Two types of pairs of primes that could be associated to Poulet numbers

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Abstract. In this paper I combine two of my objects of study, the Poulet numbers and the different types of pairs of primes and I state two conjectures about few ways in which types of Poulet numbers could be associated with types of pairs of primes.

Conjecture 1:

Any Poulet number of the form 10*n + 1 or 10*n + 9 can be written at least in one way as p*q + 10*k*h, where p and q are primes or powers of primes of the same form from the following four ones: 10*m + 1, 10*m + 3, 10*m + 7 or 10*m + 9, k and h are non-null positive integers and q – p = 10*k.

Verifying the conjecture:
(for the first six such Poulet numbers)

: 341 = 9*(9 + 20) + 4*20 = 9*(9 + 10) + 17*10, so [p, q] = [3^2, 29] or [3^2, 19];
: 561 = 19*(29 + 10) + 1*10 = 9*(9 + 50) + 3*10, so [p, q] = [19, 29] or [3^2, 59];
: 1729 = 23*(23 + 50) + 1*50 = 17*(17 + 80) + 1*80 = 23*(23 + 30) + 17*30 = 27*(27 + 10) + 73*10 = 23*(23 + 20) + 37*20 = 13*(13 + 60) + 13*60 = 7*(7 + 120) + 7*120 = 17*(17 + 30) + 31*30 = 13*(13 + 40) + 26*40, so [p, q] = [23, 73] or [17, 97] or [23, 53] or [3^3, 37] or [23, 43] or [13, 73] or [7, 127] or [17, 47] etc.;
: 2701 = 29*(29 + 60) + 2*60, so [p, q] = [29, 89] etc.;
: 2821 = 29*(29 + 60) + 4*60, so [p, q] = [29, 89] etc.;
: 4369 = 27*(27 + 130) + 1*130, so [p, q] = [3^3, 157] etc.

Note 1:

Some such Poulet numbers can be written as p*q + (q – p), where p, q primes; for instance, the Hardy-Ramanujan number 1729 can be written in two different ways like this: 1729 = 23*53 + (53 – 23) = 17*97 + (97 – 17).

Note 2:

Probably this conjecture can stipulate for h to be equal to 1 or prime or power of prime (in the examples above, we found that h is equal to: 2^2 or 17; 1 or 3; 1 or 17 or 73 or 37 or 13 or 7 or 31; 2; 2^2; 1.

Conjecture 2:

For any Poulet number N not divisible by 3 there exist at least a pair of numbers [p, q], where p is prime and q is prime or square of prime, such that $N = p^2 + q - 1$.

Verifying the conjecture:
(for the first six such Poulet numbers)

: 341 = 7^2 + 293 – 1 = 13^2 + 173 – 1 = 17^2 + 53 – 1, so [p, q] = [7, 293] or [13, 173] or [17, 53];
: 1105 = 13^2 + 937 – 1 = 23^2 + 577 – 1, so [p, q] = [13, 937] or [23, 577];
: 1387 = 23^2 + 859 – 1 = 29^2 + 547 – 1 = 37^2 + 19 – 1, so [p, q] = [23, 859] or [29, 547] or [37, 19];
: 1729 = 7^2 + 41^2 – 1 = 11^2 + 1609 – 1 = 19^2 + 37^2 – 1 = 23^2 + 1201 – 1 = 31^2 + 769 – 1, so [p, q] = [7, 41^2] or [41, 7^2] or [11, 1609] or [19, 37^2] or [37, 19^2] or [23, 1201] or [31, 769].

Note:

Some such Poulet numbers can be written as p^2 + q^2 – 1, where p, q are primes; for instance, the Hardy-Ramanujan number 1729 can be written in two different ways like this: 1729 = 7^2 + 41^2 - 1 = 19^2 + 37^2 - 1.