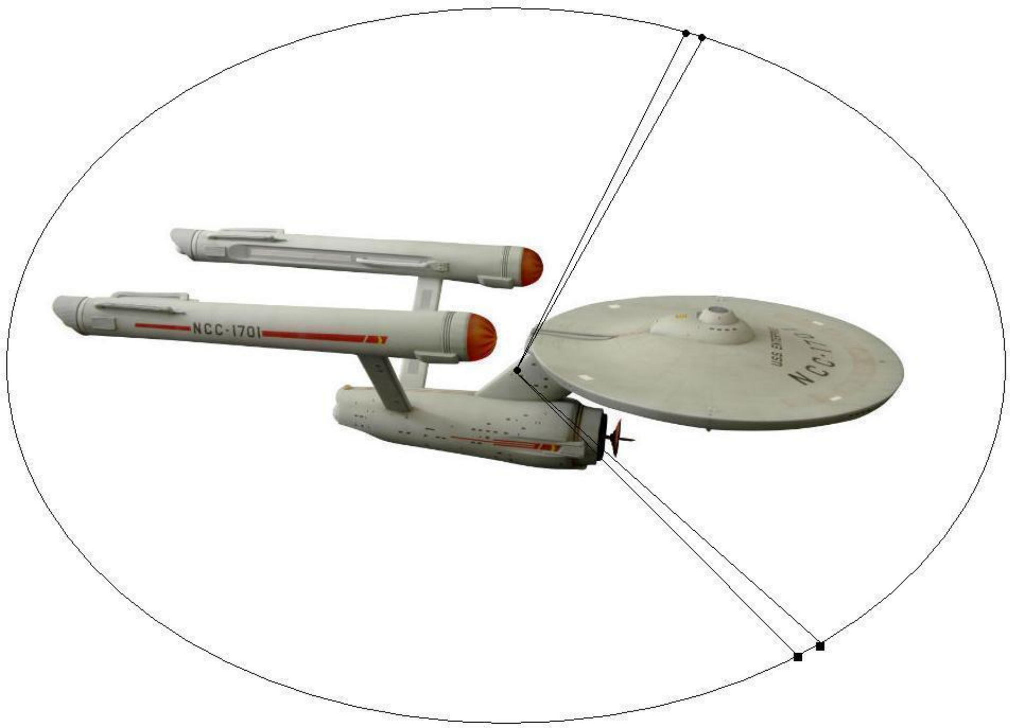

Figure 2: Artistic representation of the Star Trek Enterprise inside a Natario warp drive bubble).(Source:Internet)

10 Appendix C:Artistic presentation of the Star Trek Enterprise inside a Natario warp drive bubble

Above is being presented a Natario warp drive carrying the Star Trek Enterprise inside as a point-like particle with all its mass centered in the center of mass CM frame placed in the center of the warp bubble $rs = 0$.The point in the center of the bubble is the point C where the positive Enterprise mass M resides and the points $P1$,$P2$ above the Enterprise or the points $P3$,$P4$ below the Enterprise are the locations of the differential elements of negative mass $-n1$,$-n2$,$-n3$ and $-n4$ respectively and placed exactly over the bubble radius R.[35] [36]

All the differential elements possesses the same negative mass $-n1 = -n2 = -n3 = -n4$.

[35]The beginning and the end of the warped region coincides so according to section 2 $R_{end} = R + \epsilon$ and $R_{beg} = R - \epsilon$ with $\epsilon = 0$ giving the result $R_{end} = R_{beg}$

[36]The total negative mass mass can be obtained by a contour integral of all the differential elements of negative mass over the circumference length $L = 2\pi R$

The geometrical description of the system presented for the Star Trek Enterprise is exactly equal to the one presented for the NASA Space Shuttle except for the fact that the mass of the Enterprise is 32.500 times bigger than the mass of the Shuttle.

- Below is being presented a plot of the mass of the Star Trek Enterprise in the center of the Natario warp bubble and differential elements of negative mass placed over the bubble warped region.All masses are given in kilograms.

M	$n1$	$n2$	$n3$	$n4$
$3,2500000000E+09$	$1,0000000000E-07$	$1,0000000000E-07$	$1,0000000000E-07$	$1,0000000000E-07$

- Below is being presented a numerical plot for the integrity of the Natario warp drive bubble with the Star Trek Enterprise mass in the center of the bubble. $G = 6,6700000000E-11$ in SI units.All the forces are given in Newtons and the distances in meters.$F1$ is always negative while $F2$ is always positive.

