Solutions of Navier-Stokes Equations
plus
Solutions of Magnetohydrodynamic Equations

Abstract

After nearly 150 years of patience, the Navier-Stokes equations in 3-D for incompressible fluid flow have been analytically solved by two different methods. In fact, it is shown that these equations can be solved in 4-dimensions or *n*-dimensions. The author has proposed and applied a new law, the law of definite ratio for fluid flow. This law states that in incompressible fluid flow, the other terms of the fluid flow equation divide the gravity term in a definite ratio and each term utilizes gravity to function. The sum of the terms of the ratio is always unity. This law evolved from the author's earlier solutions of the Navier-Stokes equations. By applying the above law, the hitherto unsolved magnetohydrodynamic equations were routinely solved. It is also shown that without gravity forces on earth, there will be no incompressible fluid flow on earth as is known. In addition to the usual method of solving these equations, the N-S equations have also been solved by a second method in which the three equations are added to produce a single equation which is then integrated. The solutions by the two methods are identical, except for the constants involved. Ratios were used to split the equations; and the resulting sub-equations were readily integrable, and even, the nonlinear sub-equations were readily integrated. The preliminaries reveal how the ratio technique evolved as well as possible applications of the solution method in mathematics, science, engineering, business, economics, finance, investment and personnel management decisions. The x–direction Navier-Stokes equation will be linearized, solved, and the solution analyzed. The linearized equation represents, except for the numerical coefficient of the acceleration term, the linear part of the Navier-Stokes equation. This solution will be followed by the solution of the Euler equation of fluid flow. The Euler equation represents the nonlinear part of the Navier-Stokes equation. The Euler equation was solved in the author's previous paper. Following the Euler solution, the Navier-Stokes equation will be solved, essentially by combining the solutions of the linearized equation and the Euler solution. For the Navier-Stokes equation, the linear part of the relation obtained from the integration of the linear part of the equation satisfied the linear part of the equation; and the relation from the integration of the non-linear part satisfied the non-linear part of the equation. For the linearized equation, different terms of the equation were made subjects of the equation, and each such equation was integrated by first splitting-up the equation, using ratio, into sub-equations. The integration results were combined. Six equations were integrated. The relations obtained using these terms as subjects of the equations were checked in the corresponding equations. Only the equation with the gravity term as subject of the equation satisfied its corresponding equation, and this unique solution led to the law of definite ratio for fluid flow, stated above. This equation which satisfied its corresponding equation will be defined as the driver equation; and each of the other equations which did not satisfy its corresponding equation will be called a supporter equation. A supporter equation does not satisfy its corresponding equation completely but provides useful information which is not apparent in the solution of the driver equation. The solutions revealed the role of each term of the Navier-Stokes equations in fluid flow. The gravity term is the indispensable term in fluid flow, and it is involved in the parabolic and forward motion of fluids. The pressure gradient term is also involved in the parabolic motion. The viscosity terms are involved in the parabolic, periodic and decreasingly exponential motion. Periodicity increases with viscosity. The variable acceleration term is also involved in the periodic and decreasingly exponential motion. The convective acceleration terms produce square root function behavior and fractional terms containing square root functions with variables in the denominators and consequent turbulence behavior. For a spin-off, the smooth solutions from above are specialized and extended to satisfy the requirements of the CMI Millennium Prize Problems, and thus prove the existence of smooth solutions of the Navier-Stokes equations.

Options

The Navier-Stokes equations in three dimensions are three simultaneous equations in Cartesian coordinates for the flow of incompressible fluids. The equations are presented below:

$$\begin{cases} \mu(\frac{\partial^2 V_x}{\partial x^2}+\frac{\partial^2 V_x}{\partial y^2}+\frac{\partial^2 V_x}{\partial z^2})-\frac{\partial p}{\partial x}+\rho g_x=\rho(\frac{\partial V_x}{\partial t}+V_x\frac{\partial V_x}{\partial x}+V_y\frac{\partial V_x}{\partial y}+V_z\frac{\partial V_x}{\partial z}) & (\mathrm{N}_x) \\ \mu(\frac{\partial^2 V_y}{\partial x^2}+\frac{\partial^2 V_y}{\partial y^2}+\frac{\partial^2 V_y}{\partial z^2})-\frac{\partial p}{\partial y}+\rho g_y=\rho(\frac{\partial V_y}{\partial t}+V_x\frac{\partial V_y}{\partial x}+V_y\frac{\partial V_y}{\partial y}+V_z\frac{\partial V_y}{\partial z}) & (\mathrm{N}_y) \\ \mu(\frac{\partial^2 V_z}{\partial x^2}+\frac{\partial^2 V_z}{\partial y^2}+\frac{\partial^2 V_z}{\partial z^2})-\frac{\partial p}{\partial z}+\rho g_z=\rho(\frac{\partial V_z}{\partial t}+V_x\frac{\partial V_z}{\partial x}+V_y\frac{\partial V_z}{\partial y}+V_z\frac{\partial V_z}{\partial z}) & (\mathrm{N}_z) \end{cases}$$

Equation (N_x) will be the first equation to be solved; and based on its solution, one will be able to write down the solutions for the other two equations, (N_y), and (N_z).

Dimensional Consistency

The Navier-Stokes equations are dimensionally consistent as shown below:

$$\mu(\frac{\partial^2 V_x}{\partial x^2} + \frac{\partial^2 V_x}{\partial y^2} + \frac{\partial^2 V_x}{\partial z^2}) - \frac{\partial p_x}{\partial x} + \rho g_x = \rho(\frac{\partial V_x}{\partial t} + V_x\frac{\partial V_x}{\partial x} + V_y\frac{\partial V_x}{\partial y} + V_z\frac{\partial V_x}{\partial z})$$

Using MLT

$$\boxed{M(L^{-2}T^{-2}+L^{-2}T^{-2}+L^{-2}T^{-2}-L^{-2}T^{-2}+L^{-2}T^{-2})=M(L^{-2}T^{-2}+L^{-2}T^{-2}+L^{-2}T^{-2}+L^{-2}T^{-2})}$$

Using $kg\text{–}m\text{–}s$

$$\boxed{kg(m^{-2}s^{-2}+m^{-2}s^{-2}+m^{-2}s^{-2}-m^{-2}s^{-2}+m^{-2}s^{-2}=kg(m^{-2}s^{-2}+m^{-2}s^{-2}+m^{-2}s^{-2}+m^{-2}s^{-2}}$$

Option 1

Back to Options

Solution of 3-D Linearized Navier-Stokes Equation in the *x*-direction

The equation will be linearized by redefinition. The nine-term equation will be reduced to six terms.

Given: $$\mu\left(\frac{\partial^2 V_x}{\partial x^2}+\frac{\partial^2 V_x}{\partial y^2}+\frac{\partial^2 V_x}{\partial z^2}\right)-\frac{\partial p_x}{\partial x}+\rho g_x=\rho\left(\frac{\partial V_x}{\partial t}+V_x\frac{\partial V_x}{\partial x}+V_y\frac{\partial V_x}{\partial y}+V_z\frac{\partial V_x}{\partial z}\right) \qquad (A)$$

$$-\mu\frac{\partial^2 V_x}{\partial x^2}-\mu\frac{\partial^2 V_x}{\partial y^2}-\mu\frac{\partial^2 V_x}{\partial z^2}+\frac{\partial p_x}{\partial x}+\rho\frac{\partial V_x}{\partial t}+\rho V_x\frac{\partial V_x}{\partial x}+\rho V_y\frac{\partial V_x}{\partial y}+\rho V_z\frac{\partial V_x}{\partial z}=\rho g_x \qquad (B)$$

$$-\mu\left(\frac{\partial^2 V_x}{\partial x^2}+\frac{\partial^2 V_x}{\partial y^2}+\frac{\partial^2 V_x}{\partial z^2}\right)+\frac{\partial p_x}{\partial x}+4\rho\left(\frac{\partial V_x}{\partial t}\right)=\rho g_x \qquad (C)$$

Plan: One will split-up equation (C) into five equations, solve them, and combine the solutions. On splitting-up the equations and proceeding to solve them, the non linear terms could be redefined and made linear. This linearization is possible if the gravitational force term is the subject of the equation as in equation (B). After converting the non-linear terms to linear terms by redefinition, one will have only six terms as in equation (C). One will show logically how equation (C) was obtained from equation (B), using a method which will be called the multiplier method.

Three main steps are covered.
In main Step 1, one shows how equation (C) was obtained from equation (B)
In main Step 2, equation (C) will be split-up into five equations.
In main Step 3, each equation will be solved.
In main Step 4, the solutions from the five equations will be combined.
In main Step 5, the combined relation will be checked in equation (C). for identity.

Preliminaries

Here, one covers examples to illustrate the mathematical validity of how one splits-up equation (C). Let one think like a child - Albert Einstein. Actually, one can think like an eighth or a ninth grader. Suppose one performs the following operations:

Example 1: $10+20+25=55 \qquad (1)$

$$10=55\times\tfrac{10}{55}=55\times\tfrac{2}{11} \qquad (2)$$

$$20=55\times\tfrac{20}{55}=55\times\tfrac{4}{11} \qquad (3)$$

$$25=55\times\tfrac{25}{55}=55\times\tfrac{5}{11} \qquad (4)$$

Equations (2), (3), and (4) can be written as follows:

$$10=55a \qquad (5)$$

$$20=55b \qquad (6)$$

$$25=55c \qquad (7)$$

One will call a, b and c multipliers.

Above, $a=\frac{2}{11}$, $b=\frac{4}{11}$, $c=\frac{5}{11}$

Observe also that $a+b+c=1$

$\left(\frac{2}{11}+\frac{4}{11}+\frac{5}{11}=\frac{11}{11}=1\right)$

Example 2: Addition of only two numbers

$$20 + 25 = 45 \qquad (8)$$

$$20 = 45 \times \tfrac{20}{45} = 45 \times \tfrac{4}{9} \qquad (9)$$

$$25 = 45 \times \tfrac{25}{45} = 45 \times \tfrac{5}{9} \qquad (10)$$

Equations (9), and (10), can be written as follows:

$$20 = 45a \qquad (11)$$

$$25 = 45b \qquad (12)$$

Rewrite (8) by transposition.

If $20 - 45 = -25$

Then $20 = -25d$ (d is a multiplier)

$-45 = -25f$ (f is a multiplier)

Above, $d = \dfrac{20}{-25} = -\tfrac{4}{5}$, $f = \dfrac{-45}{-25} = \dfrac{9}{5}$,

Observe also here that $d + f = 1$ $\quad (-\tfrac{4}{5} + \tfrac{9}{5} = \tfrac{5}{5} = 1)$

$a + b = 1$ $\quad (\tfrac{4}{9} + \tfrac{5}{9} = \tfrac{9}{9} = 1)$

One can conclude that the sum of the multipliers is always 1.

More formally:

Let $A + B + C = S$, where A, B, C and S. are real numbers. (for the moment), and one excludes 0.

Let a, b, c be respectively, multipliers of the sum S corresponding to A, B, C.

Then $A = Sa$, $B = Sb$, $C = Sc$; and $a + b + c = 1$

To show that $a + b + c = 1$,

$Sa + Sb + Sc = S$.

$S(a + b + c) = S$ (factoring out the S)

$a + b + c = 1$. (Dividing both sides of the equation by S)

Example 3: Solve the quadratic equation; $6x^2+11x-10=0$
Method 1 (a common and straightforward method)

By factoring, $6x^2+11x-10=0$

$(3x-2)(2x+5)=0$ and solving,

$(3x-2)=0$ or $(2x+5)=0$

$x=\frac{2}{3}, x=-\frac{5}{2}$. Solution set: $\{-\frac{5}{2}, \frac{2}{3}\}$

Method 2: One applies the discussion in Example 2
One will call this method the **multiplier method.**

Step 1**:** From $6x^2+11x-10=0$ (1)

$6x^2+11x=10$

$6x^2=10a$; (Here, a is a multiplier)

$3x^2=5a$ (2)

$11x=10b$ (Here, b is a multiplier)

$11x=10(1-a)$ $(a+b)=1$

$11x=10-10a$

$x=\frac{10-10a}{11}$

$3(\frac{10-10a}{11})^2=5a$ (Substituting for x in (2)

$3(\frac{100-200a+100a^2)}{121}=5a$

Step 2: $300a^2-1205a+300=0$

$60a^2-241a+60=0$

$a=\frac{241\pm\sqrt{241^2-4(60)(60)}}{120}$

$a=\frac{241\pm\sqrt{43681}}{120}$

$a=\frac{241\pm 209}{120}$

$a=\frac{241\pm 209}{120}=\frac{241+209}{120}$ or $\frac{241-209}{120}$

$=\frac{450}{120}$ or $\frac{32}{120}$

$=\frac{15}{4}$ or $\frac{4}{15}$

Step 3: Since $a+b=1$, when $a=\frac{15}{4}$ or $3\frac{3}{4}$

$b=1-3\frac{3}{4}=-2\frac{3}{4}$ or $-\frac{11}{4}$

when $a=\frac{4}{15}$, $b=1-\frac{4}{15}=\frac{11}{15}$

Step 4: When $b=-\frac{11}{4}$, $11x=10(-\frac{11}{4})$

$x=-\frac{5}{2}$

When $b=\frac{11}{15}$, $11x=10(\frac{11}{15})$

$x=\frac{10}{11}(\frac{11}{15})$; $x=\frac{2}{3}$

Again, one obtains the same solution set $\{-\frac{5}{2}, \frac{2}{3}\}$ as by the factoring method.

About the multipliers

The values of the multipliers obtained were $a=\frac{15}{4}$ or $3\frac{3}{4}$, $b=-2\frac{3}{4}$ or $-\frac{11}{4}$; $a=\frac{4}{15}$. $b=\frac{11}{15}$.

It easy to understand, say, in $20=45\times\frac{20}{45}=45\times\frac{4}{9}$, that the multiplier $\frac{4}{9}$ can be viewed as the fraction of the multiplicand, 45 .

Later, one will learn that the multipliers are ratio terms as in Examples 5, 6 and 7, below.

Example 4 Solve $ax^2+bx+c=0$ by completing the square and incorporating the multiplier method.

Step 1: From $ax^2+bx+c=0$

$ax^2+bx=-c$

Let $ax^2=-cd$; (d is a multiplier) (1)

Let $bx=-cf$ (f is a multiplier) (2)

(and $d+f=1$)

$ax^2+bx=-cd-cf$ (Adding equations (1) and (2)

$$x^2+\frac{b}{a}x=\frac{-c}{a}d-\frac{c}{a}f$$

$$x^2+\frac{b}{a}x+\left(\frac{b}{2a}\right)^2-\left(\frac{b}{2a}\right)^2=-\frac{c}{a}(d+f)$$

(completing the square on the left-hand side))

$$\left(x+\frac{b}{2a}\right)^2=\left(\frac{b}{2a}\right)^2-\frac{c}{a}\qquad (d+f=1)\qquad (3)$$

One's interest is in equations (1), (2) and (3).

