Seven conjectures on a certain way to write primes including two generalizations of the twin primes conjecture

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Abstract. In this paper I make few conjectures about a way to write an odd prime $p$, id est $p = q - r + 1$, where $q$ and $r$ are also primes; two of these conjectures can be regarded as generalizations of the twin primes conjecture, which states that there exist an infinity of pairs of twin primes.

Conjecture 1
(Which can be regarded as a generalization of the twin primes conjecture)

Any odd prime $p$ can be written in an infinity of distinct ways like $p = q - r + 1$, where $q$ and $r$ are also primes; in other words, there exist an infinity of pairs of primes $(q, r)$ such that $q - r = p - 1$, for any odd prime $p$ (it can be seen that for $p = 3$ the conjecture states the same thing with the twin primes conjecture).

Conjecture 2

Any prime $p$ of the form $p = 6k + 1$, where $k$ is positive integer, can be written in an infinity of distinct ways like $p = q - r + 1$, where $q$ is a prime of the form $q = 6h - 1$ and $r$ is a prime of the form $q = 6i - 1$ and, where $h$ and $i$ are positive integers.

Example: the prime $p = 7$ can be written as $11 - 5 + 1; 17 - 11 + 1; 23 - 17 + 1$ etc.; in fact, for $p = 7$ the conjecture states that there exist an infinity of pairs of sexy primes $(q, r)$, both of the form $6k - 1$ (sexy primes are the primes that differ by each other by six).

Conjecture 3

Any prime $p$ of the form $p = 6k + 1$, where $k$ is positive integer, can be written in an infinity of distinct ways like $p = q - r + 1$, where $q$ is a prime of the form $q = 6h + 1$ and $r$ is a prime of the form $q = 6i + 1$ and, where $h$ and $i$ are positive integers.
Example: the prime $p = 7$ can be written as $13 - 7 + 1; 19 - 13 + 1; 37 - 31 + 1$ etc.; in fact, for $p = 7$ the conjecture states that there exist an infinity of pairs of sexy primes $(q, r)$, both of the form $6k + 1$.

**Conjecture 4**

Any prime $p$ of the form $p = 6k - 1$, where $k$ is a positive integer, can be written in an infinity of distinct ways like $p = q - r + 1$, where $q$ is a prime of the form $q = 6h - 1$ and $r$ is a prime of the form $q = 6i + 1$ and, where $h$ and $i$ are positive integers.

**Conjecture 5**

(Which can be regarded as a generalization of the twin primes conjecture)

There exist an infinity of pairs of primes $(p, q)$, where $p$ is of the form $6k - 1$ and $q$ is of the form $6h + 1$, such that $q - p + 1 = 3^n$, for any $n$ non-null positive integer (it can be seen that for $n = 1$ the conjecture states the same thing with the twin primes conjecture).

Example: for $n = 2$ we have the pairs of primes $(p, q)$: $(11, 19); (23, 31)$ etc.; for $n = 3$ we have the pairs of primes $(5, 31); (11, 37)$ etc.

**Conjecture 6**

Any square of prime $p^2$, $p \geq 5$, can be written in an infinity of distinct ways like $p^2 = q - r + 1$, where $q$ is a prime of the form $q = 6h + 1$ and $r$ is a prime of the form $q = 6i + 1$.

Example: the number $49 = 7^2$ can be written as $61 - 13 + 1; 67 - 19 + 1; 79 - 31 + 1$ etc.

**Conjecture 7**

Any square of prime $p^2$, $p \geq 5$, can be written in an infinity of distinct ways like $p^2 = q - r + 1$, where $q$ is a prime of the form $q = 6h - 1$ and $r$ is a prime of the form $q = 6i - 1$.

Example: the number $49 = 7^2$ can be written as $53 - 5 + 1; 59 - 11 + 1; 71 - 23 + 1$ etc.