On the alleged 'Incompleteness' of General Relativity

Stephen J. Crothers
Queensland, Australia
thenarmis@gmail.com

3rd June 2014

ABSTRACT

Although no theory is complete in that it is perfect and never needing update or modification with the discovery of new experimental facts, that is very different to a theory being logically inconsistent and also different to a theory being in conflict with a fundamental principle in physics that has been established by a vast array of experiments. Relying upon the incompleteness of a theory as an argument to keep it despite inconsistencies in logic and experiment is therefore unjustifiable, and this is particularly so in the case of General Relativity for which proponents thereof plead incompleteness to ignore inconsistencies. Logical inconsistency alone invalidates a theory. General Relativity is logically inconsistent and so that alone invalidates it. It is also in conflict with a fundamental principle of physics, determined by many experiments, to wit, in General Relativity the usual conservation of energy and momentum for a closed system is violated.

I. Introduction

I present nothing new herein as I have already dealt with these issues in my papers and other articles [1], and so I refer readers to them for more details as I provide here an overview. In doing so I shall still remain within the ambit of the tenets of General Relativity itself.

This article arose as a response to a critic who wrote to me in support of Einstein's General Theory of Relativity, before a forum of interested scientists and thinkers. After my response was circulated it was suggested that my arguments would be of interest to a broader audience, in the form of an article; this one.

II. Some Inconsistencies of General Relativity

According to Einstein, matter is everything except his alleged gravitational field:

"*We make a distinction hereinafter between 'gravitational field' and 'matter' in this way, that we denote everything but the gravitational field as 'matter'. Our use of the word therefore includes not only matter in the ordinary sense, but the electromagnetic field as well.*" [2]

However, he also maintained that his gravitational field possesses energy and momentum [2].

Consider Einstein's field equations in the following form,

$$R_{uv} = -\kappa\left(T_{uv} - \frac{1}{2}Tg_{uv}\right) \qquad (1)$$

According to Einstein when T_{uv} = 0 this reduces to,

$$R_{uv} = 0 \qquad (2)$$

The solution to (2) is called the 'Schwarzschild solution'. It is routine amongst cosmologists to consider a 'weak' gravitational field and a very slow moving 'particle' in relation to the 'Schwarzschild solution' to finally obtain an expression for the component of the metric tensor g_{00} in terms of the Newtonian potential function φ. The inclusion of φ in $g_{00,}$ although standard, is *ad hoc*, by means of a false analogy with Newton's theory. Equations (2) are Einstein's analogue to the Laplace equation.

Eventually the divergence of the Newtonian potential function is often equated to R_{00} to obtain the Poisson equation by assuming a particular form for T_{00}. One can't use the 'Schwarzschild solution' to effect this analogue of the Poisson equation since (2) allegedly pertains to an analogue of the Laplace equation. When Einstein developed his analogue of the Poisson equation he had no 'Schwarzschild solution' to work with. Instead he began with his alleged analogue of the Laplace equation and attributed energy and momentum to his gravitational field, the latter he described by the following form of (2), with a constraint [2],

$$\frac{\partial \Gamma^{a}_{uv}}{\partial x^{a}} + \Gamma^{a}_{ub}\Gamma^{b}_{va} = 0 \qquad (3)$$

$$\sqrt{-g} = 1$$

Now it is easily proven that (2), and hence (3), are physically meaningless because neither contains any matter and therefore does not describe a gravitational field. This is sufficient to render all the standard claims for (2) completely false.

The energy-momentum tensor allegedly describes the matter that causes Einstein's gravitational field. Matter by its presence induces curvature in spacetime (Einstein's gravitational field). Since (2) is obtained from (1) by setting T_{uv} = 0, there is no matter present. All matter terms are removed from equations (1) by setting T_{uv} = 0. Nonetheless Einstein claimed that a material source (a mass) is still present. He does so by asserting that equations (2), and (3), describe his gravitational field *outside a body such as a star*. Thus, all matter is removed by mathematical construction on the one hand, and then immediately reinstated on the other hand, not in the equations (2) or (3), but by the words *outside a body such as a star*. This is merely linguistic legerdemain. Equations (2) and (3) contain no matter by mathematical construction. Equations (1) show that Einstein's field equations couple his gravitational field, manifest in the curvature of spacetime, to its material sources. Without matter to cause it there is no gravitational field. That's precisely why Einstein claimed that equations (2) and (3) contain a massive source despite no matter terms appearing in the equations. Einstein's argument is inconsistent and therefore false. Thus the so-called 'Schwarzschild solution' has no physical meaning.