Step 2 $x+\dfrac{b}{2a}=\pm\sqrt{\left(\dfrac{b}{2a}\right)^2-\dfrac{c}{a}}$

$$x+\frac{b}{2a}=\pm\sqrt{\frac{b^2}{4a^2}-\frac{c}{a}}$$

$$x+\frac{b}{2a}=\pm\sqrt{\frac{b^2}{4a^2}-\frac{4ac}{4a^2}}$$

$$=\pm\sqrt{\frac{b^2-4ac}{4a^2}}$$

$$x=-\frac{b}{2a}\pm\frac{\sqrt{b^2-4ac}}{2a}$$

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

Example 5: A grandmother left \$45,000 in her will to be divided between eight grandchildren, Betsy, Comfort, Elaine, Ingrid, Elizabeth, Maureen, Ramona, Marilyn, in the ratio $\frac{1}{36}:\frac{1}{18}:\frac{1}{12}:\frac{1}{9}:\frac{5}{36}:\frac{1}{6}:\frac{7}{36}:\frac{2}{9}$. (**Note**: $\frac{1}{36}+\frac{1}{18}+\frac{1}{12}+\frac{1}{9}+\frac{5}{36}+\frac{1}{6}+\frac{7}{36}+\frac{2}{9}=1$)
How much does each receive?

Solution:

Betsy's share of \$45,000 $=\frac{1}{36}\times\$45{,}000=\$1{,}250$

Comfort's share of \$45,000 $=\frac{1}{18}\times\$45{,}000=\$2{,}500$

Elaine's share of \$45,000 $=\frac{1}{12}\times\$45{,}000=\$3{,}750$

Ingrid's share of \$45,000 $=\frac{1}{9}\times\$45{,}000=\$5{,}000$

Elizabeth's share of \$45,000 $=\frac{5}{36}\times\$45{,}000=\$6{,}250$

Maureen's share of \$45,000 $=\frac{1}{6}\times\$45{,}000=\$7{,}500$

Ramona's share of \$45,000 $=\frac{7}{36}\times\$45{,}000=\$8{,}750$

Marilyn's share of \$45,000 $=\frac{2}{9}\times\$45{,}000=\$10{,}000$

Check; Sum of shares $\boxed{=\$45{,}000}$
Sum of the fractions = 1

Example 6: Sir Isaac Newton left ρg_x units in his will to be divided between $-\mu\frac{\partial^2 V_x}{\partial x^2}$, $-\mu\frac{\partial^2 V_x}{\partial y^2}$, $-\mu\frac{\partial^2 V_x}{\partial z^2}$, $\frac{\partial p}{\partial x}$, $\rho\frac{\partial V_x}{\partial t}$, $\rho V_x\frac{\partial V_x}{\partial x}$, $\rho V_y\frac{\partial V_x}{\partial y}$, $\rho V_z\frac{\partial V_x}{\partial z}$ in the ratio $a:b:c:d:f:h:m:n$.
where $a+b+c+d+f+h+m+n=1$. How much does each receive?

Solution $-\mu\frac{\partial^2 V_x}{\partial x^2}$'s share of ρg_x units $= a\rho g_x$ units

$-\mu\frac{\partial^2 V_x}{\partial y^2}$'s share of ρg_x units $= b\rho g_x$ units

$-\mu\frac{\partial^2 V_x}{\partial z^2}$'s share of ρg_x units $= c\rho g_x$ units

$\frac{\partial p}{\partial x}$'s share of ρg_x units $= d\rho g_x$ units

$\rho\frac{\partial V_x}{\partial t}$'s share of ρg_x units $= f\rho g_x$ units

$\rho V_x\frac{\partial V_x}{\partial x}$'s share of ρg_x units $= h\rho g_x$ units

$\rho V_y\frac{\partial V_x}{\partial y}$'s share of ρg_x units $= m\rho g_x$ units

$\rho V_z\frac{\partial V_x}{\partial z}$'s share of ρg_x units $= n\rho g_x$ units

Sum of shares = $\boxed{\rho g_x \text{ units}}$ **Note**: $a+b+c+d+f+h+m+n=1$

Example 7: The returns on investments A, B, C, D are in the ratio $a:b:c:d$. If the total return on these four investments is P dollars, what is the return on each of these investments? $(a+b+c+d=1)$

Solution Return on investment $A = aP$ dollars
Return on investment $B = bP$ dollars
Return on investment $C = cP$ dollars
Return on investment $D = dP$ dollars

Check $aP+bP+cP+dP = P$
$P(a+b+c+d) = P$
$a+b+c+d=1$ (dividing both sides by P)

The objective of presenting examples 1, 2, 3, 4, 5, 6, and 7 was to convince the reader that the principles to be used in splitting the Navier-Stokes equations are valid.
In Examples 3 and 4, one could have used the quadratic formula directly to solve for x, without finding a and b first. The objective was to convince the reader that the introduction of a and b did not change the solution sets of the original equations.

For the rest of the coverage in this paper, a multiplier is the same as a ratio term
The multiplier method is the same as the ratio method.

Main Step 1
Linearization of the Non-Linear Terms

Step 1: The main principle is to multiply the right side of the equation by the ratio terms
This step is critical to the removal of the non-linearity of the equation.
ρg_x is to be divided by the terms on the left-hand--side of the equation in the ratio
$a:b:c:d:f:h:m:n \quad (a+b+c+d+f+h+m+n=1$

$$-\mu\frac{\partial^2 v_x}{\partial x^2}-\mu\frac{\partial^2 v_x}{\partial y^2}-\mu\frac{\partial^2 v_x}{\partial z^2}+\frac{\partial p_x}{\partial x}+\underbrace{\rho\frac{\partial V_x}{\partial t}+\overbrace{\rho V_x\frac{\partial V_x}{\partial x}+\rho V_y\frac{\partial V_x}{\partial y}+\rho V_z\frac{\partial V_x}{\partial z}}^{\text{nonlinear terms}}}_{\text{all acceleration terms}}=\rho g_x \qquad (1)$$

Apply the principles involved in the ratio method covered in the preliminaries, to the nonlinear terms (the last three terms.)

Then $\rho V_z\frac{\partial V_x}{\partial z}=n\rho g_x$, where n is the ratio term corresponding to $\rho V_z\frac{\partial V_x}{\partial z}$.

$$V_z\frac{\partial v_x}{\partial z}=ng_x \qquad (2)$$

$V_z\frac{dV_x}{dz}=ng_x$ (One drops the partials symbol, since a single independent variable is involved)

$\frac{dz}{dt}\frac{dV_x}{dz}=ng_x \quad (V_z=\frac{dz}{dt}$, by definition)

$$\frac{dv_x}{dt}=ng_x \qquad (3)$$

Therefore, $\boxed{V_z\frac{\partial V_x}{\partial z}=\frac{dV_x}{dt}=ng_x}$ (4)

Step 2: Similarly, Let $\rho V_y\frac{\partial V_x}{\partial y}=m\rho g_x$ (m is the ratio term corresponding to $\rho V_y\frac{\partial V_x}{\partial y}$) (5)

$V_y\frac{dV_x}{dy}=mg_x$ (One drops the partials symbol, since a single independent variable is involved)

$\frac{dy}{dt}\frac{dV_x}{dy}=mg_x \quad (V_y=\frac{dy}{dt})$

$$\frac{dv_x}{dt}=mg_x \qquad (6)$$

Therefore, $\boxed{V_y\frac{dV_x}{dy}=\frac{dV_x}{dt}=mg_x}$ (7)

Step 3: Let $\rho V_x\frac{\partial v_x}{\partial x}=h\rho g_x$ where h is the ratio term corresponding to $\rho V_x\frac{\partial v_x}{\partial x}$.

$$V_x\frac{\partial V_x}{\partial x}=hg_x \qquad (8)$$

$V_x\frac{dV_x}{dx}=hg_x$ (One drops the partials symbol, since a single independent variable is involved)

$\frac{dx}{dt}\frac{dV_x}{dx}=hg_x \quad (V_x=\frac{dx}{dt})$

$\frac{dv_x}{dt}=hg_x$ (9) Therefore, $\boxed{V_x\frac{\partial V_x}{\partial x}=\frac{dV_x}{dt}=hg_x}$ (10)

From equations (4), (7), (10), $V_x \frac{\partial v_x}{\partial x} = V_y \frac{\partial v_x}{\partial y} = V_z \frac{\partial v_x}{\partial z} = \frac{dv_x}{dt}$ and

$$V_x \frac{\partial v_x}{\partial x} + V_y \frac{\partial v_x}{\partial y} + V_z \frac{\partial v_x}{\partial z} = \boxed{3\frac{dv_x}{dt}} \qquad (11)$$

Thus, the ratio of the linear term $\frac{\partial V_x}{\partial t}$ to the nonlinear sum $V_x \frac{\partial v_x}{\partial x} + V_y \frac{\partial v_x}{\partial y} + V_z \frac{\partial v_x}{\partial z}$ in equation (1) is 1 to 3. Unquestionably, there is a ratio between the sum of the nonlinear terms and the linear term $\frac{\partial v_x}{\partial t}$. This ratio must be verified experimentally.

Note: One could have obtained equation (C) from equation (A) by redefining the nonlinear terms by **carelessly** disregarding the partial derivatives of the nonlinear terms in equation (1). However, the author did not do that, but logically, the terms became linearized.

Note also that the above linearization is possible only if ρg_x is the subject of the equation, and it will later be learned that a solution to the logically linearized Navier-Stokes equation is obtained only if ρg_x is the subject of the equation.

Step 4: Substitute the right side of equation (11) for the nonlinear terms on the left- side of

$$-\mu\frac{\partial^2 V_x}{\partial x^2} - \mu\frac{\partial^2 V_x}{\partial y^2} - \mu\frac{\partial^2 V_x}{\partial z^2} + \frac{\partial p_x}{\partial x} + \underbrace{\rho\frac{\partial V_x}{\partial t} + \overbrace{\rho V_x\frac{\partial V_x}{\partial x} + \rho V_y\frac{\partial V_x}{\partial y} + \rho V_z\frac{\partial V_x}{\partial z}}^{\text{nonlinear terms}}}_{\text{all acceleration terms}} = \rho g_x \qquad (12)$$

Then one obtains $-\mu\frac{\partial^2 V_x}{\partial x^2} - \mu\frac{\partial^2 V_x}{\partial y^2} - \mu\frac{\partial^2 V_x}{\partial z^2} + \frac{\partial p_x}{\partial x} \underbrace{+\rho\frac{\partial V_x}{\partial t} + 3\rho\frac{\partial V_x}{\partial x}}_{\text{all acceleration terms}} = \rho g_x$

$$\boxed{-\mu\frac{\partial^2 V_x}{\partial x^2} - \mu\frac{\partial^2 V_x}{\partial y^2} - \mu\frac{\partial^2 V_x}{\partial z^2} + \frac{\partial p_x}{\partial x} + 4\rho\frac{\partial V_x}{\partial t} = \rho g_x} \quad \text{(simplifying)} \qquad (13)$$

Now, instead of solving equation (1), previous page, one will solve the following equation

$$\boxed{-K\frac{\partial^2 V_x}{\partial x^2} - K\frac{\partial^2 V_x}{\partial y^2} - K\frac{\partial^2 V_x}{\partial z^2} + \frac{1}{\rho}\frac{\partial p}{\partial x} + 4\frac{\partial V_x}{\partial t} = g_x} \quad (k = \frac{\mu}{\rho}) \qquad (14)$$

Main Step 2

Step 5: In equation (14) divide g_x by the terms on the left side in the ratio $a : b : c : d : f$.

$$\boxed{-K\frac{\partial^2 V_x}{\partial x^2} = ag_x; \quad -K\frac{\partial^2 V_x}{\partial y^2} = bg_x; \quad -K\frac{\partial^2 V_x}{\partial z^2} = cg_x; \quad \frac{1}{\rho}\frac{\partial p}{\partial x} = dg_x; \quad 4\frac{\partial V_x}{\partial t} = fg_x}$$

(a, b, c, d, f are the ratio terms and $a + b + c + d + f = 1$).

As proportions: $\frac{-K\frac{\partial^2 V_x}{\partial x^2}}{a} = \frac{g_x}{1}; \quad \frac{-K\frac{\partial^2 V_x}{\partial y^2}}{b} = \frac{g_x}{1}; \quad \frac{-K\frac{\partial^2 V_x}{\partial z^2}}{c} = \frac{g_x}{1}; \quad \frac{\frac{1}{\rho}\frac{\partial p}{\partial x}}{d} = \frac{g_x}{1}; \quad \frac{4\frac{\partial V_x}{\partial t}}{f} = \frac{g_x}{1}$

One can view each of the ratio terms a, b, c, d, f as a fraction (a real number) of $\boxed{g_x}$ contributed by each expression on the left-hand side of equation (14) above

Main Step 3

Step 6: Solve the differential equations in Step 5.

