The Bohr Postulate, the De Broglie Condition and the Scale Principle

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Earlier this year I wrote a paper entitled Scale Factors and the Scale Principle. In that paper I formulated a new law which describes nature at both quantum and cosmic scales. This article shows that both the Bohr postulate and the De Broglie condition are special cases of the abovementioned formulation.

Keywords: *Bohr postulate, De Broglie wavelength, Planck length, Planck mass, Planck Momentum.*

1. Introduction

In a previous article [1] I introduced the scale principle or scale law through the following mathematical relationship

(1)

Scale principle or scale law
$\left(\frac{Q_1}{Q_2}\right)^n \left[\le \middle| = \middle| \ge \right] S \left(\frac{Q_3}{Q_4}\right)^m$

Where

a) Q_1, Q_2, Q_3 and Q_4 are physical quantities of identical dimension (such as Length, Time, Mass, Temperature, etc), or

b) Q_1 and Q_2 are physical quantities of dimension 1 or dimensionless constants while Q_3 and Q_4 are physical quantities of dimension 2 or dimensionless constants. However, if Q_1 and Q_2 are dimensionless constants then Q_3 and Q_4 must have dimensions and viceversa.
The physical quantities can be variables, constants, dimensionless constants, differentials, derivatives, integrals, etc.
(e.g.: Q_1 and Q_2 could be quantities of Mass while Q_3 and Q_4 could be quantities of Length).

c) The relationship is one of three possibilities: **less than or equal to** inequation ($\leq$), or an equation (=), or a **greater than or equal to** inequation ($\geq$).

d) S is a dimensionless scale factor. This factor could be a real number, a complex number, a real function or a complex function (strictly speaking real numbers are a particular case of complex numbers). The scale factor could have more than one value for the same relationship.

e) n and m are integers 0, 1, 2, 3,… (In general these two numbers are different. e.g. 1: n =1 and m = 1. e.g. 2: n = 1 and m = 2. n and m cannot be both zero in the same relationship).

2. The Bohr Postulate

I shall show, through very simple mathematics, that the Bohr postulate

$$mvr = n\frac{h}{2\pi} \qquad \text{(Bohr postulate)} \qquad (1)$$

obeys the scale principle.

We shall start the analysis by drawing the following scale table

Momentum (electron)	Length (orbit radius)	Length (Planck Scale)	Momentum (Planck Scale)
mv	r	L_P	$M_P\,c$

***TABLE 1**: This simple scale table is used to show that the Bohr postulate obeys the scale law.*

Where

mv = momentum of the electron "orbiting" the proton
m = mass of the electron
v = velocity of the electron "orbiting" the proton
r = orbit radius
L_P = Planck length
$M_P c$ = Planck momentum
M_P = Planck mass
c = speed of light in vacuum

In order to get the right relationship we need to balance the table so that we place the Planck mass on one half of the table while the Planck length is placed on the other half. We also notice that there is symmetry about the vertical axis (shown in green) that divides the table into two halves:

Momentum	Length	Length	Momentum

Thus, according to the above scale table we write the following relationship

$$mvr = S\, L_P\, M_P c \tag{2}$$

As always we have introduced the scale factor *S* on the second side of the equation to complete the relationship.

Now we rewrite equation (2) in the form of the scale principle to get

$$\frac{mv}{M_P c} = S\, \frac{L_P}{r} \tag{3}$$

Comparing equation (3) with relationship (1) we find that equation (3) has the following form

$$\frac{Q_1}{Q_2} = S\, \frac{Q_3}{Q_4} \tag{4}$$

Where

$n = m = 1$

$Q_1 = mv$

$Q_2 = M_P c$

$Q_3 = L_P$

$Q_4 = r$

$S =$ 1, 2, 3, etc.

The Planck length and the Planck mass are defined respectively by

$$L_P = \sqrt{\frac{h\, G}{2\pi\, c^3}} \tag{5}$$

$$M_P = \sqrt{\frac{h\, c}{2\pi\, G}} \tag{6}$$

Substituting L_P and M_P in equation (3) with equations (5) and (6) yields

$$L_P M_P = \sqrt{\frac{hG}{2\pi\, c^3}\frac{hc}{2\pi\, G}} = \frac{h}{2\pi\, c} \tag{7}$$

$$mvr = S\frac{h}{2\pi\, c}c \tag{8}$$

$$mvr = S\frac{h}{2\pi} \tag{9}$$

The scale factor in this equation is in fact a quantum number and the values are

S = 1, 2, 3, 4, 5, etc.

Equation (9) is the Bohr postulate. Thus we have proved that the Bohr postulate obeys the scale law. It is worthy to remark that in quantum mechanics scale factors can be quantum numbers and therefore they can have more than one value.

3. The De Broglie Condition based on the De Broglie Wavelength

I shall show, through very simple mathematics, that the De Broglie condition

$2\pi r = n\lambda$ (De Broglie condition or "postulate") (10)

Where
n = 1, 2, 3, 4, 5, etc.

obeys the scale principle.

We shall start the analysis by drawing a table very similar to Table 1

Momentum (electron)	Length (orbit radius)	Length (Planck Scale)	Momentum (Planck Scale)
$\frac{h}{\lambda}$	r	L_P	$M_P\, c$

***TABLE 2**: This simple scale table is used to show that the above De Broglie condition obeys the scale law.*

Where

h = Planck's constant
λ = De Broglie wavelength associated to a particle with momentum *mv*
r = orbit radius
L_P = Planck length
$M_P c$ = Planck momentum
M_P = Planck mass
c = speed of light in vacuum

The only difference between Table 1 and Table 2 is that we have replaced the momentum of the electron *mv* by the De Broglie expression h/λ which we obtained from the De Broglie relationship which associates a wavelength λ to a particle of momentum *mv*

$$\lambda = \frac{h}{mv} \tag{11}$$

Where

mv = momentum of the electron "orbiting" the proton

Thus, according to the above scale table we write

$$\frac{h}{\lambda} r = S\, L_P\, M_P c \tag{11}$$

Following a similar method as the one we applied in the previous section we can demonstrate that

$$2\pi\, r = S\lambda \tag{12}$$

When the values of the scale factor are

S = 1, 2, 3, etc

we obtain the De Broglie condition (equation 10) which is equivalent to the Bohr postulate. Thus we have proved that the De Broglie condition obeys the scale law.

4. Conclusions

Taking into account that the scale law describes several known laws of physics as I have shown both on previous papers [1, 2, 3, 4] and on this paper, we can consider the scale law as a more universal law than the specific laws it describes. Therefore the scale principle is a *law model* (I prefer to call it *Meta law* or *hyper law)* nature applies to a wide range of phenomena.

REFERENCES

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