Black Holes Have No Interior Singularities

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Abstract
The paper describes a simple solution based on a relativistic extension of Newton-Galileo physics to the purely gravitational, spherical supermassive black hole. The solution yields a black hole size that equals the Schwarzschild radius, but without an interior singularity. For a supermassive black hole residing at the center of a galaxy, the theory yields a simple expression for the dynamics of the host galaxy, according to which the black hole is part of a binary system, together with a naked spatial singularity at redshift \( z = 2^{-\frac{1}{2}} \approx 0.707 \), suspected to be a quasar with extreme velocity offsets or an active galactic nucleus. Another redshift, \( z \approx 2.078 \), is also predicted to be associated with quasars and AGNs. The derived results are contrasted with observational data and with a recent ΛCDM model.

Taken together, the derived galaxy dynamics, and the aforementioned results, could shed some light on the role played by supermassive black holes in the evolution of the galaxies in which they reside.

Keywords: Black hole, Singularity, Schwarzschild radius, Relativity, Binary system, Galaxy.

1. Black Holes - A Brief History
The term “black hole” was coined by John Wheeler in 1964, but the possibility of its existence within the framework of Newtonian physics was conjectured by John Michell in 1784, who argued for the possible existence of an object massive enough to have an escape velocity greater than the velocity of light [1]. Twelve years later, Simon Pierre LaPlace also predicted the existence of black holes. Laplace argued that “It is therefore possible that the largest luminous bodies in the universe may, through this cause, be invisible” [2].
A better understanding of black holes, and how gravity and waves intermingle, had to wait until 1915, when Albert Einstein delivered a lecture on his theory of General Relativity (GR) to the German Academy of Science in Berlin. Within a month of the publication of Einstein’s work, Karl Schwarzschild, while serving in the German Army on the Russian front, solved Einstein’s field equations for a non-rotating, uncharged, spherical black hole [3, 4]. For a star of a given mass, M, Schwarzschild found the critical radius \( R = \frac{2GM}{c^2} \), where \( G \) is the gravitational constant and \( c \) is the velocity of light, at which light emitted from the surface would have an infinite gravitational redshift, and thereby infinite time dilation. Such a star, Schwarzschild concluded, would be undetectable by an external observer at any distance from the star.

Our understanding of the processes involved in the evolution and decay of black holes is largely due to quantum mechanical and thermodynamic theories. Early in 1974, Stephen Hawking predicted that a black hole should radiate like a hot, non-black (“gray”) body [5]. Hawking’s theory of black holes, is consistent with Bekenstein's generalized second law of thermodynamics [6], stating that the sum of the black-hole entropy and the ordinary thermal entropy outside the black hole cannot decrease. According to this prediction, black holes should have a finite, non-zero, and non-decreasing temperature and entropy.

The first X-ray source, widely accepted to be a black hole, was Cygnus X-1 [7]. Since 1994, The Hubble Space Telescope, and other space-crafts and extremely large ground telescopes [see, e.g., 8, 9], have detected numerous black holes of different sizes and redshifts. We now know that black holes exist in two mass ranges: small ones of \( (M \lesssim 10 M_\odot) \) \( (M_\odot, \text{ solar mass}) \), believed to be the evolutionary end points of the gravitational collapse of massive stars, and supermassive black holes of \( M \gtrsim 10^6 M_\odot \), responsible for the powering of quasars and active galactic nuclei (AGN) [10, 12]. Supermassive black holes, residing at the centers of most galaxies, are believed to be intimately related to the formation and evolution of their galaxies [10- 14].
2. Pathology and Previous Solutions

As mentioned above, the solution to Einstein’s field equations \([3, 4]\) yields a critical hole radius of \(R = \frac{2GM}{c^2}\). However, Schwarzschild’s solution suffers from a serious pathology, because it predicts a singularity whereby the fabric of spacetime is torn, causing all matter and radiation passing the event horizon to be ejected out to an undefined spacetime, leaving the black hole empty, thus, in violation of the laws of thermodynamics and contradiction with quantum mechanics [e.g., 15-16]. Many believe that the black holes (and the Big Bang) singularities mark a breakdown in GR.

