

A very exhaustive generalization of de Polignac's conjecture

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Abstract. In a previous paper I made a generalization of de Polignac's conjecture. In this paper I extend that generalization as much as is possible.

Conjecture:

For any n even positive integer and for any i and j non-null positive integers there exist an infinity of distinct sets of i primes p_1, p_2, \dots, p_i and also an infinity of distinct sets of j primes q_1, q_2, \dots, q_j such that $p_1 * p_2 * \dots * p_i - q_1 * q_2 * \dots * q_j = n$.

Case $[i, j, n] = [1, 1, 2]$:

In this case we have $p - q = 2$, which gave us the twin primes conjecture.

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Case $[i, j, n] = [2, 1, 2]$:

In this case we have $p_1 * p_2 - q = 2$.

Such triplets of primes $[p_1, p_2, q]$, are: $[7, 13, 89]$, $[7, 19, 131]$, $[7, 37, 257]$...Note that the conjecture can be further extended in this case to: for any p_1 odd prime there exist an infinity of pairs of primes $[p_2, q]$ such that $p_1 * p_2 - q = 2$.

Case $[i, j, n] = [1, 2, 2]$:

In this case we have $p - q_1 * q_2 = 2$.

Such triplets of primes $[p, q_1, q_2]$, are: $[79, 11, 7]$, $[163, 23, 7]$, $[331, 47, 7]$...Note that the conjecture can be further extended in this case to: for any q_1 odd prime there exist an infinity of pairs of primes $[p, q_2]$ such that $p - q_1 * q_2 = 2$.

Conjecture:

(the most exhaustive generalization of de Polignac's conjecture)

For any n even positive integer and for any i, j, k, l non-null positive integers, for any k given primes a_1, a_2, \dots, a_k and for any l given primes b_1, b_2, \dots, b_l , there exist an infinity of distinct sets of i primes p_1, p_2, \dots, p_i and also an infinity of distinct sets of j primes q_1, q_2, \dots, q_j such that $p_1 * p_2 * \dots * p_i * a_1 * a_2 * \dots * a_k - q_1 * q_2 * \dots * q_j * b_1 * b_2 * \dots * b_l = n$.