A wave function and quantum state vector in indefinite metric Minkowski space

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Indefinite metric vectors are absolutely required as the physical states in Minkowski space because that is indefinite metric space and the physical space-time. For example, Maxwell equations are wave equations in Minkowsky space. However, traditional Quantum theory ordinarily has been studied only in definite metric space, i.e., Hilbert space. There are no clear expression for indefinite metric vectors. Here we show a wave function example using Dirac's delta function for indefinite metric vectors in Minkowski space. In addition, we show the vectors can interfere with itself.

I. INTRODUCTION

When we take advantage of non-observable potentials that can be identified as indefinite metric vectors, we can interpret single photon and electron interferences and entanglement without quantum-superposition.[1, 2]

First, we deal with the definition of metric space. Arbitrary state vectors $|\varphi\rangle$ and $|\psi\rangle$ satisfy following conditions in definite metric space.

$$\begin{aligned} \langle\varphi|\psi\rangle &= \langle\psi|\varphi\rangle^* \\ \langle\varphi|\varphi\rangle &\geq 0 \\ \langle\psi|\psi\rangle = 0 &\Leftrightarrow |\psi\rangle = 0 \end{aligned} \tag{1}$$

In contrast, the second and third relations are replaced with $\langle\varphi|\varphi\rangle = \alpha$ and $\langle\psi|\psi\rangle = 0 \Leftrightarrow |\psi\rangle \neq 0$ or 0 in indefinite metric space. Where α is an arbitrary number.

Minkowski space is divided into time-like (T), light-like (L), space-like (S) parts and point of the origin P ($\mathbf{x} = 0, t = 0$). Traditional quantum theory has been studied in T$\oplus$P. Where $\oplus$ stands for direct sum. However the non-observable potentials which can be identified as indefinite metric vectors propagate at the speed of light. Hence the vectors will be on the surface of the light cone, i.e., L. Of course photons in free space are on L. Because there exist some entity related to the non-observable potentials and the photons in free space on the light cone but can not be observe any entities, there must be the vector $|\psi\rangle_{\rm L} \neq 0$ with the norm $\langle\psi|\psi\rangle_{\rm L} = 0$. Although the vectors exist on the light cone, P does not belong to L, i.e., $|\psi\rangle_{\rm P} \neq 0 \Rightarrow \langle\psi|\psi\rangle_{\rm P} > 0$.

The space-like part S is estimated to have the rest of the characteristics, i.e., negative norm $\langle\psi|\psi\rangle_{\rm S} < 0$.

Therefore we should study the states in T$\oplus$P$\oplus$L as the physical states instead of (1). In this letter, we show a wave function example using Dirac's delta function for indefinite metric vectors in Minkowski space satisfied with the following conditions.

$$\begin{aligned} \langle\varphi|\psi\rangle_{\rm T\oplus P\oplus L} &= \langle\psi|\varphi\rangle^*_{\rm T\oplus P\oplus L} \\ \langle\psi|\psi\rangle_{\rm T\oplus P} &\geq 0 \\ \langle\psi|\psi\rangle_{\rm T\oplus P\oplus L} &\geq 0 \\ \langle\psi|\psi\rangle_{\rm T\oplus P} = 0 &\Leftrightarrow |\psi\rangle_{\rm T\oplus P} = 0 \\ \langle\psi|\psi\rangle_{\rm L} = 0 &\Leftrightarrow |\psi\rangle_{\rm L} \neq 0 \end{aligned} \tag{2}$$

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II. AN EXAMPLE EXPRESSION

Let $|\varphi\rangle$ is an observable physical state in T$\oplus$P, and $|\psi\rangle$ is an indefinite metric vector in L. Note that $|\psi\rangle$ can be observed in T$\oplus$P. These stats are expressed by wave functions $\varphi(x)$ and $\psi(x)$ as follows.