Attempts to solve the singularity problem are aplenty. Bardeen was the first to propose a regular black hole model [17]. In 1968, he produced a famous model, conventionally interpreted as a counterexample to the possibility that the existence of singularities may be proved in black hole spacetimes without assuming either a global Cauchy hyper-surface or the strong energy condition. Other regular “Bardeen black holes” models have been also proposed [e.g., 18-23], but none of these models is an exact solution to Einstein equations [24]. Other solutions to produce singularity-free black hole come from string theory [e.g., 25, 26], and quantum mechanics [e.g., 27-31]. As examples, Ashtekar and others [27-28] proposed a loop quantum gravity model that avoids the singularities of black holes and the Big Bang. Their strategy was to utilize a regime that keeps GR intact, except at the singularity point, at which the classical spacetime is bridged by a discrete quantum one. Although the solution is mathematically difficult, its strategy is simple. It begins with semi-classical state at large late times (“now”), and evolves it back in time, while keeping it semi-classical until one encounters the deep Planck regime near the classical singularity. In this regime, it allows the quantum geometry effects to dominate. As the state becomes semi-classical again on the other side, the deep Planck region serves as a quantum bridge between two large, classical spacetimes [27].

3. The Proposed Solution

Here I propose another solution to the spherical supermassive, gravitational black hole. The solution is based on a new relativity theory, termed Newtonian Relativity (NR). First, I present the theory.
Then I utilize it to derive a dynamical equation for a typical galaxy with a supermassive black hole (e.g., the Milky Way), and a solution to the black hole radius. The resulting radius turns out to be equal to the Schwarzschild radius \( R = \frac{2GM}{c^2} \), but *with no singularity at the interior*. Moreover, the proposed solution predicts that supermassive black holes, residing at the center of galaxies, are part of binary systems, with naked singularities at redshift \( z = 2^{-\frac{1}{2}} \approx 0.707 \), suspected to be quasars with extreme velocity offsets or active galactic nuclei (AGNs).

### 4. Theory

The theory, detailed elsewhere [32], is termed Newtonian Relativity. It constitutes a straightforward extension of Galileo-Newton mechanics to the domain of relativistic velocities. The theory has no postulates, except the well accepted principle that sufficiently low (non-relativistic) velocities, the laws of physics in all internal frames reduce to the classical Galileo-Newton mechanics. For cosmological applications I also assume that information regarding physical entities is translated from one frame of reference to another via light and electromagnetic waves of equal velocity.

The requirement that at low velocities the laws of physics are classical Galileo-Newton laws has a profound implication on the strategy used in the proposed theory. Not only must we expect that all laws should converge at low velocities to the laws of classical mechanics; we must also require that any relativistic effect should be *uniquely* a function of relative velocities. A major advantage of Newtonian relativity lies in the fact that it maintains a smooth and natural continuity between relativistic and classical (non-relativistic) physics.

It is worth noting that the assumption that information from one frame of reference to another is translated by light or other electromagnetic waves is motivated by practical considerations, not by theoretical necessity. In fact, the proposed theory could be applied to any information carrier. The only theoretical requirement is that the velocity of the carrier is isotropic with regard to the observer’s frame. Nonetheless, for a theory of cosmology, light and other electromagnetic waves are the only practical choice. An immediate consequence of the above assumption is that the proposed
theory is limited to the observable universe. Like in General Relativity, Newtonian Relativity does not require that the recession velocities of cosmological objects, relative to an observer on Earth, do not exceed the velocity of light. The theory allows superluminal velocities, but information emitted from them will never reach an observer on Earth.

