From quantum mechanics to intelligent particle.

Michail Zak[a]

Jet Propulsion Laboratory California Institute of Technology, Pasadena, CA 91109, USA

The challenge of this work is to connect quantum mechanics with the concept of intelligence. By intelligence we understand a capability to move from disorder to order without external resources, i.e. in violation of the second law of thermodynamics. The objective is to find such a mathematical object described by ODE that possesses such a capability. The proposed approach is based upon modification of the Madelung version of the Schrodinger equation by replacing the force following from quantum potential with non-conservative forces that link to the concept of information. A mathematical formalism suggests that a hypothetical intelligent particle, besides the capability to move against the second law of thermodynamics, acquires such properties like self-image, self-awareness, self- supervision, etc. that are typical for Livings. However since this particle being a quantum-classical hybrid acquires non-Newtonian and non-quantum properties, it does not belong to the physics matter as we know it: the modern physics should be complemented with the concept of an *information force* that represents a bridge to intelligent particle. It has been suggested that quantum mechanics should be complemented by the intelligent particle as an independent entity, and that will be the necessary step to physics of Life. At this stage, the intelligent particle is introduced as an ***abstract mathematical concept*** that is satisfied only mathematical rules and assumptions, and its physical representation is still an open problem.

1. Introduction.

The recent statement about completeness of the physical picture of our Universe made in Geneva raised many questions, and one of them is the ability to create Life and Mind out of physical matter without any additional entities. The main difference between living and non-living matter is in directions of their evolution: it has been recently recognized that the evolution of livings is progressive in a sense that it is directed to the highest levels of complexity if the complexity is measured by an irreducible number of different parts that interact in a well-regulated fashion. Such a property is not consistent with the behavior of *isolated* Newtonian systems that cannot increase their complexity without external forces. That difference created so called Schrödinger paradox: in a world governed by the second law of thermodynamics, all isolated systems are expected to approach a state of maximum *disorder;* since life approaches and maintains a highly *ordered* state – one can argue that this violates the Second Law implicating a paradox,[1].

[a] Senior research Scientist (Emeritus). Electronic mail:michail.zak@gmail.com

But livings are not isolated due to such processes as metabolism and reproduction: the increase of order inside an organism is compensated by an increase in disorder outside this organism, and that removes the paradox. Nevertheless it is still tempting to find a mechanism that drives livings from disorder to order. The purpose of this paper is to demonstrate that moving from a disorder to order is not a prerogative of open systems: an isolated system can do it without help from outside. However such system cannot belong to the world of the modern physics: it belongs to the world of living matter, and that lead us to a concept of an intelligent particle – the first step to physics of livings. In order to introduce such a particle, we start with an idealized mathematical model of livings by addressing only one aspect of Life: a ***biosignature,*** i.e. ***mechanical*** invariants of Life, and in particular, the ***geometry and kinematics of intelligent behavior*** disregarding other aspects of Life such as metabolism and reproduction. By narrowing the problem in this way, we are able to extend the mathematical formalism of physics' First Principles to include description of intelligent behavior. At the same time, by ignoring metabolism and reproduction, we can make the system isolated, and it will be a challenge to show that it still can move from a disorder to the order.

2. Starting with quantum mechanics.

The starting point of our approach is the Madelung equation that is a hydrodynamical version of the Schrödinger equation

$$\frac{\partial \rho}{\partial t} + \nabla \bullet \left(\frac{\rho}{m} \nabla S\right) = 0 \qquad (1)$$

$$\frac{\partial S}{\partial t} + (\nabla S)^2 + F - \frac{\hbar^2 \nabla^2 \sqrt{\rho}}{2m\sqrt{\rho}} = 0 \qquad (2)$$

Here ρ and S are the components of the wave function $\psi = \sqrt{\rho} e^{iS/\hbar}$, and $\hbar$ is the Planck constant divided by 2π. The last term in Eq. (2) is known as quantum potential. From the viewpoint of Newtonian mechanics, Eq. (1) expresses continuity of the flow of probability density, and Eq. (2) is the Hamilton-Jacobi equation for the action S of the particle. Actually the quantum potential in Eq. (2), as a feedback from Eq. (1) to Eq. (2), represents the difference between the Newtonian and quantum mechanics, and therefore, it is solely responsible for fundamental quantum properties.

The Madelung equations (1), and (2) can be converted to the Schrödinger equation using the ansatz

$$\sqrt{\rho} = \Psi \exp(-iS / h) \qquad (3)$$

where ρ and S being real function.

In order to associate quantum potential with the concept of information, recall that information is an indirectly observed quantity that is defined via entropy as a measure of unpredictability: For a random variable *X* with *n* outcomes, the Shannon information denoted by *H(X),* is

$$H(X) = -\sum_{i=1}^{n} \rho(x_i) \log_b \rho(x_i) \tag{4}$$

In our further applications we will use the continuous version of this formula

$$H(X) = -\int_{-\infty}^{\infty} \rho(x) \ln \rho(x)\, dx \tag{5}$$

Actually our approach is based upon a modification of the Madelung equation, and in particular, upon replacing the quantum potential with a different Liouville feedback, Fig.1

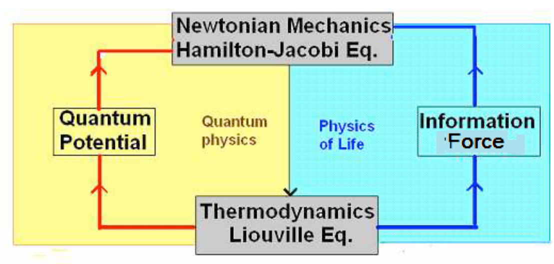

Figure 1. Classic Physics, Quantum Physics and Physics of Life.

In Newtonian physics, the concept of probability ρ is introduced via the Liouville equation

$$\frac{\partial \rho}{\partial t} + \nabla \bullet (\rho \mathbf{F}) = 0 \tag{6}$$

generated by the system of ODE

$$\frac{d\mathbf{v}}{dt} = \mathbf{F}[\mathbf{v}_1(t), \ldots \mathbf{v}_n(t), t] \tag{7}$$

where **v** is velocity vector.

It describes the continuity of the probability density flow originated by the error distribution

$$\rho_0 = \rho(t = 0) \tag{8}$$

in the initial condition of ODE (8).

Let us rewrite Eq. (2) in the following form

$$\frac{d\mathbf{v}}{dt} = \mathbf{F}[\rho(\mathbf{v})] \tag{9}$$

where **v** is a velocity of a hypothetical particle.

This is a fundamental step in our approach: in Newtonian dynamics, the probability never explicitly enters the equation of motion, [2,3]. In addition to that, the Liouville equation generated by Eq. (9) is nonlinear with respect to the probability density ρ

$$\frac{\partial \rho}{\partial t} + \nabla \bullet \{\rho \mathbf{F}[\rho(\mathbf{V})]\} = 0 \tag{10}$$

and therefore, the system (9),(10) departs from Newtonian dynamics. However although it has the same topology as quantum mechanics (since now the equation of motion is coupled with the equation of continuity of probability density), it does not belong to it either. Indeed Eq. (9) is more general than the Hamilton-Jacoby equation (2): it is not necessarily conservative, and **F** is not necessarily the quantum potential although further we will impose some restriction upon it that links **F** to the concept of information, [3]. The relation of the system (9), (10) to Newtonian and quantum physics is illustrated in Fig.1.

Remark. Here and below we make distinction between the random *variable v(t)* and its *values V* in probability space.

Prior to considering a specific form of the force **F,** we will make a comment concerning the normalization constrain satisfaction

$$\int_V \rho \, dV = 1 \tag{11}$$

in which V is the volume where Eqs. (9) and (10) are defined.
Turning to Eq. (10) and integrating it over the volume V

$$\frac{\partial}{\partial t}\int_V \rho \, dV = -\int_V dV \nabla \bullet \{\rho \mathbf{F}[\rho(\mathbf{V})]\} = -\oint_\Phi d\Phi \nabla \bullet (\rho \mathbf{F}) = 0 \tag{12}$$

if

$$\rho = 0, \quad |\mathbf{F}| < \infty \quad at \quad \Phi \tag{13}$$

where Φ is the surface bounding the volume V.
Therefore, if the normalization constraint (9) is satisfied at $t = 0$, it is satisfied for all the times.