The falsity of equations (2) and (3) is reaffirmed and amplified by comparing equations (2) to the 'field' equations,

$$R_{uv} = \lambda g_{uv} \qquad (4)$$

wherein λ is the so-called 'cosmological constant'. The solution to equations (4) is the so-called de Sitter empty universe. The reason why the de Sitter universe is empty is because it contains no matter, by mathematical construction, i.e. T_{uv} = 0.

Thus, Einstein and his followers assert by (2) and (4) that material sources of a gravitational field are both present and absent by the very same constraint, T_{uv} = 0. That's impossible. Since (4) contains no matter by virtue of T_{uv} = 0, equations (2) (and (3) also) contains no matter by the very same condition. Consequently, equations (2) and (3) are physically meaningless, as is equation (4). This also means that the 'Schwarzschild solution' has no physical meaning, and hence no other black hole solution has any physical meaning since they subsume the 'Schwarzschild solution'.

The inclusion of a massive source in the so-called 'Schwarzschild solution' is obtained by inserting Newton's expression for escape velocity, in disguise. Consider Hilbert's metric (the so-called 'Schwarzschild solution'), where c and G are both set to unity,

$$ds^2 = \left(1 - \frac{\alpha}{r}\right)dt^2 - \left(1 - \frac{\alpha}{r}\right)^{-1} dr^2 - r^2 d\Omega^2$$

$$0 \le r \qquad (5)$$

$$d\Omega^2 = \left(d\theta^2 + \sin^2\theta\, d\varphi^2\right)$$

Note that this solution to (2) and (3) is alleged to apply to a body such as a star, as Einstein has claimed for equations (2) and (3). It is also used to conjure a black hole. By using a false analogy with Newton's theory the cosmologists assign $\alpha = 2m$ and call m the massive source of the alleged associated gravitational field. Rewrite (5) in terms of c and G explicitly so that nothing is hidden,

$$ds^2 = c^2\left(1 - \frac{2Gm}{c^2 r}\right)dt^2 - \left(1 - \frac{2Gm}{c^2 r}\right)^{-1} dr^2 - r^2 d\Omega^2$$

$$0 \le r \qquad (6)$$

$$d\Omega^2 = \left(d\theta^2 + \sin^2\theta\, d\varphi^2\right)$$

At $r = 2Gm/c^2 = r_s$ metric (6) is undefined, but the cosmologists claim the event horizon of a black hole there; the so-called 'Schwarzschild radius'. Let's rearrange this expression as follows,

$$c = \sqrt{\frac{2Gm}{r_s}} \qquad (7)$$

We immediately recognise this as Newton's expression for escape velocity. It is from this expression that the cosmologists allege that a black hole has an escape velocity $\geq c$. It is also from this expression that they claim Newton's theory predicts black holes. However, the theoretical Michell-Laplace dark body does not possess any of the characteristics of the alleged black hole, other than mass, and so it is not a black hole. The very appearance of (7) in (6) is because the cosmologists put it there, *ad arbitrium*!

However, (6) contains no matter on account of (2) and (4). Moreover, although (2) and hence (6) is alleged to contain a massive source, Newton's expression for escape velocity is actually a 2-body relation; one body escapes from another body. It cannot therefore appear in what is alleged to be a solution for a 1-body universe.

Recall that the alleged black hole event horizon at the 'Schwarzschild radius' is said to be a one-way membrane by the cosmologists; things can go into a black hole but nothing can come out. In other words, not only can things not escape from the event horizon, nothing can even leave it. However, escape velocity does not mean that things can't leave, only that they can't escape if propelled initially with less than the escape velocity. Throw a ball into the air. Did it leave Earth's surface? Yes! Did it escape? No! Thus, proponents of the black hole thoughtlessly attribute to their black holes an escape velocity and no escape velocity simultaneously. That's impossible too.

The quantity r in (6) is not the radius, although it is always treated as the radius by the cosmologists, manifest in their alleged 'Schwarzschild radius'. In fact, r is not even a distance in (6). Cosmologists have no idea as to the geometric identity of r in (6). It is easily proven [1] that this r is the inverse square root of the Gaussian curvature of the spherically symmetric geodesic surface in the spatial section of (6), and is therefore neither a radius nor a distance in (6), contrary to Einstein and his followers. One cannot therefore treat r in (6) as radius or distance. However, in Newton's expression for escape velocity, r

is the radius from the centre of mass of the body from which another escapes. This amplifies yet another falsity in the standard cosmology.

Let's now compare all black hole universes with all alleged big bang universes.

All alleged black hole universes:

(1) Are spatially infinite
(2) Are eternal
(3) Contain only one mass
(4) Are not expanding
(5) And are either asymptotically flat or asymptotically curved.