To derive the term for the Newtonian relativistic time, consider two observers who synchronize their watches just before one of them starts to move in +x direction with constant velocity \( v \). Assume that a certain event started exactly at the time of departure \( t = t' = 0 \). Suppose the event ended when the moving frame was at distance \( x = d \) (in the rest frame of the “staying” observer). If the “moving” observer sends a signal to indicate the termination of the event, the signal will arrive at the “staying” observer after time dilation of \( \Delta t = \frac{d}{c} \), where \( c \) is the velocity of the wave signal relative to “staying” observer. Thus we can write:

\[
t = t' + \Delta t = t' + \frac{d}{c}
\]

\[\text{…… (1)}\]

But \( d = v t \), where \( v \) is the velocity of the “moving” frame relative to the “stationary” frame. Substitution the value of \( d \) in Eq. 1 yields:

\[
t = t' + \frac{v t}{c} = t' + \beta t
\]

\[\text{…… (2)}\]

Where \( \beta = \frac{v}{c} \).

Or:

\[
\frac{t}{t'} = \frac{1}{1-\beta}
\]

\[\text{…… (3)}\]

Note that eq. (3) is similar to the Doppler formula, except that the Doppler Effect describes frequency shifts of waves propagating from a departing or approaching wave source, whereas the result above describes the time “shifts” of moving bodies. For two frames that depart from each other \( \beta > 0 \), and
thus \( \frac{1}{1-\beta} \) is larger than one, implying a *time dilation* (comparable to redshift), whereas for two frames which approach each other, \( \beta < 0 \), and thus \( \frac{1}{1-\beta} \) is smaller than one, implying a *time contraction* (comparable to blue-shift). In a cosmology of the evolution of the universe, generally only positive \( \beta \) values are encountered. For cosmological applications of the theory, the relationship between the Doppler formula and Eq. 3 is not metaphoric. In [32], I show that the velocity \( \beta \) could be expressed in terms of the redshift \( z \) as:

\[
\beta = \frac{z}{1+z} \quad \ldots (4)
\]

Derivations of the distance, mass density, and energy transformations are also detailed in [32]. Table 1 summarizes these transformations in terms of velocity \( \beta \) (first column) and redshift (second column).

### Table 1

**Transformations**

<table>
<thead>
<tr>
<th>Physical Term</th>
<th>Relativistic Expression</th>
<th>In Velocity</th>
<th>In Redshift</th>
</tr>
</thead>
<tbody>
<tr>
<td>Time (sec)</td>
<td>( \frac{t}{t'} = \frac{1}{1-\beta} )</td>
<td>( z + 1 )</td>
<td>(5)</td>
</tr>
<tr>
<td>Time (round trip)</td>
<td>( \frac{t}{t'} = \frac{2}{1-\beta^2} )</td>
<td>( \frac{2(z+1)^2}{2z+1} )</td>
<td>(6)</td>
</tr>
<tr>
<td>Distance (m)</td>
<td>( \frac{x}{x'} = \frac{1+\beta}{1-\beta} )</td>
<td>( 2z + 1 )</td>
<td>(7)</td>
</tr>
<tr>
<td>Mass density (kg/m(^3))</td>
<td>( \frac{\rho}{\rho_0} = \frac{1-\beta}{1+\beta} )</td>
<td>( \frac{1}{2z+1} )</td>
<td>(8)</td>
</tr>
<tr>
<td>Kinetic energy density ( e_k )</td>
<td>( \frac{1}{2} \rho_0 c^2 \frac{1-\beta}{1+\beta} \beta^2 )</td>
<td>( \frac{1}{2} \rho_0 c^2 \frac{z^2}{(z+1)^2(2z+1)} )</td>
<td>(9)</td>
</tr>
</tbody>
</table>
5. Black holes in Newtonian Relativity

Figure 1 depicts a schematic representation of a supermassive black hole with mass $M$ and radius $R$ residing at the center of its host galaxy.