Solutions of the five sub-equations

$$\boxed{-K\frac{\partial^2 V_x}{\partial x^2} = ag}$$
$$k\frac{\partial^2 V_x}{\partial x^2} = -ag$$
$$\frac{\partial^2 V_x}{\partial x^2} = -\frac{a}{k}g$$
$$\frac{\partial V_x}{\partial x} = -\frac{ag}{k}x + C_1$$
$$V_{x1} = -\frac{ag}{2k}x^2 + C_1 x + C_2$$

$$\boxed{-K\frac{\partial^2 V_x}{\partial y^2} = bg}$$
$$K\frac{\partial^2 V_x}{\partial y^2} = -bg$$
$$\frac{\partial^2 V_x}{\partial y^2} = -\frac{b}{k}g$$
$$\frac{\partial V_x}{\partial y} = -\frac{bg}{k}y + C_3$$
$$V_{x2} = -\frac{bg}{2k}y^2 + C_3 y + C_4$$

$$\boxed{-K\frac{\partial^2 V_x}{\partial z^2} = cg}$$
$$K\frac{\partial^2 V_x}{\partial z^2} = -cg$$
$$\frac{\partial^2 V_x}{\partial z^2} = -\frac{c}{k}g$$
$$\frac{\partial V_x}{\partial z} = -\frac{cg}{k}z + C_5$$
$$V_{x3} = -\frac{cg}{2k}z^2 + C_5 z + C_6$$

$$\boxed{\frac{1}{\rho}\frac{\partial p}{\partial x} = dg}$$
$$\frac{1}{\rho}\frac{\partial p}{\partial x} = dg$$
$$\frac{\partial p}{\partial x} = d\rho g$$
$$p = d\rho g x + C_7$$
$$\boxed{4\frac{\partial V_x}{\partial t} = fg}$$
$$\frac{\partial V_x}{\partial t} = \frac{f}{4}g$$
$$V_{x4} = \frac{fg}{4}t$$

Main Step 4

Step 7: One combines the above solutions

$$\begin{aligned}
V_x &= V_{x1} + V_{x2} + V_{x3} + V_{x4} \\
&= -\frac{ag}{2k}x^2 + C_1 x + C_2 - \frac{bg}{2k}y^2 + C_3 y + C_4 - \frac{cg}{2k}z^2 + C_5 z + C_6 + \frac{fg}{4}t + C_7 \\
&= -\frac{ag}{2k}x^2 + C_1 x - \frac{bg}{2k}y^2 + C_3 y - \frac{cg}{2k}z^2 + C_5 z + \frac{fg}{4}t + C_9 \\
&= -\frac{ag}{2k}x^2 - \frac{bg}{2k}y^2 - \frac{cg}{2k}z^2 + C_1 x + C_3 y + C_5 z + \frac{fg}{4}t + C_9 \\
&= -\frac{ag}{2k}x^2 - \frac{bg}{2k}y^2 - \frac{cg}{2k}z^2 + C_1 x + C_3 y + C_5 z + \frac{fg}{4}t + C_9 \\
&= -\frac{g_x}{2k}(ax^2 + by^2 + cz^2) + C_1 x + C_3 y + C_5 z + \frac{fg_x}{4}t + C_9
\end{aligned}$$

$$V_x = -\frac{\rho g_x}{2\mu}(ax^2 + by^2 + cz^2) + C_1 x + C_3 y + C_5 z + \frac{fg_x}{4}t + C_9$$

$$P(x) = d\rho g_x x$$

$$\boxed{\begin{aligned}
&V_x = V_{x1} + V_{x2} + V_{x3} + V_{x4} \\
&V_x(x,y,z,t) = -\frac{\rho g_x}{2\mu}(ax^2 + by^2 + cz^2) + C_1 x + C_3 y + C_5 z + \frac{fg_x}{4}t + C_9 \\
&P(x) = d\rho g_x x
\end{aligned}}$$

For $V_x(x,t)$, let $y = 0,\ z = 0$

Then $\boxed{V_x(x,t) = -\frac{\rho g_x}{2\mu}ax^2 + C_1 x + \frac{fg_x}{4}t + C_9}$ $\boxed{P(x) = d\rho g_x x}$

$$\boxed{V_x(x,0) = V_x^0(x) = -\frac{\rho g_x}{2\mu}ax^2 + C_{10}x + C_9}$$

Main Step 5

Checking in equation (C)

Step 8: Find the derivatives, using

$$V_x = -\frac{\rho g_x}{2\mu}(ax^2 + by^2 + cz^2) + C_1 x + C_3 y + C_5 z + \frac{f g_x}{4} t + C_9$$

$$\boxed{P(x) = d\rho g_x x}$$

$$\frac{\partial V_x}{\partial x} = -\frac{\rho g_x}{2\mu}(2ax) + C_1$$

1. $\boxed{\frac{\partial^2 V_x}{\partial x^2} = -\frac{a\rho g_x}{\mu}}$

4. $\boxed{\frac{\partial p}{\partial x} = d\rho g}$; 5. $\boxed{\frac{\partial V_x}{\partial t} = \frac{f g_x}{4}}$

$$\frac{\partial V_x}{\partial y} = -\frac{\rho g_x}{\mu}(by) + C_3$$

2. $\boxed{\frac{\partial^2 V_x}{\partial y^2} = -\frac{b\rho g_x}{\mu}}$

$$\frac{\partial V_x}{\partial z} = -\frac{\rho g_x}{\mu}(cz)$$

3. $\boxed{\frac{\partial^2 V_x}{\partial z^2} = -\frac{c\rho g_x}{\mu}}$;

Step 9: Substitute the derivatives from Step 8 in $-\mu(\frac{\partial^2 V_x}{\partial x^2} + \frac{\partial^2 V_x}{\partial y^2} + \frac{\partial^2 V_x}{\partial z^2}) + \frac{\partial p_x}{\partial x} + 4\rho\frac{\partial V_x}{\partial t} = \rho g_x$ to check for identity (to determine if the relation obtained satisfies the original equation).

$$-\mu(\frac{\partial^2 V_x}{\partial x^2} + \frac{\partial^2 V_x}{\partial y^2} + \frac{\partial^2 V_x}{\partial z^2}) + \frac{\partial p}{\partial x} + 4\rho\frac{\partial V_x}{\partial t} = \rho g_x$$

$$-\mu(-\frac{a\rho g_x}{\mu} - \frac{b\rho g_x}{\mu} - \frac{c\rho g_x}{\mu}) + d\rho g_x + 4\rho\frac{f}{4}g_x \stackrel{?}{=} \rho g_x$$

$$a\rho g_x + b\rho g_x + c\rho g_x + d\rho g_x + \rho f g_x \stackrel{?}{=} \rho g_x$$

$$a g_x + b g_x + c g_x + d g_x + f g_x \stackrel{?}{=} g_x$$

$$g_x(a + b + c + d + f) \stackrel{?}{=} g_x$$

$$g_x(1) \stackrel{?}{=} g_x \quad (a + b + c + d + f = 1)$$

$$g_x \stackrel{?}{=} g_x \quad \text{Yes}$$

Scrapwork

$$\boxed{\begin{aligned} \frac{\partial^2 V_x}{\partial x^2} &= -\frac{a\rho g_x}{\mu}; \\ \frac{\partial^2 V_x}{\partial y^2} &= -\frac{b\rho g_x}{\mu}; \\ \frac{\partial^2 V_x}{\partial z^2} &= -\frac{c\rho g_x}{\mu}; \end{aligned}}$$

$$\boxed{\frac{\partial p}{\partial x} = d\rho g;} \quad \boxed{\frac{\partial V_x}{\partial t} = \frac{f g_x}{4}}$$

An identity is obtained and therefore, the solution of equation (C), p.96, is given by

$$\boxed{V_x(x,y,z,t) = -\frac{\rho g_x}{2\mu}(ax^2 + by^2 + cz^2) + C_1 x + C_3 y + C_5 z + \frac{f g_x}{4} t + C_9; \;\; P(x) = d\rho g_x x}$$

The above solution is unique, because all possible equations were integrated but only a single equation, the equation with the gravity term as the subject of the equation produced the solution.

Solution Summary for V_x, V_y and V_z

For V_x $\quad a+b+c+d+f=1$

$$\mu(\frac{\partial^2 V_x}{\partial x^2}+\frac{\partial^2 V_x}{\partial y^2}+\frac{\partial^2 V_x}{\partial z^2})-\frac{\partial p_x}{\partial x}+\rho g_x=\rho(\frac{\partial V_x}{\partial t}+V_x\frac{\partial V_x}{\partial x}+V_y\frac{\partial V_x}{\partial y}+V_z\frac{\partial V_x}{\partial z})$$

$$-K\frac{\partial^2 V_x}{\partial x^2}-K\frac{\partial^2 V_x}{\partial y^2}-K\frac{\partial^2 V_x}{\partial z^2}+\frac{1}{\rho}\frac{\partial p}{\partial x}+4\frac{\partial V_x}{\partial t}=g_x$$

$$\begin{aligned}V_x &= V_{x1}+V_{x2}+V_{x3}+V_{x4}\\ &=-\frac{ag}{2k}x^2+C_1x+C_2-\frac{bg}{2k}y^2+C_3y+C_4-\frac{cg}{2k}z^2+C_5z+C_6+\frac{fg}{4}t+C_7+C_8\\ &=-\frac{ag}{2k}x^2+C_1x-\frac{bg}{2k}y^2+C_3y-\frac{cg}{2k}z^2+C_5z+\frac{fg}{4}t+C_9\\ &=-\frac{ag}{2k}x^2-\frac{bg}{2k}y^2-\frac{cg}{2k}z^2+C_1x+C_3y+C_5z+\frac{fg}{4}t+C_9\end{aligned}$$

$$V_y(x,y,z,t)=-\frac{\rho g_x}{2\mu}(ax^2+by^2+cz^2)+C_1x+C_3y+C_5z+\frac{fg_x}{4}t+C_9$$

$$P(x)=d\rho gx$$

For V_y $\quad h+j+m+n+q=1$

$$\mu(\frac{\partial^2 V_y}{\partial x^2}+\frac{\partial^2 V_y}{\partial y^2}+\frac{\partial^2 V_y}{\partial z^2})-\frac{\partial p}{\partial y}+\rho g_y=\rho(\frac{\partial V_y}{\partial t}+V_x\frac{\partial V_y}{\partial x}+V_y\frac{\partial V_y}{\partial y}+V_z\frac{\partial V_y}{\partial z})$$

$$-K\frac{\partial^2 V_y}{\partial x^2}-K\frac{\partial^2 V_y}{\partial y^2}-K\frac{\partial^2 V_y}{\partial z^2}+\frac{1}{\rho}\frac{\partial p}{\partial y}+4\frac{\partial V_y}{\partial t}=g_y$$

$$\boxed{\begin{aligned}&V_y=-\frac{hg_y}{2k}x^2+C_1x-\frac{jg_y}{2k}y^2+C_3y-\frac{mg_y}{2k}z^2+C_5z+\frac{ng_y}{4}t\\ &V_y(x,y,z,t)=-\frac{\rho g_y}{2\mu}(hx^2+jy^2+mz^2)+C_1x+C_3y+C_5z+\frac{qg_y}{4}t+C\\ &P(y)=n\rho g_y y\end{aligned}}$$

For V_z $\quad r+s+u+v+w=1$

$$\mu(\frac{\partial^2 V_z}{\partial x^2}+\frac{\partial^2 V_z}{\partial y^2}+\frac{\partial^2 V_z}{\partial z^2})-\frac{\partial p}{\partial z}+\rho g_z=\rho(\frac{\partial V_z}{\partial t}+V_x\frac{\partial V_z}{\partial x}+V_y\frac{\partial V_z}{\partial y}+V_z\frac{\partial V_z}{\partial z})$$

$$-k\frac{\partial^2 V_z}{\partial x^2}-k\frac{\partial^2 V_z}{\partial y^2}-k\frac{\partial^2 V_z}{\partial z^2}+\frac{1}{\rho}\frac{\partial p}{\partial z}+4\frac{\partial V_z}{\partial t}=g_z$$

$$\boxed{\begin{aligned}&V_z=-\frac{rg_z}{2k}x^2+C_1x-\frac{sg_z}{2k}y^2+C_3y-\frac{ug_z}{2k}z^2+C_5z+\frac{wg_z}{4}t\\ &V_z(x,y,z,t)=-\frac{\rho g_z}{2\mu}(rx^2+sy^2+uz^2)+C_1x+C_3y+C_5z+\frac{wg_z}{4}t+C\\ &P(z)=v\rho g_z z\end{aligned}}$$

Discussion About Solutions

A solution to equation $-\mu(\frac{\partial^2 v_x}{\partial x^2}+\frac{\partial^2 v_x}{\partial y^2}+\frac{\partial^2 v_x}{\partial z^2})+\frac{\partial p}{\partial x}+4\rho(\frac{\partial v_x}{\partial t})=\rho g_x$ (C) is

$$\boxed{\begin{array}{l} V_x(x,y,z,t)=-\frac{\rho g_x}{2\mu}(ax^2+by^2+cz^2)+C_1x+C_3y+C_5z+\frac{fg_x}{4}t+C_9 \\ P(x)=d\rho g_x x; \quad (a+b+c+d+f=1) \end{array}}.$$

This relation gives an identity when checked in Equation (C) above.

One observes above that the most important insight of the above solution is the indispensability of the gravity term in incompressible fluid flow. Observe that if gravity, g, were zero, the first three terms, the seventh term, and $P(x)$ would all be zero be. This result can be stated emphatically that without gravity forces on earth, there will be no incompressible fluid flow on earth as is known. The above result will be the same when one covers the general case, Option 4.

The above parabolic solution is also encouraging. It reminds one of the parabolic curve obtained when a stone is projected vertically upwards at an acute angle to the horizontal..
The author also tried the following possible approaches: (D), (E) and (F), but none of the possible solutions completely satisfied the corresponding original equations (D), (E) or (F) .

$\mu\frac{\partial^2 v_x}{\partial x^2}+\mu\frac{\partial^2 v_x}{\partial y^2}+\mu\frac{\partial^2 v_x}{\partial z^2}+\rho g_x-4\rho\frac{\partial v_x}{\partial t}=\frac{\partial p}{\partial x}$ (D) (One uses the subject $\boxed{\frac{\partial p}{\partial x}}$

$\frac{K}{4}\frac{\partial^2 V_x}{\partial x^2}+\frac{K}{4}\frac{\partial^2 V_x}{\partial y^2}+\frac{K}{4}\frac{\partial^2 V_x}{\partial z^2}-\frac{1}{4\rho}\frac{\partial p}{\partial x}+\frac{g_x}{4}=\frac{\partial V_x}{\partial t}$ (E), (One uses the subject $\boxed{\frac{\partial V_x}{\partial t}}$

$-\frac{\partial^2 v_x}{\partial y^2}-\frac{\partial^2 v_x}{\partial z^2}-\frac{\rho g_x}{\mu}+\frac{4\rho}{\mu}\frac{\partial v_x}{\partial t}+\frac{1}{\mu}\frac{\partial p}{\partial x}=\frac{\partial^2 v_x}{\partial x^2}$ (F) (One uses subject $\boxed{\frac{\partial^2 V_x}{\partial x^2}}$

Integration Results Summary

Case 1: $-\mu(\frac{\partial^2 V_x}{\partial x^2}+\frac{\partial^2 V_x}{\partial y^2}+\frac{\partial^2 V_x}{\partial z^2})+\frac{\partial p}{\partial x}+4\rho(\frac{\partial V_x}{\partial t})=\rho g_x$ (C)

$$\boxed{\begin{array}{l} V_x(x,y,z,t)=-\frac{\rho g_x}{2\mu}(ax^2+by^2+cz^2)+C_1x+C_3y+C_5z+\frac{fg_x}{4}t+C_9 \\ P(x)=d\rho g_x x; \quad (a+b+c+d+f=1) \end{array}}$$ **<----Solution**

Case 2: $\mu\frac{\partial^2 v_x}{\partial x^2}+\mu\frac{\partial^2 v_x}{\partial y^2}+\mu\frac{\partial^2 v_x}{\partial z^2}+\rho g_x-4\rho\frac{\partial v_x}{\partial t}=\frac{\partial p}{\partial x}$ (D). (One uses the subject $\boxed{\frac{\partial p}{\partial x}}$

$$\boxed{\begin{array}{l} V_x(x,y,z.t)=\frac{\lambda_x}{2\mu}(ax^2+by^2+cz^2)+C_1x+\lambda_p x+C_3y+C_5z-\frac{f\lambda}{4\rho}t+C \\ P(x)=\frac{1}{d}\rho g_x x \end{array}}$$

