Proof of Beal's Conjecture

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Abstract : Using a functional equation and different proofs for its existence, we are able to prove and show that A,B and C will always have a common prime factor.

Introduction :

The Beal Conjecture

Let A, B, C, x, y, and z be positive integers with x, y, z > 2. If $A^x + B^y = C^z$, then A, B, and C have a common factor. [1]

Let A, B, C, x, y, and z be positive integers with x, y, z > 2. Then the equation

$$A^x + B^y = C^z$$

follows

$$A^x = p^n u = pu$$
$$B^y = p^n v = pv$$
$$C^z = p^n (u + v) = p (u + v)$$

Wherein p represents the factor (p^n or just p to clearly represent the prime factor) and u and v are positive integers.

Direct Proof

$$A^x + B^y = C^z$$
$$pu + pv = p (u + v)$$

Simplifying Left Hand Side

$$C^z = C^z$$
$$pu + pv = p (u + v)$$
$$p (u + v)= p (u + v)$$

Simplifying Right Hand Side

$$A^x + B^y = A^x + B^y$$
$$pu + pv = p (u + v)$$
$$pu + pv = pu + pv$$

Thus the equality holds.

Proof by Induction

Let p = u = v = 2;

$$pu + pv = p (u + v)$$
$$2.2 + 2.2 = 2(2 + 2)$$

$4 + 4 = 4 + 4$
$8 = 8$

Thus equality holds.

Let $p = p_1 + 1$, $u = u_1 + 1$ and $v = v_1 + 1$

$(p_1+1)(u_1+1) + (p_1+1)(v_1+1) = (p_1+1)((u_1+1) + (v_1+1))$
$(p_1u_1+ u_1 + p_1 + 1) + (p_1v_1+ v_1 + p_1 + 1) = (p_1+1)((u_1+1) + (v_1+1))$
$(p_1u_1+ u_1 + p_1 + 1) + (p_1v_1+ v_1 + p_1 + 1) = (p_1u_1+ p_1) + (p_1 v_1+ p_1) + (u_1+1) + (v_1+1)$
$p_1u_1+ p_1v_1+ u_1 + v_1 + 2p_1 +2 = p_1u_1+ p_1 v_1 + u_1+ v_1+ 2p_1+2$

Thus equality holds.

To visualize our induction with exponents, we raise our prime number *p* to *n* power. Let $p = u = v = 2$; $n = 3$

$p^nu + p^nv = p^n (u + v)$
$(2^3 * 2) + (2^3 * 2) = 2^3 (2 + 2)$
$2^4 + 2^4 = 2^4 + 2^4$
$2^5 = 2^5$

Thus equality holds.

Proof by Contradiction and Example

Assume that our equation is wrong and that the equation will not hold further with different bases and exponents.

$7^3 + 7^4 = 14^3$
$14^3 = 7^3 * 2^3 = 7^3 * 8 = 7^3 (1 + 7) = 7^3 * 1 + 7^3 * 7$
$14^3 = 7^3 + 7^4$
$7^3 + 7^4 = 7^3 + 7^4$
$4^3 = 14^3$

Then our assumption is false and still the equality holds.

Conclusion:

We have clearly shown using the equation in different proofs that the equation holds its equality with a common prime factor. Since A, B and C are positive integers, and have a common prime factor, as stated previously, therefore we can conclude that the conjecture is true.

References :

[1] R. Daniel Muldin, ***A Generalization of Fermat's Last Theorem: The Beal Conjecture and Prize Problem***, Notices of the AMS Volume 44 Number 11, 1436.