3. Information force instead of quantum potential.

In this section we propose the structure of the force **F** that plays the role of a feedback from the Liouville equation (10) to the equation of motion (9). Turning to one-dimensional case, let us specify this feedback as

$$F = c_0 + \frac{1}{2}c_1\rho - \frac{c_2}{\rho}\frac{\partial \rho}{\partial v} + \frac{c_3}{\rho}\frac{\partial^2 \rho}{\partial v^2} \tag{14}$$

$$c_0 > 0, c_1 > 0, c_3 > 0 \tag{15}$$

Then Eq.(9) can be reduced to the following:

$$\dot{v} = c_0 + \frac{1}{2}c_1\rho - \frac{c_2}{\rho}\frac{\partial\rho}{\partial v} + \frac{c_3}{\rho}\frac{\partial^2\rho}{\partial v^2} \tag{16}$$

and the corresponding Liouville equation will turn into the following PDE

$$\frac{\partial\rho}{\partial t} + (c_0 + c_1\rho)\frac{\partial\rho}{\partial V} - c_2\frac{\partial^2\rho}{\partial v^2} + c_3\frac{\partial^3\rho}{\partial V^3} = 0 \tag{17}$$

This equation is known as the KdV-Bergers' PDE. The mathematical theory behind the KdV equation became rich and interesting, and, in the broad sense, it is a topic of active mathematical research. A homogeneous version of this equation that illustrates its distinguished properties is nonlinear PDE of parabolic type. However a fundamental difference between the standard KdV-Bergers equation and Eq. (17) is that Eq. (17) ***dwells in the probability space***, and therefore, it must satisfy the normalization constraint

$$\int_{-\infty}^{\infty} \rho\, dV = 1 \tag{18}$$

However as shown in [4], this constraint is satisfied: in physical space it expresses conservation of mass, and it can be easily scale-down to the constraint (18) in probability space. That allows one to apply all the known results directly to Eq. (17). However it should be noticed that all the conservation invariants have different physical meaning: they are not related to conservation of momentum and energy, but rather impose constraints upon the Shannon information.

In physical space, Eq. (17) has many applications from shallow waves to shock waves and solitons. However, application of solutions of the same equations in probability space is fundamentally different. In the next sections we present two phenomena that exist neither in Newtonian nor in quantum physics.

4. Emergence of randomness.

In this section we discuss a fundamentally new phenomenon: transition from determinism to randomness in ODE that coupled with their Liouville PDE.

In order to complete the solution of the system (16), (17), one has to substitute the solution of Eq. (17):

$$\rho = \rho(V, t) \quad at \quad V = v \tag{19}$$

into Eq.(16). Since the transition from determinism to randomness occurs at $t \to 0$, let us turn to Eq. (17) with sharp initial condition

$$\rho_0(V) = \delta(V)\, at \qquad t = 0, \tag{20}$$

Then applying one of the standard analytical approximations of the delta-function, one obtains the asymptotic solution

$$\rho = \frac{1}{t\sqrt{\pi}} e^{-\frac{V^2}{t^2}} \quad at \quad t \to 0 \tag{21}$$

Substitution this solution into Eq. (14) shows that

$$O(c_0+\frac{1}{2}c_1\rho)=\frac{1}{t},\quad O(\frac{c_2}{\rho}\frac{\partial\rho}{\partial v})=\frac{1}{t^2},$$
$$and\quad O(\frac{c_3}{\rho}\frac{\partial^2\rho}{\partial v^2})=\frac{1}{t^4}\quad at\quad t\to 0,\quad v\neq 0 \tag{22}$$

i.e.

$$c_0+\frac{1}{2}c_1\rho << \frac{c_2}{\rho}\frac{\partial\rho}{\partial v} << \frac{c_3}{\rho}\frac{\partial^2\rho}{\partial v^2}\quad at\quad t\to 0,\quad v\neq 0 \tag{23}$$

and therefore, the first three terms in Eq. (16) can be ignored

$$\dot{v}=\frac{c_3}{\rho}\frac{\partial^2\rho}{\partial v^2}\quad at\quad t\to 0,\quad v\neq 0 \tag{24}$$

or after substitution of eq. (21)

$$\dot{v}=\frac{4c_3v^2}{t^4}\quad at\quad t\to 0,\quad v\neq 0 \tag{25}$$

Eq. (35) has the following solution (see Fig. 2)

$$v=\frac{t^3}{4c_3+Ct^3}\quad at\quad t\to 0,\quad v\neq 0 \tag{26}$$

where C is an arbitrary constant.
This solution has the following property: the Lipchitz condition at $t\to 0$ fails

$$\frac{\partial\dot{v}}{\partial v}=\frac{8c_3v}{t^4}=\frac{8c_3t^3}{t^4(4c_3+Ct^3)}\to\infty\quad at\quad t\to 0,\quad v\neq 0 \tag{27}$$

and as a result of that, the uniqueness of the solution is lost. Indeed, as follows from Eq. (36), for any value of the arbitrary constant C, the solutions are different, but they satisfy the same initial condition

$$v\to 0\quad at\quad t\to 0 \tag{28}$$

Due to violation of the Lipchitz condition (27), the solution becomes unstable. That kind of instability when infinitesimal errors lead to finite deviations from basic motion (the Lipchitz instability) has been discussed in [2,3,5]. This instability leads to unpredictable shift of solution from one value of *C* to another. It means that appearance of any specified solution out of the whole family is random, and that randomness is controlled by the feedback (14) from the Liouville equation (17). Indeed if the solution (26) runs independently many times with the same initial conditions, and the statistics is collected, the probability density will satisfy the Liouville equation (17), Fig.3.

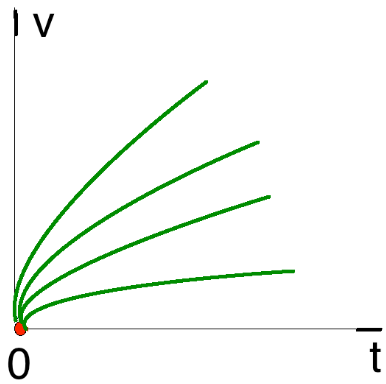

Figure 2. Family of random solutions describing transition from determinism to stochastisity.

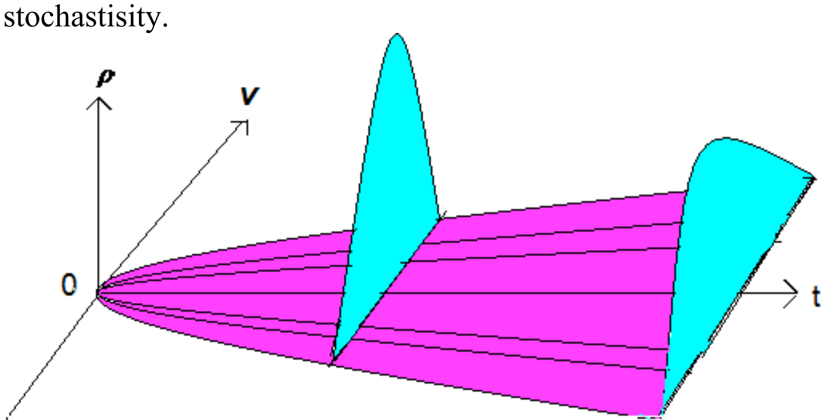

Figure 3. Stochastic process and probability density.

Remark. It should be emphasized that with the probability density defined by Eq. (20), the point $v = 0$ must be excluded from consideration since at this point Eq. (16) in meaningless.

5. Departure from Newtonian and quantum physics.

In this section we will derive a distinguished property of the system (16),(17) that is associated with violation of the second law of thermodynamics i.e. with the capability of moving from disorder to order without help from outside. That property can be predicted qualitatively even prior to analytical proof: due to the nonlinear term in Eq. (17), the solution form shock waves and solitons in probability space, and that can be interpreted as "concentrations" of probability density, i.e. departure from disorder. In order to demonstrate it analytically, let us turn to Eq. (17) at

$$c_1 >> |c_2|, c_3 \tag{29}$$

and find the change of entropy H

$$\frac{\partial H}{\partial t}=-\frac{\partial}{\partial t}\int_{-\infty}^{\infty}\rho\ln\rho\, dV=-\int_{-\infty}^{\infty}\frac{1}{c_1}\dot{\rho}(\ln\rho+1)\,dV=\int_{-\infty}^{\infty}\frac{1}{c_1}\frac{\partial}{\partial V}(\rho^2)\ln(\rho+1)\,dV$$
$$=\frac{1}{c_1}[\,\Big|_{-\infty}^{\infty}\rho^2(\ln\rho+1)-\int_{-\infty}^{\infty}\rho\, dV]=-\frac{1}{c_1}<0 \tag{30}$$

At the same time, the original system (16), (17) is isolated: it has no external interactions. Indeed the information force Eq. (14) is generated by the Liouville equation that, in turn, is generated by the equation of motion (16). Therefore the solution of Eqs. (16), and (17) can violate the second law of thermodynamics, and that means that this class of dynamical systems does not belong to physics as we know it. This conclusion triggers the following question: are there any phenomena in Nature that can be linked to dynamical systems (16), (17)? The answer will be discussed bellow.

