Newton's Second Law Revisited

Ramzi Suleiman
University of Haifa

Abstract

The present paper utilized a Complete Relativity Theory (CR), in which the common assumption of speed of light invariance is relaxed. Under the simplifying assumption of collinear motion, I derive a new relativistic expression for Newton's Second Law, according to which the acceleration $\boldsymbol{a}$ produced by a force $\boldsymbol{F}$ is an increasing function of the velocity $\boldsymbol{\beta}$, satisfying $\boldsymbol{a}\ (\boldsymbol{\beta}) \geq \boldsymbol{a}_N$ and $\boldsymbol{a}\ (\boldsymbol{0}) = \boldsymbol{a}_N$, where $\boldsymbol{a}_N$ is the classical Newtonian acceleration. This result may account for the observed increase of acceleration at large distances, without altering Newton's second law, except for the modification implied by relativity.

Keywords: Relativity, Newton's second Law, Acceleration, Anomalies,

Introduction

Newton's second law is perhaps the most commonly known law of physics. Its popularity is well deserved, not only due to its elegant simplicity but also because it is the law that tells as what force **F** is needed to bestow to a body of mass *m* with acceleration $\boldsymbol{a}$**,** and how much work should be invested to increase the body's momentum by **P**. Alternatively, it is often sought as a definition of the body's mass, being the ratio between the applied force and the body's acceleration. In Newtonian mechanics this ratio is assumed to be constant; an inherent property of the object, which is independent of the observer.

Any deviation from Newton's second law would have profound consequences, since it implies violation of the laws of conservation of energy and momentum in their conventional definition. Consequently, considerable effort has been directed to the study of possible violations. In astrophysics and cosmology, several anomalous observations justify reexamination of the validity for the law **F**=m**a**, at least for astrophysical and cosmological systems. One anomaly is the observed flatness of galactic rotation curves, indicating that the velocity of stars as a function of distance from the galactic center rises first and then flattens for larger distances (e.g., [1], [2]). Another anomaly is the acceleration of the universe at large distances, which is attributed to dark energy (e.g., [3], [4]). A third anomaly is the yet unexplained Doppler-tracking data of Pioneer 10/11, Galileo, and Ulysses space-crafts, indicating anomalous constant accelerations directed towards the Sun, acting on the space-crafts at distances of >15 AU (e.g., [5], [6]). For Pioneer 10, the measured acceleration was $\approx 8 \text{ x } 10^{-10}\ m/s^2$ [6]. A forth anomaly, known as the *flyby anomaly*, refers to observations indicating that satellites after an Earth swing–by possess a significant unexplained velocity increase by a few mm/s [7, 8]. For a detailed discussion of the last two anomalies, and of less documented anomalies see [8].

A major attempt to explain some of the mentioned anomalies was undertaken by Milgrom and Bekenstein [9, 10], who proposed a classical modification of Newton’s second law. In their Modified Newtonian Dynamics (MOND) a parameter a_0 is added to Newton's law, such that for a << a_0, a force would yield a larger acceleration than predicted by the law, while for a ≈ a_0, the relationship $\frac{F}{m} = a$ is recovered. The characteristic acceleration a_0 was determined from fits to galactic rotation curves to be $a_0 = 1.2 \text{ x}10^{-10}\ m/s^2$ [11]. Several relativistic theories incorporating MOND have been also discussed in the literature, but none is satisfactory (see, e.g. [12] and [13]).

Accepting MOND's approach, whether for non-relativistic [9, 10] or relativistic scales [14, 15], implies a substantive change of Newton's law (e.g., the acceleration parameter a_0 in MOND), as well as of current cosmology, since it entails abandoning the notions of dark matter and dark energy; two entities which are believed to constitute 95% of our universe, and whose tremendous effects on the universe are well documented (e.g. [3],[4], [16]–[24]).

In this paper, I present a new relativistic modification to Newton's second law based on a Complete Relativity Theory (CR) [25]. In CR, I proposed a new physics of relativity based on the assumption that all measurements of physical entities, *including the velocity of light*, are relativistic. Allowing light anisotropy is the only meaningful difference between CR and SR. Nonetheless, it is a fundamental difference, resulting in a totally different set of transformations. With regard to Newton's second law, while CR predicts that a body's mass *increases* with velocity (approaching ∞ as v ⟶ c), resulting in decrease in acceleration, CR predicts that mass *decreases* with velocity, resulting in increased acceleration.

