A wave function and quantum state vector in indefinite metric Minkowski space

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Indefinite metric vectors are absolutely required as the physical states in Minkowski space because that is indefinite metric space and the physical space-time. For example, Maxwell equations are wave equations in Minkowski space. However, traditional Quantum theory ordinarily has been studied only in definite metric space, i.e., Hilbert space. There are no clear expression for indefinite metric vectors. Here we show a wave function example using Dirac's delta function for indefinite metric vectors in Minkowski space. In addition, we show the vectors can interfere with itself. This example also suggests indefinite metric will be absolutely required.

I. INTRODUCTION

When we take advantage of unobservable potentials that can be identified as indefinite metric vectors, we can interpret single photon and electron interferences and entanglement without quantum-superposition.[1, 2]

First, we deal with the definition of metric space. Arbitrary state vectors $|\varphi\rangle$ and $|\psi\rangle$ satisfy following conditions in definite metric space.

$$\begin{aligned}&\langle\varphi|\psi\rangle = \langle\psi|\varphi\rangle^*\\ &\langle\psi|\psi\rangle \geq 0\\ &\langle\psi|\psi\rangle = 0 \Leftrightarrow |\psi\rangle = 0\end{aligned} \tag{1}$$

In contrast, the second and third relations are replaced with $\langle\psi|\psi\rangle = \alpha$ and $\langle\psi|\psi\rangle = 0 \Leftrightarrow |\psi\rangle \neq 0$ or 0 in indefinite metric space. Where α is an arbitrary number.

Minkowski space is divided into time-like (T), light-like (L), space-like (S) parts and point of the origin P ($\mathbf{x} = 0, t = 0$). Traditional quantum theory has been studied in $\mathrm{T} \oplus \mathrm{P}$. Where $\oplus$ stands for direct sum. However the unobservable potentials which can be identified as indefinite metric vectors propagate at the speed of light. Hence the vectors will be on the origin P and surface of the light cone L, i.e., $\mathrm{P} \oplus \mathrm{L}$. Of course photons in free space are on $\mathrm{P} \oplus \mathrm{L}$. Because there exist some entity related to the unobservable potentials but can not be observe any entities, there must be the vector $|\psi\rangle_{\mathrm{P}\oplus\mathrm{L}} \neq 0$ with the norm $\langle\psi|\psi\rangle_{\mathrm{P}\oplus\mathrm{L}} = 0$. Where $|\psi\rangle$ is the state expressing the unobservable potentials. However there always exist the unobservable potentials on $\mathrm{T} \oplus \mathrm{P} \oplus \mathrm{L}$. Then $\langle\psi|\psi\rangle_{\mathrm{L}} = \alpha$.

When there are physical observable entity $|\varphi\rangle$ on $\mathrm{T} \oplus \mathrm{P}$, $|\varphi\rangle_{\mathrm{T}\oplus\mathrm{P}} + |\psi\rangle_{\mathrm{T}\oplus\mathrm{P}\oplus\mathrm{L}} \neq 0 \Rightarrow ((\langle\psi| + \langle\varphi|)(|\varphi\rangle + \psi\rangle)_{\mathrm{T}\oplus\mathrm{P}\oplus\mathrm{L}} \geq 0$. The space-like part S is estimated to have the rest of the characteristics, i.e., negative norm $((\langle\psi| + \langle\varphi|)(|\varphi\rangle + \psi\rangle)_{\mathrm{S}} < 0$.

Therefore we should study the states in $\mathrm{T}\oplus\mathrm{P}\oplus\mathrm{L}$ as the physical states instead of (1). In this letter, we show a wave function example using Dirac's delta function for indefinite metric vectors in Minkowski space satisfied with the following conditions.

$$\begin{aligned}&\langle\varphi|\psi\rangle_{\mathrm{T}\oplus\mathrm{P}\oplus\mathrm{L}} = \langle\psi|\varphi\rangle^*_{\mathrm{T}\oplus\mathrm{P}\oplus\mathrm{L}}\\ &\langle\psi|\psi\rangle_{\mathrm{T}\oplus\mathrm{P}\oplus\mathrm{L}} = \alpha\\ &\langle\psi|\psi\rangle_{\mathrm{T}\oplus\mathrm{P}\oplus\mathrm{L}} = 0 \Leftrightarrow |\psi\rangle_{\mathrm{T}\oplus\mathrm{P}\oplus\mathrm{L}} = 0 \text{ or } \neq 0\end{aligned} \tag{2}$$

with

$$((\langle\psi| + \langle\varphi|)(|\varphi\rangle + \psi\rangle)_{\mathrm{T}\oplus\mathrm{P}\oplus\mathrm{L}} \geq 0 \tag{3}$$

