A wave function and quantum state vector in indefinite metric Minkowski space

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Indefinite metric vectors are absolutely required as the physical states in Minkowski space because that is indefinite metric space and the physical space-time. For example, Maxwell equations are wave equations in Minkowski space. However, traditional Quantum theory ordinarily has been studied only in definite metric space, i.e., Hilbert space. There are no clear expression for indefinite metric vectors. Here we show a wave function example using Dirac's delta function for indefinite metric vectors in Minkowski space. In addition, we show the vectors can interfere with itself.

I. INTRODUCTION

When we take advantage of non-observable potentials that can be identified as indefinite metric vectors, we can interpret single photon and electron interferences and entanglement without quantum-superposition.[1, 2]

First, we deal with the definition of metric space. Arbitrary state vectors $|\varphi\rangle$ and $|\psi\rangle$ satisfy following conditions in definite metric space.

$$\begin{aligned} \langle\varphi|\psi\rangle &= \langle\psi|\varphi\rangle^* \\ \langle\varphi|\varphi\rangle &\geq 0 \\ \langle\psi|\psi\rangle = 0 &\Leftrightarrow |\psi\rangle = 0 \end{aligned} \tag{1}$$

In contrast, the second and third relations are replaced with $\langle\varphi|\varphi\rangle = \alpha$ and $\langle\psi|\psi\rangle = 0 \Leftrightarrow |\psi\rangle \neq 0$ or 0 in indefinite metric space. Where α is an arbitrary number.

Minkowski space is divided into time-like (T), light-like (L), space-like (S) parts and point of the origin P ($\mathbf{x} = 0, t = 0$). Traditional quantum theory has been studied in T$\oplus$P. Where $\oplus$ stands for direct sum. However the non-observable potentials which can be identified as indefinite metric vectors propagate at the speed of light. Hence the vectors will be on the surface of the light cone, i.e., L. Of course photons in free space are on L. Because there exist some entity related to the non-observable potentials and the photons in free space on the light cone but can not be observe any entities, there must be the vector $|\psi\rangle_{\mathrm{L}} \neq 0$ with the norm $\langle\psi|\psi\rangle_{\mathrm{L}} = 0$. Although the vectors exist on the light cone, P does not belong to L, i.e., $|\psi\rangle_{\mathrm{P}} \neq 0 \Rightarrow \langle\psi|\psi\rangle_{\mathrm{P}} > 0$.

The space-like part S is estimated to have the rest of the characteristics, i.e., negative norm $\langle\psi|\psi\rangle_{\mathrm{S}} < 0$.

Therefore we should study the states in T$\oplus$P$\oplus$L as the physical states instead of (1). In this letter, we show a wave function example using Dirac's delta function for indefinite metric vectors in Minkowski space satisfied with the following conditions.

$$\begin{aligned} \langle\varphi|\psi\rangle_{\mathrm{T\oplus P\oplus L}} &= \langle\psi|\varphi\rangle^*_{\mathrm{T\oplus P\oplus L}} \\ \langle\psi|\psi\rangle_{\mathrm{T\oplus P}} &\geq 0 \\ \langle\psi|\psi\rangle_{\mathrm{T\oplus P\oplus L}} &\geq 0 \\ \langle\psi|\psi\rangle_{\mathrm{T\oplus P}} = 0 &\Leftrightarrow |\psi\rangle_{\mathrm{T\oplus P}} = 0 \\ \langle\psi|\psi\rangle_{\mathrm{L}} = 0 &\Leftrightarrow |\psi\rangle_{\mathrm{L}} \neq 0 \end{aligned} \tag{2}$$

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II. AN EXAMPLE EXPRESSION

Let $|\varphi\rangle$ is an observable physical state in T$\oplus$P, and $|\psi\rangle$ is an indefinite metric vector in L. Note that $|\psi\rangle$ can be observed in T$\oplus$P. These stats are expressed by wave functions $\varphi(x)$ and $\psi(x)$ as follows.