Relativistic mass

The relativistic mass according to CR (see [25]) is given by:

$$m = m_0 \ \frac{1-\beta}{1+\beta} \quad \left(\beta = \frac{v}{c}\right) \qquad \text{……(1)}$$

Eq. 1 implies that bodies moving away from an observer with normalized velocity β, will suffer a relativistic mass loss of $\frac{1-\beta}{1+\beta}$. Obviously, this result contradicts the prediction of SR, which prescribes that regardless of the direction of its motion, the measured mass will increase with velocity, by a factor γ, where $\gamma = \frac{1}{\sqrt[2]{1-\beta^2}}$ is the Lorentz Factor (See Fig.1). Note that in both theories for $\beta \rightarrow 0$, the Newtonian mass is recovered.

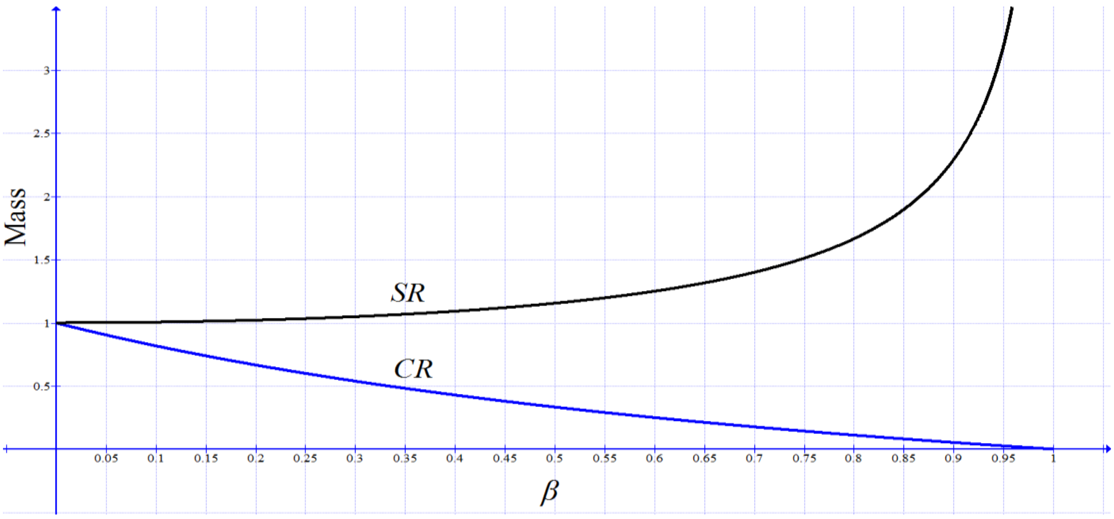

Figure 1. Relativistic mass in Special and Complete Relativities

The relativistic mass loss is explained by CR by the transformation of observable matter to non-observable, *dark matter*. The amount of dark matter is given by:

$$m_d = m_0 - m_0 \frac{1-\beta}{1+\beta} = (1 - \frac{1-\beta}{1+\beta})\, m_0 = \frac{2\beta}{1+\beta}\, m_0 \qquad \ldots\ldots(2)$$

For more on CR's definitions of dark matter and dark energy see [275]. Figure 2 depicts the observable and the dark mass as functions of β.