II. AN EXAMPLE EXPRESSION

Let $|\varphi\rangle$ is an observable physical state in $\mathrm{T}\oplus\mathrm{P}$, and $|\psi\rangle$ is an indefinite metric vector in $\mathrm{T} \oplus \mathrm{P} \oplus \mathrm{L}$. These stats are expressed by wave functions $\varphi(x)$ and $\psi(x)$ as follows.

$$\begin{aligned}\langle\varphi|\varphi\rangle_{\mathrm{T}\oplus\mathrm{P}} &= \int \varphi^*(x)\varphi(x)dx \geq 0\\ \langle\varphi|\psi\rangle_{\mathrm{T}\oplus\mathrm{P}\oplus\mathrm{L}} &= \int \varphi^*(x)\psi(x)dx = \langle\psi|\varphi\rangle^*_{\mathrm{T}\oplus\mathrm{P}\oplus\mathrm{L}}\\ \langle\psi|\psi\rangle_{\mathrm{T}\oplus\mathrm{P}\oplus\mathrm{L}} &= \int \psi^*(x)\psi(x)dx = \alpha\end{aligned} \tag{4}$$

We can easily confirm the following $\varphi(x)$ and $\psi(x)$ satisfy the above relations.

$$\begin{aligned}\varphi(x) &= f(x) = |f(x)|e^{-i\phi}\\ \psi(x) &= a(x)\left\{\frac{1}{2}e^{i\theta/2} - \frac{1}{2}e^{-i\theta/2}\right\}\sqrt{\delta(x)}\end{aligned} \tag{5}$$

where $f(x)$, $\delta(x)$, $a(x)$, α and θ are a traditional wave function, i.e., $\langle\varphi|\varphi\rangle_{\mathrm{T}\oplus\mathrm{P}} = \langle f|f\rangle = \int |f(x)|^2 dx \geq 0$, Dirac's delta function, an arbitrary complex function of x, an arbitrary complex number of dimension $[x]^{-1}$ and phase difference between P and arbitrary space-time point of $\psi(x)$ respectively.

The dimension of $\langle f|f\rangle = \int |f(x)|^2 dx$ and $\langle\psi|\psi\rangle = \int |\psi(x)|^2 dx$ are $[x]^0$ which means the dimensions of $f(x)$ and $\psi(x)$ are $[x]^{-\frac{1}{2}}$. Because the dimension of $\sqrt{\delta(x)}$ is $[x]^{-\frac{1}{2}}$, the expression of (5) using $\sqrt{\delta(x)}$ is valid from the point of view of dimension. From (5), followings can be

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calculated.

$$\langle\psi|\psi\rangle_{\mathrm{T}\oplus\mathrm{P}\oplus\mathrm{L}} = \int \psi^*(x)\psi(x)dx = |a(0)|^2 \left\{\frac{1}{2} - \frac{1}{4}e^{i\theta} - \frac{1}{4}e^{-i\theta}\right\} = \frac{1}{2}|a(0)|^2 (1-\cos\theta) \tag{6}$$

When $\theta = 0$ on P, $\theta = 0$ on $\mathrm{P}\oplus\mathrm{L}$, because the unobservable potentials move at light speed, i. e., static on L. Hence $\langle\psi|\psi\rangle_{\mathrm{P}\oplus\mathrm{L}} = 0$. and $\theta \neq 0$ is on T, i. e., $\langle\psi|\psi\rangle_{\mathrm{T}} = \frac{1}{2}|a(0)|^2 (1-\cos\theta) \geq 0$. Hence the second and third relations of (2) are obtained, though $\alpha \geq 0$.

In addition,

$$\langle\varphi|\psi\rangle = \int \varphi^*(x)\psi(x)dx = \int a(x)f^*(x)\left\{\frac{1}{2}e^{i\theta/2} - \frac{1}{2}e^{-i\theta/2}\right\}\sqrt{\delta(x)}dx \tag{7}$$

$$\langle\psi|\varphi\rangle = \int \psi^*(x)\varphi(x)dx = \int a^*(x)f(x)\left\{\frac{1}{2}e^{-i\theta/2} - \frac{1}{2}e^{i\theta/2}\right\}\sqrt{\delta(x)}dx \tag{8}$$

Therefore $\langle\varphi|\psi\rangle_{\mathrm{T}\oplus\mathrm{P}\oplus\mathrm{L}} = \langle\psi|\varphi\rangle^*_{\mathrm{T}\oplus\mathrm{P}\oplus\mathrm{L}}$. Hence the all relations of (2) are obtained.