$$\begin{aligned} \langle\varphi|\varphi\rangle_{\mathrm{T\oplus P}} &= \int \varphi^*(x)\varphi(x)dx \geq 0 \\ \langle\psi|\psi\rangle_{\mathrm{L}} &= \int \psi^*(x)\psi(x)dx = 0 \\ \langle\psi|\psi\rangle_{\mathrm{T\oplus P}} &\geq 0 \\ \langle\varphi|\psi\rangle_{\mathrm{T\oplus P\oplus L}} &= \int \varphi^*(x)\psi(x)dx = \langle\psi|\varphi\rangle^*_{\mathrm{T\oplus P\oplus L}} \end{aligned} \tag{3}$$

We can easily confirm the following $\varphi(x)$ and $\psi(x)$ satisfy the above relations.

$$\begin{aligned} \varphi(x) &= f(x) = |f(x)|e^{-i\theta} \\ \psi(x) &= ae^{i\alpha x}\sqrt{\delta(x)} = |a|e^{i(\alpha x+\phi)}\sqrt{\delta(x)} \end{aligned} \tag{4}$$

where $f(x)$, $\delta(x)$, a and α are a traditional wave function, i.e., $\langle\varphi|\varphi\rangle_{\mathrm{T\oplus P}} = \langle f|f\rangle = \int |f(x)|^2 dx \geq 0$, Dirac's delta function, an arbitrary complex number and an arbitrary complex number of dimension $[x]^{-1}$ respectively.

The dimension of $\langle f|f\rangle = \int |f(x)|^2 dx$ and $\langle\psi|\psi\rangle = \int |\psi(x)|^2 dx$ are $[x]^0$ which means the dimensions of $f(x)$ and $\psi(x)$ are $[x]^{-\frac{1}{2}}$. Because the dimension of $\sqrt{\delta(x)}$ is $[x]^{-\frac{1}{2}}$, the expression of (4) using $\sqrt{\delta(x)}$ is valid from the point of view of dimension. From (4), followings can be calculated.

$$\begin{aligned} \langle\psi|\psi\rangle_{\mathrm{T\oplus P}} &= \int |a|^2 e^{-i(\alpha x+\phi)}e^{i(\alpha x+\phi)}\delta(x)dx = |a|^2 \geq 0 \\ \langle\psi|\psi\rangle_{\mathrm{T\oplus P}} &= 0 \Rightarrow |a| = 0 \Rightarrow \psi(x) = 0 \\ \langle\psi|\psi\rangle_{\mathrm{L}} &= \int |a|^2 e^{-i(\alpha\epsilon+\phi)}e^{i(\alpha\epsilon+\phi)}\delta(\epsilon)dx = 0 \\ \langle\psi|\psi\rangle_{\mathrm{L}} &= 0 \Leftrightarrow |a| \neq 0 \end{aligned} \tag{5}$$

where $\epsilon \neq 0$ ($\neq$ P).

$$\begin{aligned} \langle\varphi|\psi\rangle_{\mathrm{T\oplus P\oplus L}} &= \int \varphi^*(x)\psi(x)dx \\ &= \int af^*(x)e^{i\alpha x}\sqrt{\delta(x)}dx \\ &= \int |a||f(x)|e^{i(\alpha x+\theta+\phi)}\sqrt{\delta(x)}dx \end{aligned} \tag{6}$$

$$\begin{aligned}\langle\psi|\varphi\rangle_{\mathrm{T\oplus P\oplus L}} &= \int \psi^*(x)\varphi(x)dx \\ &= \int a^* f(x) e^{-i\alpha x}\sqrt{\delta(x)}dx \\ &= \int |a||f(x)| e^{-i(\alpha x+\theta+\phi)}\sqrt{\delta(x)}dx \quad (7)\end{aligned}$$

In order to clarify the characteristics of $\int\sqrt{\delta(x)}dx$, here we consider $\psi(x) = f(x)\sqrt{\delta(x)}$, $f(0) \neq 0$.

$$\begin{aligned}\int \psi^*(x)\psi(x)dx &= \int |f(x)|^2\sqrt{\delta(x)}\sqrt{\delta(x)}dx \\ &= \int |f(x)|^2\delta(x)dx \\ &= |f(0)|^2 \neq 0 \quad (8)\end{aligned}$$

Therefore $\int af(x)\sqrt{\delta(x)}dx$ is estimated to be $\neq 0$.