$$\begin{aligned} \langle\varphi|\varphi\rangle_{\rm T\oplus P} &= \int \varphi^*(x)\varphi(x)dx \geq 0 \\ \langle\psi|\psi\rangle_{\rm L} &= \int \psi^*(x)\psi(x)dx = 0 \\ \langle\psi|\psi\rangle_{\rm T\oplus P} &\geq 0 \\ \langle\varphi|\psi\rangle_{\rm T\oplus P\oplus L} &= \int \varphi^*(x)\psi(x)dx = \langle\psi|\varphi\rangle^*_{\rm T\oplus P\oplus L} \end{aligned} \tag{3}$$

We can easily confirm the following $\varphi(x)$ and $\psi(x)$ satisfy the above relations.

$$\begin{aligned} \varphi(x) &= f(x) = |f(x)|e^{-i\theta} \\ \psi(x) &= ae^{i\alpha x}\sqrt{\delta(x)} = |a|e^{i(\alpha x+\phi)}\sqrt{\delta(x)} \end{aligned} \tag{4}$$

where $f(x)$, $\delta(x)$, a and α are a traditional wave function, i.e., $\langle\varphi|\varphi\rangle_{\rm T\oplus P} = \langle f|f\rangle = \int |f(x)|^2dx \geq 0$, Dirac's delta function, an arbitrary complex number and an arbitrary complex number of dimension $[x]^{-1}$ respectively.

The dimension of $\langle f|f\rangle = \int |f(x)|^2dx$ and $\langle\psi|\psi\rangle = \int |\psi(x)|^2dx$ are $[x]^0$ which means the dimensions of $f(x)$ and $\psi(x)$ are $[x]^{-\frac{1}{2}}$. Because the dimension of $\sqrt{\delta(x)}$ is $[x]^{-\frac{1}{2}}$, the expression of (4) using $\sqrt{\delta(x)}$ is valid from the point of view of dimension. From (4), followings can be calculated.

$$\begin{aligned} \langle\psi|\psi\rangle_{\rm T\oplus P} &= \int |a|^2e^{-i(\alpha x+\phi)}e^{i(\alpha x+\phi)}\delta(x)dx = |a|^2 \geq 0 \\ \langle\psi|\psi\rangle_{\rm T\oplus P} &= 0 \Rightarrow |a| = 0 \Rightarrow \psi(x) = 0 \\ \langle\psi|\psi\rangle_{\rm L} &= \int |a|^2e^{-i(\epsilon+\phi)}e^{i(\epsilon+\phi)}\delta(\epsilon)dx = 0 \\ \langle\psi|\psi\rangle_{\rm L} &= 0 \Leftrightarrow |a| \neq 0 \end{aligned} \tag{5}$$

where $\epsilon \neq 0$ ($\neq$ P).

$$\begin{aligned} \langle\varphi|\psi\rangle_{\rm T\oplus P\oplus L} &= \int \varphi^*(x)\psi(x)dx \\ &= \int af^*(x)e^{i\alpha x}\sqrt{\delta(x)}dx \\ &= \int |a||f(x)|e^{i(\alpha x+\theta+\phi)}\sqrt{\delta(x)}dx \end{aligned} \tag{6}$$

$$\begin{aligned}\langle\psi|\varphi\rangle_{\mathrm{T\oplus P\oplus L}} &= \int \psi^*(x)\varphi(x)dx \\ &= \int a^* f(x)e^{-i\alpha x}\sqrt{\delta(x)}dx \\ &= \int |a||f(x)|e^{-i(\alpha x+\theta+\phi)}\sqrt{\delta(x)}dx \end{aligned} \quad (7)$$

In order to clarify the characteristics of $\int\sqrt{\delta(x)}dx$, here we consider $\psi(x) = f(x)\sqrt{\delta(x)}$, $f(0) \neq 0$.

$$\begin{aligned}\int \psi^*(x)\psi(x)dx &= \int |f(x)|^2\sqrt{\delta(x)}\sqrt{\delta(x)}dx \\ &= \int |f(x)|^2\delta(x)dx \\ &= |f(0)|^2 \neq 0 \end{aligned} \quad (8)$$

Therefore $\int af(x)\sqrt{\delta(x)}dx$ is estimated to be $\neq 0$.