As for (3),

$$(\langle\psi| + \langle\varphi|)(|\varphi\rangle + \psi\rangle) = \langle\varphi|\varphi\rangle + \langle\varphi|\psi\rangle + \langle\psi|\varphi\rangle + \langle\psi|\psi\rangle = \langle\varphi|\varphi\rangle + \frac{1}{2}|a(0)|^2(1-\cos\theta) + \langle\varphi|\psi\rangle + \langle\psi|\varphi\rangle \tag{9}$$

This means special forms must be chosen for $\psi(x)$ in order to satisfy (3).

III. APPLICATION - SELF-INTERFERENCE

When we choose $a(x) = if(x)$ and $\varphi(x) = f(x) + \psi(x)$ in (5), i.e., $|\varphi\rangle = |f\rangle + |\psi\rangle$ as a physical state, then $\langle\varphi|\psi\rangle = \langle\psi|\varphi\rangle^* = -\langle\psi|\varphi\rangle$ and the norm is calculated to be as follows.

$$\langle\varphi|\varphi\rangle = \langle f|f\rangle + \langle\psi|f\rangle + \langle f|\psi\rangle + \langle\psi|\psi\rangle = \langle f|f\rangle + \langle\psi|\psi\rangle = \langle f|f\rangle + \frac{1}{2}|a(0)|^2(1-\cos\theta) \tag{10}$$

Therefore $|\varphi\rangle = |f\rangle + |\psi\rangle$ can make self interference. However $\langle\varphi|\varphi\rangle > 0$. We will never satisfy (3) unless

$$|a(0)|^2 = -\langle f|f\rangle \tag{11}$$

IV. GENERALIZATION FOR VECTOR SPACE

Here we generalize the above discussion to vector space. Let's consider zero norm vectors set. The set becomes vector space $\mathfrak{V}_0$under following conditions.

A) For arbitrarily $|\gamma\rangle$, $|\gamma'\rangle \in \mathfrak{V}_0$, i.e., $\langle\gamma|\gamma\rangle = \langle\gamma'|\gamma'\rangle = 0$, $\mathfrak{V}_0$ is closed under multiplication, i.e., $|\gamma\rangle + |\gamma'\rangle \in \mathfrak{V}_0$ with associative law, existence of zero element, existence of inverse element and commutative law of addition.

B) For arbitrarily $|\gamma\rangle \in \mathfrak{V}_0$ and $\lambda \in F$ (field of scalars), $\mathfrak{V}_0$ is closed under multiplication of the scalar, i.e., $\lambda|\gamma\rangle \in \mathfrak{V}_0$ with associative law of scalar multiple, $1|\gamma\rangle = |\gamma\rangle$, distributive law of scalar and elements.

From A), the norm $|\gamma\rangle + |\gamma'\rangle$ is calculate to be $\langle\gamma|\gamma\rangle + \langle\gamma'|\gamma'\rangle + 2\mathrm{Re}\langle\gamma|\gamma'\rangle = 2\mathrm{Re}\langle\gamma|\gamma'\rangle = 0$ then $\langle\gamma|\gamma'\rangle = i\mu$, where μ is a real number, i.e., $\langle\gamma|\gamma'\rangle$ is a purely imaginary number.

From B), the norm $|\gamma\rangle + \lambda|\gamma'\rangle$ is calculate to be $\langle\gamma|\gamma\rangle + |\lambda|^2\langle\gamma'|\gamma'\rangle + 2\mathrm{Re}\lambda\langle\gamma|\gamma'\rangle = 2\mathrm{Re}\lambda\langle\gamma|\gamma'\rangle = 0$ then $\lambda\langle\gamma|\gamma'\rangle = i\mu'$, where μ' is a real number. Because $\langle\gamma|\gamma'\rangle$ is a purely imaginary number, then $\lambda \in F$ (field of scalars) must be a real number. Therefore, $\mathfrak{V}_0$ is a real vector space.

In contrast, a complex vector space $\mathfrak{H}_{phys}$ which is positive semidefinite space, has been studied as the physical states in traditional quantum theory.

However we should study the vectors formed by the direct sum $\mathfrak{V}_{phys} = \mathfrak{H}_{phys} \oplus \mathfrak{V}_0$ as the real physical states whose norms are positive semidefinite.