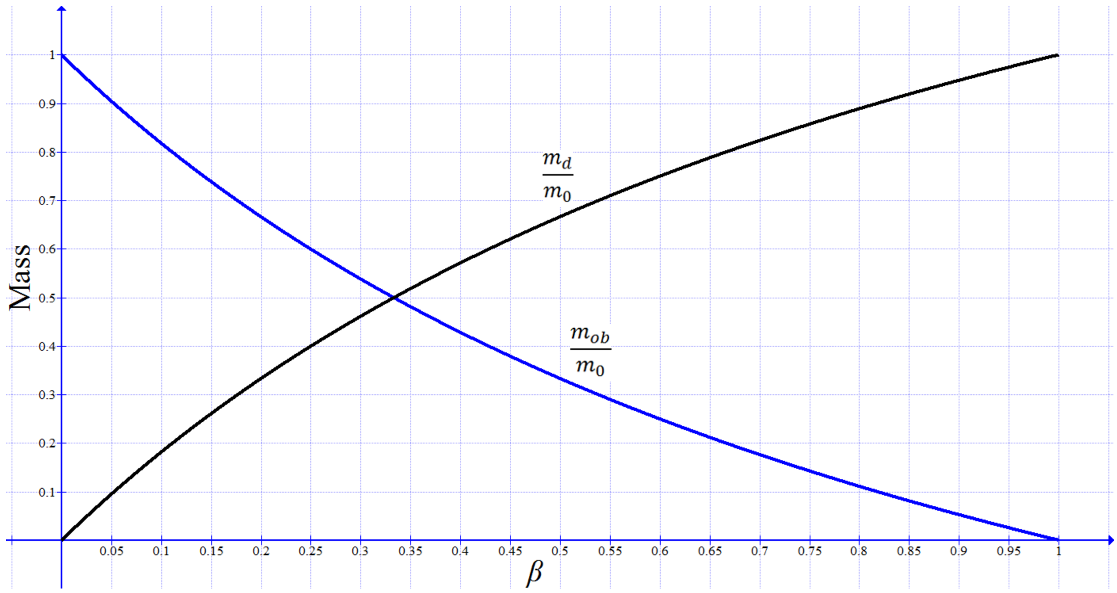

Figure 2. Ratios of observable and the dark mass as functions of velocity

As shown in the figure, the observable mass *decreases* and the dark mass *increases*, nonlinearly, with β. The rates of decrease and increase are $\pm \frac{2}{(1+\beta)^2}$. The velocity at which the two components become equal is obtained from solving for $m = m_d$:

$$m = m_0 \frac{1-\beta}{1+\beta} = m_d = \frac{2\beta}{1+\beta}\, m_0 \qquad \ldots\ldots(3)$$

Which yields:

$$\beta = \frac{1}{3} \qquad \ldots\ldots(4)$$

This implies that up to velocities of one third of the velocity of light, a physical system will be dominated by observable matter, whereas above this critical value, it will be dominated by dark matter. The ratio of dark to observable matter as a function of β is equal to:

$$\frac{m_d}{m} = \frac{2\beta}{1-\beta} \qquad \text{......(5)}$$

Relativistic Newton's Second Law

For relativistic velocities, Newton's second law is given by:

$$\boldsymbol{F} = \frac{\partial \boldsymbol{P}}{\partial t} = \frac{\partial (m\boldsymbol{v})}{\partial t} = m\,\frac{\partial (\boldsymbol{v})}{\partial t} + \boldsymbol{v}\,\frac{\partial (m)}{\partial t}$$

$$= m\;a + \boldsymbol{v}\,\frac{\partial (m)}{\partial v}\,\frac{\partial (\boldsymbol{v})}{\partial t} = m\;a + \boldsymbol{v}\,\boldsymbol{a}\,\frac{\partial (m)}{\partial v} \qquad \text{......(6)}$$

Or:

$$\mathrm{F} = (m + \boldsymbol{v}\,\frac{\partial (m)}{\partial v})\,a \qquad \text{......(7)}$$

Assuming Complete Relativity, *m* is given by Eq. 1. Substitution in Eq. 6 and derivation with respect to *v* yields:

$$\mathrm{F} = \frac{1-2\beta-\beta^2}{(1+\beta)^2}\;\; m_0\;\; \mathrm{a} = (\frac{1-2\beta-\beta^2}{(1+\beta)^2})\;\; F_N \qquad \text{......(8)}$$

Where $F_N = m_0$ a is the classical Newtonian force. Similarly, for SR we have:

$$F_{SR} = (m + \boldsymbol{v}\,\frac{\partial (m)}{\partial v})\,a = (\frac{m_0}{\sqrt[2]{1-\beta^2}} + v\;\frac{\partial}{\partial v}\,(\frac{m_0}{\sqrt[2]{1-\beta^2}})\;)\;\mathrm{a} = (\frac{1}{\sqrt[2]{1-\beta^2}} + \frac{\beta^2}{(1-\beta^2)^{\frac{3}{2}}})\;m_0\;\mathrm{a}$$

$$= (\frac{1-\beta^2+\beta^2}{(1-\beta^2)^{\frac{3}{2}}})\;m_0\;\mathrm{a} = (\frac{1}{(1-\beta^2)^{\frac{3}{2}}})\;m_0\;\mathrm{a} = \gamma^3\;F_N \qquad \text{...... (9)}$$

Where γ is the Lorentz Factor.