Case 3: $\frac{K}{4}\frac{\partial^2 V_x}{\partial x^2}+\frac{K}{4}\frac{\partial^2 V_x}{\partial y^2}+\frac{K}{4}\frac{\partial^2 V_x}{\partial z^2}-\frac{1}{4\rho}\frac{\partial p}{\partial x}+\frac{g_x}{4}=\frac{\partial V_x}{\partial t}$ (E). (One uses the subject $\boxed{\frac{\partial V_x}{\partial t}}$

$$\boxed{\begin{array}{l} V_x(x,y,z,t)=(C_1\cos\lambda_x x+C_2\sin\lambda_x x)e^{-(\lambda^2/\beta)t}+(C_3\cos\lambda_y y+C_4\sin\lambda_y y)e^{-(\lambda_y{}^2/\omega)t} \\ +(C_5\cos\lambda_z z+C_6\sin\lambda_z z)e^{-(\lambda_z{}^2/\varepsilon)t}+\frac{g}{4f}t+\lambda x+C_8 \\ P(x)=\lambda x=d\rho g_x x \end{array}}$$

Case 4: $-\frac{\partial^2 v_x}{\partial y^2}-\frac{\partial^2 v_x}{\partial z^2}-\frac{\rho g_x}{\mu}+\frac{4\rho}{\mu}\frac{\partial v_x}{\partial t}+\frac{1}{\mu}\frac{\partial p}{\partial x}=\frac{\partial^2 v_x}{\partial x^2}$ (F). (One uses the subject $\boxed{\frac{\partial^2 V_x}{\partial x^2}}$

$$\boxed{\begin{aligned}&V_x(x,y,z,t)=(A\cos\lambda y+B\sin\lambda y)\left(Ce^{(\frac{\lambda\sqrt{a}}{a})x}+De^{-(\frac{\lambda\sqrt{a}}{a})x}\right)\\&+(E\cos\lambda z+F\sin\lambda z)\left(He^{(\frac{\lambda\sqrt{b}}{b})x}+Le^{(-\frac{\lambda\sqrt{b}}{b})x}\right)-\frac{\rho g_x x^2}{2c\mu}+Ax+B+(A_1\cos\lambda x+B_1\sin\lambda x)e^{-(\lambda^2/\alpha)t}\\&+\frac{\lambda}{2\mu f}x^2+C_2x+C_3);\qquad P(x)=d\rho g_x x\end{aligned}}$$

Note: Relations for equations with subjects g_x and $\frac{\partial p}{\partial x}$ are almost identical.

By comparing possible solutions for equations (C) and (D), $\lambda_x=-\rho g_x$ in relation for (D).

$$\boxed{V_x(x,y,z,t)=\frac{\lambda_x}{2\mu}(ax^2+by^2+cz^2)+C_1x+\lambda_p x+C_3y+C_5z-\frac{f\lambda}{4\rho}t+C;\quad P(x)=\frac{1}{d}\rho g_x x}$$

Comparative analysis of the possible solutions when checked in each corresponding equation

Equation	Equation Subject	Number of terms of possible solutions **not** satisfying original equation
Case 1: $-\mu(\frac{\partial^2 V_x}{\partial x^2}+\frac{\partial^2 V_x}{\partial y^2}+\frac{\partial^2 V_x}{\partial z^2})+\frac{\partial p}{\partial x}+4\rho(\frac{\partial V_x}{\partial t})=\rho g_x$	ρg_x	None Case 1 yields the solution
Case 2: $\mu\frac{\partial^2 v_x}{\partial x^2}+\mu\frac{\partial^2 v_x}{\partial y^2}+\mu\frac{\partial^2 v_x}{\partial z^2}+\rho g_x-4\rho\frac{\partial v_x}{\partial t}=\frac{\partial p}{\partial x}$	$\frac{\partial p}{\partial x}$	One term
Case 3: $\frac{K}{4}\frac{\partial^2 V_x}{\partial x^2}+\frac{K}{4}\frac{\partial^2 V_x}{\partial y^2}+\frac{K}{4}\frac{\partial^2 V_x}{\partial z^2}-\frac{1}{4\rho}\frac{\partial p}{\partial x}+\frac{g_x}{4}=\frac{\partial V_x}{\partial t}$	$\frac{\partial V_x}{\partial t}$	At least 2 terms
Case 4: $-\frac{\partial^2 v_x}{\partial y^2}-\frac{\partial^2 v_x}{\partial z^2}-\frac{\rho g_x}{\mu}+\frac{4\rho}{\mu}\frac{\partial v_x}{\partial t}+\frac{1}{\mu}\frac{\partial p}{\partial x}=\frac{\partial^2 v_x}{\partial x^2}$	$\frac{\partial^2 V_x}{\partial x^2}$	At least 2 terms
Case 5: $-\frac{\partial^2 v_x}{\partial x^2}-\frac{\partial^2 v_x}{\partial z^2}-\frac{\rho g_x}{\mu}+\frac{4\rho}{\mu}\frac{\partial v_x}{\partial t}+\frac{1}{\mu}\frac{\partial p}{\partial x}=\frac{\partial^2 v_x}{\partial y^2}$	$\frac{\partial^2 v_x}{\partial y^2}$	At least 2 terms
Case 6: $-\frac{\partial^2 V_x}{\partial y^2}-\frac{\partial^2 V_x}{\partial x^2}-\frac{\rho g_x}{\mu}+\frac{4\rho}{\mu}\frac{\partial V_x}{\partial t}+\frac{1}{\mu}\frac{\partial p}{\partial x}=\frac{\partial^2 V_x}{\partial z^2}$	$\frac{\partial^2 V_x}{\partial z^2}$	At least 2 terms

Note above that only Case 1 is the solution, and this may imply that the solution to the Navier-Stokes equation is unique. Out of six possible subjects, only one subject produced a solution. The above results show that a relation obtained by the integration of a partial differential equation must be checked in the corresponding equation for identity before claiming that the relation is a solution, Cases 2, 3, 4, 5 and 6, are not solutions but integration relations. For example, it would be incorrect to say that the equation in Case 3 has a periodic solution; but it would be correct to say that the equation in Case 3 has a periodic relation, since the relation obtained by integration does not satisfy its corresponding equation. It would be correct to say that the equation in Case 1 has a parabolic solution or a parabolic relation.

Below are detailed explanation of results of the identity checking process.

Outcome 1: With g_x included and with g_x as the subject of the equation. The solution is straightforward and the possible solution checks well in the original equation (C). Also, if g_x or ρg_x is not the subject of the equation, the linearization of the nonlinear terms could not be justified.

Outcome 2: With g_x included but with $\frac{\partial V_x}{\partial t}$ as the subject of the equation.

There are two problems when checking . **1.** For $\frac{\partial V_x}{\partial t} = -\frac{1}{4\rho}\frac{\partial p}{\partial x} \to -\frac{\lambda t}{4\rho d}$**;** **2.** $\frac{g}{4} = \frac{\partial V_x}{\partial t} \to \frac{gt}{4f}$

With d and f in the denominators, the multipliers sum $a+b+c+d+f=1$ is false.

Outcome 3 : With g_x excluded, and $\frac{\partial V_x}{\partial t}$ as the subject of the equation, there is one problem:

$-\frac{1}{4\rho}\frac{\partial p}{\partial x} = \frac{\partial V_x}{\partial t} \to -\frac{\lambda t}{4\rho d}$.With d in the denominator $a+b+c+d+f=1$ is false

Outcome 4 : With g_x included, and $\frac{\partial^2 V_x}{\partial x^2}$ as the subject of the equation, there are at least, two problems in the checking with the multipliers c and f in the denominators.
Checking for $a+b+c+d+f=1$ is impossible.

Outcomes 5 and 6 are similar to Outcome 4.

Characteristic curves of the integration results

Equations	Equation Subject	Curve characteristics
Case 1: $-\mu(\frac{\partial^2 V_x}{\partial x^2}+\frac{\partial^2 V_x}{\partial y^2}+\frac{\partial^2 V_x}{\partial z^2})+\frac{\partial p}{\partial x}+4\rho(\frac{\partial V_x}{\partial t})=\rho g_x$	ρg_x	Parabolic and Inverted
Case 2: $\mu\frac{\partial^2 V_x}{\partial x^2}+\mu\frac{\partial^2 V_x}{\partial y^2}+\mu\frac{\partial^2 V_x}{\partial z^2}+\rho g_x-4\rho\frac{\partial V_x}{\partial t}=\frac{\partial p}{\partial x}$	$\frac{\partial p}{\partial x}$	Parabolic
Case 3: $\frac{K}{4}\frac{\partial^2 V_x}{\partial x^2}+\frac{K}{4}\frac{\partial^2 V_x}{\partial y^2}+\frac{K}{4}\frac{\partial^2 V_x}{\partial z^2}-\frac{1}{4\rho}\frac{\partial p}{\partial x}+\frac{g_x}{4}=\frac{\partial V_x}{\partial t}$	$\frac{\partial V_x}{\partial t}$	Periodic and decreasingly exponential
Case 4: $-\frac{\partial^2 V_x}{\partial y^2}-\frac{\partial^2 V_x}{\partial z^2}-\frac{\rho g_x}{\mu}+\frac{4\rho}{\mu}\frac{\partial V_x}{\partial t}+\frac{1}{\mu}\frac{\partial p}{\partial x}=\frac{\partial^2 V_x}{\partial x^2}$	$\frac{\partial^2 V_x}{\partial x^2}$	Periodic, parabolic, and decreasingly exponential
Case 5: $-\frac{\partial^2 V_x}{\partial x^2}-\frac{\partial^2 V_x}{\partial z^2}-\frac{\rho g_x}{\mu}+\frac{4\rho}{\mu}\frac{\partial V_x}{\partial t}+\frac{1}{\mu}\frac{\partial p}{\partial x}=\frac{\partial^2 V_x}{\partial y^2}$	$\frac{\partial^2 V_x}{\partial y^2}$	Periodic, parabolic, and decreasingly exponential
Case 6: $-\frac{\partial^2 V_x}{\partial y^2}-\frac{\partial^2 V_x}{\partial x^2}-\frac{\rho g_x}{\mu}+\frac{4\rho}{\mu}\frac{\partial V_x}{\partial t}+\frac{1}{\mu}\frac{\partial p}{\partial x}=\frac{\partial^2 V_x}{\partial z^2}$	$\frac{\partial^2 V_x}{\partial z^2}$	Periodic, parabolic, and decreasingly exponential

The following are possible interpretations of the roles of the terms based on the types of curves produced when using the terms as subjects of the equations.

1. g_x and $\frac{\partial p}{\partial x}$ are involved in the parabolic motion; g_x is responsible for the forward motion.
2. $\frac{\partial V_x}{\partial t}$ is involved in the periodic and decreasingly exponential bevavior.
3. $\frac{\partial^2 V_x}{\partial x^2}$, $\frac{\partial^2 V_x}{\partial y^2}$ and $\frac{\partial^2 V_x}{\partial z^2}$ are involved in the parabolic, periodic and decreasingly exponential motion. As μ increases, the periodicity increases

Definitions and Classification of Equations

$$\boxed{-K\frac{\partial^2 V_x}{\partial x^2} - K\frac{\partial^2 V_x}{\partial y^2} - K\frac{\partial^2 V_x}{\partial z^2} + \frac{1}{\rho}\frac{\partial p}{\partial x} + 4\frac{\partial V_x}{\partial t} = g_x} \qquad (k = \frac{\mu}{\rho})$$

One may classify the equations involved in Option 1 according to the following:

Driver Equation: A differential equation whose integration relation satisfies its corresponding equation.

Supporter equation: A differential equation which contains the same terms as the driver equation but whose integration relation does not satisfy its corresponding equation but provides useful information about the driver equation.

Note that the driver equation and a supporter equation differ only in the subject of the equation.

Equation	Equation Subject	Type of equation	# of terms of relation not satisfying original equation
Case 1: $-\mu(\frac{\partial^2 V_x}{\partial x^2} + \frac{\partial^2 V_x}{\partial y^2} + \frac{\partial^2 V_x}{\partial z^2}) + \frac{\partial p}{\partial x} + 4\rho(\frac{\partial V_x}{\partial t}) = \rho g_x$	ρg_x	Driver Equation	None
Case 2: $\mu\frac{\partial^2 V_x}{\partial x^2} + \mu\frac{\partial^2 V_x}{\partial y^2} + \mu\frac{\partial^2 V_x}{\partial z^2} + \rho g_x - 4\rho\frac{\partial V_x}{\partial t} = \frac{\partial p}{\partial x}$	$\frac{\partial p}{\partial x}$	Supporter equation	One term
Case 3: $\frac{K}{4}\frac{\partial^2 V_x}{\partial x^2} + \frac{K}{4}\frac{\partial^2 V_x}{\partial y^2} + \frac{K}{4}\frac{\partial^2 V_x}{\partial z^2} - \frac{1}{4\rho}\frac{\partial p}{\partial x} + \frac{g_x}{4} = \frac{\partial V_x}{\partial t}$	$\frac{\partial V_x}{\partial t}$	Supporter equation	At least 2 terms
Case 4: $-\frac{\partial^2 V_x}{\partial y^2} - \frac{\partial^2 V_x}{\partial z^2} - \frac{\rho g_x}{\mu} + \frac{4\rho}{\mu}\frac{\partial V_x}{\partial t} + \frac{1}{\mu}\frac{\partial p}{\partial x} = \frac{\partial^2 V_x}{\partial x^2}$	$\frac{\partial^2 V_x}{\partial x^2}$	Supporter equation	At least 2 terms
Case 5: $-\frac{\partial^2 V_x}{\partial x^2} - \frac{\partial^2 V_x}{\partial z^2} - \frac{\rho g_x}{\mu} + \frac{4\rho}{\mu}\frac{\partial V_x}{\partial t} + \frac{1}{\mu}\frac{\partial p}{\partial x} = \frac{\partial^2 V_x}{\partial y^2}$	$\frac{\partial^2 V_x}{\partial y^2}$	Supporter equation	At least 2 terms
Case 6: $-\frac{\partial^2 V_x}{\partial y^2} - \frac{\partial^2 V_x}{\partial x^2} - \frac{\rho g_x}{\mu} + \frac{4\rho}{\mu}\frac{\partial V_x}{\partial t} + \frac{1}{\mu}\frac{\partial p}{\partial x} = \frac{\partial^2 V_x}{\partial z^2}$	$\frac{\partial^2 V_x}{\partial z^2}$	Supporter equation	At least 2 terms

One can apply the above definitions in solving the magnetohydrodynamic equations (Option 6)

Applications of the splitting technique in science, engineering, business fields

The approach used in solving the equations allows for how the terms interact with each other. The author has not seen this technique anywhere, but the results are revealing and promising.