Thus despite the mathematical similarity between Eq.(17) and the KdV-Bergers equation, the physical interpretation of Eq.(17) is fundamentally different: it is a part of the dynamical system (16),(17) in which Eq. (17) plays the role of the Liouville equation generated by Eq. (16). As follows from Eq. (30), this system being isolated has a capability to decrease entropy, i.e. to move from disorder to order without external resources. In addition to that, the system displays transition from deterministic state to randomness (see Eq. (27)).

This property represents departure from classical and quantum physics, and, as shown in [2,3], provide a link to behavior of livings. That suggests that this kind of dynamics requires extension of modern physics to include physics of life.

Remark. The system (16), (17) displays transition from deterministic state to randomness (see Eq. (27))., and this property can be linked to the similar property of the Madelung equation, although strictly speaking, Eq.(1) is a “truncated” version of the Liouvile equation: it does not include the contribution of the quantum potential. Nevertheless the origin of randomness in quantum mechanics is the same as in the system (16), (17) as demonstrated in [3,8,9].

6. Hypothetical particle with a diffusion feedback.

In this Section we concentrate on a specific form of the system (16), (17) by choosing the Liouville feedback (12) in the form

$$F=-\sigma^2\frac{\partial}{\partial v}\ln\rho, \tag{31}$$

to obtain the following equation of motion

$$\dot{v}=-\sigma^2\frac{\partial}{\partial v}\ln\rho, \tag{32}$$

The feedback (31) is a particular case of the feedback (14) when

$$c_0=0,\, c_1=0,\, c_2>0,\, c_3=0 \tag{33}$$

This equation should be complemented by the corresponding Liouville equation (in this particular case, the Liouville equation takes the form of the Fokker-Planck equation)

$$\frac{\partial\rho}{\partial t}=\sigma^2\frac{\partial^2\rho}{\partial V^2} \qquad (34)$$

Here *v* stands for a particle velocity, and σ^2 is the diffusion coefficient.

A. Emergence of randomness.

In this sub-section we describe the random solution not only at $t\to 0$, but also in whole time interval.

If $\sigma^2 = const.$ (35)

the solution of Eq. (34) subject to the sharp initial condition

$$\rho=\frac{1}{2\sigma\sqrt{\pi t}}\exp\left(-\frac{V^2}{4\sigma^2 t}\right) \qquad (36)$$

describes diffusion of the probability density, and that is why the feedback (31) can be called a diffusion feedback.

Substituting this solution into Eq. (32) at ***V = v,*** one arrives at the differential equation with respect to *v (t)*

$$\dot{v}=\frac{v}{2t} \qquad (37)$$

and therefore,

$$v=C\sqrt{t} \qquad (38)$$

where *C* is an arbitrary constant. Since *v = 0* at *t = 0* for any value of *C*, the solution (38) is consistent with the sharp initial condition for the solution (36) of the corresponding Liouvile equation (34). The solution (38) describes the simplest irreversible motion: it is characterized by the "beginning of time" where all the trajectories intersect (that results from the violation of Lipcsitz condition at *t =0,* Fig.6), while the backward motion obtained by replacement of *t* with *(-t)* leads to imaginary values of velocities. One can notice that the probability density (36) possesses the same properties.

It is easily verifiable that the solution (36) has the same structure as the solution (27).

Further analysis of the solution (38) demonstrates that this solution is *unstable* since

$$\frac{d\dot{v}}{dv}=\frac{1}{2t}>0 \qquad (39)$$

and therefore, an initial error always grows generating *randomness.* Initially, at *t=0*, this growth is of infinite rate since the Lipchitz condition at this point is violated

$$\frac{\partial\dot{v}}{\partial v}\to\infty \qquad at \qquad t\to 0 \qquad (40)$$

This type of instability has been introduced and analyzed in [5]. The unstable equilibrium point ($v=0$) has been called a terminal repeller, and the instability triggered by the violation of the Lipchitz condition – non-Lipchitz, or terminat instability. The basic property of the non- Lipchitz instability is the following: if the initial condition is infinitely close to the repeller, the transient solution will escape the repeller during a *bounded* time while for a regular repeller the time would be *unbounded*. Indeed, an escape from the simplest regular repeller can be described by the exponent $v=v_0e^t$.

Obviously $v \to 0$ if $v_0 \to 0$, unless the time period is unbounded. On the contrary, the period of escape from the terminal repeller (38) is bounded (and even infinitesimal) if the initial condition is infinitely small, (see Eq. (40)).

Considering first Eq. (38) at fixed C as a sample of the underlying stochastic process (36), and then varying C, one arrives at the whole ensemble characterizing that process, (see Fig. 6). The curves that envelope the cross-sectional blue areas at $t^* = const$ present the probability density distribution at fixed times. One can verify that, as follows from Eq. (36), [6], the expectation and the variance of this process are, respectively

$$\bar{v} = 0, \; \tilde{v} = 2\sigma^2 t \qquad (41)$$

The same results follow from the ensemble (38) at $-\infty \le C \le \infty$. Indeed, the first equality in (41) results from symmetry of the ensemble with respect to $v = 0$; the second one follows from the fact that

$$\tilde{v} \propto v^2 \propto t \qquad (42)$$

It is interesting to notice that the stochastic process (35) is an alternative to the following Langevin equation, [6]

$$\dot{v} = \Gamma(t), \qquad \bar{\Gamma} = 0, \; \tilde{\Gamma} = \sigma \qquad (43)$$

that corresponds to the *same* Fokker-Planck equation (33). Here $\Gamma(t)$ is the Langevin (random) force with zero mean and constant variance σ.

Thus, the emergence of self-generated stochasticity is the first basic non-Newtonian property of the dynamics with the Liouville feedback.

b. Second law of thermodynamics. In order to demonstrate another non-Newtonian property of the systems considered above, let us start with the dimensionless form of the Langevin equation for a one-dimensional Brownian motion of a particle subjected to a random force

$$\dot{v} = \Gamma(t), \quad <\Gamma(t)>= 0, \quad <\Gamma(t)\Gamma(t')>= 2\sigma\delta(t-t'), \; [\Gamma] = 1/s \qquad (44)$$

Here v is the dimensionless velocity of the particle (referred to a representative velocity v_0), and $\Gamma(t)$ is the Langevin (random) force per unit mass, $\sigma > 0$ is the noise strength. The representative velocity v_0 can be chosen, for instance, as the initial velocity of the motion under consideration.

The corresponding continuity equation for the probability density ρ is the following Fokker-Planck equation

$$\frac{\partial \rho}{\partial t} = \sigma \frac{\partial^2 \rho}{\partial V^2}, \int_{-\infty}^{\infty} \rho \, dV = 1 \qquad (45)$$

Obviously without external control, the particle cannot escape the Brownian motion.