If

$$\int f(x)\sqrt{\delta(x)}dx = \beta f(0) \quad (9)$$

where β is an arbitrary number, (6) and (7) are calculated to be as follows.

$$\begin{aligned}\langle\varphi|\psi\rangle_{\mathrm{T\oplus P\oplus L}} &= \beta|a||f(0)|e^{i(\theta+\phi)} \\ \langle\psi|\varphi\rangle_{\mathrm{T\oplus P\oplus L}} &= \beta|a||f(0)|e^{-i(\theta+\phi)} \end{aligned} \quad (10)$$

These relations (5) and (10) correspond to (2) except $\langle\psi|\psi\rangle_{\mathrm{T\oplus P\oplus L}} \geq 0$.

Note that, we must establish the (9) in order to justify the results.

III. APPLICATION - SELF-INTERFERENCE

When we choose $\varphi(x) = f(x) + \psi(x)$ in (4), i.e., $|\varphi\rangle_{\mathrm{T\oplus P\oplus L}}$, as a physical state, the norm is calculated to be as follows.

$$\begin{aligned}&\langle\varphi|\varphi\rangle_{\mathrm{T\oplus P\oplus L}} = \langle f|f\rangle + \langle\psi|f\rangle + \langle f|\psi\rangle + \langle\psi|\psi\rangle \\ &= \int |f(x)|^2dx \\ &+ \int |a||f(x)|\left\{e^{i(\alpha x+\theta+\phi)} + e^{-i(\alpha x+\theta+\phi)}\right\}\sqrt{\delta(x)}dx \\ &+ \int |a|^2\delta(x)dx \\ &= \langle f|f\rangle + 2\beta|a||f(0)|\cos(\theta+\phi) + |a|^2 \end{aligned} \quad (11)$$

When $|a| = \sqrt{\langle f|f\rangle}$ and $\beta = \frac{\sqrt{\langle f|f\rangle}}{|f(0)|}$ then

$$\langle\varphi|\varphi\rangle_{\mathrm{T\oplus P\oplus L}} = 4\langle f|f\rangle\left\{\frac{1}{2} + \frac{1}{2}\cos(\theta+\phi)\right\} \quad (12)$$

Therefore if (9) is

$$\int f(x)\sqrt{\delta(x)}dx = \frac{\sqrt{\langle f|f\rangle}}{|f(0)|}f(0) \quad (13)$$

then $\langle\varphi|\varphi\rangle_{\mathrm{T\oplus P\oplus L}} \geq 0$ can be satisfied by taking $|a| = \sqrt{\langle f|f\rangle}$.

When the maximum norm $\cos(\theta+\phi) = 1$ is normalized, i.e., $\langle\varphi|\varphi\rangle_{\mathrm{T\oplus P\oplus L}} = 1$, $\langle f|f\rangle = \frac{1}{4}$ will express self interference of $|\varphi\rangle_{\mathrm{T\oplus P\oplus L}}$. This relation

$$\langle\varphi|\varphi\rangle_{\mathrm{T\oplus P\oplus L}} = \frac{1}{2} + \frac{1}{2}\cos(\theta+\phi) \quad (14)$$

can describe the single photon and electron interferences.

IV. CONCLUSION

Traditional quantum theory has been studied by using $|\psi\rangle_{\mathrm{T\oplus P}}$. However we should study $|\psi\rangle_{\mathrm{T\oplus P\oplus L}}$ as the physical states instead of $|\psi\rangle_{\mathrm{T\oplus P}}$. In order to justify the above discussion, we must establish the (9) or (13).

[1] M. Morimoto, "New Insight of Single Photon and Electron Interference", http://vixra.org/abs/1312.0097, 2013

[2] M. Morimoto, "Review of Quantum Eraser", http://vixra.org/abs/1405.0006, 2014