Fluid flow design considerations:

1. Maximize the role of g_x forces, followed by**;** **2.** $\frac{\partial p}{\partial x}$ forces; then 3. $\frac{\partial V_x}{\partial t}$

(Make g_x happy by always providing a workable $mg\sin\theta$) .

For long distance flow design such as for water pipelines, water channels, oil pipelines. whenever possible, the design should facilitate and maximize the role of gravity forces, and if design is impossible to facilitate the role of gravity forces, design for $\frac{\partial p}{\partial x}$ to take over flow.

The performance of $\frac{\partial^2 V_x}{\partial x^2}$ should be studied further, since its role is the most complicated: periodic, parabolic, and decreasingly exponential.

Tornado Effect Relief

Perhaps, machines can be designed and built to chase and neutralize or minimize tornadoes during touch-downs. The energy in the tornado at touch-down can be harnessed for useful purposes.

Business and economics applications.

1. Figuratively, if g_x is the president of a company, it will have good working relationships with all the members of the board of directors, according to the solution of the Navier-Stokes equation. If g_x is present at a meeting g_x must preside over the meeting for the best outcome.

2. If g_x is absent from a meeting, let $\frac{\partial p}{\partial x}$ preside over the meeting, and everything will workout well. However, if g_x is present, g_x must preside over the meeting.

To apply the results of the solutions of the Navier-Stokes equations in other areas or fields, the properties, characteristics and functions of g_x, $\frac{\partial p}{\partial x}$, $\frac{\partial v_x}{\partial t}$ must be studied to determine analogous terms in those areas of possible applications. Other areas of applications include investments choice decisions, financial decisions, personnel management and family relationships.

Option 2
Solutions of 4-D Linearized Navier-Stokes Equations

One advantage of the pairing approach is that the above solution can easily be extended to any number of dimensions.

If one adds $\mu\frac{\partial^2 V_x}{\partial s^2}$ and $\rho V_s\frac{\partial V_x}{\partial s}$ to the 3-D x–direction equation, one obtains the 4-D Navier--Stokes equation $-\mu(\frac{\partial^2 V_x}{\partial x^2}+\frac{\partial^2 V_x}{\partial y^2}+\frac{\partial^2 V_x}{\partial z^2}+\frac{\partial^2 V_x}{\partial s^2})+\frac{\partial p}{\partial x}+4\rho(\frac{\partial v_x}{\partial t})+\rho V_s\frac{\partial V_x}{\partial s}=\rho g_x$

After linearization, $-\mu(\frac{\partial^2 V_x}{\partial x^2}+\frac{\partial^2 V_x}{\partial y^2}+\frac{\partial^2 V_x}{\partial z^2}+\frac{\partial^2 V_x}{\partial s^2})+\frac{\partial p}{\partial x}+5\rho(\frac{\partial v_x}{\partial t})=\rho g_x$ and its solution is

$$\boxed{\begin{array}{l} V_x(x,y,z,s,t)=-\frac{\rho g_x}{2\mu}(ax^2+by^2+cz^2+es^2)+C_1x+C_3y+C_5z+C_7s+\frac{fg_x}{5}t+C_9 \\ P(x)=d\rho g_x x \qquad (a+b+c+d+e+f=1) \end{array}}.$$

For n–dimensions one can repeat the above as many times as one wishes.

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Option 3
Solutions of the Euler Equations of Fluid flow

In the Navier-Stokes equation, if $\mu = 0$, one obtains the Euler equation. From

$$\mu(\frac{\partial^2 v_x}{\partial x^2}+\frac{\partial^2 v_x}{\partial y^2}+\frac{\partial^2 v_x}{\partial z^2})-\frac{\partial p}{\partial x}+\rho g_x=\rho(\frac{\partial v_x}{\partial t}+V_x\frac{\partial v_x}{\partial x}+V_y\frac{\partial v_x}{\partial y}+V_z\frac{\partial v_x}{\partial z})$$, one obtains

Euler equation : $(\mu = 0)$ $-\frac{\partial p}{\partial x}+\rho g_x=\rho(\frac{\partial v_x}{\partial t}+V_x\frac{\partial v_x}{\partial x}+V_y\frac{\partial v_x}{\partial y}+V_z\frac{\partial v_x}{\partial z})$ or

$$\rho(\frac{\partial v_x}{\partial t}+V_x\frac{\partial V_x}{\partial x}+V_y\frac{\partial v_x}{\partial y}+V_z\frac{\partial v_x}{\partial z})+\frac{\partial p_x}{\partial x}=\rho g_x$$ <---driver equation.

Euler equation $(\mu = 0)$: $\frac{\partial v_x}{\partial t}+V_x\frac{\partial v_x}{\partial x}+V_y\frac{\partial v_x}{\partial y}+V_z\frac{\partial v_x}{\partial z}+\frac{1}{\rho}\frac{\partial p}{\partial x}=g_x$ <---driver equation

Split the equation using the ratio terms f_e, h_e, n_e, q_e, d_e,, and solve. $(f_e+h_e+n_e+q_e+d_e=1)$

1. $\frac{\partial V_x}{\partial t}=f_e g_x$ $V_{x4}=f_e g_x t$ $V_{x4}=fg_x t$	**2.** $V_x\frac{\partial V_x}{\partial x}=h_e g_x$ $V_x\frac{dV_x}{dx}=h_e g_x$ $V_x dV_x=h_e g_x dx$ $\frac{V_x^{\ 2}}{2}=h_e g_x x$ or $V_x^{\ 2}=2h_e g_x x$ $V_x=\pm\sqrt{2h_e g_x x}$	**3.** $V_y\frac{\partial V_x}{\partial y}=n_e g_x$ $V_y\frac{dV_x}{dy}=n_e g_x$ $V_y dV_x=n_e g_x \mathrm{dy}$ $V_y V_x=n_e g_x \mathrm{y}+\psi_y(V_y)$ $V_{x6}=\frac{n_e g_x \mathrm{y}}{V_y}+\frac{\psi_y(V_y)}{V_y}$ $V_y\neq 0$	**4.** $V_z\frac{\partial V_x}{\partial z}=q_e g_x$ $V_z\frac{dV_x}{dz}=q_e g_x$ $V_z dV_x=q_e g_x \mathrm{dz}$; $V_z V_x=q_e g_x \mathrm{z}+\psi_z(V_z)$ $V_{x7}=\frac{q_e g_x \mathrm{z}}{V_z}+\frac{\psi_z(V_z)}{V_z}$ $V_z\neq 0$	**5.** $\frac{1}{\rho}\frac{\partial p}{\partial x}=d_e g_x$ $\frac{1}{\rho}\frac{\partial p}{\partial x}=d_e g_x$ $\frac{\partial p}{\partial x}=d_e\rho g_x$ $p=d_e\rho g_x x+C_7$

$$\boxed{\begin{array}{l} V_x(x,y,z,t)=f_e g_x t\pm\sqrt{2h_e g_x x}+\frac{n_e g_x \mathrm{y}}{V_y}+\frac{q_e g_x \mathrm{z}}{V_z}+\frac{\psi_y(V_y)}{V_y}+\frac{\psi_z(V_z)}{V_z}+C \\ P(x)=d_e\rho g_x x \qquad (f_e+h_e+n_e+q_e+d_e=1)\ V_y\neq 0,\ V_z\neq 0 \end{array}}$$

Find the test derivatives to check in the original equation.

1. $\frac{\partial V_x}{\partial t}=f_e g_x$	**2.** $V_x^{\ 2}=2h_e g_x x$; $2V_x\frac{\partial V_x}{\partial x}=2h_e g_x x$; $\frac{\partial V_x}{\partial x}=\frac{h_e g_x}{V_x}$, $V_x\neq 0$	3. $\frac{\partial V_x}{\partial y}=\frac{n_e g_x}{V_y}$ $V_y\neq 0$	4. $\frac{\partial V_x}{\partial z}=\frac{q_e g_x}{V_z}$ $V_z\neq 0$	5. $\frac{\partial p}{\partial x}=d_e\rho g_x$

$\frac{\partial v_x}{\partial t}+V_x\frac{\partial v_x}{\partial x}+V_y\frac{\partial v_x}{\partial y}+V_z\frac{\partial v_x}{\partial z}+\frac{1}{\rho}\frac{\partial p}{\partial x}=g_x$ (Above, $\psi_y(V_y)$ and $\psi_z(V_z)$ are arbitrary functions)

$$f_e g_x+V_x\frac{h_e g_x}{V_x}+V_y\frac{n_e g_x}{V_y}+V_z\frac{q_e g_x}{V_z}+\frac{1}{\rho}d_e\rho g_x\overset{?}{=}g_x$$

$$f_e g_x+h_e g_x+\ n_e g_x+q_e g_x+d_e g_x\overset{?}{=}g_x$$

$$g_x(f_e+h_e+\ n_e+q_e+d_e)\overset{?}{=}g_x$$

$$g_x(1)\overset{?}{=}g_x \qquad (f_e+h_e+\ n_e+q_e+d_e=1)$$

$$g_x\overset{?}{=}g_x$$ Yes

The relation obtained satisfies the Euler equation. Therefore the solution to the Euler equation is

$$\boxed{\begin{array}{l} V_x(x,y,z,t) = fgt \pm \sqrt{2hg_x x} + \dfrac{ng_x y}{V_y} + \dfrac{qg_x z}{V_z} + \underbrace{\dfrac{\psi_y(V_y)}{V_y} + \dfrac{\psi_z(V_z)}{V_z}}_{\text{arbitrary functions}} + C \\ P(x) = d\rho g_x x; \quad V_y \neq 0,\ V_z \neq 0 \end{array}}$$

The above is the solution of the driver equation. There are 5 supporter equations not covered here. Let it be known that the Euler equation of fluid flow has been solved for the first time in this paper.

Note: So far as the solutions of the equations are concerned, one needs not have explicit expressions for V_x, V_y, and V_z.

However, by solving algebraically and simultaneously for V_x, V_y and V_z, the $(ng_x y/V_y)$ and $(qg_x z/V_z)$ terms would be replaced by fractional terms containing square root functions with variables in the denominators and consequent turbulence behavior

The impediment to solving the Euler equations has been due to how to obtain sub-equations from the six-term equation. The above solution was made possible after pairing the terms of the equation using ratios (ratio terms). The author was encouraged by Lagrange's use of ratios and proportion in solving differential equations. One advantage of the pairing approach is that the above solution can easily be extended to any number of dimensions.

Extra:

Linearized Euler Equation: If one linearizes the Euler equation as was done in Option 1, one obtains $4\dfrac{\partial V_x}{\partial t} + \dfrac{1}{\rho}\dfrac{\partial p_x}{\partial x} = g_x$; whose solution is $V_x = \dfrac{fg_x}{4}t + C$; $P(x) = d\rho g_x x$. (see Option 1 results)

Results for the Euler equations are presented below: for V_x, V_y and V_z

For V_x : $\dfrac{\partial p}{\partial x} + \rho\dfrac{\partial V_x}{\partial t} + \rho V_x\dfrac{\partial V_x}{\partial x} + \rho V_y\dfrac{\partial V_x}{\partial y} + \rho V_z\dfrac{\partial V_x}{\partial z}) = \rho g_x$

$$\boxed{\begin{array}{l} V_x(x,y,z,t) = fg_x t \pm \sqrt{2hg_x x} + \dfrac{ng_x y}{V_y} + \dfrac{qg_x z}{V_z} + \underbrace{\dfrac{\psi_y(V_y)}{V_y} + \dfrac{\psi_z(V_z)}{V_z}}_{\text{arbitrary functions}};\ P(x) = d\rho g_x x \\ V_y \neq 0,\ V_z \neq 0 \end{array}} \quad x\text{-direction}$$

For V_y, $\dfrac{\partial p}{\partial y} + \rho\dfrac{\partial V_y}{\partial t} + \rho V_x\dfrac{\partial V_y}{\partial x} + \rho V_y\dfrac{\partial V_y}{\partial y} + \rho V_z\dfrac{\partial V_y}{\partial z} = \rho g_y$

$$\boxed{\begin{array}{l} V_y(x,y,z,t) = \lambda_5 g_y t \pm \sqrt{2\lambda_7 g_y y} + \dfrac{\lambda_6 g_y x}{V_x} + \dfrac{\lambda_8 g_y z}{V_z} + \dfrac{\psi_x(V_x)}{V_x} + \dfrac{\psi_z(V_z)}{V_z};\ P(y) = \lambda_4 \rho g_y y \\ V_x \neq 0,\ V_z \neq 0 \end{array}} \quad y\text{-direction}$$

For V_z: $\dfrac{\partial p}{\partial z} + \rho\dfrac{\partial V_z}{\partial t} + \rho V_x\dfrac{\partial V_z}{\partial x} + \rho V_y\dfrac{\partial V_z}{\partial y} + \rho V_z\dfrac{\partial V_z}{\partial z} = \rho g_z$

$$\boxed{\begin{array}{l} V_z(x,y,z,t) = \beta_5 g_z t \pm \sqrt{2\beta_8 g_z z} + \dfrac{\beta_6 g_z x}{V_x} + \dfrac{\beta_7 g_z y}{V_y} + \dfrac{\psi_x(V_x)}{V_x} + \dfrac{\psi_y(V_y)}{V_y};\ P(z) = \beta_4 \rho g_z z \\ V_x \neq 0,\ V_y \neq 0 \end{array}} \quad z\text{-direction}$$

Note: By comparison with Navier-Stokes equation and its relation, a relation to Euler equation can be found by deleting the Navier-Stokes relation resulting from the μ-terms.

