The Size of Fundamental Particles

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The present investigation is focused on a simple quantum mechanical model that will unveil the size of fundamental particles such as the electron. The result of this research indicates that the diameter of the electron is smaller or equal than 10 times the Planck length, approximately.

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1. Introduction

The electron is one of the basic building blocks of matter, a particle that seems to have no internal parts or structure. This means that the electron is a fundamental (or elementary) particle.

The story I will describe goes back to 1928, at time when Dirac wrote his famous paper entitled "The Quantum Theory of the Electron" [1]. This paper assumes the electron is a point with no volume. On the other hand we have the so called *classical electron radius* which according to CODATA 2010 is 2.817 940 3267 x 10^{-15} m. Before all the experimental evidence and theoretical suggestions is hard to believe that the size of the electron could be so big. The size of the electron is more likely to be much smaller than that. Most recent theories, such as the string theory, strongly suggest that the electron is a very small particle. As mentioned by Leonard Susskind [2]: *"we all expect that electrons, photons and other elementary particles to be at least as big as the Plank length, and possibly bigger".* Furthermore, since electrons are particles with all kind of properties such as mass, electric charge, spin, etc., it is hard to think of them as dimensionless objects. It is therefore reasonable to assume that the size of this particle to be dependant on the properties of space itself. Assuming space is quantized and starting from the Universal Uncertainty Principle (UUP) that I developed in 2012 and that I published online this year [3], I shall show that the electron is not a point charge but either an extremely small sphere or an extremely small string. However, recent experimental evidence seems to indicate that the electron is an almost a perfect sphere [4]. Therefore I shall assume that the electron is a small sphere and I shall calculate its diameter from the basic solution to the Schrödinger equation for a particle confined to a box of width L and infinite potential well.

2. Theory

We shall start with the universal uncertainty principle (UUP) which is a generalization of the Heisenberg uncertainty principle. The form of the UUP is

$$\Delta p \Delta x \geq \sqrt{\left(\frac{h}{4\pi}\right)^2 - \frac{h}{4\pi} \Delta p L_Z} \tag{1}$$

Where

$\Delta p =$ Uncertainty in the momentum of a particle due to its wave nature (wave-packet representing the particle). This uncertainty does not include the uncertainty P_Z in the momentum due to the quantum fluctuations of space-time.

$\Delta x =$ Uncertainty in the position of the particle due to the wave-packet representing the particle.

$L_Z =$ Uncertainty in the position of the particle due to the quantum fluctuations of space-time. This uncertainty does not include the uncertainty Δx due to the wave-packet representing the particle. The minimum value of this uncertainty cannot be measured experimentally with the present technology. Further, it seems logical to assume that this uncertainty is identical to the Planck length L_P. However, these two lengths could be different but the difference should not be significant.

$P_Z =$ Uncertainty in the momentum of a particle due to to the quantum fluctuations of space-time (uncertainty due to the zero point momentum). This uncertainty does not include the uncertainty Δp in the momentum due to the wave nature of the wave-packet representing the particle. We shall neglect the effects of P_Z in this principle. On way of extending this principle to include the zero point momentum (or zero point energy if the temporal form of the uncertainty principle is used) is to use the Schwinger formulation.

We assume that the distance the electron move due to the zero point energy is the Planck length. This is the minimum distance with physical meaning. Thus in equation (1) we substitute both Δx and L_Z with L_P

$$\Delta x = L_P \tag{2}$$

$$L_Z = L_P \tag{3}$$

$$L_P = \sqrt{\frac{h G}{2\pi\ c^3}} \tag{4}$$

This yields

$$\Delta p L_P \geq \sqrt{\left(\frac{h}{4\pi}\right)^2 - \frac{h}{4\pi}\Delta p L_P} \tag{5}$$

Now we square both sides

$$(\Delta p)^2 L_P^2 \geq \left(\frac{h}{4\pi}\right)^2 - \frac{h}{4\pi}\Delta p L_P \tag{6}$$

$$L_P^2(\Delta p)^2 + \left(\frac{h L_P}{4\pi}\right)\Delta p - \left(\frac{h}{4\pi}\right)^2 \geq 0 \tag{7}$$

This is a quadratic inequality with coefficients *A*, *B* and *C* defined by

$$A = L_P^2 \tag{8}$$

$$B = \left(\frac{h L_P}{4\pi}\right) \tag{9}$$

$$C = -\left(\frac{h}{4\pi}\right)^2 \tag{10}$$

The solution to this inequality is

$$\Delta p \geq \frac{-B \pm \sqrt{B^2 - 4AC}}{2A} \tag{11}$$

$$\Delta p \geq \frac{-\frac{h L_P}{4\pi} \pm \sqrt{\left(\frac{h L_P}{4\pi}\right)^2 + 4L_P^2\left(\frac{h}{4\pi}\right)^2}}{2L_P^2} \tag{12}$$

$$\Delta p \geq \frac{1}{2L_P^2}\left(-\frac{h L_P}{4\pi} \pm \sqrt{\left(\frac{h L_P}{4\pi}\right)^2 + 4\left(\frac{h L_P}{4\pi}\right)^2}\right) \tag{13}$$

$$\Delta p \geq \frac{1}{2L_P^2}\left(-\frac{h L_P}{4\pi} \pm \sqrt{5\left(\frac{h L_P}{4\pi}\right)^2}\right) \tag{14}$$

$$\Delta p \geq \frac{1}{2L_P^2}\left(- \frac{h\,L_P}{4\pi} \pm \frac{h\,L_P}{4\pi}\sqrt{5}\right) \tag{15}$$