Let us now introduce a new force (referred to unit mass and divided by v_0) as a Liouville feedback

$$f = \sigma \exp\sqrt{D}\frac{\partial}{\partial v}\ln\rho, \qquad [f] = 1/s \qquad (46)$$

Here D is the dimensionless variance of the stochastic process $D(t) = \int_{-\infty}^{\infty} \rho V^2 \, dV$,

Then the new equation of motion takes the form

$$\dot{v} = \Gamma(t) + \sigma \exp\sqrt{D} \frac{\partial}{\partial v} \ln \rho, \tag{47}$$

and the corresponding Fokker-Planck equation becomes ***nonlinear***

$$\frac{\partial \rho}{\partial t} = \sigma(1 - \exp\sqrt{D}) \frac{\partial^2 \rho}{\partial V^2}, \quad \int_{-\infty}^{\infty} \rho \, dV = 1 \tag{48}$$

Obviously the diffusion coefficient in Eq. (48) is *negative.* Multiplying Eq. (48) by V^2, then integrating it with respect to V over the whole space, one arrives at ODE for the variance D

$$\dot{D} = 2[\sigma(1 - \exp\sqrt{D})] \tag{49}$$

Thus, as a result of *negative* diffusion, the variance D monotonously vanishes regardless of the initial value $D(0)$. It is interesting to note that the time T of approaching the point $D = 0$ is finite

$$T = \frac{1}{2\sigma} \int_0^{\infty} \frac{dD}{\exp\sqrt{D} - 1} = \frac{\pi}{6\sigma} \tag{50}$$

This terminal effect is due to violation of the Lipchitz condition, at $D = 0$, [5].

Let us review the structure of the force (46): it is composed only out of the probability density and its variance, i.e. out of the components of the conservation equation (47); at the same time, Eq. (47) itself is generated by the equation of motion (46). Consequently, the force (45) is *not* an external force. Nevertheless, it allows the particle to escape from the Brownian motion using its own "internal effort". It would be reasonable to call the force (45) an *information force* since it links to information rather than to energy.

Thus, we came across the phenomenon that violates the second law of thermodynamics when the dynamical system moves from disorder to order without external interactions due to a feedback from the equation of conservation of the probability to the equation of conservation of the momentum. One may ask why the negative diffusion was chosen to be nonlinear. Let us turn to a linear version of Eq. (49)

$$\frac{\partial \rho}{\partial t} = -\sigma^2 \frac{\partial^2 \rho}{\partial V^2}, \qquad \int_{-\infty}^{\infty} \rho \, dV = 1 \tag{51}$$

and discuss the negative diffusion in more details. As follows from the linear equivalent of Eq. (49)

$$\dot{D} = -2\sigma, i.e.\ D = D_0 - 2\sigma t < 0 \quad at \quad t > D_0 / (2\sigma) \tag{52}$$

Thus, eventually the variance becomes negative, and that disqualifies Eq. (3.30) from being meaningful. As shown in [3], the initial value problem for this equation is ill-posed: its solution is not differentiable at any point. Therefore, a *negative diffusion must be nonlinear* in order to protect the variance from becoming negative, Fig.4.

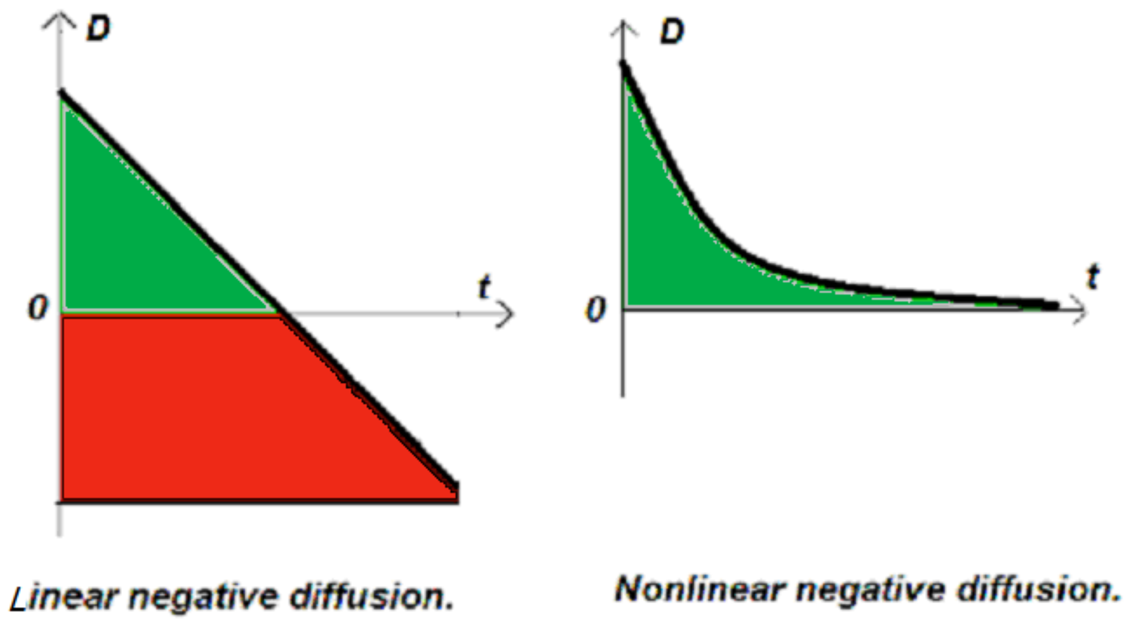

Figure 4. Negative diffusion.

It should be emphasized that negative diffusion represents a major departure from both Newtonian mechanics and classical thermodynamics by providing a progressive evolution of complexity against the Second Law of thermodynamics.

Next we will demonstrate again that formally the dynamics introduced above does not belong to the Newtonian world; nevertheless its self-supervising capability may associate such a dynamics with a potential model for intelligent behavior. For that purpose we will turn to even simpler version of this dynamics by removing the external Langevin force and simplifying the information force:

$$\dot{v} = \sigma\sqrt{D}\frac{\partial}{\partial v}\ln\rho, \qquad (53)$$

$$\frac{\partial\rho}{\partial t} = -\sigma\sqrt{D}\frac{\partial^2\rho}{\partial V^2}, \quad \int_{-\infty}^{\infty}\rho\, dV = 1 \qquad (54)$$

Removal of the Langevin forces makes the particle *isolated*. Nevertheless the particle has a capability of moving from disorder to order. For demonstration of this property we will assume that the Langevin force was suddenly removed at $t = 0$ so that the initial variance $D_0 > 0$. Then

$$\dot{D} = -2\sigma\sqrt{D} \tag{55}$$

whence $$D = (\sqrt{D_0} - \sigma t)^2 \tag{56}$$

As follows from Eq. (56), as a result of *internal, self-generated* force

$$F = \sigma\sqrt{D}\frac{\partial}{\partial v}\ln\rho, \tag{57}$$

the Brownian motion gradually disappears and then vanishes abruptly:

$$D \to 0, \qquad \dot{D} \to 0, \qquad \frac{d\dot{D}}{dD} \to \infty \qquad at \qquad t \to \frac{\sqrt{D_0}}{\sigma} \tag{58}$$

Thus the probability density shrinks to a delta-function at $t = \frac{\sqrt{D_0}}{\sigma}$. Consequently, the entropy $H(t) = -\int_V \rho \ln \rho dV$ decreases down to zero, and that violates the second law of thermodynamics, Fig. 5.