The accelerations predicted by CR is given by:

$$a = \frac{(1+\beta)^2}{(1-2\beta-\beta^2)}\;\frac{\mathrm{F}}{\mathrm{m}_0} \qquad \text{...... (10)}$$

Figure 3 depicts $\frac{F}{F_N}$ as a function of β according to CR and SR.

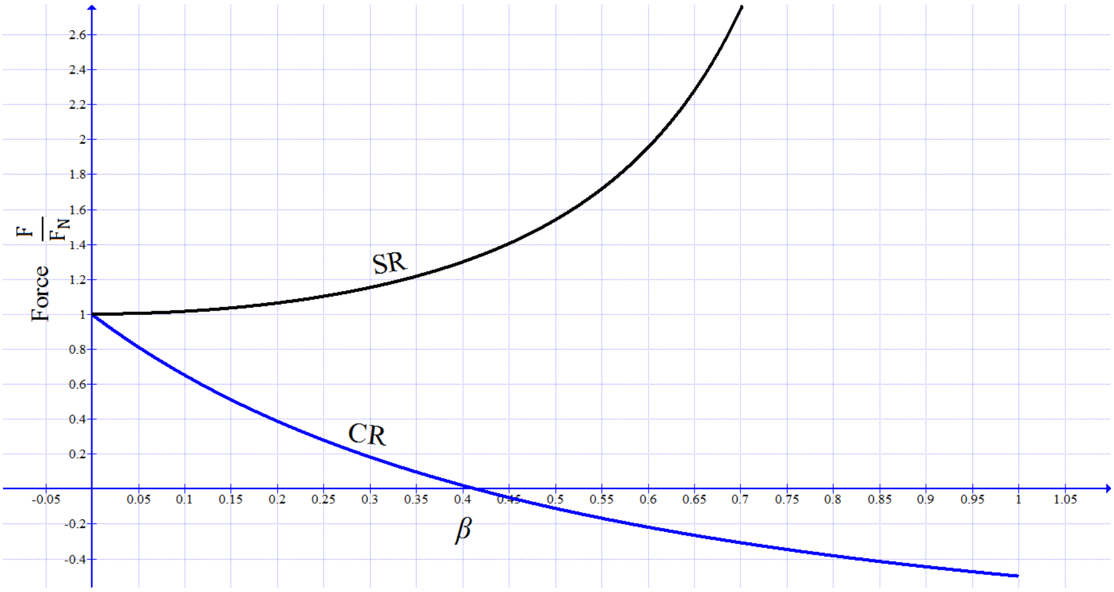

Figure 3. Relativistic force in Special and Complete Relativities

The figure shows that the two theories yield contradictory predictions. SR predicts that compared with Newton's second law, the force needed to bestow a mass with acceleration *a* should *increase* with velocity in an *accelerating rate* from 1 at β = 0, approaching ∞ as β→1. In contrast, CR predicts that the force should *decrease*, in a *decelerating rate* from 1 at β = 0 to zero and below. This is an unintuitive result, like many others in relativity, quantum mechanics, and cosmology. It implies that beyond a critical velocity, a force acting in opposite direction to the velocity vector is needed to maintain a constant Newtonian acceleration. Within the framework of Complete Relativity, this state of affairs is justified by the relativistic mass loss, through which the observable matter diminishes below a critical value. The point of flip in $\frac{F}{F_N}$ (see Eq. 8) is found solving the equality:

$$1 - 2\beta - \beta^2 = 0 \qquad \text{...... (10)}$$

Yielding a positive solution of

$$\beta = \sqrt[2]{2} - 1 \approx 0.41421356 \ \text{(or v} \approx 124{,}178.101 \tfrac{km}{s}\text{).} \qquad \text{...... (11)}$$