If

$$\int f(x)\sqrt{\delta(x)}dx = \beta f(0) \quad (9)$$

where β is a coefficient, (6) and (7) are calculated to be as follows.

$$\begin{aligned}\langle\varphi|\psi\rangle_{\mathrm{T\oplus P\oplus L}} &= \beta|a||f(0)|e^{i(\theta+\phi)} \\ \langle\psi|\varphi\rangle_{\mathrm{T\oplus P\oplus L}} &= \beta|a||f(0)|e^{-i(\theta+\phi)} \quad (10)\end{aligned}$$

These relations (5) and (10) correspond to (2) except $\langle\psi|\psi\rangle_{\mathrm{T\oplus P\oplus L}} \geq 0$.

Note that, we must establish the (9) in order to justify the results.

III. APPLICATION - SELF-INTERFERENCE

When we choose $\varphi(x) = f(x) + \psi(x)$ in (4), i.e., $|\varphi\rangle_{\mathrm{T\oplus P\oplus L}} = |f\rangle_{\mathrm{T\oplus P}} + |\psi\rangle_{\mathrm{L}}$, as a physical state, the norm is calculated to be as follows. Where $\langle f|f\rangle = \langle f|f\rangle_{\mathrm{T\oplus P}}$, $\langle\psi|f\rangle = \langle\psi|f\rangle_{\mathrm{T\oplus P\oplus L}}$, $\langle f|\psi\rangle = \langle f|\psi\rangle_{\mathrm{T\oplus P\oplus L}}$ and $\langle\psi|\psi\rangle = \langle\psi|\psi\rangle_{\mathrm{L}}$.

$$\begin{aligned}\langle\varphi|\varphi\rangle_{\mathrm{T\oplus P\oplus L}} &= \langle f|f\rangle + \langle\psi|f\rangle + \langle f|\psi\rangle + \langle\psi|\psi\rangle \\ &= \langle f|f\rangle + 2\beta|a||f(0)|\cos(\theta+\phi) \quad (11)\end{aligned}$$

When $|a| = \frac{\langle f|f\rangle}{2}$ and $\beta = \frac{1}{|f(0)|}$ then

$$\langle\varphi|\varphi\rangle_{\mathrm{T\oplus P\oplus L}} = 2\langle f|f\rangle\left\{\frac{1}{2} + \frac{1}{2}\cos(\theta+\phi)\right\} \quad (12)$$

Therefore if $|a| = \frac{\langle f|f\rangle}{2}$ and (9) is

$$\int f(x)\sqrt{\delta(x)}dx = \frac{f(0)}{|f(0)|} \quad (13)$$

then $\langle\varphi|\varphi\rangle_{\mathrm{T\oplus P\oplus L}} \geq 0$ can be satisfied.

When the maximum norm $\cos(\theta+\phi) = 1$ is normalized, i.e., $\langle\varphi|\varphi\rangle_{\mathrm{T\oplus P\oplus L}} = 1$, $\langle f|f\rangle = \frac{1}{2}$ will express self interference of $|\varphi\rangle_{\mathrm{T\oplus P\oplus L}}$. This relation

$$\langle\varphi|\varphi\rangle_{\mathrm{T\oplus P\oplus L}} = \frac{1}{2} + \frac{1}{2}\cos(\theta+\phi) \quad (14)$$

can describe the single photon and electron interferences.

IV. GENERALIZATION FOR VECTOR SPACE

Here we generalize the above discussion to vector space. Let's consider zero norm vectors set. The set becomes vector space $\mathfrak{V}_0$under following conditions.