Taking the positive square root and doing some mathematical work we obtain the solution

$$\Delta p \geq \left(\frac{\sqrt{5} - 1}{8\pi}\right)\frac{h}{L_P} \tag{16}$$

The uncertainty in the momentum of the electron due to the zero point energy is

$$\Delta p = m_e \Delta v \tag{17}$$

Where

$m_e =$ electron rest mass
$\Delta v =$ uncertainty in the speed of the electron due to the quantum fluctuations of the vacuum

Substituting Δp in equation (17) with the second side of inequality (17) and then solving for Δv yields

$$\Delta v \geq \left(\frac{\sqrt{5} - 1}{8\pi}\right)\frac{h}{m_e L_P} \tag{18}$$

On the other hand the uncertainty in the kinetic energy (due to the zero point energy of the vacuum) of the electron is

$$\Delta E_Z \geq \frac{1}{2} m_e (\Delta v)^2 \tag{19}$$

Substituting Δv in inequality (19) with the second side of inequality (18) gives

$$\Delta E_Z \geq \frac{1}{2} m_e \left(\frac{\sqrt{5} - 1}{8\pi}\right)^2 \left(\frac{h}{m_e L_P}\right)^2 \tag{20}$$

$$\Delta E_Z \geq \frac{\left(\sqrt{5} - 1\right)^2}{128\,\pi^2}\frac{h^2}{m_e L_P^2} \tag{21}$$

Now we shall introduce a quantum model of the electron based on the solution of the Schrödinger equation for a particle in a rectangular box of width L with an infinite potential well. The infinite potential well means that the electron is confined to an imaginary region of length L (in fact the electron will be confined to a cube of size L). This model is used to calculate the size of the electron and it does not mean that the electron is not able to move through space. To illustrate this point we can imagine the box representing the electron as a box that will accelerate when a force is applied to it. Thus the box will respond to both electrical and gravitational forces as the electron does. The allowed energy levels for an electron confined to this imaginary box is

$$E_n = \left(\frac{\pi^2 \hbar^2}{2 m_e L^2} \right) n^2 \tag{22}$$

This result could be found in any good book on quantum mechanics for university students.

Where
E_n = permitted energy levels
$\hbar$ = reduced Planck constant $(h/2\pi)$
L = width of the box
n = quantum number (1, 2, 3, 4, 5, …)

The fundamental energy level is the minimum energy of the electron and corresponds to *n=1.* Thus substituting n in equation (22) with 1 yields

$$E_1 = \frac{\pi^2 \hbar^2}{2 m_e L^2} \tag{23}$$

Taking into account that

$$\hbar = \frac{h}{2\pi} \tag{24}$$

and after doing some work we re-write equation (23) as follows

$$E_1 = \frac{h^2}{8 m_e L^2} \tag{25}$$

Now we assume that when the width L of the box is reduced until it equals the diameter d_e of the electron, the minimum energy E_1 of the electron in the box must be identical to the energy of the electron at the zero point energy E_Z, thus

$$E_1 = E_Z = \Delta E_Z \tag{26}$$

The reason of using the infinite potential well model is that the box represents the "physical boundary" of the electron itself, a boundary the electron cannot penetrate.

If we substitute E_1 in equation (25) with ΔE_Z, we must also substitute the width of the box with the diameter of the electron, this gives

$$E_Z = \frac{h^2}{8m_e(2r_e)^2} \tag{27}$$

Where

r_e = radius of the electron

Solving for r_e

$$r_e = \frac{h}{4\sqrt{2m_e\, E_Z}} \tag{28}$$

Remembering that E_Z is identical to ΔE_Z we substitute E_Z in equation (28) with inequality (21) thus we get the following inequality for the radius of the electron

$$r_e \le \frac{h}{4\sqrt{2m_e \frac{\left(\sqrt{5}-1\right)^2}{128\pi^2}\frac{h^2}{m_e L_P^2}}} \tag{29}$$

And after doing some mathematical work inequality (29) transforms into

$$r_e \le \frac{(1+\sqrt{5})}{2}\pi\ L_P \qquad (\textit{radius of the electron}) \tag{30}$$

$$r_e \le \varphi\ \pi\ L_P \tag{31}$$

Where

$$\varphi = \frac{1+\sqrt{5}}{2} \tag{32}$$

φ = divine ratio = 1.618 033 989

$r_e \le 1.618033989 \times 3.141592654 \times 1.616199255 \times 10^{-35}\, m$

$r_e \le 8.215470024 \times 10^{-35}\, m$

$r_e \le 5.083\, L_P$

And the diameter d_e of the electron is given by the following inequality

$$d_e \le (1+\sqrt{5})\pi \; L_P \qquad \textit{(diameter of the electron)} \qquad (33)$$

Where

d_e = diameter of the electron

Or approximately

$$d_e \le 10.166 \, L_P \qquad (34)$$

Thus the diameter of the electron is smaller or equal than 10 times the Planck length, approximately. Inequality (33) shows that the diameter of the electron does not depend on its rest mass. When I first saw this result I thought that something was wrong. But I could not find anything wrong and after thinking it over, I realized that the reason the electron size is independent of the mass is because the electron is a fundamental particle. But then what would be the size of the other fundamental particles such as quarks (assuming they are fundamental)? Apparently, and despite the confinement they live in it seems that quarks and electrons should have the same size (unless there are other variables I did not take into account).

3. Conclusions

The formulation presented here assumes that the electron can be represented by an infinite potential well of width *L*. This well simulates the interface of the electron with the external world. In other words it simulates the electron itself. When the fundamental energy level of this electronic model equals the electron's zero point energy, then we assume that the size of the box equals the size of the electron. The result of this model yields an electron diameter not bigger than 10 times the Planck length. This result is qualitatively in agreement with the string theory. However the abovementioned experiment [4] suggests the electron is almost a perfect sphere.

REFERENCES

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