# Numerological Formula for the Electron Spin g-factor

*The present article introduces a numeric expression for the electron spin g-factor. This formula is accurate, at least, to twelve decimal places. I also introduce a generalized version of this formula that may be used to adjust the calculated value to future measurements.*

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*April 30th 2017*

***Keywords***: *spin of the electron, magnetic moment, spin angular momentum, electron spin g-factor, electron g-factor, electric charge, fine structure constant, electromagnetic coupling constant, atomic structure constant, NIST, Dirac equation, Schrödinger equation.*

## 1. Introduction

The electron spin g-factor, $g_e$, or simply the electron g-factor is part of the proportionality constant of the equation relating the magnetic moment, $\vec{\mu}_s$, due to the spin of the electron with the spin angular momentum, $\vec{S}$. The relationship between these two variables is

$$\vec{\mu}_s = g_e \frac{q_e}{2\mathrm{m}_e} \vec{S}$$

The other constants are the electric charge, $q_e$, of the electron and the electron rest mass, $m_e$. In 1928 Dirac developed his quantum theory of the electron [1] based on a relativistic version of the Schrödinger equation. Dirac's equation predicted an electron g-factor of 2. However, the measured value was about 2.002319304. Behind this small difference in absolute terms there was a giant difference in concepts. To overcome the limitations of Dirac's formulation, Richard Feynman developed the quantum electrodynamics theory (QED) in which he introduced the concept of virtual particles. This theory predicted a value for the electron g-factor which was in agreement with the experiment to 12 decimal places. This was an unprecedented success. Thus QED became humanity's most accurate formulation of reality. The next section introduces a formula with only four terms inside a 4096 root. The formula is accurate, at least, to 12 decimal places, matching the corresponding QED's theoretical result.

## 2. The Formula

The formula for the electron g-factor I developed is

$$g_e = 2\left( \sqrt[4096]{\frac{1}{\alpha} - \frac{2}{\alpha^{0.5}} + \frac{1}{\alpha^{0.1}} + \frac{0.00002}{\alpha^{0.09}}} \right) \tag{2.1}$$

Where

$g_e$ = electron spin g-factor
$\alpha$ = fine structure constant or electromagnetic coupling constant or atomic structure constant.

It is interesting to see that $2^{12} = 4096$. This formula yields the following value

$$g_e = 2.\underline{002\,319\,304\,361}17 \qquad \text{(R1)}$$
$$\textit{accurate, at least, to} \uparrow 12 \textit{ decimal places}$$

NIST gives the following experimental value for the electron spin g-factor

$$g_{e\ \exp} = 2.002\,319\,304\,361\,53(53) \qquad \text{(R2)}$$

Considering the error we have the following minimum and maximum values

$$g_{e\_\exp\_\min} = 2.002\,319\,304\,361\,00 \qquad \text{(R3)}$$

$$g_{e\_\exp\_\max} = 2.002\,319\,304\,362\,06 \qquad \text{(R4)}$$

Comparing the result (R1) with the experiment (R2) we see that formula (2.1) is accurate, at least, to 12 decimal places. Note that, due to the error, the experimental value does not allow us to determine the accuracy of the formula presented in this paper beyond 12 decimal places. It is also worthwhile to observe that the theoretical value falls within the uncertainty of the measured value (R2).

Finally, the magnetic moment $\vec{\mu}_s$ due to the spin of the electron can be written as

$$\vec{\mu}_s = 2\left( \sqrt[4096]{\frac{1}{\alpha} - \frac{2}{\alpha^{0.5}} + \frac{1}{\alpha^{0.1}} + \frac{0.00002}{\alpha^{0.09}}} \right) \frac{q_e}{2\mathrm{m}_e} \vec{S} \tag{2.2}$$

Considering that the electric charge of the electron, $q_e$ , is negative, we can substitute it with $-e$ . Thus

$$q_e = -e \tag{2.3}$$

where $e$ is the absolute value of the electric charge. Finally, from equations (2.2) and (2.3) we get

$$\vec{\mu}_s = -\left( \sqrt[4096]{\frac{1}{\alpha} - \frac{2}{\alpha^{0.5}} + \frac{1}{\alpha^{0.1}} + \frac{0.00002}{\alpha^{0.09}}} \right) \frac{e}{m_e} \vec{S} \tag{2.4}$$

It is worthwhile to remark that the vectors $\vec{\mu}_s$ and $\vec{S}$ point in opposite directions.

# 3. The General Formula

I want a formula that will withstand the passage of time. In other words a I want a formula that we may be able to update in the future when the electron g-factor to be known with more precision (accuracy). This is important because the experimental methodology improves from time to time. Thus, I shall generalize formula (2.1) as follows

$$g_e = 2 \left( \sqrt[4096]{\frac{1}{\alpha} + \sum_{i=1}^{n} \frac{R_i}{\alpha^{b_i}}} \right) \tag{3.1a}$$

or, equivalently

$$g_e = 2 \left( \sqrt[4096]{\frac{1}{\alpha} + \frac{R_1}{\alpha^{b_1}} + \frac{R_2}{\alpha^{b_2}} + \frac{R_3}{\alpha^{b_3}} + \frac{R_4}{\alpha^{b_4}} + \frac{R_5}{\alpha^{b_5}} + \ldots} \right) \tag{3.1b}$$

where

$R_i$ = real number (real numbers could be either positive or negative). In particular, some of these real numbers could be integers (positive or negative)) (all integers are real numbers).
$b_i$ = positive real number smaller than 1.

Since, the quantity

$$g_e \approx 2 \left( \sqrt[4096]{\frac{1}{\alpha}} \right) \approx 2.002\ 403\ 906 \tag{3.2}$$

is a reasonably good approximation to the observed value of the electron spin g-factor, I shall define a correction term, *C,* as follows

$$C \equiv \sum_{i=1}^{n} \frac{R_i}{\alpha^{b_i}} \tag{3.3}$$

This correction term allows to write formula (3.1a) in a more compact way

$$g_e = 2\left(\sqrt[4096]{\frac{1}{\alpha} + C}\right) \tag{3.4}$$

It is interesting to note that *C* is a negative real number. For example, the value of *C* in formula (2.1) is –21.776 820 11, this is

$$C = -\frac{2}{\alpha^{0.5}} + \frac{1}{\alpha^{0.1}} + \frac{0.00002}{\alpha^{0.09}} = -21.776\,820\,11 \tag{3.5}$$

# 4. Conclusions

The importance of the general formula (3.1a)/(3.1b) is that allow us to add more terms in the future to adjust the calculate value of the electron spin g-factor to future experimental data with more that 12 decimals of accuracy (for example, 30 decimal places of accuracy). Thus, the general formula is a formula that can be updated in the future to reflect the values of new measurements, and this means that the formula will always be a correct numerological equation.

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**REFERENCES**

[1] P. A. M. Dirac, *The Quantum Theory of the Electron*. Proc. Royal Society. Lond. A. 1928.