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Option 4
Solutions of 3-D Navier-Stokes Equations (Original)
Mehod 1

As in Option 1 for solving these equations, the first step here, is to split-up the equation into eight sub-equations using the ratio method. One will solve **only** the driver equation, based on the experience gained in solving the linearized equation. There are 8 supporter equations.

$$-\mu\frac{\partial^2 V_x}{\partial x^2}-\mu\frac{\partial^2 V_x}{\partial y^2}-\mu\frac{\partial^2 V_x}{\partial z^2}+\frac{\partial p_x}{\partial x}+\rho\frac{\partial V_x}{\partial t}+\overbrace{\rho V_x\frac{\partial V_x}{\partial x}+\rho V_y\frac{\partial V_x}{\partial y}+\rho V_z\frac{\partial V_x}{\partial z}}^{\text{nonlinear terms}}=\rho g_x \qquad \text{(A)}$$

$$-K\frac{\partial^2 V_x}{\partial x^2}-K\frac{\partial^2 V_x}{\partial y^2}-K\frac{\partial^2 V_x}{\partial z^2}+\frac{1}{\rho}\frac{\partial p}{\partial x}+\frac{\partial V_x}{\partial t}+V_x\frac{\partial V_x}{\partial x}+V_y\frac{\partial V_x}{\partial y}+V_z\frac{\partial V_x}{\partial z}=g_x \qquad (K=\frac{\mu}{\rho}) \quad \text{(B)}$$

Step 1: Apply the ratio method to equation (B) to obtain the following equations:

$$1.\ -K\frac{\partial^2 V_x}{\partial x^2}=ag_x;\ 2.\ -K\frac{\partial^2 V_x}{\partial y^2}=bg_x;\ 3.\ -K\frac{\partial^2 V_x}{\partial z^2}=cg_x;\ 4.\ \frac{1}{\rho}\frac{\partial p}{\partial x}=dg_x;\ 5.\ \frac{\partial V_x}{\partial t}=fg_x$$

$$6.\ V_x\frac{\partial V_x}{\partial x}=hg_x;\ 7.\ V_y\frac{\partial V_x}{\partial y}=qg_x;\ 8.\ V_z\frac{\partial V_x}{\partial z}=ng_x$$

where $a,\ b,\ c,\ d,\ f,\ h,\ n,\ q$ are the ratio terms and $a+b+c+d+f+h+n+q=1$

Step 2: Solve the differential equations in Step 1.

Note that after splitting the equations, the equations can be solved using techniques of ordinary differential equations.

One can view each of the ratio terms $a,\ b,\ c,\ d,\ f,\ h,\ n,\ q$ as a fraction (a real number) of $\boxed{g_x}$ contributed by each expression on the left-hand side of equation (B) above.

Solutions of the eight sub-equations

$\boxed{1.\ -k\frac{\partial^2 V_x}{\partial x^2}=ag}$

$$k\frac{\partial^2 V_x}{\partial x^2}=-ag$$
$$\frac{\partial^2 V_x}{\partial x^2}=-\frac{a}{k}g$$
$$\frac{\partial V_x}{\partial x}=-\frac{ag}{k}x+C_1$$
$$V_{x1}=-\frac{ag}{2k}x^2+C_1x+C_2$$

$\boxed{2.\ -K\frac{\partial^2 V_x}{\partial y^2}=bg}$

$$K\frac{\partial^2 V_x}{\partial y^2}=-bg$$
$$\frac{\partial^2 V_x}{\partial y^2}=-\frac{b}{k}g$$
$$\frac{\partial V_x}{\partial y}=-\frac{bg}{k}y+C_3$$
$$V_{x2}=-\frac{bg}{2k}y^2+C_3y+C_4$$

$\boxed{3.\ -K\frac{\partial^2 V_x}{\partial z^2}=cg}$

$$K\frac{\partial^2 V_x}{\partial z^2}=-cg$$
$$\frac{\partial^2 V_x}{\partial z^2}=-\frac{c}{k}g$$
$$\frac{\partial V_x}{\partial z}=-\frac{cg}{k}z+C_5$$
$$V_{x3}=-\frac{cg}{2k}z^2+C_5z+C_6$$

$\boxed{4.\ \frac{1}{\rho}\frac{\partial p}{\partial x}=dg}$

$$\frac{1}{\rho}\frac{\partial p}{\partial x}=dg$$
$$\frac{\partial p}{\partial x}=d\rho g$$
$$p=d\rho gx+C_7$$

$\boxed{5.\ \frac{\partial V_x}{\partial t}=fg}$

$$V_{x4}=fgt$$

$\boxed{6.\ V_x\frac{\partial V_x}{\partial x}=hg_x}$

$$V_x\frac{dV_x}{dx}=hg_x$$
$$V_x dV_x=hg_x dx$$
$$\frac{V_x^{\ 2}}{2}=hg_x x$$
$$V_{x5}=\pm\sqrt{2hg_x x}+C_7$$

$\boxed{7.\ V_y\frac{\partial V_x}{\partial y}=ng_x}$

$$V_y\frac{dV_x}{dy}=ng_x$$
$$V_y dV_x=ng_x\mathrm{dy}$$
$$V_yV_x=ng_x\mathrm{y}+\psi_y(V_y)$$
$$V_{x6}=\frac{ng_x\mathrm{y}}{V_y}+\frac{\psi_y(V_y)}{V_y}$$

$\boxed{8.\ V_z\frac{\partial V_x}{\partial z}=qg_x}$

$$V_z\frac{dV_x}{dz}=qg_x$$
$$V_z dV_x=qg_x\mathrm{dz};$$
$$V_zV_x=qg_x\mathrm{z}+\psi_z(V_z)$$
$$V_{x7}=\frac{qg_x\mathrm{z}}{V_z}+\frac{\psi_z(V_z)}{V_z}$$

Note:
$\psi_y(V_y),\ \psi_z(V_z)$ are arbitrary functions, (integration constants)
$V_y\neq 0$
$V_z\neq 0$

Step 3: One combines the above solutions

$$V_x(x,y,z,t) = V_{x1} + V_{x2} + V_{x3} + V_{x4} + V_{x5} + V_{x6} + V_{x7}$$

$$= -\frac{ag_x}{2k}x^2 + C_1x - \frac{bg_x}{2k}y^2 + C_3y - \frac{cg_x}{2k}z^2 + C_5z + fg_xt \pm \sqrt{2hg_xx} + \frac{ng_xy}{V_y} + \frac{qg_xz}{V_z} + \frac{\psi_y(V_y)}{V_y} + \frac{\psi_z(V_z)}{V_z}$$

$$\overbrace{-\frac{\rho g_x}{2\mu}(ax^2 + by^2 + cz^2) + C_1x + C_3y + C_5z + fg_xt}^{\text{relation for linear terms}} \pm \overbrace{\sqrt{2hg_xx} + \frac{ng_xy}{V_y} + \frac{qg_xz}{V_z} + \frac{\psi_y(V_y)}{V_y} + \frac{\psi_z(V_z)}{V_z}}^{\text{relation for non - linear terms}} + C_9$$

arbitrary functions: $\underbrace{\frac{\psi_y(V_y)}{V_y} + \frac{\psi_z(V_z)}{V_z}}_{\text{arbitrary functions}}$

$$P(x) = d\rho g_xx; \quad (a+b+c+d+f+h+n+q=1) \quad V_y \neq 0,\ V_z \neq 0$$

Step 4: Find the test derivatives

Test derivatives for the linear part					Test derivatives for the non-linear part		
$\frac{\partial^2 V_x}{\partial x^2} = -\frac{a\rho g_x}{\mu}$	$\frac{\partial^2 V_x}{\partial y^2} = -\frac{b\rho g_x}{\mu}$	$\frac{\partial^2 V_x}{\partial z^2} = -\frac{c\rho g_x}{\mu}$	$\frac{\partial p}{\partial x} = d\rho g_x$	$\frac{\partial V_x}{\partial t} = fg_x$	$V_x^2 = 2hg_xx$ $2V_x\frac{\partial V_x}{\partial x} = 2hg_xx$ $\frac{\partial V_x}{\partial x} = \frac{hg_x}{V_x}, V_x \neq 0$	$\frac{\partial V_x}{\partial y} = \frac{ng_x}{V_y}$	$\frac{\partial V_x}{\partial z} = \frac{qg_x}{V_z}$

Step 5: **S**ubstitute the derivatives from Step 4 in equation (A) for the checking.

$$-\mu\frac{\partial^2 V_x}{\partial x^2} - \mu\frac{\partial^2 V_x}{\partial y^2} - \mu\frac{\partial^2 V_x}{\partial z^2} + \frac{\partial p_x}{\partial x} + \rho\frac{\partial V_x}{\partial t} + \rho V_x\frac{\partial V_x}{\partial x} + \rho V_y\frac{\partial V_x}{\partial y} + \rho V_z\frac{\partial V_x}{\partial z} = \rho g_x \qquad \textbf{(A)}$$

$$-\mu\left(-\frac{a\rho g_x}{\mu} - \frac{b\rho g_x}{\mu} - \frac{c\rho g_x}{\mu}\right) + d\rho g_x + f\rho g_x + \rho\left(V_x\frac{hg_x}{V_x}\right) + \rho V_y\left(\frac{ng_x}{V_y}\right) + \rho V_z\left(\frac{qg_x}{V_z}\right) \stackrel{?}{=} \rho g_x$$

$$a\rho g_x + b\rho g_x + c\rho g_x + d\rho g_x + f\rho g_x + h\rho g_x + n\rho g_x + q\rho g_x \stackrel{?}{=} \rho g_x$$

$$ag_x + bg_x + cg_x + dg_x + fg_x + hg_x + ng_x + qg_x \stackrel{?}{=} g_x$$

$$g_x(a+b+c+d+f+h+n+q) \stackrel{?}{=} g_x$$

$$g_x(1) \stackrel{?}{=} g_x \quad \text{Yes} \quad (a+b+c+d+f+h+n+q=1)$$

Step 6: The linear part of the relation satisfies the linear part of the equation; and the non-linear part of the relation satisfies the non-linear part of the equation.(B) below is the solution.

Analogy for the Identity Checking Method: If one goes shopping with American dollars and Japanese yens (without any currency conversion) and after shopping, if one wants to check the cost of the items purchased, one would check the cost of the items purchased with dollars against the receipts for the dollars; and one would also check the cost of the items purchased with yens against the receipts for the yens purchase. However, if one converts one currency to the other, one would only have to check the receipts for only a single currency, dollars or yens. This conversion case is similar to the linearized equations, where there was no partitioning in identity checking.

Summary of solutions for V_x V_y, V_z ($P(x)=d\rho g_x x$; $P(y)=\lambda_4\rho g_y y$, $P(z)=\beta_4\rho g_z z$)

$$V_x = -\frac{\rho g_x}{2\mu}(ax^2+by^2+cz^2)+C_1x+C_3y+C_5z+fg_xt\pm\sqrt{2hg_xx}+\frac{ng_xy}{V_y}+\frac{qg_xz}{V_z}+\frac{\psi_y(V_y)}{V_y}+\frac{\psi_z(V_z)}{V_z}+C_9 \quad \textbf{(B)}$$

$P(x)=d\rho g_x x$; $(a+b+c+d+h+n+q=1)$ $V_y\neq 0$, $V_z\neq 0$

$$V_y=-\frac{\rho g_y}{2\mu}(\lambda_1x^2+\lambda_2y^2+\lambda_3z^2)+C_1x+C_3y+C_5z+\lambda_5g_yt\pm\sqrt{2\lambda_7g_yy}+\frac{\lambda_6g_yx}{V_x}+\frac{\lambda_8g_yz}{V_z}+\frac{\psi_x(V_x)}{V_x}+\frac{\psi_z(V_z)}{V_z}$$

$P(y)=\lambda_4\rho g_y y$ $V_x\neq 0$, $V_z\neq 0$

$$V_z=-\frac{\rho g_z}{2\mu}(\beta_1x^2+\beta_2y^2+\beta_3z^2)+C_1x+C_3y+C_5z+\beta_5g_zt\pm\sqrt{2\beta_8g_zz}+\frac{\beta_6g_zx}{V_x}+\frac{\beta_7g_zy}{V_y}+\frac{\psi_x(V_x)}{V_x}+\frac{\psi_y(V_y)}{V_y}$$

$V_x\neq 0$, $V_y\neq 0$

The above solutions are unique, because from the experience in Option 1, only the equations with the gravity terms as the subjects of the equations produced the solutions.

Option 5

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Solutions of 4-D Navier-Stokes Equations

In the above method, the solution can easily be extended to any number of dimensions..

Adding $\mu\frac{\partial^2V_x}{\partial s^2}$ and $\rho V_s\frac{\partial V_x}{\partial s}$ to the 3-D x–direction equation yields the 4-D N-S equation

$$-\mu\left(\frac{\partial^2V_x}{\partial x^2}+\frac{\partial^2V_x}{\partial y^2}+\frac{\partial^2V_x}{\partial z^2}+\frac{\partial^2V_x}{\partial s^2}\right)+\frac{\partial p}{\partial x}+\rho\frac{\partial V_x}{\partial t}+\rho V_x\frac{\partial V_x}{\partial x}+\rho V_y\frac{\partial V_x}{\partial y}+\rho V_z\frac{\partial V_x}{\partial z}+\rho V_s\frac{\partial V_x}{\partial s}=\rho g_x$$

whose solution is given by

$$V_x(x,y,z,s,t)= -\frac{\rho g_x}{2\mu}(ax^2+by^2+cz^2+es^2)+C_1x+C_3y+C_5z+C_6s+fg_xt\pm\sqrt{2hg_xx}+\frac{ng_xy}{V_y}+\frac{qg_xz}{V_z}+\frac{rg_xs}{V_s}+\underbrace{\frac{\psi_y(V_y)}{V_y}+\frac{\psi_z(V_z)}{V_z}+\frac{\psi_s(V_s)}{V_s}}_{\text{arbitrary functions}}+C_9$$

$P(x)=d\rho g_x x$ $(a+b+c+d+e+f+h+n+q+r=1)$ $V_x\neq 0$, $V_y\neq 0$, $V_s\neq 0$,

For n–dimensions one can repeat the above as many times as one wishes.

Extra: Two-term Linearization of the Navier-Stokes Equation
(Equation contains one nonlinear term)

By linearization as in Option 1, if one replaces $\rho V_y\frac{\partial V_x}{\partial y}+\rho V_z\frac{\partial V_x}{\partial z}$ by $2\rho\frac{\partial V_x}{\partial t}$ in

$$-\mu\frac{\partial^2V_x}{\partial x^2}-\mu\frac{\partial^2V_x}{\partial y^2}-\mu\frac{\partial^2V_x}{\partial z^2}+\frac{\partial p_x}{\partial x}+\rho\frac{\partial V_x}{\partial t}+\rho V_x\frac{\partial V_x}{\partial x}+\rho V_y\frac{\partial V_x}{\partial y}+\rho V_z\frac{\partial V_x}{\partial z}=\rho g_x$$ one obtains

$$-\mu\left(\frac{\partial^2V_x}{\partial x^2}+\frac{\partial^2V_x}{\partial y^2}+\frac{\partial^2V_x}{\partial z^2}+\frac{\partial^2V_x}{\partial s^2}\right)+\frac{\partial p}{\partial x}+3\rho\left(\frac{\partial V_x}{\partial t}\right)+\rho V_x\frac{\partial V_x}{\partial x}=\rho g_x,$$ whose solution is

$$V_x(x,y,z,t)=-\frac{\rho g_x}{2\mu}(ax^2+by^2+cz^2)+C_1x+C_3y+C_5z+\frac{fg_xt}{3}\pm\sqrt{2hg_xx}+C_6$$

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Conclusion (for Option 4)

Since one began solving the Navier-Stokes equations by thinking like an eighth grader, and one was able to find a ratio technique for splitting the equations and solving them, perhaps, it is appropriate, after a few months of aging, to think like a ninth grader in the conclusion. One will reverse the coverage approach and begin from the general case and end with the special cases.