A) For arbitrarily $|\gamma\rangle$, $|\gamma'\rangle \in \mathfrak{V}_0$, i.e., $\langle\gamma|\gamma\rangle = \langle\gamma'|\gamma'\rangle = 0$, $\mathfrak{V}_0$ is closed under multiplication, i.e., $|\gamma\rangle + |\gamma'\rangle \in \mathfrak{V}_0$ with associative law, existence of zero element, existence of inverse element and commutative law of addition.

B) For arbitrarily $|\gamma\rangle \in \mathfrak{V}_0$ and $\lambda \in F$ (field of scalars), $\mathfrak{V}_0$ is closed under multiplication of the scalar, i.e., $\lambda|\gamma\rangle \in \mathfrak{V}_0$ with associative law of scalar multiple, $1|\gamma\rangle = |\gamma\rangle$, distributive law of scalar and elements.

From A), the norm $|\gamma\rangle + |\gamma'\rangle$ is calculate to be $\langle\gamma|\gamma\rangle + \langle\gamma'|\gamma'\rangle + 2\mathrm{Re}\langle\gamma|\gamma'\rangle = 2\mathrm{Re}\langle\gamma|\gamma'\rangle = 0$ then $\langle\gamma|\gamma'\rangle = i\mu$, where μ is a real number, i.e., $\langle\gamma|\gamma'\rangle$ is a purely imaginary number.

From B), the norm $|\gamma\rangle + \lambda|\gamma'\rangle$ is calculate to be $\langle\gamma|\gamma\rangle + |\lambda|^2\langle\gamma'|\gamma'\rangle + 2\mathrm{Re}\lambda\langle\gamma|\gamma'\rangle = 2\mathrm{Re}\lambda\langle\gamma|\gamma'\rangle = 0$ then $\lambda\langle\gamma|\gamma'\rangle = i\mu'$, where μ' is a real number. Because $\langle\gamma|\gamma'\rangle$ is a purely imaginary number, then $\lambda \in F$ (field of scalars) must be a real number. Therefore, $\mathfrak{V}_0$ is a real vector space.

In contrast, a complex vector space $\mathfrak{H}_{phys}$ which is positive semidefinite space, has been studied as the physical states in traditional quantum theory.

However we should study the vectors formed by the direct sum $\mathfrak{V}_{phys} = \mathfrak{H}_{phys} \oplus \mathfrak{V}_0$ as the real physical states whose norms are positive semidefinite.

For example, the norm of

$$|\varphi\rangle(\in \mathfrak{V}_{phys}) = |f\rangle(\in \mathfrak{H}_{phys}) + |\gamma\rangle(\in \mathfrak{H}_0) \quad (15)$$

is calculated to be

$$\begin{aligned}\langle\varphi|\varphi\rangle &= \langle f|f\rangle + \langle\gamma|f\rangle + \langle f|\gamma\rangle + \langle\gamma|\gamma\rangle \\ &= \langle f|f\rangle + \langle\gamma|f\rangle + \langle f|\gamma\rangle \quad (16)\end{aligned}$$

Therefore $\langle\gamma|f\rangle$ should be $\frac{\langle f|f\rangle}{2}e^{i\theta}$ because of the positive semidefinite norm $\langle\varphi|\varphi\rangle = 2\langle f|f\rangle\left\{\frac{1}{2} + \frac{1}{2}\cos\theta\right\} \geq 0$.

V. CONCLUSION

Traditional quantum theory has been studied by using $|\psi\rangle_{\mathrm{T\oplus P}}$. However we should study $|\psi\rangle_{\mathrm{T\oplus P\oplus L}}$ as the physical states instead of $|\psi\rangle_{\mathrm{T\oplus P}}$. In order to justify the above example expression, we must establish the (9) or (13).

[1] M. Morimoto, "New Insight of Single Photon and Electron Interference", http://vixra.org/abs/1312.0097, 2013
[2] M. Morimoto, "Review of Quantum Eraser", http://vixra.org/abs/1405.0006, 2014