All alleged big bang universes:

(1) Are either spatially finite (k = 1) or spatially infinite (k = 0 and k = -1)
(2) Are of finite age (~13.8 billion years)
(3) Contain radiation and many masses
(4) Are expanding
(5) And are not asymptotically anything.

Note also that none of the alleged black hole universes possesses any big bang k-curvature.

It is immediately apparent that none of the foregoing defining characteristics of black hole universes are compatible with those of the big bang universes. Consequently black holes and big bangs are mutually exclusive. To combine then with one another or with themselves violates their very definitions. Black holes and big bangs are nonsense.

All alleged black hole universes are no less a universe than are all the alleged big bang universes. There is no bound on asymptotic, for otherwise it would not be asymptotic. Thus the black hole universe is not contained within its event horizon; it is a spatially infinite universe that is also eternal. Furthermore, black hole universes and big bang universes pertain to entirely different sets of Einstein field equations and so they have nothing whatsoever to do with one another.

Einstein [2] proceeded from his analogue of the Laplace equation, equations (3), to his analogue of the Poisson equation. Using equations (3) he first alleged the conservation of the energy-momentum of his gravitational field by introducing his so-called 'pseudotensor', $t^{\alpha}{}_{\sigma}$, via a Hamiltonian form of equations (3). His conservation law for his gravitational field alone is by means of an ordinary divergence of $t^{\alpha}{}_{\sigma}$, not a tensor divergence, since $t^{\alpha}{}_{\sigma}$ is not a tensor, and therefore in conflict with his tenet that all the equations of physics be covariant tensor expressions. He and his followers to this day justify this procedure on the basis that $t^{\alpha}{}_{\sigma}$ acts 'like a tensor' under linear transformations of coordinates. Nevertheless, this does not make $t^{\alpha}{}_{\sigma}$ a tensor. After a long-winded set of calculations Einstein [2] gets the ordinary divergence,

$$\frac{\partial t_{\sigma}^{\alpha}}{\partial x_{\alpha}} = 0 \qquad (8)$$

and proclaims a conservation law, but only for the energy and momentum of his gravitational field,

"*This equation expresses the law of conservation of momentum and energy for the gravitational field*." [2]

Einstein then replaces equations (3) with the following,

$$\frac{\partial}{\partial x_{\alpha}}\left(g^{\sigma\beta}\Gamma_{\mu\beta}^{\alpha}\right) = -\kappa\left(t_{\mu}^{\sigma} - \frac{1}{2}\delta_{\mu}^{\sigma} t\right) \qquad (9)$$

$$\sqrt{-g} = 1$$

Equations (9) are still Einstein's alleged analogue of the Laplace equation. To get his alleged analogue of the Poisson equation he simply adds a term for the material sources of his gravitational field, namely, his energy-momentum tensor $T^{\sigma}{}_{\mu}$, thus,

$$\frac{\partial}{\partial x_\alpha}\left(g^{\sigma\beta}\Gamma^{\alpha}_{\mu\beta}\right)=-\kappa\left[\left(t^{\sigma}_{\mu}+T^{\sigma}_{\mu}\right)-\frac{1}{2}\delta^{\sigma}_{\mu}\left(t+T\right)\right] \tag{10}$$

$$\sqrt{-g}=1$$

and says,

"… *in place of the energy-components of the gravitational field alone, the sums* $t^{\sigma}{}_{\mu}$ + $T^{\sigma}{}_{\mu}$ *of the energy-components of matter and of gravitational field.*" [2]

Einstein [3] also says,

"*It must be remembered that besides the energy density of the matter there must also be given an energy density of the gravitational field, so that there can be no talk of principles of conservation of energy and momentum for matter alone*."

Thus the total energy-momentum of his gravitational field and its material sources, $\mathcal{E}$, is $\mathcal{E} = (t^{\sigma}{}_{\mu} + T^{\sigma}{}_{\mu})$. This is still not a tensor expression, so Einstein can't take a tensor divergence. He then takes the ordinary divergence to get,

$$\frac{\partial\left(t^{\sigma}_{\mu}+T^{\sigma}_{\mu}\right)}{\partial x_\alpha}=0 \tag{11}$$

and proclaims the usual conservation laws of energy and momentum for a closed system,

"*Thus it results from our field equations of gravitation that the laws of conservation of momentum and energy are satisfied.*" [2]