R	d	$F1$	$F2$
$1,0000000000E+03$	$1,0000000000E-06$	$2,1677500000E-14$	$6,6700000000E-13$
$9,0000000000E+02$	$1,0000000000E-06$	$2,6762345679E-14$	$6,6700000000E-13$
$8,0000000000E+02$	$1,0000000000E-06$	$3,3871093750E-14$	$6,6700000000E-13$
$7,0000000000E+02$	$1,0000000000E-06$	$4,4239795918E-14$	$6,6700000000E-13$
$6,0000000000E+02$	$1,0000000000E-06$	$6,0215277778E-14$	$6,6700000000E-13$
$5,0000000000E+02$	$1,0000000000E-06$	$8,6710000000E-14$	$6,6700000000E-13$
$4,0000000000E+02$	$1,0000000000E-06$	$1,3548437500E-13$	$6,6700000000E-13$
$3,0000000000E+02$	$1,0000000000E-06$	$2,4086111111E-13$	$6,6700000000E-13$
$2,5000000000E+02$	$1,0000000000E-06$	$3,4684000000E-13$	$6,6700000000E-13$
$2,0000000000E+02$	$1,0000000000E-06$	$5,4193750000E-13$	$6,6700000000E-13$
$1,5000000000E+02$	$1,0000000000E-06$	$9,6344444444E-13$	$6,6700000000E-13$
$1,0000000000E+02$	$1,0000000000E-06$	$2,1677500000E-12$	$6,6700000000E-13$
$9,0000000000E+01$	$1,0000000000E-06$	$2,6762345679E-12$	$6,6700000000E-13$
$1,0000000000E+01$	$1,0000000000E-06$	$2,1677500000E-10$	$6,6700000000E-13$
$1,0000000000E+00$	$1,0000000000E-06$	$2,1677500000E-08$	$6,6700000000E-13$

Above are the plots for a Natario warp bubble with the mass of the Enterprise in the CM frame in the center of the bubble $rs = 0$ reacting with differential elements of negative mass each element with $|10^{-7}|$ kilograms.

Note that in this case for a warp bubble of 100 meters radius the repulsive force $|F1|$ have a magnitude of 10^{-12} Newtons and the attractive force $|F2|$ have a magnitude of 10^{-13} Newtons. Since $10^{-12} > 10^{-13}$ this bubble not stable and would collapse.

Note that for a warp bubble of 500 meters radius the repulsive force $|F1|$ have a magnitude of 10^{-14} Newtons and the attractive force $|F2|$ have a magnitude of 10^{-13} Newtons. Since $10^{-13} > 10^{-14}$ this bubble is stable however $|10^{-7}|$ kilograms may deflect incoming blueshifted photons from interstellar space but cannot cope with large asteroids.

Like we said before:

-)-In order to keep $|F2| >> |F1|$ the radius of the bubble must be enormous in order to weak the force$|F1|$ when $|M| >> (|n1| + |n2| + |n3| + |n4|)$.An enormous radius will keep $|F1|$ always weaker than $|F2|$.

So the Shuttle can support a stable bubble of 100 meters of radius with differential elements of negative mass of $|10^{-7}|$ kilograms while the Enterprise cannot because the positive mass of the Enterprise is 32.500 times the mass of the Shuttle and according to Lobo and Visser the negative mass of the bubble must be a reasonable fraction of the positive mass of the spaceship (see summary page 13 in [10]).Although $|10^{-7}|$ kilograms cannot be considered an appreciable fraction of the positive mass of the Shuttle it is more closer to be a reasonable fraction of the ship mass in the Shuttle case when compared to the Enterprise case.

- Below is presented a plot of the mass of the Star Trek Enterprise in the center of the Natario warp bubble and differential elements of negative mass placed over the bubble warped region.All masses are given in kilograms.

M	$n1$	$n2$	$n3$	$n4$
$3,2500000000E+09$	$4,8185631766E-21$	$4,8185631766E-21$	$4,8185631766E-21$	$4,8185631766E$ -

- Blow is presented a numerical plot for the integrity of the Natario warp drive bubble with the Star Trek Enterprise mass in the center of the bubble. $G = 6,6700000000E-11$ in SI units.All the forces are given in Newtons and the distances in meters.$F1$ is always negative while $F2$ is always positive.

R	d	$F1$	$F2$
$1,0000000000E+10$	$1,0000000000E-06$	$1,0445440326E-41$	$1,5486773575E-39$
$1,0000000000E+09$	$1,0000000000E-06$	$1,0445440326E-39$	$1,5486773575E-39$
$1,0000000000E+08$	$1,0000000000E-06$	$1,0445440326E-37$	$1,5486773575E-39$
$1,0000000000E+06$	$1,0000000000E-06$	$1,0445440326E-33$	$1,5486773575E-39$
$1,0000000000E+03$	$1,0000000000E-06$	$1,0445440326E-27$	$1,5486773575E-39$
$1,0000000000E+02$	$1,0000000000E-06$	$1,0445440326E-25$	$1,5486773575E-39$

Above are the plots for a Natario warp bubble with the mass of the Enterprise in the CM frame in the center of the bubble $rs = 0$ reacting with differential elements of negative mass each element with $|10^{-21}|$ kilograms exactly the mass of Lamoreaux for the Casimir Effect.