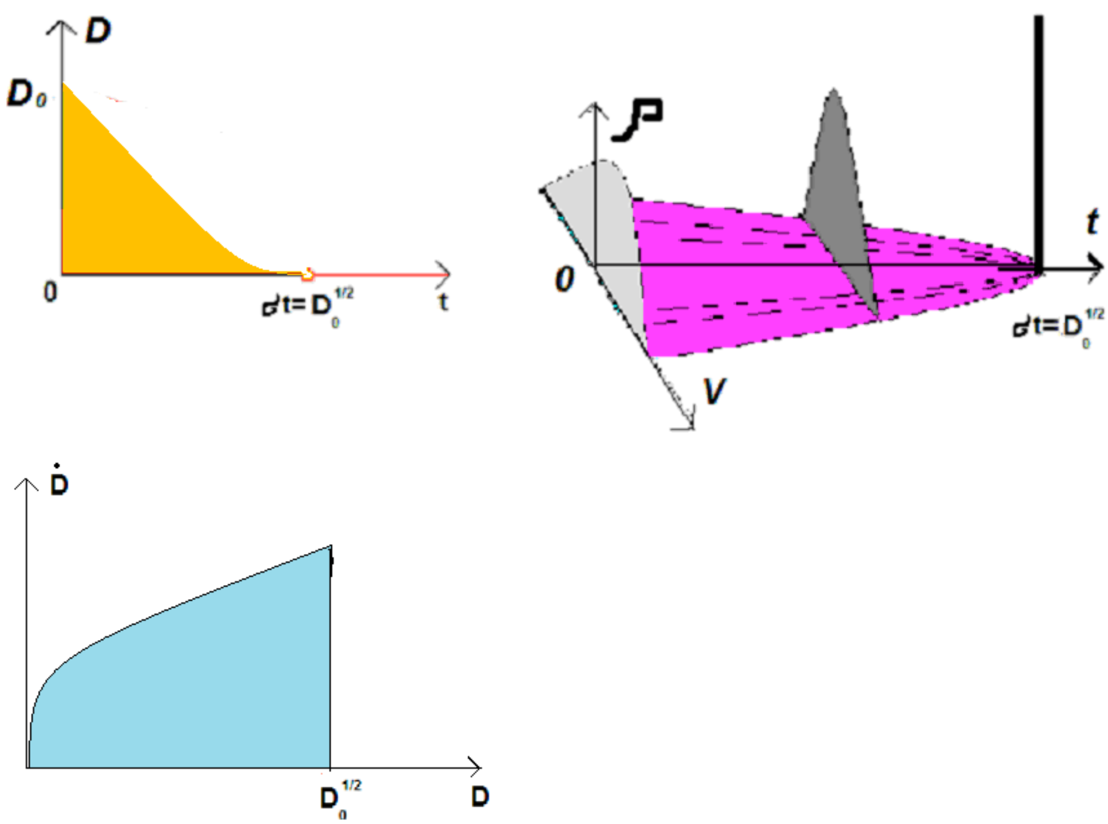

Figure 5. Vanishing Brownian motion.

C. Violation of the first law of thermodynamics.

Let us turn to the general case described by Eq. (9). As follows from this equation, the particle under consideration possesses only kinetic energy

$$W = v^2 / 2 \tag{59}$$

However this energy is not conserved although the particle is isolated. Indeed,

$$dW = \mathbf{v} \cdot \mathbf{F}[\rho(\mathbf{v})]dt \tag{60}$$

i.e. change of the kinetic energy is equal to the work done by the *self-generated information* force $\mathbf{F}[\rho(\mathbf{v})]$. But in contradistinction to dissipative systems, this work can be positive, i.e. an information force can increase the kinetic energy of the particle. In particular, that would happen in case of *negative* diffusion.

The significance of Eq. (60) is fundamental: it relates the change of energy to change of information.

7. Hypothetical particle with soliton feedback.

In this section we introduce the structure of the force **F** that is a particular case of the feedback (14) at

$$c_2 = 0 \tag{61}$$

i.e.

$$F = c_0 + \frac{1}{2}c_1\rho + \frac{c_3}{\rho}\frac{\partial^2\rho}{\partial v^2} \tag{62}$$

Then Eq.(16) can be reduced to the following:

$$\dot{v} = c_0 + \frac{1}{2}c_1\rho + \frac{c_3}{\rho}\frac{\partial^2\rho}{\partial v^2} \tag{63}$$

and the corresponding Liouville equation will turn into the following PDE

$$\frac{\partial\rho}{\partial t} + (c_0 + c_1\rho)\frac{\partial\rho}{\partial V} + c_3\frac{\partial^3\rho}{\partial V^3} = 0 \tag{64}$$

that is a celebrated Korteweg-de Vries (KdV) equation.

However a fundamental difference between the standard KdV equation and Eq. (64) is that Eq. (64) ***dwells in the probability space***, and therefore, it must satisfy the normalization constraint

$$\int_{-\infty}^{\infty} \rho\, dV = 1 \tag{65}$$

But since the KdV equation has the conservation invariants, [7]

$$\int_{-\infty}^{\infty} \rho\, dV = Const., \tag{66}$$

$$\int_{-\infty}^{\infty} \rho^2\, dV = Const., \text{ etc.} \tag{67}$$

the constraint (65) becomes a particular case of the invariant (66); consequently, if the normalization constraint is satisfied at *t = 0*, it is satisfied all the time. That allows one to apply all the known result directly to Eq. (64). However it should be noticed that the conservation invariants (66) and (67) have different physical meaning: they are not related to conservation of momentum and energy, but rather impose constraints upon the Shannon information.

We will start the analysis of the equation (64) with consideration of its linear version when $c_1 = 0$

$$\frac{\partial \rho}{\partial t} + c_0 \frac{\partial \rho}{\partial V} + b \frac{\partial^3 \rho}{\partial V^3} = 0 \qquad (68)$$

The first applications of linear (parabolic) version of KdV equation appear in models of shallow water waves [7]. The equation is also conservative, and its solution is represented by a train of traveling waves

$$\rho(v,t) = Ae^{ikv - \omega t} \qquad (69)$$

where ω is the frequency, and k is the wave number. For KdV equation, these two constants are connected by the following dispersion relation

$$\omega = c_0 k - bk^3 \qquad (70)$$

If the initial profile $\rho = u(v,0)$ is represented as a sum of the Fourier harmonics, then each of this harmonic will propagate with the phase speed

$$C = \omega / k \,. \qquad (71)$$

Comparing equations (70) and (71), one can see that each Fourier harmonics will propagate with different phase speed that depends upon its wave number k. Therefore any initial profile eventually is dispersed over the whole positive subspace, Fig.6.

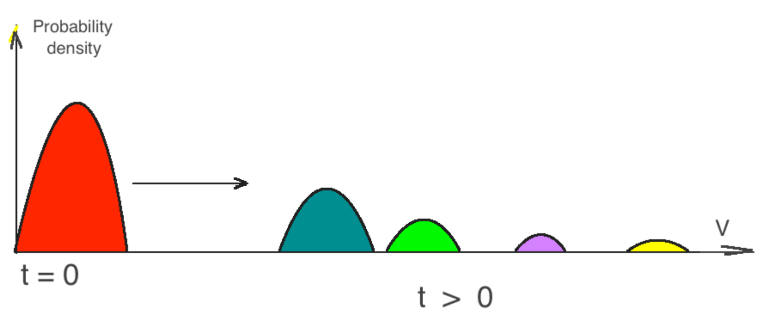

Figure 6. Linear dispersion of initial profile.

An important property of the linear version of the KdV equation is the dependence of its solution on the initial conditions for all times.

Let us assume now that

$$b = 0, \quad c_0 = 0 \qquad (72)$$

We get the equation

$$\frac{\partial \rho}{\partial t}+c_1\rho\frac{\partial \rho}{\partial V}=0 \tag{73}$$

Unlike the previous versions of the KdV equation, this is a nonlinear PDE of hyperbolic type. It appears in models of free particles flow, traffic jam, etc. This is the simplest equation that describes formation of shock waves. Its closed analytical solution can be written only in an implicit form, and here we will analyze it only qualitatively. We will start our analysis with studying a propagation of an initial profile $\rho=\rho(v,0)$. As follows from Equation (73), the higher values of ρ propagate faster than lower ones. As a result, the moving front becomes steeper and steeper, and finally a strong discontinuity representing a shock emerges, see Fig.7.

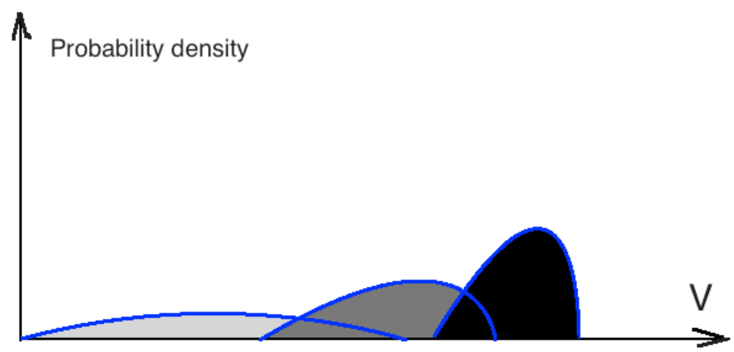

Figure 7. Formation of shock waves in probability space.