Solutions of the Navier--Stokes equations (general case)

x–direction **Navier-Stokes Equation** (also driver equation)

$$\boxed{-\mu\frac{\partial^2 V_x}{\partial x^2}-\mu\frac{\partial^2 V_x}{\partial y^2}-\mu\frac{\partial^2 V_x}{\partial z^2}+\frac{\partial p_x}{\partial x}+\rho\frac{\partial V_x}{\partial t}+\rho V_x\frac{\partial V_x}{\partial x}+\rho V_y\frac{\partial V_x}{\partial y}+\rho V_z\frac{\partial V_x}{\partial z}=\rho g_x}\quad x\text{–direction}$$

$$V_x(x,y,z,t)=$$

$$\overbrace{-\frac{\rho g_x}{2\mu}(ax^2+by^2+cz^2)+C_1x+C_3y+C_5z+fg_xt}^{\text{solution for linear terms}}\pm\overbrace{\sqrt{2hg_xx+\frac{ng_xy}{V_y}+\frac{qg_xz}{V_z}+\underbrace{\frac{\psi_y(V_y)}{V_y}+\frac{\psi_z(V_z)}{V_z}}_{\text{arbitrary functions}}}}^{\text{solution for non - linear terms}}+C_9$$

$$P(x)=d\rho g_x x;\quad (a+b+c+d+h+n+q=1)\quad V_y\neq 0,\ V_z\neq 0$$

One observes above that the most important insight of the above solution is the indispensability of the gravity term in incompressible fluid flow. Observe that if gravity, g, were zero, the first three terms, the 7th term, the 8th term, the 9th term, the 10th term and $P(x)$ would all be zero.
This result can be stated emphatically that without gravity forces on earth, there will be no incompressible fluid flow on earth as is known. The above is a very important new insight, because in posing problems on incompressible fluid flow, it is sometimes suggested that the gravity term is zero. Such a suggestion would guarantee a no solution to the problem, according to the above solution of the Navier-Stokes equation.
The author proposed and applied a new law, the law of definite ratio for incompressible fluid flow. This law states that in incompressible fluid flow, the other terms of the fluid flow equation divide the gravity term in a definite ratio and also each term utilizes gravity to function. This law was applied in splitting-up the Navier-Stokes equations. The resulting sub-equations were readily integrable, and even the nonlinear sub-equations were readily integrated.
The x–direction Navier-Stokes equation was split-up into sub-equations using ratios. The sub-equations were solved and combined. The relation obtained from the integration of the linear part of the equation satisfied the linear part of the equation and the relation obtained from integrating the nonlinear part of the equation satisfied the nonlinear part of the equation. By solving algebraically and simultaneously for V_x, V_y and V_z, the (ng_xy/V_y) and (qg_xz/V_z) terms would be replaced by fractional terms containing square root functions with variables in the denominators and consequent turbulence behavior.. One may note that in checking the relations obtained for integrating the equations for possible solutions, one needs not have explicit expressions for V_x, V_y, and V_z, since these behave as constants in the checking process. The above solution is the solution to the driver equation. There are eight supporter equations (see below and see also Option 1 solution, p110). Only the solution to the driver equation completely satisfies its corresponding Navier-Stokes equation.. A supporter equation does not completely satisfy its corresponding Navier-Stokes equation. The above x–direction solution is the solution everyone has been waiting for, for nearly 150 years. It was obtained in two simple steps, namely, splitting the equation using ratios and integrating. The task for the future is to solve the equations for V_x, V_y and V_z simultaneously. and algebraically, in order to replace two implicit terms of the solution.

Supporter Equations

$$1.\ -\mu\frac{\partial^2 V_x}{\partial x^2}-\mu\frac{\partial^2 V_x}{\partial y^2}-\mu\frac{\partial^2 V_x}{\partial z^2}+\frac{\partial p_x}{\partial x}+\rho\frac{\partial V_x}{\partial t}+\rho V_y\frac{\partial V_x}{\partial y}+\rho V_z\frac{\partial V_x}{\partial z}+\rho g_x=\rho V_x\frac{\partial V_x}{\partial x}$$

$$2.\ -\mu\frac{\partial^2 V_x}{\partial x^2}-\mu\frac{\partial^2 V_x}{\partial y^2}-\mu\frac{\partial^2 V_x}{\partial z^2}+\frac{\partial p_x}{\partial x}+\rho V_x\frac{\partial V_x}{\partial x}+\rho V_y\frac{\partial V_x}{\partial y}+\rho V_z\frac{\partial V_x}{\partial z}+\rho g_x=\rho\frac{\partial V_x}{\partial t}$$

$$3.\ -\mu\frac{\partial^2 V_x}{\partial x^2}-\mu\frac{\partial^2 V_x}{\partial y^2}-\mu\frac{\partial^2 V_x}{\partial z^2}+\rho\frac{\partial V_x}{\partial t}+\rho V_x\frac{\partial V_x}{\partial x}+\rho V_y\frac{\partial V_x}{\partial y}+\rho V_z\frac{\partial V_x}{\partial z}+\rho g_x=\frac{\partial p_x}{\partial x}$$

$$4.\ -\mu\frac{\partial^2 V_x}{\partial y^2}-\mu\frac{\partial^2 V_x}{\partial z^2}+\frac{\partial p_x}{\partial x}+\rho\frac{\partial V_x}{\partial t}+\rho V_x\frac{\partial V_x}{\partial x}+\rho V_y\frac{\partial V_x}{\partial y}+\rho V_z\frac{\partial V_x}{\partial z}+\rho g_x=-\mu\frac{\partial^2 V_x}{\partial x^2}$$

Explicit Functions for V_x, V_y, and V_z,

For explicit functions for V_x, V_y, and V_z, one has to solve (algebraically) the simultaneous system of solutions for V_x, V_y, and V_z.

System of Navier − Stokes relations to solve for V_x, V_y, V_z simultaneously (algebraically).

$$V_x=\frac{\left(-\frac{\rho g_x}{2\mu}(ax^2+by^2+cz^2)+C_1x+C_3y+C_5z+fg_xt\pm\sqrt{2hg_xx}\right)V_yV_z+[qg_xz+\psi_z(V_z)]V_y+[ng_xy+\psi_y(V_y)]V_z}{V_yV_z}$$

$$V_y=\frac{\left(-\frac{\rho g_y}{2\mu}(\lambda_1x^2+\lambda_2y^2+\lambda_3z^2)+C_1x+C_3y+C_5z+\lambda_5g_yt\pm\sqrt{2\lambda_7g_yy}\right)V_xV_z+[\lambda_8g_yz+\psi_z(V_z)]V_x+[\lambda_6g_yx+\psi_x(V_x)]V_z}{V_xV_z}$$

$$V_z=\frac{\left(-\frac{\rho g_z}{2\mu}(\beta_1x^2+\beta_2y^2+\beta_3z^2)+C_1x+C_3y+C_5z+\beta_5g_zt\pm\sqrt{2\beta_8g_zz}\right)V_xV_y+[\beta_6g_zx+\psi_x(V_x)]V_y+[\beta_7g_zy+\psi_y(V_y)]V_x}{V_xV_y}$$

Special Cases of the Navier-Stokes Equations

1. Linearized Navier--Stokes equations

One may note that there are six linear terms and three nonlinear terms in the Navier-Stokes equation. The linearized case was covered before the general case, and the experience gained in the linearized case guided one to solve the general case efficiently. In particular, the gravity term must be the subject of the equation for a solution. When the gravity term was the subject of the equation, the equation was called the driver equation. A splitting technique was applied to the linearized Navier-Stokes equations (Option 1). Twenty sub-equations were solved. (Four sets of equations with different equation subjects). The integration relations of one of the sets satisfied the linearized Navier-Stokes equation; and this set was from the equation with g_x as the subject of the equation. In addition to finding a solution, the results of the integration revealed the roles of the terms of the Navier-Stokes equations in fluid flow. In particular, the gravity forces and $\partial p/\partial x$ are involved mainly in the parabolic as well as the forward motion of fluids; $\partial V_x/\partial t$ and $\partial^2 V_x/\partial x^2$ are involved in the periodic motion of fluids, and one may infer that as μ increases, the periodicity increases. One should determine experimentally, if the ratio of the linear term $\partial V_x/\partial t$ to the nonlinear sum $V_x(\partial V_x/\partial x)+V_y(\partial V_x/\partial y)+V_z(\partial V_x/\partial z)$ is 1 to 3.

$$V_x(x,y,z,t) = \overbrace{-\frac{\rho g_x}{2\mu}(ax^2+by^2+cz^2)+C_1x+C_3y+C_5z+\frac{fg_x}{4}t+C_9}^{\textbf{Solution to linearized Navier–Stokes equation}} \; ; \; P(x)=d\rho g_x x$$

$$\boxed{\overbrace{-\mu\frac{\partial^2 v_x}{\partial x^2}-\mu\frac{\partial^2 v_x}{\partial y^2}-\mu\frac{\partial^2 v_x}{\partial z^2}+\frac{\partial p_x}{\partial x}+4\rho\frac{\partial v_x}{\partial t}=\rho g_x}^{\textbf{Linearized Equation}}}$$

2. Solutions of the Euler equation

Since one has solved the Navier-Stokes equation, one has also solved the Euler equation.

$$\text{Euler equation } (\mu=0): \quad \frac{\partial v_x}{\partial t}+V_x\frac{\partial v_x}{\partial x}+V_y\frac{\partial v_x}{\partial y}+V_z\frac{\partial v_x}{\partial z}+\frac{1}{\rho}\frac{\partial p}{\partial x}=g_x$$

$$\boxed{\begin{array}{l} V_x(x,y,z,t)=f_e g_x t \pm\sqrt{2h_e g_x x}+\dfrac{n_e g_x y}{V_y}+\dfrac{q_e g_x z}{V_z}+\underbrace{\dfrac{\psi_y(V_y)}{V_y}+\dfrac{\psi_z(V_z)}{V_z}}_{\text{arbitrary functions}}+C \\ P(x)=d_e\rho g_x x \qquad (f_e+h_e+n_e+q_e+d_e=1)\; V_y\neq 0,\; V_z\neq 0 \end{array}} \quad x\text{-direction}$$

A **Euler solution system to solve for** V_x, V_y, V_z

$$V_x=\frac{(f_e g_x t\pm\sqrt{2h_e g_x x})V_yV_z+[q_e g_x z+\psi_z(V_z)]V_y+[n_e g_x y+\psi_y(V_y)]V_z}{V_yV_z}$$

$$V_y=\frac{(\lambda_5 g_y t\pm\sqrt{2\lambda_7 g_y y})V_xV_z+[\lambda_8 g_y z+\psi_z(V_z)]V_x+[\lambda_6 g_y x+\psi_x(V_x)]V_z}{V_xV_z}$$

$$V_z=\frac{(\beta_5 g_z t\pm\sqrt{2\beta_8 g_z z})V_xV_y+[\beta_6 g_z x+\psi_x(V_x)]V_y+[\beta_7 g_z y+\psi_y(V_y)]V_x}{V_xV_y}$$

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Option 6

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Solutions of the Magnetohydrodynamic Equations

This system consists of four equations and one is to solve for $V_x, V_y, V_z, B_x, B_y, B, P(x)$

Magnetohydrodynamic Equations

1. $$\frac{\partial V_x}{\partial x}+\frac{\partial V_y}{\partial y}+\frac{\partial V_z}{\partial z}=0 <\text{-- continuity equation}$$

2. $$\overbrace{\rho\frac{\partial V_x}{\partial t}+\rho V_x\frac{\partial V_x}{\partial x}+\rho V_y\frac{\partial V_x}{\partial y}+\rho V_z\frac{\partial V_x}{\partial z}}^{\text{Navier–Stokes}}=\overbrace{-\frac{\partial p}{\partial x}+\frac{1}{\mu}(\nabla\times B)\times B+\rho g_x}^{\text{Lorentz force}}$$

3. $$\rho\frac{\partial B}{\partial t}=\nabla\times(V\times B)+\eta\nabla^2 B$$
$$\rho\frac{\partial B}{\partial t}=\nabla\times(V\times B)+\eta\left(\frac{\partial^2 B}{\partial x^2}+\frac{\partial^2 B}{\partial y^2}+\frac{\partial^2 B}{\partial z^2}\right)$$
(η = magnetic diffusivity)

4. $$\nabla\bullet B=0$$
$$\frac{\partial B_x}{\partial x}+\frac{\partial B_y}{\partial y}+\frac{\partial B_z}{\partial z}=0$$

Step 1:

1. If ρ is constant: (for incompressible fluid)
$$\frac{\partial V_x}{\partial x}+\frac{\partial V_y}{\partial y}+\frac{\partial V_z}{\partial z}=0 <\text{-- continuity equation}$$

2. $$\overbrace{\rho\frac{\partial V_x}{\partial t}+\rho V_x\frac{\partial V_x}{\partial x}+\rho V_y\frac{\partial V_x}{\partial y}+\rho V_z\frac{\partial V_x}{\partial z}}^{\text{Navier - Stokes}}=\overbrace{-\frac{\partial p}{\partial x}+\frac{1}{\mu}(\nabla\times B)\times B+\rho g_x}^{\text{Lorentz force}}$$
$$\rho\frac{\partial V_x}{\partial t}+\rho V_x\frac{\partial V_x}{\partial x}+\rho V_y\frac{\partial V_x}{\partial y}+\rho V_z\frac{\partial V_x}{\partial z}=-\frac{\partial p}{\partial x}+\frac{1}{\mu}\left(B_z\left(\frac{\partial B_x}{\partial z}-\frac{\partial B_z}{\partial x}\right)-B_y\left(\frac{\partial B_y}{\partial x}-\frac{\partial B_x}{\partial y}\right)\right)+\rho g_x$$
$$\boxed{\rho\frac{\partial V_x}{\partial t}+\rho V_x\frac{\partial V_x}{\partial x}+\rho V_y\frac{\partial V_x}{\partial y}+\rho V_z\frac{\partial V_x}{\partial z}=-\frac{\partial p}{\partial x}+\frac{1}{\mu}\left(B_z\frac{\partial B_x}{\partial z}-B_z\frac{\partial B_z}{\partial x}-B_y\frac{\partial B_y}{\partial x}+B_y\frac{\partial B_x}{\partial y}\right)+\rho g_x}$$