![Diagram of particles near a black hole](image)

**Figure 1.** Three particles near a black hole

The figure shows three particles, with equal masses and velocities, at different distances from the center of the black hole. As depicted in the figure, the more distant particle will be deflected toward the black hole, but will escape it due to its large distance, and continue its travel in space. By contrast, the closest particle to the black hole will experience a strong enough gravitational force to cause its absorption into the black hole. Now consider the third particle, which rotates around the black hole at radius $r$. Such particle could be a baryon or wave quanta entrapped at a critical distance, ensuring that it rotates around the black hole. For such particle, the acceleration $|\vec{a}|$ supporting a uniform radial motion with radius $r$ should satisfy

$$a = |\vec{a}| = \frac{v^2}{r} = \frac{c^2}{r} \beta^2$$

... (10)

The force supporting such motion, according to Newton's second law, could be expressed as:

$$F = \frac{\partial P}{\partial t} = \frac{\partial (mv)}{\partial t} = m \frac{\partial (v)}{\partial t} + v \frac{\partial (m)}{\partial t}$$

$$= m a + v \frac{\partial (m)}{\partial v} \frac{\partial (v)}{\partial t} = m a + v a \frac{\partial (m)}{\partial v} = (m + v \frac{\partial (m)}{\partial v}) a$$

... (11)
Substitution the term for \( m \) from Eq. 8 (see Table 1), and deriving \( m \) with respect to \( v \) yields:

\[
F = \frac{1-2\beta - \beta^2}{(1+\beta)^2} m_0 a
\]  
....(12)

Substitution the value of \( a \), from Eq. 12 in Eq. 10 yields:

\[
F = \frac{1-2\beta - \beta^2}{(1+\beta)^2} m_0 a = \frac{1-2\beta - \beta^2}{(1+\beta)^2} m_0 \frac{v^2}{r} = m_0 c^2 \frac{1-2\beta - \beta^2}{(1+\beta)^2} \beta^2 \frac{1}{r}
\]  
.... (13)

Using Newton’s general law of gravitation, we get:

\[
G \frac{m_0 M}{r^2} = m_0 c^2 \frac{1-2\beta - \beta^2}{(1+\beta)^2} \beta^2 \frac{1}{r}
\]  
.... (14)

Solving for \( r \) yields:

\[
r = \frac{G M}{c^2} \frac{(1+\beta)^2}{1-2\beta - \beta^2} \beta^2
\]  
..... (15)

Assuming spherical symmetry, eq. 15 describes the dynamics of the host galaxy as a function of velocity. For a light photon \((\beta = 1)\), we have:

\[
r ((\beta = 1)) = R = \frac{2 G M}{c^2}
\]  
..... (16)

Which exactly equals the Schwarzschild radius, but with no singularity in the hole’s interior.

Interestingly, the solution (eq. 15) has a naked spatial singularity at \( \beta \) satisfying:

\[
1 - 2\beta - \beta^2 = 0
\]  
.... (17)

\textbf{Solving for } \beta, \textbf{ we have:}

\[\beta \approx 0.4142\]

..... (18)

With corresponding redshift of \( z = \frac{\beta}{1-\beta} = \frac{1}{\sqrt{2}} \approx 0.707 \).
It is important to stress that the predicted singularity is in space and not in spacetime, as prescribed by the Schwarzschild's solution of General Relativity's field equations. In fact, Newtonian Relativity in general, including in its present application to the black hole problem, does not require reference to the notion of spacetime. As explained in section 4, the theory is a straightforward relativistic extension of Galileo-Newton's physics, and as such, it treats space and time independently of each other.

To express the derived radius in terms of redshift, we substitute the value of $\beta$ from Eq. 3 in Eq. 15 and solve for $r$, yielding:

$$
    r = \left(\frac{GM}{c^2}\right) \frac{z^2(1+2z)^2}{(1+z)^2(1-2z^2)}
$$

Figure 2 depicts the ratio $r$, normalized by $\frac{GM}{c^2}$, as a function of $z$.

![Figure 2](image)

Figure 2. $r/\left(\frac{GM}{c^2}\right)$ as a function of redshift

As shown by the figure, for very high redshifts $r$ converges to $2\frac{GM}{c^2}$ (the Schwarzschild radius).

Moreover, the result in Eq. 19 has some interesting properties. (1) $r$ has a naked *spatial singularity*,
at \( z = \frac{1}{\sqrt{2}} \approx 0.707 \), (2) It displays a striking Golden Ratio symmetry, such that for \( z = \varphi \approx 1.618 \), \( \frac{G M}{c^2} \approx 1.618 \), (3) It has a point of minimum in the range between the above mentions redshifts. To find the point of minimum we derive \( r' / \left( \frac{G M}{c^2} \right) \) with respect to \( z \) and equate the result to zero, yielding:

\[
4 z^4 - 2 z^3 - 10 z^2 - 6 z - 1 = 0
\]

Which solves at \( z_m \approx 2.078 \), yielding \( r_m \approx 1.5867 \left( \frac{G M}{c^2} \right) \).