Since closed form solution of Eq. (73) is not available, we will continue with the solution for large time. The rationale for that is the assumption that eventually the solution tends to a stationary shape as a result of a balance between dispersion and shock wave formation. Therefore we will seek the solution in the form of a stationary motion

$$\rho(v,t)=f(v-Ut)=u(\zeta) \quad at \quad t\to\infty \tag{74}$$

Substituting Eq.(74) into Eq.(73) one obtains

$$-U\frac{\partial \rho}{\partial \zeta}+(c_0+c_1\rho)\frac{\partial \rho}{\partial \zeta}+b\frac{\partial^3 \rho}{\partial \zeta^3}=0 \tag{75}$$

Integrating this equation with respect to ζ and setting the arbitrary constant to zero, one arrives at the ODE in its final form

$$b\frac{\partial^2 \rho}{\partial \zeta^2}+(c_0-U)\rho+\frac{c_1}{2}\rho^2=0 \tag{76}$$

The solution of this equation is a soliton moving with the speed U

$$\rho = aSech^2[\frac{\sqrt{c_1 a}}{\sqrt{12b}}(v - Ut)] \quad (77)$$

where

$$U = c_0 + \frac{1}{3}c_1 a \quad (78)$$

see Fig. 8. It should be emphasized that the soliton (77) does not depend upon initial conditions, and consequently it can be considered as a static attractor ***in probability space***. This means that in physical space, a solution of Eq. (63) eventually approaches a stochastic attractor. The analytical form of this solution at $t \to 0$ was derived in Section 4, (see Eq. (26), and Fig. 2).

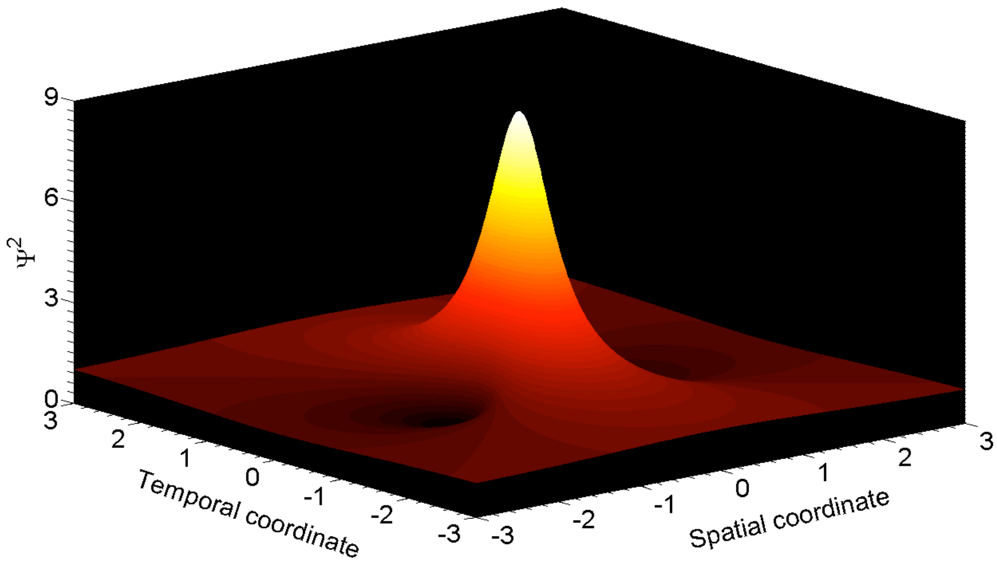

Figure 8. Soliton as an attractor of KdV solution.

It should be emphasized that the dynamics system (63), (64) is ***isolated,*** but despite of that, its entropy decreases in the course of the soliton wave formation.

7. Origin of intelligence.

A. Relevance to model of intelligent particle. The proposed model illuminates the "border line" between living and non-living systems. The model introduces an intelligent particle that, in addition to Newtonian properties, possesses the ability to process information. The probability density can be associated with the *self-image* of the intelligent particle as a member of the class to which this particle belongs, while its ability to convert the density into the information force - with the *self-awareness* (both these concepts are adopted from psychology). Continuing this line of associations, the equation of motion (such as Eqs (16) or (32)) can be identified with a motor dynamics, while the evolution of density (see Eqs. (17) or (34) –with a mental dynamics. Actually the mental dynamics plays the role of the Maxwell sorting demon: it rearranges the probability distribution by creating the information potential and converting it into a force that is applied to the particle. One should notice that mental dynamics describes evolution of the whole class of state variables (differed from each other only by initial conditions), and that can be associated with the ability to generalize that is a privilege of intelligent systems. Continuing our biologically inspired interpretation, it should be recalled that the second law of thermodynamics states that the entropy of an isolated system can only increase. This law has a clear probabilistic interpretation: increase of entropy corresponds to the passage of the system from less probable to more probable states, while the highest probability of the most disordered state (that is the state with the highest entropy) follows from a simple combinatorial analysis. However, this statement is correct only if there is no Maxwell' sorting demon, i.e., nobody inside the system is rearranging the probability distributions. But this is precisely what the Liouville feedback is doing: it takes the probability density ρ from Equation (17), creates functionals and functions of this density, converts them into the information force and applies this force to the equation of motion (16). As already mentioned above, because of that property of the model, the evolution of the probability density can become nonlinear, and the entropy may decrease "against the second law of thermodynamics", Fig.10. Actually the proposed model represents governing equations for interactions of intelligent agents. In order to emphasize the autonomy of the agents' decision-making process, we will associate the proposed models with ***self-supervised (SS) active systems.*** By an active system we will understand here a set of interacting intelligent agents capable of processing information, while an intelligent agent is an autonomous entity, which observes and acts upon an environment and directs its activity towards achieving goals. The active system is not derivable from the Lagrange or Hamilton principles, but it is rather created for information processing. One of specific differences between active and physical systems is that the former are supposed to act in uncertainties originated from incompleteness of information. Indeed, an intelligent agent almost never has access to the whole truth of its environment. Uncertainty can also arise because of incompleteness and incorrectness in the agent's understanding of the properties of the environment. That is why *quantum-inspired SS* systems represented by the particles under consideration are well suited for representation of active systems, and the hypothetical particle introduced above can be associated with the term "intelligent" particle. It is important to emphasize that self-supervision is implemented by the feedback from mental dynamics, i.e. by internal force, since the mental dynamics is generated by intelligent particle itself.

B. Comparison with control systems. In this sub-section we will establish a link between the concepts of intelligent control and phenomenology of behavior of intelligent particle.

*Example**.*** One of the limitations of classical dynamics, and in particular, neural networks, is inability to change their structure without an external input. As will be shown below, an intelligent particle can change the locations and even the type of the attractors being triggered only by information forces i.e. by an internal effort. We will start with a simple dynamical system

$$\dot{v} = 0, \quad v = 0 \quad at\ t = 0 \tag{79}$$

and than apply the following control

$$F = -k\bar{v} + a\bar{\bar{v}} - \sigma \frac{\partial}{\partial v} \ln \rho , \qquad (80)$$

where $\bar{\bar{V}} = \int_{-\infty}^{\infty} \rho (V - \bar{V})^2 dV, \qquad \bar{V} = \int_{-\infty}^{\infty} \rho V dV ,$ (81)

and k, a, σ are constant coefficients.

Then the controlled version of the motor dynamics (79) is changed to

$$\dot{v} = -k\bar{v} + a\bar{\bar{v}} - \sigma \frac{\partial}{\partial v} \ln \rho \qquad (82)$$

while F represents the information forces that play the role of ***internal*** actuator.

Let us notice that the internal actuator (80) is a particular case of the information force (14) at

$$c_0 = -k\bar{v} + a\bar{\bar{v}}, \quad c_1 = 0, \quad c_2 = \sigma, \quad c_3 = 0 \qquad (83)$$

For a closure, Eq. (82) is complemented by the corresponding Liouville equation

$$\frac{\partial \rho}{\partial t} = k\bar{V} \frac{\partial \rho}{\partial V} - a\bar{\bar{V}} \frac{\partial \rho}{\partial V} + \sigma \frac{\partial^2 \rho}{\partial V^2} , \qquad (84)$$

to be solved subject to sharp initial condition

$$\rho_0(V) = \delta(V)\, at \qquad t = 0, \qquad (85)$$

As shown in Section 4, the solution of Eq.(82) is random, (see Eq. (26) and Fig. 2) while this randomness is controlled by Eq. (84). Therefore in order to describe it, we have to transfer to the mean values $\bar{v}$ and $\bar{\bar{v}}$. For that purpose, let us multiply Eq.(84) by V. Then integrating it with respect to V over the whole space, one arrives at ODE for the expectation $\bar{v}(t)$

$$\dot{\bar{v}} = -k\bar{v} + a\bar{\bar{v}} \qquad (86)$$

Multiplying Eq.(84) by V^2, then integrating it with respect to V over the whole space, one arrives at ODE for the variance $\bar{\bar{v}}(t)$

$$\dot{\bar{\bar{v}}} = -2k\bar{\bar{v}} + 2a\bar{v}\,\bar{\bar{v}} + 2\sigma \qquad (87)$$