Note that in this case for a warp bubble of 100 meters radius the repulsive force $|F1|$ have a magnitude of 10^{-25} Newtons and the attractive force $|F2|$ have a magnitude of 10^{-39} Newtons. Since $10^{-25} >> 10^{-39}$ this bubble not stable and would collapse.

Note that for a warp bubble of 10^{10} meters radius the repulsive force $|F1|$ have a magnitude of 10^{-41} Newtons and the attractive force $|F2|$ have a magnitude of 10^{-39} Newtons. Since $10^{-39} > 10^{-41}$ this bubble is stable however $10^{10} = 10.000.000.000$ a bubble of 10.000.000 kilometers:Not practical at all!!!!!

Like we said before:

-)-In order to keep $|F2| >> |F1|$ the radius of the bubble must be enormous in order to weak the force$|F1|$ when $|M| >> (|n1| + |n2| + |n3| + |n4|)$.An enormous radius will keep $|F1|$ always weaker than $|F2|$.

Note that a warp bubble of 100 meters radius in some of the cases previously presented for the Shuttle and the Enterprise can carry the Shuttle (or not) depending on the magnitude of the value of the differential elements of negative mass but the same differential elements of negative mass that sustains a warp bubble able to carry the Shuttle cannot support a warp bubble able to carry the Enterprise because its mass is 32.500 bigger than the mass of the Shuttle.

A warp bubble designed to carry a light spaceship cannot carry a heavy spaceship because the positive mass of the spaceship will disrupt the differential elements of negative mass of the bubble.

If we want to carry the Enterprise in a warp bubble of 100 meters radius we must provide an amount of negative energy density much bigger than the amount required to transport the Shuttle in a warp bubble of the same radius otherwise due to the weight of the Enterprise the bubble will collapse.

- Below is presented a plot of the mass of the Star Trek Enterprise in the center of the Natario warp bubble and differential elements of negative mass placed over the bubble warped region.All masses are given in kilograms.

M	$n1$	$n2$	$n3$	$n4$
$3,2500000000E+09$	$1,0000000000E+01$	$1,0000000000E+01$	$1,0000000000E+01$	$1,0000000000E$ -

-)-Below is presented a numerical plot for the integrity of the Natario warp drive bubble with the Star Trek Enterprise mass in the center of the bubble. $G = 6,6700000000E-11$ in SI units.All the forces are given in Newtons and the distances in meters.$F1$ is always negative while $F2$ is always positive.

R	d	$F1$	$F2$
$1,0000000000E+02$	$1,0000000000E-06$	$2,1677500000E-04$	$6,6700000000E+03$

Above are the plots for a Natario warp bubble with the mass of the Enterprise in the CM frame in the center of the bubble $rs = 0$ reacting with differential elements of negative mass each element with $|10|$ kilograms.

Note that in this case for a warp bubble of 100 meters radius the repulsive force $|F1|$ have a magnitude of 10^{-4} Newtons and the attractive force $|F2|$ have a magnitude of 10^3 Newtons. Since $10^3 > 10^{-4}$ this bubble is stable and $|10|$ kilograms of negative mass can generate repulsive gravitational fields able to deflect not only Doppler blueshifted photons but also can deflect interstellar dust molecules of gas and micrometeorites.[37]

[37] See Appendices H and M in [4] and [5]

We just finished to demonstrate according to Lobo and Visser the relation between the following elements:

- 1)-The negative mass of the warp bubble
- 2)-The positive mass of the spaceship
- 3)-The warp bubble radius

According to Lobo and Visser the negative mass of the bubble must be a reasonable fraction of the positive mass of the spaceship (see summary page 13 in [10]) for a warp bubble of a radius R.Although $|10|$ kilograms cannot be considered an appreciable fraction of the positive mass of the Enterprise it is more closer to be a reasonable fraction of the ship mass than the amount of $|10^{-7}|$ kilograms.

A warp bubble of 100 meters radius with differential elements of negative mass of $|10^{-7}|$ kilograms can carry the Shuttle but cannot carry the Enterprise due to a weight 32.500 times bigger.

A warp bubble of 100 meters radius with differential elements of negative mass of $|10|$ kilograms can carry the Enterprise because possesses more negative mass able to support the extra weight.

