

Numerological Formula for the Electron Spin g-factor

The present article introduces a numeric expression for the electron spin g-factor. This formula is accurate, at least, to twelve decimal places.

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1. Introduction

The electron spin g-factor, g_e , or simply the electron g-factor is part of the proportionality constant of the equation relating the magnetic moment, $\vec{\mu}_s$, due to the spin of the electron with the spin angular momentum, \vec{S} . The relationship between these two variables is

$$\vec{\mu}_s = g_e \frac{q_e}{2m_e} \vec{S}$$

The other constants are the electric charge, q_e , of the electron and the electron rest mass, m_e . In 1928 Dirac developed his quantum theory of the electron [1] based on a relativistic version of the Schrödinger equation. Dirac's equation predicted an electron g-factor of 2. However, the measured value was about 2.002319304. Behind this small difference in absolute terms there was a giant difference in concepts. To overcome the limitations of Dirac's formulation, Richard Feynman developed the quantum electrodynamics theory (QED) in which he introduced the concept of virtual particles. This theory predicted a value for the electron g-factor which was in agreement with the experiment to 12 decimal places. This was an unprecedented success. Thus QED became humanity's most accurate formulation of reality. The next section introduces a formula

with only four terms inside a 4096 root. The formula is accurate, at least, to 12 decimal places, matching the corresponding QED's theoretical result.

2. The Formula

The formula for the electron g-factor I developed is

$$g_e = 2 \left(\sqrt[4096]{\frac{1}{\alpha} - \frac{2}{\alpha^{0.5}} + \frac{1}{\alpha^{0.1}} + \frac{0.00002}{\alpha^{0.09}}} \right) \quad (1)$$

Where

g_e = electron spin g-factor

α = fine structure constant or electromagnetic coupling constant or atomic structure constant.

It is interesting to see that $2^{12} = 4096$

This formula yields the following value

$$g_e = \underline{2.002\ 319\ 304\ 361\ 17} \quad (R1)$$

accurate, at least, to ↑ 12 decimal places

NIST gives the following experimental value for the electron spin g-factor

$$g_{e\ \text{exp}} = 2.002\ 319\ 304\ 361\ 53(53) \quad (R2)$$

Considering the error we have the following minimum and maximum values

$$g_{e_exp_min} = 2.002\ 319\ 304\ 361\ 00 \quad (R3)$$

$$g_{e_exp_max} = 2.002\ 319\ 304\ 362\ 06 \quad (R4)$$

Comparing the result (R1) with the experiment (R2) we see that formula (1) is accurate, at least, to 12 decimal places. Note that, due to the error, the experimental value does not allow us to determine the accuracy of the formula beyond 12 decimal places. It is also worthwhile to observe that the theoretical value falls within the uncertainty of the measured value (R2).

Finally, the magnetic moment $\vec{\mu}_s$ due to the spin of the electron can be written as

$$\vec{\mu}_s = 2 \left(\sqrt[4096]{\frac{1}{\alpha} - \frac{2}{\alpha^{0.5}} + \frac{1}{\alpha^{0.1}} + \frac{0.00002}{\alpha^{0.09}}} \right) \frac{q_e}{2m_e} \vec{S} \quad (2)$$

Considering that the electric charge of the electron, q_e , is negative, we can substitute it with $-e$. Thus

$$q_e = -e \quad (3)$$

where e is the absolute value of the electric charge. Finally, from equations (2) and (3) we get

$$\vec{\mu}_s = - \left(\sqrt[4096]{\frac{1}{\alpha} - \frac{2}{\alpha^{0.5}} + \frac{1}{\alpha^{0.1}} + \frac{0.00002}{\alpha^{0.09}}} \right) \frac{e}{m_e} \vec{S} \quad (4)$$

It is worthwhile to remark that the vectors $\vec{\mu}_s$ and \vec{S} point in opposite directions.

REFERENCES

- [1] P. A. M. Dirac, *The Quantum Theory of the Electron*. Proc. Royal Society. Lond. A. 1928.