The prediction of an extreme galactic activity at \( z \approx 0.707 \) is supported by many observational studies, which reported the detection of quasars, blazars and other AGNs at \( z \approx 0.707 \) [e.g., 33-36]. For example, a recent study by Steinhardt et al. [34] reported the discovery of a Type 1 quasar, SDSS 0956+5128, with extreme velocity offsets at redshifts \( z = 0.690, 0.714, \) and \( 0.707 \). The prediction of AGNs at \( z \approx 2.078 \) is also confirmed by observations [e.g. 37, 38].

I also compared the dynamical dependence of \( r \) on redshift (Eq. 19) with the dynamics reported in [39] for a cosmology of \( \Omega_m = 0.3 \) and \( \Omega_\Lambda = 0.7 \), \( H_0 = 70 \text{ km s}^{-1} \text{ Mpc}^{-1} \). Figure 3a depicts the predicted radius \( r \) (in Km) as a function of redshift for intermediate and massive black holes and Figure 3b depicts comparable results reported in [39]. Comparison of the two figures, despite differences in scaling, reveals a remarkable similarity between the results of the two models.

6. Summary and Concluding Remarks

The singularity problem of the Schwarzschild's solution to the black hole radius has prompted many attempts to produce singularity-free or singularity-avoiding solutions. Such attempts include what is known as the “Bardeen black hole” models [e.g., 16-23], as well as quantum mechanical models [e.g., 26-30] and string theory models [e.g., 24, 25]. A minor portion of this literature was discussed in section 2.
Figure 3. Predicted $r$ as a function of $z$ (Fig. 3a) and comparable results based on $\Lambda$CDM model ($\Omega_M = 0.3$, $\Omega_A = 0.7$, $H_0 = 70$ km s$^{-1}$ Mpc$^{-1}$) reported by Hook (2005) [37] (Fig 3b).

An additional solution was proposed here, based on a straightforward extension of Galileo-Newton mechanics, to the range of relativistic velocities. For the non-rotating, purely gravitational, spherical black hole Newtonian Relativity predicts a black hole radius equaling the Schwarzschild radius, but with no interior singularity. No less important, the theory yields a simple equation for the dynamics of
a typical galaxy with a supermassive black hole. Investigation of the emerging dynamics suggests that a galaxy's supermassive black hole is part of a binary system, comprised of the black hole and a spatial singularity at redshift \( z \approx 0.707 \), suspected to be a quasar with extreme velocity offsets, or an active galactic nucleus (AGN).

The produced model is successful in making several interesting predictions. The point of singularity at \( z \approx 0.707 \) confirms with several observations reporting the detection of quasars and AGNs at the predicted redshift [e.g., 34-36]. The prediction of galactic activity at \( z \approx 2.078 \) is also confirmed by observations reporting the existence of quasars and AGNs at \( z = 2.078 \) and \( z = 2.08 \) [e.g. 35, 36, 39]. Taken together, the derived dynamics, and the aforementioned results, shed some light on the intimate relationship between supermassive black holes, and the evolution of the galaxies in which they reside.

A major advantage of the proposed solution and of Newtonian Relativity in general, lies in the fact that they present a model of the universe that maintains a smooth and natural continuity, between relativistic and classical (non-relativistic) physics. For sufficiently low velocities, all the relativistic results discussed above, reduce, by setting \( \beta = 0 \), to the corresponding classical terms. Obviously, the proposed solution is very simple, certainly when compared to other black hole solutions. It is argued that the simplicity of the present model should not be taken as a liability. In science it is well accepted that when confronted with two theories that are equal in their predictability and explanatory power, we should prefer the simpler one, i.e., the theory containing the reduced parameter set or based on a less complicated model [40].

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