Astronomical and cosmological investigations of Newton's second law, involve considerably lower velocities and extremely small accelerations (e.g., [5]-[8]). Figure

4 plots the difference $a - a_N$ (on log. scale) between the predicted and Newtonian acceleration, for various β values, as a function of force per unit mass. As could be seen, for all depicted β values, $a - a_N$ first increases rapidly, but becomes almost flat starting from small $\frac{F}{m_0}$ values. As an example, for velocities in the magnitude of $50{,}000\ \frac{km}{h}$, we have $\beta = \frac{v}{c} \approx 4.6 \times 10^{-5}$, with a corresponding difference of a - a_N in the flattened section ranging between $\approx 10^{-11}$ for $\frac{F}{m_0} = 5 \times 10^{-8}$ to $\approx 10^{-10}$ for $\frac{F}{m_0} = 6 \times 10^{-7}$ (see dotted redline in Figure 4).

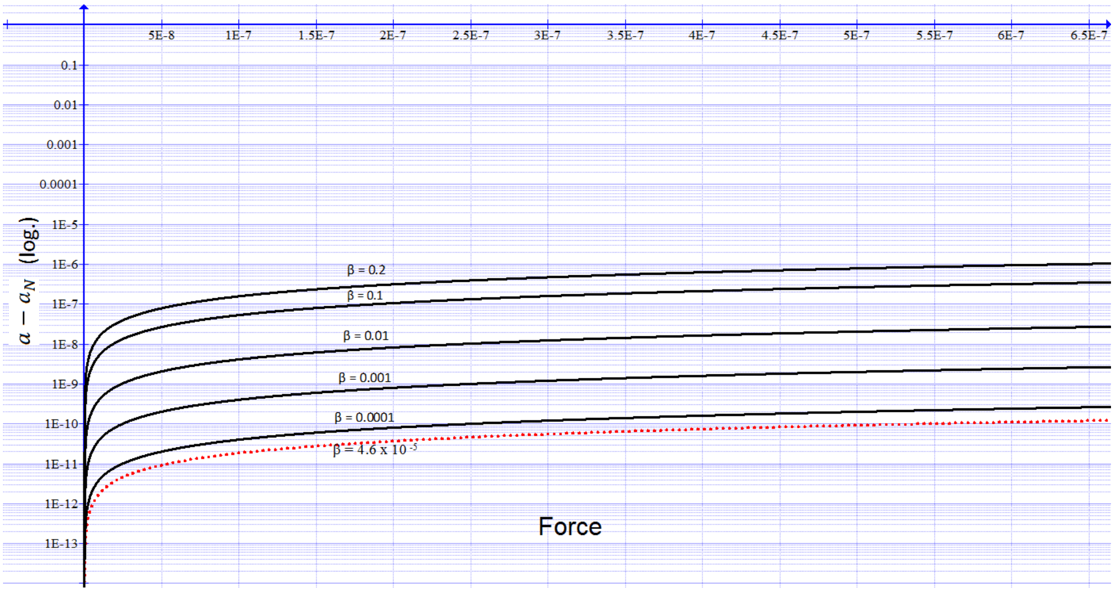

Figure 4. Excess in acceleration (a - a_N) as a function of force for various velocities

Concluding remarks

The present paper utilized Complete Relativity Theory (CR), in which the common assumption of the speed of light invariance is relaxed, to derive a new relativistic expression for Newton's Second Law. The force and acceleration expressions (Equations 8 and 10) reveal that at relativistic velocities, the predicted acceleration resulting from applying a force F on an object with rest mass m_0 increases with velocity. This result is explained by a relativistic mass loss (Eq. 1, Fig.1), which increases with velocity. The gradual loss in normal mass is explained as the fact that observable (normal) mass is gradually transformed to unobservable (dark) mass. Put differently, the acceleration a produced by a force F is an increasing function of the velocity β, satisfying $a(\beta) \geq a_N$ and $a\,(0) = a_N$, where a_N is the classical Newtonian acceleration. This result may account for the observed increase of acceleration at large distances, without altering Newton's second law, except for the modification implied by relativity.

