A set of Poulet numbers and generalizations of the twin primes and de Polignac’s conjectures inspired by this

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Abstract. In this paper I show a set of Poulet numbers, each one of them having the same interesting relation between its prime factors, and I make four conjectures, one about the infinity of this set, one about the infinity of a certain type of duplets respectively triplets respectively quadruplets and so on of primes and finally two generalizations, of the twin primes conjecture respectively of de Polignac’s conjecture.

Conjecture 1:

There exist an infinity of Poulet numbers of the form \( n^2 + 120n \), where \( n \) is prime or a composite positive integer.

Note:

In the first case, obviously \( n \) is a prime factor of such a Poulet number and the product of the other prime factors is equal to \( n + 120 \); for instance, the number 1729 is a part of this set of Poulet numbers because 1729 = 7\*13\*19 can be written as 13\(^2\) + 13\*120 and implicitly 7\*19 = 13 + 120. First few such Poulet numbers are:

\[
\begin{align*}
1729 &= 7\*13\*19 = 13^2 + 13\*120; \\
4681 &= 31\*151 = 31^2 + 31\*120; \\
6601 &= 7\*23\*41 = 41^2 + 41\*120.
\end{align*}
\]

Note:

In the second case, obviously \( n \) is a product of few prime factors of such a Poulet number and the product of the other prime factors is equal to \( n + 120 \). Such a Poulet number is 75361 = 11\*13\*17\*31 = 221\(^2\) + 221\*120 and implicitly 11\*31 = 13\*17 + 120.
Conjecture 2:

There exist an infinity of duplets of primes \([p, q]\) such that \(p - q = 120\); there also exist an infinity of triplets of primes \([p_1, p_2, q]\) such that \(p_1*p_2 - q = 120\); there also exist an infinity of quadruplets of primes \([p_1, p_2, p_3, q]\) such that \(p_1*p_2*p_3 - q = 120\); generally, for any non-null positive integer \(i\) there exist \(i\) primes \(p_1, p_2, ..., p_i\) and a prime \(q\) such that \(p_1*p_2*...*p_i - q = 120\).

Examples:

: \(151 - 31 = 120\);
: \(7*19 - 13 = 120\);
: \(7*17*37 - 4283 = 120\).

Conjecture 3:

(generalization of the twin primes conjecture)

For any non-null positive integer \(i\) there exist an infinity of sets of \(i + 1\) primes \(p_1, p_2, ..., p_i, q\) such that \(p_1*p_2*...*p_i - q = 2\).

Conjecture 4:

(generalization of de Polignac’s conjecture)

For any \(n\) even positive integer and for any \(i\) non-null positive integer there exist an infinity of sets of \(i + 1\) primes \(p_1, p_2, ..., p_i, q\) such that \(p_1*p_2*...*p_i - q = n\).