For example, the norm of

$$|\varphi\rangle(\in \mathfrak{V}_{phys}) = |f\rangle(\in \mathfrak{H}_{phys}) + |\gamma\rangle(\in \mathfrak{H}_0) \tag{12}$$

is calculated to be

$$\langle\varphi|\varphi\rangle = \langle f|f\rangle + \langle\gamma|f\rangle + \langle f|\gamma\rangle + \langle\gamma|\gamma\rangle = \langle f|f\rangle + \langle\gamma|f\rangle + \langle f|\gamma\rangle \tag{13}$$

Therefore $\langle\gamma|f\rangle$ should be $\frac{\langle f|f\rangle}{2}e^{i\theta}$ because of the positive semidefinite norm $\langle\varphi|\varphi\rangle = 2\langle f|f\rangle\left\{\frac{1}{2} + \frac{1}{2}\cos\theta\right\} \geq 0$.

V. CONCLUSION

Traditional quantum theory has been studied by using $|\varphi\rangle_{\mathrm{T}\oplus\mathrm{P}}$. However we should study $|\varphi\rangle_{\mathrm{T}\oplus\mathrm{P}\oplus\mathrm{L}}$, i. e., $|\varphi\rangle_{\mathrm{T}\oplus\mathrm{P}\oplus\mathrm{L}} \equiv |\varphi\rangle_{\mathrm{T}\oplus\mathrm{P}} + |\psi\rangle_{\mathrm{T}\oplus\mathrm{P}\oplus\mathrm{L}}$, as the physical states instead of $|\varphi\rangle_{\mathrm{T}\oplus\mathrm{P}}$. In order to justify the discussion in this letter, we must establish the (11).

VI. APPENDIX - CORRECTION UTILIZING INDEFINITE METRIC

In the above context, indefinite metric vector is expressed in consideration of the point of light-cone coordinates and Dirac's delta function. Unfortunately (11) could not be satisfied with definite metric which suggest requirement of a special number such as imaginary number without complex conjugate. This kind of number corresponds to indefinite metric. When we directly introduce indefinite metric $\gamma^2 = -1$, the above discussion will be dramatically simplified as follows.

$$\psi(x) = \frac{1}{2}\gamma e^{i\phi/2} g(x) - \frac{1}{2}\gamma e^{-i\phi/2} g(x) \tag{14}$$

Then

$$\begin{aligned}\langle\psi|\psi\rangle &= \int -\frac{1}{4}g(x)^* g(x) - \frac{1}{4}g(x)^* g(x) \\ &\quad + \frac{1}{4}e^{i\phi} g(x)^* g(x) + \frac{1}{4}e^{-i\phi} g(x)^* g(x) dx \\ &= -\frac{1}{4}\langle g|g\rangle - \frac{1}{4}\langle g|g\rangle + \frac{1}{4}\langle g|g\rangle e^{i\phi} + \frac{1}{4}\langle g|g\rangle e^{-i\phi} \\ &= -\frac{1}{2}\langle g|g\rangle + \frac{1}{2}\langle g|g\rangle \cos\phi \end{aligned} \tag{15}$$

When $\phi = \pm N\pi$ (N : even number), $\langle\psi|\psi\rangle = 0$. Because ϕ will be correspond to a phase of the unobservable potentials. Then $\phi = 0$ and $\phi \neq 0$ will correspond to P $\oplus$ L and T respectively, which means ϕ can act as a switching parameter of the point of light-cone coordinates. By introducing γ, (2) is naturally satisfied.

We can easily calculate self interaction as described the above using this expression replacing $g(x)$ with $f(x)$ as follows.

$$\begin{aligned}\varphi(x) &\equiv f(x) + \frac{1}{2}\gamma e^{i\phi/2} f(x) - \frac{1}{2}\gamma e^{-i\phi/2} f(x) \\ \langle\varphi|\varphi\rangle &= \langle f|f\rangle - \frac{1}{2}\langle f|f\rangle + \frac{1}{2}\langle f|f\rangle \cos\phi \\ &= \frac{1}{2}\langle f|f\rangle + \frac{1}{2}\langle f|f\rangle \cos\phi \\ &= \langle f|f\rangle \left\{ \frac{1}{2} + \frac{1}{2}\cos\theta \right\} \end{aligned} \tag{16}$$

As can be seen from the simplified discussion, we should introduce indefinite metric from the beginning. Then we can leave intricate discussion using consideration of the point of light-cone coordinates and Dirac's delta function.

[1] M. Morimoto, "Unobservable Gauge Fields to Explain Single Photon and Electron Interference", http://vixra.org/abs/1312.0097, 2013

[2] M. Morimoto, "Unobservable Potentials to Explain a Quantum Eraser and a Delayed-Choice Experiment", http://vixra.org/abs/1405.0006, 2014