Let us find fixed points of the system (86) and (87) by solving the system of algebraic equations:

$$0 = -k\bar{v} + a\bar{\bar{v}} \qquad (88)$$

$$0 = -2k\bar{\bar{v}} + 2a\bar{v}\,\bar{\bar{v}} + 2\sigma \qquad (89)$$

By selecting

$$\sigma = \frac{k^3}{2a^2} \tag{90}$$

we arrive at the following single fixed point

$$\bar{v}^* = \frac{k}{2a}, \quad \bar{\bar{v}}^* = \frac{k^2}{2a^2} \tag{91}$$

In order to establish whether this fixed point is an attractor or a repeller, we have to analyze stability of the homogeneous version of the system (86), (87) linearized with respect to the fixed point (91)

$$\dot{\bar{v}} = -k\bar{v} + a\bar{\bar{v}} \tag{92}$$

$$\dot{\bar{\bar{v}}} = -k\bar{\bar{v}} + \frac{k^2}{a}\bar{v} \tag{93}$$

Analysis of its characteristic equation shows that it has non-positive roots:

$$\lambda_1 = 0, \quad \lambda_2 = -2k < 0 \tag{94}$$

and therefore, the fixed point (91) is a stochastic attractor with stationary mean and variance. However the higher moments of the probability density are not necessarily stationary: they can be found from the original PDE (84).

Thus as a result of a ***mental*** control, an ***isolated*** dynamical system (79) that prior to control was at rest, moves to the stochastic attractor (91) having the expectation $\bar{v}^*$ and the variance $\bar{\bar{v}}^*$.

The distinguished property of the particle introduced above definitely fits into the concept of intelligence. Indeed, the evolution of intelligent living systems is directed toward the highest levels of complexity if the complexity is measured by an irreducible number of different parts that interact in a well-regulated fashion. At the same time, the solutions to the models based upon dissipative Newtonian dynamics eventually approach attractors where the evolution stops while these attractors dwell on the subspaces of lower dimensionality, and therefore, of the lower complexity (until a "master" reprograms the model). Therefore, such models fail to provide an autonomous progressive evolution of intelligent systems (i.e. evolution leading to increase of complexity). At the same time, a self-controlled particle can create its own complexity based only upon an *internal* effort.

Thus the actual source of intelligent behavior of the particle introduced above is a new type of force - the information force - that contributes its work into the Law of conservation of energy. However this force is internal: it is generated by the particle itself with help of the Liouvile equation. The machinery of the intelligence is similar to that of control system with the only difference that control systems are driven by external actuators while the intelligent particle is driven by a feedback from the Liouvile equation without any external resources.

8. Comparison with quantum mechanics.

a. Mathematical Viewpoint. The model of intelligent particle is represented by a nonlinear ODE (9) and a nonlinear parabolic PDE (10) coupled in a master-slave fashion: Eq. (10) is to be solved independently, prior to solving Eq. ((9). The coupling is implemented by a feedback that includes the probability density and its space derivatives, and that converts the first order PDE (the Liouville equation) to the second or higher order nonlinear PDE. As a result of the nonlinearity, the solutions to PDE can have attractors (static, periodic, or chaotic) in probability space. The solution of ODE (9) represents another major departure from classical ODE: due to violation of Lipchitz conditions at states where the probability density has a sharp value, the solution loses its uniqueness and becomes random. However, this randomness is controlled by the PDE (10) in such a way that each random sample occurs with the corresponding probability, Fig.6.

b. Physical Viewpoint. The model of intelligent particle represents a fundamental departure from both Newtonian and quantum mechanics. The fundamental departure of all the modern physics is the violation of the first and the second laws of thermodynamics,(see Eqs.(60), (30), (58), Figs. 5 and 7). However the model has some similarity to quantum mechanics, and these similarities are outlined below.

α.Superposition. In quantum mechanics, any observable quantity corresponds to an eigenstate of a Hermitian linear operator. The linear combination of two or more eigenstates results in quantum superposition of two or more values of the quantity. If the quantity is measured, the projection postulate states that the state will be randomly collapsed onto one of the values in the superposition (with a probability proportional to the square of the amplitude of that eigenstate in the linear combination). Let us compare the behavior of the model of intelligent particle from that viewpoint. As follows from Eq. (38), all the particular solutions intersect at the same point $v = 0$ at $t = 0$, and that leads to non-uniqueness of the solution due to violation of the Lipcshitz condition (see Eq. (40). Therefore, the same initial condition $v = 0$ at $t = 0$ yields infinite number of different solutions forming a family (38); each solution of this family appears with a certain probability guided by the corresponding Fokker-Planck equation. For instance, in case of Eq. (38), the "winner" solution is $v \equiv 0$ since it passes through the maxima of the probability density (36). However, with lower probabilities, other solutions of the family (38) can appear as well. Obviously, this is a non-classical effect. Qualitatively, this property is similar to those of quantum mechanics: the system keeps all the solutions simultaneously and displays each of them "by a chance", while that chance is controlled by the evolution of probability density (36).

β. Entanglement. Quantum entanglement is a phenomenon in which the quantum states of two or more objects have to be described with reference to each other, even though the individual objects may be spatially separated. This leads to correlations between observable physical properties of the systems. For example, it is possible to prepare two particles in a single quantum state such that when one is observed to be spin-up, the other one will always be observed to be spin-down and vice versa, this despite the fact that it is impossible to predict, according to quantum mechanics, which set of measurements will be observed. As a result, measurements performed on one system seem to be instantaneously influencing other systems entangled with it.

Qualitatively similar effect can be found in the model of intelligent particle. In order to demonstrate that, we start with Eqs.(32) and (34) and generalize them to the two-dimensional case

$$\dot{v}_1 = -a_{11}\frac{\partial}{\partial v_1}\ln\rho - a_{12}\frac{\partial}{\partial v_2}\ln\rho, \tag{95}$$

$$\dot{v}_2 = -a_{21}\frac{\partial}{\partial v_1}\ln\rho - a_{22}\frac{\partial}{\partial v_2}\ln\rho, \tag{96}$$

$$\frac{\partial\rho}{\partial t} = a_{11}\frac{\partial^2\rho}{\partial V^2} + (a_{12}+a_{21})\frac{\partial^2\rho}{\partial V_1 \partial V_2} + a_{22}\frac{\partial^2\rho}{\partial V_2}, \tag{97}$$

As in the one- dimensional case, this system describes diffusion without a drift
The solution of Eq. (97) has a closed form

$$\rho = \frac{1}{\sqrt{2\pi\det[\hat{a}_{ij}]t}}\exp\left(-\frac{1}{4t}b'_{ij}V_iV_j\right), \qquad i = 1,2. \tag{98}$$

Here

$$[b'_{ij}]=[\hat{a}_{ij}]^{-1},\ \hat{a}_{11} = a_{11}, \hat{a}_{22} = a_{22}, \hat{a}_{12} = \hat{a}_{21} = a_{12} + a_{21},\ \hat{a}_{ij} = \hat{a}_{ji}, b'_{ij} = b'_{ji}, \tag{99}$$

Substituting the solution (98) into Eqs. (95) and (96), one obtains

$$\dot{v}_1 = \frac{b_{11}v_1 + b_{12}v_2}{2t} \tag{100}$$

$$\dot{v}_2 = \frac{b_{21}v_1 + b_{22}v_2}{2t}, \qquad b_{ij} = b'_{ij}\hat{a}_{ij} \tag{101}$$

Eliminating *t* from these equations, one arrives at the ODE in configuration space

$$\frac{dv_2}{dv_1} = \frac{b_{21}v_1 + b_{22}v_2}{b_{11}v_1 + b_{12}v_2},\ v_2 \to 0 \qquad at \qquad v_1 \to 0, \tag{102}$$

This is a classical singular point treated in textbooks on ODE.
Its solution depends upon the roots of the characteristic equation

$$\lambda^2 - 2b_{12}\lambda + b_{12}^2 - b_{11}b_{22} = 0 \tag{103}$$

Since both the roots are real in our case, let us assume for concreteness that they are of the same sign, for instance, $\lambda_1 = 1, \quad \lambda_2 = 1$. Then the solution of Eq. (102) is presented by the family of straight lines

$$v_2 = \tilde{C}v_1, \qquad \tilde{C} = const. \tag{104}$$

Substituting this solution into Eq. (100) yields

$$v_1 = Ct^{\frac{1}{2}(b_{11}+\tilde{C}b_{12})} \qquad v_2 = \tilde{C}Ct^{\frac{1}{2}(b_{11}+\tilde{C}b_{12})} \tag{105}$$