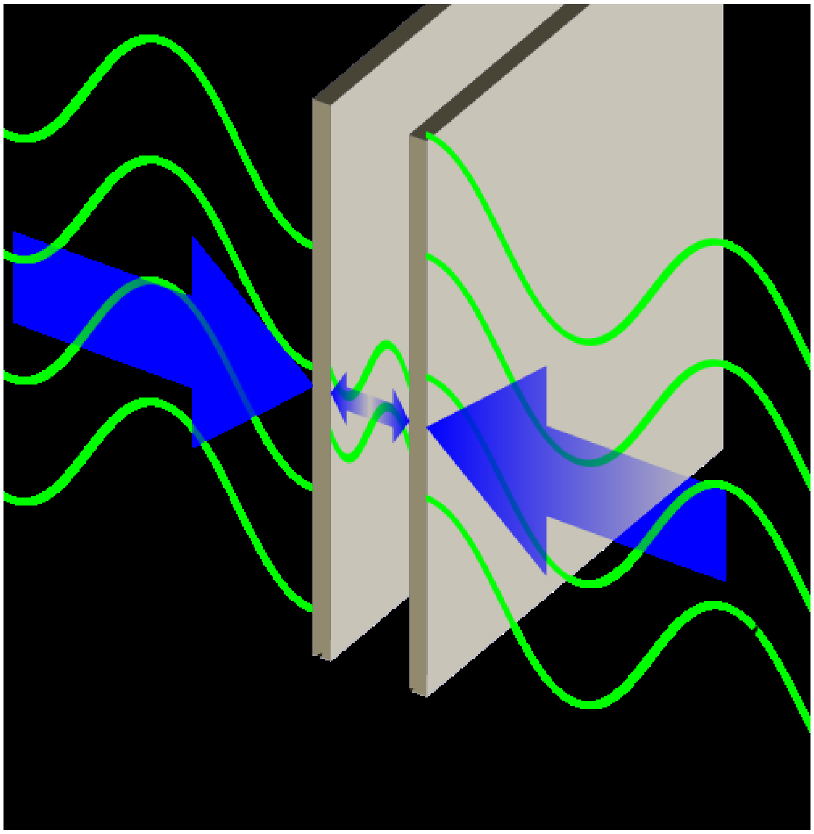

Figure 3: Artistic Presentation of the Casimir Effect

11 Appendix D:Artistic presentation of the Casimir Effect

The Casimir Effect states that the vacuum energy density between two parallel conduction plates separated by a distance d is given by:(pg 42 in [8])

$$\rho = -\frac{\pi^2}{720}\frac{\hbar}{d^4} \tag{61}$$

The equation above was written in the Geometrized System of Units $c = G = 1$.In the International System of Units the same equation would be:

$$\rho = -\frac{\pi^2}{720}\frac{\hbar c}{d^4} \tag{62}$$

As the Casimir plates are placed close together at a very small distance d a repulsive force(the small blue arrows in the inner region between the plates) appears.The repulsive behavior is due to the negative energy density that appears between the plates.

It was first theoretically predicted by Casimir in 1948 [6] but was experimentally demonstrated almost 50 years later by Lamoreaux in 1997 [7]

12 Epilogue

- "The only way of discovering the limits of the possible is to venture a little way past them into the impossible."-Arthur C.Clarke[38]
- "The supreme task of the physicist is to arrive at those universal elementary laws from which the cosmos can be built up by pure deduction. There is no logical path to these laws; only intuition, resting on sympathetic understanding of experience, can reach them"-Albert Einstein[39][40]

13 Remarks

Reference 8 can be obtained from the web pages of Professor Eric Poisson at University of Guelph Ontario Canada as long as the site remains on-line. [41] [42] [43]

Although the main references of this work were taken from scientific sites available to the general public for consultation (eg:arXiv,HAL) we can provide a copy in PDF Acrobat reader of all our references for those interested.

[38]special thanks to Maria Matreno from Residencia de Estudantes Universitas Lisboa Portugal for providing the Second Law Of Arthur C.Clarke

[39]"Ideas And Opinions" Einstein compilation, ISBN $0-517-88440-2$, on page 226."Principles of Research" ([Ideas and Opinions],pp.224-227), described as "Address delivered in celebration of Max Planck's sixtieth birthday (1918) before the Physical Society in Berlin"

[40]appears also in the Eric Baird book Relativity in Curved Spacetime ISBN $978-0-9557068-0-6$

[41]http://www.physics.uoguelph.ca/poisson/research/

[42]http://www.physics.uoguelph.ca/poisson/research/notes.html

[43]http://www.physics.uoguelph.ca/poisson/research/agr.pdf

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[10] *Lobo F.,Visser M.,(*2004*).Class.Quant.Grav. 21 5871-5892,gr-qc/0406083*

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