Other modifications of Newton's second law, whether at non-relativistic scales as in MOND (e.g., [9], [11]), or relativistic scales as in TeVes ([15], [16]), require not only a fundamental change in Newtonian dynamics, but also abandonment of the notions of dark matter and dark energy, two corner stones of contemporary cosmology. In contrast, the increase in acceleration, which serves as the *raison d'être* for constructing MOND, is explained by CR *specifically* by evoking the notion of dark matter.

It is worth noting that the analysis brought above should be taken within its limitations. First, it accounts for a simple mechanics, of collinear mass motion with no angular momentum. Second, it does not probe the nature of the acting force. Employment of the derived results to accounting for observational or experimental data requires generalization of the model for more complex physical systems. Although more cumbersome, such generalization does not seem to be particularly intractable.

References

[1] Rubin, V. C. (1982). *Science*, 220, 1339.

[2] Sofue, Y., Ruben, V. (2001). *Annual Review of Astronomy & Astrophysics.* 39, 137.

[3] Riess, A. G., et al. (1998). *The Astrophysical Journal*, 116, 1009.

[4] Reiss, A. G. et al. (2004). *The Astrophysical Journal*, 607, 665–687.

[5] P.G. Antreasian and J.R. Guinn (1998). Investigations into the unexpected delta–v increase during the Earth Gravity Assist of GALILEO and NEAR. Paper no. 98-4287 presented at the AIAA/AAS Astrodynamics Specialist Conference and Exhibition, Boston, 1998.

[6] John D. Anderson, Philip A. Laing, Eunice L. Lau, Anthony S. Liu, Michael Martin Nieto, & Slava G. Turyshev. (2002). *Physical Review D*, 65, 082004.

[7] J.D. Anderson and J.G.Williams. Long-range tests of the equivalence principle. *Classical Quantum Gravity.*, 18:2447, 2001.

[8] C. Lämmerzahl, O. Preuss, and H. Dittus. (2008). In H. Dittus, C. Lämmerzahl, & Tutyshev, S. (Eds.). Lasers, Clocks and Drag-Free Control: *Exploration of relativistic Gravity in Space. Astrophysics and Space* (pp. 75-101). Astrophysics and Space Science Library 349. Springer.

[9] Bekenstein, J., & Milgrom, M. (1984). *The Astrophysical Journal*, 286, 7-14.

[10] Milgrom, M. (2002). MOND–Theoretical aspects. *New Astronomy Reviews*, 46, 741–753

[11] R. H. Sanders and S. S. McGaugh. (2002). *Annual Reviews of Astronomy & Astrophysics*. **40**, 263 [36 pages].

[12] Bekenstein, J. 1988, *Physical Letters B*, 202, 497

[13] Sanders, R.H. 1997. *The Astrophysical Journal*, 480, 492.

[14] Bekenstein J. , 2004, *Physical Review D*, 70, 3509

[15] Luz Maria Diaz-Rivera, Lado Samushia, and Bharat Ratra (2006). *Physical Review D*, 73, 083503 [8 pages].

[16] Blumenthal, G. R., Faber, S. M., Primack, J. R., & Rees, M. J. 1984, *Nature*, 311, 517.

[17] White, S. D. M., & Rees, M. J. 1978, *Monthly Notices of the Royal Astronomical Society*, 183, 341.

[18] Percival, W. J., et al. 2002, *Monthly Notices of the Royal Astronomical Society*, 337, 1068.

[19] Spergel, D. N., et al. 2003. *The Astrophysical Journal Supplement Series*, 148, 175–194.

[20] Clowe, D., Gonzalez, A., & Markevitch, M. (2004). *The Astrophysics Journal*, 606, 596-603.

[21] Tegmark, M., et al. 2004, *Physical Review D*, 69, 10350 [26 pages]

[22] Adam S. Bolton, Scott Burles, Le´on V. E. Koopmans, Tommaso Treu, and Leonidas A. Moustakas (2006). *The Astrophysical Journal*, 638:703–724, 2006.

[23] Clowe, D. Brada, M. Gonzalez, A. H., Markevitch, M., Randall, S. W., Jones, C., & Zaritsky, D. (2006). *The Astrophysical Journal,* 648*, L109–L113*.

[24] Massay, R. et al. (2007*). Nature*, 455, 286-290.

[25] Suleiman, R. (2013). *Progress in Physics*, *4,* 34-40.