Thus, the solutions of Eqs. (95) and (96) are represented by two-parametrical families of random samples, as expected, while the randomness enters through the time-independent parameters C and $\tilde{C}$ that can take any real numbers. Let us now find such a combination of the variables that is deterministic. Obviously, such a combination should not include the random parameters C or $\tilde{C}$. It easily verifiable that

$$\frac{d}{dt}(\ln v_1) = \frac{d}{dt}(\ln v_2) = \frac{b_{11} + \tilde{C}b_{12}}{2t} \tag{106}$$

and therefore,

$$(\frac{d}{dt}\ln v_1)/(\frac{d}{dt}\ln v_2) \equiv 1 \tag{107}$$

Thus, the ratio (107) is deterministic although both the numerator and denominator are random,(see Eq.(106). This is a fundamental non-classical effect representing a global constraint. Indeed, in theory of stochastic processes, two random functions are considered statistically equal if they have the same statistical invariants, but their point-to-point equalities are not required (although it can happen with a vanishingly small probability). As demonstrated above, the *diversion of determinism into randomness via instability (due to a Liouville feedback), and then conversion of randomness to partial determinism (or coordinated randomness) via entanglement* is the fundamental non-classical paradigm.
γ. Decoherence. In quantum mechanics, decoherence is the process by which quantum systems in complex environments exhibit classical behavior. It occurs when a system interacts with its environment in such a way that different portions of its wavefunction can no longer interfere with each other.

Qualitatively similar effects are displayed by the intelligent particle. In order to illustrate that, let us turn to Eqs. (32), (34), and notice that, as soon as the feedback (31) disappears, the system becomes classical, i.e. fully deterministic, while the deterministic solution is a continuation of the corresponding "chosen" random solution.

δ. Uncertainty Principle. In quantum physics, the Heisenberg uncertainty principle states that one cannot measure values (with arbitrary precision) of certain conjugate quantities that are pairs of observables of a single elementary particle. These pairs include the position and momentum. Similar (but not identical) relationship follows from Eq. (38):

$$v\dot{v} = C^2 / 2 \tag{108}$$

i.e. the product of the velocity and the acceleration is constant along a fixed trajectory. In particular, at $t = 0$, v and $\dot{v}$ can not be defined separately.

ε. *Wave-particle duality.* In physics, wave–particle duality is a conceptualization that all objects in our universe exhibit properties of both waves (such as non-locality) and of particles (such as quantization of some of their properties). As shown by Max Born, the wave associated with the electron is not a tangible 'matter wave', but one that determines the *probability* of scattering of the electron in different directions. Similar “duality” follows from the model of intelligent particle. Indeed, Eq. (32) describes the “trajectories” of particles, while Eq. (34) represents the wave of probability that captures the particle “scattering”.

η. *Interference of probabilities.* In Newtonian physics, the probability is introduced via the Liouville equation describing the continuity of the probability density flow. This equation is linear with respect to the probability density, and therefore, according to the superposition principle, the probabilities are combined by summation: when an event can occur in several alternative ways, the probability of the event is the sum of the probabilities for each way considered separately, i.e.

$$\rho = \rho_1 + \rho_2 \tag{109}$$

In quantum physics, the probability is introduced via the Schrödinger equation that is linear with respect to probability amplitude, i.e. with respect to the square root of the probability density. Therefore, when an event can occur in several alternative ways, the probability amplitude of the event is the sum of the probability amplitudes for each way considered separately

The probability interference in quantum mechanics follows from the ***linearity*** of the Schrödinger equation with respect to the probability amplitudes ψ_i as state variables. Due to linear superposition of these amplitudes, the following rule can be formulated

$$\psi = \psi_1 + \psi_2, \ \rho_i = |\psi_i|^2, \quad \rho = |\psi_1 + \psi_2|^2 \neq \rho_1 + \rho_2 \tag{110}$$

and this phenomenon is known as interference of probabilities: the probabilities are combined as the intensities of waves.

The situation with interference of probabilities in the model of intelligent particle is more complex: it depends upon the type of information forces. Indeed, in the diffusion and the integral feedbacks cases, Eqs.(32), and (34) are linear with respect to the probability density, and the probabilities are combined according to Eq. (109). i.e. without interference. But in the shock/solpton feedback, the Liouville equation is nonlinear with respect to the probability density, and consequently, the probabilities interfere, (see Eqs.(64) and (73)). However, this interference is different from the quantum one and it will be discussed below.

Indeed, following [7] and reinterpreting confluence of shock waves in physical space to confluence of densities in probability space obtain the rule of combining the probabilities

$$\rho = \frac{\rho_1 f_1 + \rho_2 f_2}{f_1 + f_2}, \qquad \rho_2 > \rho_1 \tag{111}$$

where $f_i = \exp(-\frac{\rho_i V}{2\sigma} + \frac{\rho_i^2 t}{4\sigma}), \qquad i = 1, 2.$ (112)

This means that when an event can occur in several alternative ways, the probability of the event is the sum of nonlinear combinations of the probabilities for each way considered separately.

10. Discussion and Conclusion.

The discovery of the Higgs boson and the following from it completeness of the physical picture of our Universe roused many questions, and one of them is the ability to create Life and Mind out of physical matter without any additional entities. The primary objective of this paper is to presents a ***mathematical*** answer to the ancient philosophical question, "How mind is related to matter" in connection with this outstanding accomplishment in physics. The paper is inspired by analysis of the Madelung equation and discovery of the origin of randomness in quantum mechanics, [3,8,9]. It turns out that replacement of the quantum potential by the information force, while preserving some quantum properties, introduces fundamental changes in the first and the second laws of thermodynamics, and that leads to a mathematical model that captures behavior of livings. The idea of an intelligent particle has been introduced as a first step of physics of life since it does not include such properties as metabolism and reproduction. Instead it concentrates attention to intelligent behavior. At the same time, by ignoring metabolism and reproduction, we can make the system isolated, and it will be a challenge to show that it still can move from a disorder to the order.

Thus the paper introduces and discusses a possible extension of modern physics to include a concept of intelligent particle as the first step to physics of Life since all attempts to create livings from non-living matter failed. It has been proven that there exists a fundamentally new type of dynamical systems (represented by intelligent particles) that can evolve from disorder to order without external forces thereby violating the second law of thermodynamics. It has been demonstrated that these systems belong neither to Newtonian, nor to quantum mechanics. Their departure from Newtonian mechanics is due to a feedback from the underlying Liouville equation to the equations of motion that represents an additional (internal) information force. Topologically this feedback shifts intelligent particles towards quantum mechanics. However since the information force is different from forces produced by quantum potential, the intelligent particles are not quantum, and they can be identified as quantum-classical hybrids. Therefore intelligent particles dwell in an abstract mathematical world rather than in the physical world, as we know it. This means that intelligent particles, in principle, cannot be composed out of physical particles. It also means that their behavior can be computed, but not simulated using Newtonian or quantum resources.

Since the model of intelligent particle fits well into the mathematical formalism of modern physics, it can be consider as a new branch of quantum mechanics, and that rouses a belief that intelligent particle is not only a mathematical abstraction, but a reality as well.

References.

1. Schrödinger, E**.** What is life, Cambridge University Press, 1944
2. Zak, M., 2013, Information: Forgotten Variable in Physics Models, EJTP 10, No. 28, 199–220,
3. Zak, M., 2013, Particle of life: mathematical abstraction or reality? , Nova publishers, NY.
4. G. Whitham, *Linear and Nonlinear Waves* Wiley-Interscience, New York, 1974.
5. Zak, M., "Terminal Model of Newtonian Dynamics," Int. J. of Theoretical Physics, No.32, 159-190, 1992.
6. Risken,H., 1989, The Fokker-Planck Equation, Springer, NY.
7. G. Whitham, *Linear and Nonlinear Waves* Wiley-Interscience, New York, 1974.
8. Zak, M., 2009, Hidden statistics of Schrödinger equation, Physics essays, 22, 2

9. Zak, M., 2014, The origin of randomness in quantum mechanics, EJTP, (in press).