However, Einstein's argument is patently false. His pseudotensor is defined as [4],

$$\sqrt{-g}\,t^{u}_{v}=\frac{1}{2}\left[\delta^{u}_{v}L-\left(\frac{\partial L}{\partial g^{sb}_{\ ,u}}\right)g^{sb}_{\ ,v}\right]$$

wherein,

$$L=-g^{\alpha\beta}\left(\Gamma^{\gamma}_{\alpha\kappa}\Gamma^{\kappa}_{\beta\gamma}-\Gamma^{\gamma}_{\alpha\beta}\Gamma^{\kappa}_{\gamma\kappa}\right)$$

$$g^{sb}_{\ ,v}\equiv\frac{\partial g^{sb}}{\partial x^{v}}$$

$$\Gamma^{a}_{bc}=\frac{1}{2}g^{ad}\left(\frac{\partial g_{dc}}{\partial x^{b}}-\frac{\partial g_{cb}}{\partial x^{d}}+\frac{\partial g_{bd}}{\partial x^{c}}\right)$$

and g is the determinant of the metric tensor. We now contract Einstein's pseudotensor, and noting that

$$\left(\frac{\partial L}{\partial g^{sb}_{\ ,u}}\right)g^{sb}_{\ ,u}=2L$$

it obtains that

$$t=\frac{L}{\sqrt{-g}}$$

Thus, by the definition of L and g, t is a first-order intrinsic differential invariant, i.e. it is an invariant that depends solely upon the components of the metric tensor and their first derivatives. However, the pure mathematicians, G. Ricci-Curbastro and T. Levi-Civita [5], inventors of the tensor calculus, proved in 1900 that such invariants do not exist. Hence, by *reductio ad absurdum*, Einstein's pseudotensor is a meaningless concoction of mathematical symbols, and therefore, contrary to Einstein and his followers, it can't be used to make any calculations, to represent any physical quantity, or to model any physical phenomena.

Now compare eq. (1) above with an equivalent form, thus

$$R_{uv} = -\kappa\left(T_{uv} - \frac{1}{2}Tg_{uv}\right) \qquad (12)$$

$$T_{uv} = -\frac{1}{\kappa}\left(R_{uv} - \frac{1}{2}Rg_{uv}\right) \qquad (13)$$

Thus by (12), according to Einstein, if $T_{uv} = 0$ then $R_{uv} = 0$. But by (13), if $R_{uv} = 0$ then $T_{uv} = 0$. In other words, R_{uv} and T_{uv} must vanish identically – no material sources then no gravitational field, and no universe. Bearing this in mind, and in view of (2) and (4), consideration of the conservation of energy and momentum, and tensor relations, Einstein's field equations must take the following form [5],

$$\frac{G_{uv}}{\kappa} + T_{uv} = 0 \qquad (14)$$

Comparing this to expression (11) it is clear that the G_{uv}/κ constitute the energy-momentum components of Einstein's gravitational field, which is rather natural since the Einstein tensor G_{uv} describes the curvature of Einstein's spacetime (i.e. his gravitational field), and that (14) constitutes the total energy-momentum of Einstein's gravitational field and its material sources. Unlike (11), expression (14) is a tensor expression. The tensor (covariant derivative) divergence of the left side of (14) is zero. Thus, (14) constitutes a conservation law for Einstein's gravitational field and its material sources T_{uv}.

However, the total energy-momentum of (14) is always zero, the G_{uv}/κ and the T_{uv} must vanish identically (i.e. when $T_{uv} = 0$, $G_{uv} = 0$, and vice-versa, producing the identity $0 = 0$), and gravitational energy can't be localised. Moreover, since the total energy-momentum is always zero the usual conservation laws for energy and momentum of a closed system can't be satisfied. Thus, General Relativity violates the usual conservation of energy and momentum and is therefore in conflict with experiment on a fundamental level.

DEDICATION

For my late brother,

Paul Raymond Crothers

12th May 1968 – 25th December 2008

REFERENCES

[1] Crothers, S. J., http://vixra.org/author/stephen_j_crothers

[2] Einstein, A., The Foundation of the General Theory of Relativity, *Annalen der Physik*, 49, (1916)

[3] Einstein, A., The Meaning of Relativity, expanded Princeton Science Library Edition, (2005)

[4] Ricci-Curbastro, G., Levi-Civita, T., Méthodes de calcul différentiel absolu ET leurs applications", *Matematische Annalen*, B. 54, p.162, (1900), https://eudml.org/doc/157997

[5] Levi-Civita, T., Mechanics. – On the Analytical Expression that Must be Given to the Gravitational Tensor in Einstein's Theory, *Rendiconti della Reale* Accadmeia dei Lincei 26: 381, (1917), http://arxiv.org/pdf/physics/9906004