3. $$\rho\frac{\partial B}{\partial t}=\nabla\times(V\times B)+\eta\nabla^2 B$$
$$\rho\frac{\partial B}{\partial t}=\frac{\partial}{\partial y}(V_xB_y-V_yB_x)-\frac{\partial}{\partial z}(V_zB_x-V_xB_z)+\eta\left(\frac{\partial^2 B}{\partial x^2}+\frac{\partial^2 B}{\partial y^2}+\frac{\partial^2 B}{\partial z^2}\right)$$
$$\boxed{\rho\frac{\partial B}{\partial t}=\frac{\partial}{\partial y}V_xB_y-\frac{\partial}{\partial y}V_yB_x-\frac{\partial}{\partial z}V_zB_x+\frac{\partial}{\partial z}V_xB_z+\eta\frac{\partial^2 B_x}{\partial x^2}+\eta\frac{\partial^2 B_x}{\partial y^2}+\eta\frac{\partial^2 B_x}{\partial z^2}}$$

4. $$\boxed{\begin{array}{l}\nabla\bullet B=0\\ \dfrac{\partial B_x}{\partial x}+\dfrac{\partial B_y}{\partial y}+\dfrac{\partial B_z}{\partial z}=0\end{array}}$$

Step 2:
After the "vector juggling" one obtains the following system of equations which one will solve.

$$\begin{cases} 1.\ \dfrac{\partial V_x}{\partial x}+\dfrac{\partial V_y}{\partial y}+\dfrac{\partial V_z}{\partial z}=0 \\ 2.\ \rho\dfrac{\partial V_x}{\partial t}+\rho V_x\dfrac{\partial V_x}{\partial x}+\rho V_y\dfrac{\partial V_x}{\partial y}+\rho V_z\dfrac{\partial V_x}{\partial z}+\dfrac{\partial p}{\partial x}-\dfrac{1}{\mu}B_z\dfrac{\partial B_x}{\partial z}+\dfrac{1}{\mu}B_z\dfrac{\partial B_z}{\partial x}+\dfrac{1}{\mu}B_y\dfrac{\partial B_y}{\partial x}-\dfrac{1}{\mu}B_y\dfrac{\partial B_x}{\partial y}=\rho g_x \\ 3. \\ \dfrac{\rho\partial B_x}{\partial t}-V_x\dfrac{\partial B_y}{\partial y}-B_y\dfrac{\partial V_x}{\partial y}+V_y\dfrac{\partial B_x}{\partial y}+B_x\dfrac{\partial V_y}{\partial y}+V_z\dfrac{\partial B_x}{\partial z}+B_x\dfrac{\partial V_z}{\partial z}-V_x\dfrac{\partial B_z}{\partial z}-B_z\dfrac{\partial V_x}{\partial z}-\dfrac{\eta\partial^2 B_x}{\partial x^2}-\dfrac{\eta\partial^2 B_x}{\eta\partial y^2}-\dfrac{\eta\partial^2 B_x}{\eta\partial z^2}=0 \\ 4.\ \dfrac{\partial B_x}{\partial x}+\dfrac{\partial B_y}{\partial y}+\dfrac{\partial B_z}{\partial z}=0 \end{cases}$$

At a glance, and from the experience gained in solving the Navier-Stokes equations, one can identify equation (2) as the driver equation, since it contains the gravity term, and the gravity term is the subject of the equation. However, since the system of equations is to be solved simultaneously and there is only a single "driver", the gravity term, all the terms in the system of equations will be placed in the driver equation, Equation 2. As suggested by Albert Einstein, Friedrich Nietzsche, and Pablo Picasso, one will think like a child at the next step.

Step 3: Thinking like a ninth grader, one will apply the following axiom:

If $a=b$ and $c=d$, then $a+c=b+d$; and therefore, add the left sides and add the right sides of the above equations . That is, $(1)+(2)+(3)+(4)=\rho g_x$

$$\begin{cases} \dfrac{\partial V_x}{\partial x}+\dfrac{\partial V_y}{\partial y}+\dfrac{\partial V_z}{\partial z}+\rho\dfrac{\partial V_x}{\partial t}+\rho V_x\dfrac{\partial V_x}{\partial x}+\rho V_y\dfrac{\partial V_x}{\partial y}+\rho V_z\dfrac{\partial V_x}{\partial z}+\dfrac{\partial p}{\partial x}-\dfrac{1}{\mu}B_z\dfrac{\partial B_x}{\partial z}+\dfrac{1}{\mu}B_z\dfrac{\partial B_z}{\partial x}+\dfrac{1}{\mu}B_y\dfrac{\partial B_y}{\partial x}- \\ \dfrac{1}{\mu}B_y\dfrac{\partial B_x}{\partial y}+\dfrac{\rho\partial B_x}{\partial t}-V_x\dfrac{\partial B_y}{\partial y}-B_y\dfrac{\partial V_x}{\partial y}+V_y\dfrac{\partial B_x}{\partial y}+B_x\dfrac{\partial V_y}{\partial y}+V_z\dfrac{\partial B_x}{\partial z}+B_x\dfrac{\partial V_z}{\partial z}-V_x\dfrac{\partial B_z}{\partial z}-B_z\dfrac{\partial V_x}{\partial z}- \\ \dfrac{\eta\partial^2 B_x}{\partial x^2}-\dfrac{\eta\partial^2 B_x}{\eta\partial y^2}-\dfrac{\eta\partial^2 B_x}{\eta\partial z^2}+\dfrac{\partial B_x}{\partial x}+\dfrac{\partial B_y}{\partial y}+\dfrac{\partial B_z}{\partial z}=\rho g_x \qquad \text{(Three lines per equation)} \end{cases}$$

Step 4: Writing all the linear terms first

$$\begin{cases} \dfrac{\partial V_x}{\partial x}+\dfrac{\partial V_y}{\partial y}+\dfrac{\partial V_z}{\partial z}+\rho\dfrac{\partial V_x}{\partial t}+\dfrac{\partial p}{\partial x}+\dfrac{\rho\partial B_x}{\partial t}-\dfrac{\eta\partial^2 B_x}{\partial x^2}-\dfrac{\eta\partial^2 B_x}{\eta\partial y^2}-\dfrac{\eta\partial^2 B_x}{\eta\partial z^2}+\dfrac{\partial B_x}{\partial x}+\dfrac{\partial B_y}{\partial y}+\dfrac{\partial B_z}{\partial z} \\ +\rho V_x\dfrac{\partial V_x}{\partial x}+\rho V_y\dfrac{\partial V_x}{\partial y}+\rho V_z\dfrac{\partial V_x}{\partial z}-\dfrac{1}{\mu}B_z\dfrac{\partial B_x}{\partial z}+\dfrac{1}{\mu}B_z\dfrac{\partial B_z}{\partial x}+\dfrac{1}{\mu}B_y\dfrac{\partial B_y}{\partial x}-\dfrac{1}{\mu}B_y\dfrac{\partial B_x}{\partial y}-V_x\dfrac{\partial B_y}{\partial y}-B_y\dfrac{\partial V_x}{\partial y} \\ +V_y\dfrac{\partial B_x}{\partial y}+B_x\dfrac{\partial V_y}{\partial y}+V_z\dfrac{\partial B_x}{\partial z}+B_x\dfrac{\partial V_z}{\partial z}-V_x\dfrac{\partial B_z}{\partial z}-B_z\dfrac{\partial V_x}{\partial z}=\rho g_x \qquad \text{(Three lines per equation)} \end{cases}$$

(Since all the terms are now in the same driver equation, let ρg_x "drive them" simultaneously.)

Step 5: Solve the above 28-term equation using the ratio method. (27 ratio terms)

The ratio terms to be used are respectively the following: (Sum of the ratio terms = 1)

$\beta_1, \beta_2, \beta_3, a, b, c, d, f, m, q, r, s, \omega_1, \omega_2, \omega_3, \omega_4, \omega_5, \omega_6, \lambda_1, \lambda_2, \lambda_3, \lambda_4, \lambda_5, \lambda_6, \lambda_7, \lambda_8, \lambda_9$

1. $\dfrac{\partial V_x}{\partial x}=\beta_1\rho g_x$ $\dfrac{dV_x}{dx}=\beta_1\rho g_x$ $V_x=\beta_1\rho g_x x+C_{16}$	2. $\dfrac{\partial V_y}{\partial y}=\beta_2\rho g_x$ $\dfrac{dV_y}{dy}=\beta_2\rho g_x$ $V_y=\beta_2\rho g_x y+C_{17}$	3. $\dfrac{\partial V_z}{\partial z}=\beta_3\rho g_x$ $\dfrac{dV_z}{dz}=\beta_3\rho g_x$ $V_z=\beta_3\rho g_x z+C_{18}$	4. $\rho\dfrac{\partial V_x}{\partial t}=a\rho g_x$ $\dfrac{\partial V_x}{\partial t}=ag_x$ $V_x=ag_x t+C_1$

5.	6.	7.
$\frac{\partial p}{\partial x} = b\rho g_x$	$\rho \frac{\partial B_x}{\partial t} = c\rho g_x$	$-\eta \frac{\partial^2 B_x}{\partial x^2} = d\rho g_x$
$\frac{dp}{dx} = b\rho g_x$	$\frac{\partial B_x}{\partial t} = c g_x$	$\frac{d^2 B_x}{dx^2} = -\frac{d\rho g_x}{\eta}$
$P(x) = b\rho g_x x + C$	$\frac{dB_x}{dt} = c g_x$	$\frac{dB_x}{dx} = -\frac{d\rho g_x x}{\eta} + C_2$
	$B_x = c g_x t + C_{1b}$	$B_x = -\frac{d\rho g_x x^2}{2\eta} + C_2 x + C_3$

8.	9.	10.
$-\eta \frac{\partial^2 B_x}{\partial y^2} = f\rho g_x$	$-\eta \frac{\partial^2 B_x}{\partial z^2} = m\rho g_x$	$\frac{\partial B_x}{\partial x} = q\rho g_x$
$\frac{d^2 B_x}{dy^2} = -\frac{f\rho g_x}{\eta}$	$\frac{d^2 B_x}{dz^2} = -\frac{m\rho g_x}{\eta}$	$\frac{dB_x}{dx} = q\rho g_x$
$\frac{dB_x}{dy} = -\frac{f\rho g_x y}{\eta} + C_4$	$\frac{dB_x}{dz} = -\frac{m\rho g_x z}{\eta} + C_6$	$B_x = q\rho g_x x + C_{19}$
$B_x = -\frac{f\rho g_x y^2}{2\eta} + C_4 y + C_5$	$B_x = -\frac{m\rho g_x z^2}{2\eta} + C_6 x + C_7$	

11.	12.	13.	14.
$\frac{\partial B_y}{\partial y} = r\rho g_x$	$\frac{\partial B_z}{\partial z} = s\rho g_x$	$\rho V_x \frac{\partial V_x}{\partial x} = \omega_1 \rho g_x$	$\rho V_y \frac{\partial V_x}{\partial y} = \omega_2 \rho g_x$
$\frac{dB_y}{dy} = r\rho g_x$	$\frac{dB_z}{dz} = s\rho g_x$	$V_x \frac{dV_x}{dx} = \omega_1 g_x$	$V_y dV_x = \omega_2 g_x dy$
$B_y = r\rho g_x y + C_{20}$	$B_z = s\rho g_x z + C_{21}$	$V_x dV_x = \omega_1 g_x dx$	$V_y V_x = \omega_2 g_x y + \psi_y(V_y)$
		$\frac{V_x^{\,2}}{2} = \omega_1 g_x x$	$V_x = \frac{\omega_2 g_x y}{V_y} + \frac{\psi_y(V_y)}{V_y}$
		$V_x^{\,2} = 2\omega_1 g_x x$	$V_y \neq 0$
		$V_x = \pm\sqrt{2\omega_1 g_x x} + C_2$	

15.	16	17.
$\rho V_z \frac{\partial V_x}{\partial z} = \omega_3 \rho g_x$	$B_z \frac{\partial B_x}{\partial z} = -\omega_4 \mu \rho g_x$	$B_z \frac{\partial B_z}{\partial x} = \omega_5 \mu \rho g_x$
$V_z \frac{dV_x}{dz} = \omega_3 g_x$	$B_z dB_x = -\omega_4 \mu \rho g_x dz$	$B_z \frac{dB_z}{dx} = \omega_5 \mu \rho g_x$
$V_z dV_x = \omega_3 g_x dz$	$B_z B_x = -\omega_4 \mu \rho g_x z + \psi_z(B_z)$	$B_z dB_z = \omega_5 \mu \rho g_x dx$
$V_z V_x = \omega_3 g_x z + \psi_z(V_z)$	$B_x = -\frac{\omega_4 \mu \rho g_x z}{B_z} + \frac{\psi_z(B_z)}{B_z}$	$\frac{B_z^{\,2}}{2} = \omega_5 \mu \rho g_x x$
$V_x = \frac{\omega_3 g_x z}{V_z} + \frac{\psi_z(V_z)}{V_z}$	$B_z \neq 0$	$B_z^{\,2} = 2\omega_5 \mu \rho g_x x$
$V_z \neq 0$		$B_z = \pm\sqrt{2\omega_5 \mu \rho g_x x} + C$

18.

$$B_y \frac{\partial B_y}{\partial x} = \omega_6 \mu \rho g_x$$
$$B_y \frac{dB_y}{dx} = \omega_6 \mu \rho g_x$$
$$B_y dB_y = \omega_6 \mu \rho g_x dx$$
$$\frac{B_y{}^2}{2} = \omega_6 \mu \rho g_x x$$
$$B_y{}^2 = 2\omega_6 \mu \rho g_x x$$
$$B_y = \pm\sqrt{2\omega_6 \mu \rho g_x x} + C$$

19.

$$-\frac{1}{\mu} B_y \frac{\partial B_x}{\partial y} = \lambda_1 \rho g_x$$
$$B_y \frac{dB_x}{dy} = -\lambda_1 \mu \rho g_x$$
$$B_y dB_x = -\lambda_1 \mu \rho g_x dy$$
$$B_y B_x = -\lambda_1 \mu \rho g_x y + \psi_y(B_y)$$
$$B_x = -\frac{\lambda_1 \mu \rho g_x y}{B_y} + \frac{\psi_y(B_y)}{B_y}$$
$$B_y \neq 0$$

20

$$-V_x \frac{\partial B_y}{\partial y} = \lambda_2 \rho g_x$$
$$V_x \frac{dB_y}{dy} = -\lambda_2 \rho g_x$$
$$V_x dB_y = -\lambda_2 \rho g_x dy$$
$$V_x B_y = -\lambda_2 \rho g_x y + \psi_x(V_x)$$
$$B_y = \frac{-\lambda_2 \rho g_x y}{V_x} + \frac{\psi_x(V_x)}{V_x}$$
$$V_x \neq 0$$

21.

$$-B_y \frac{\partial V_x}{\partial y} = \lambda_3 \rho g_x$$
$$B_y \frac{dV_x}{dy} = -\lambda_3 \rho g_x$$
$$B_y dV_x = -\lambda_3 \rho g_x dy$$
$$B_y V_x = -\lambda_3 \rho g_x y + \psi_y(B_y)$$
$$V_x = -\frac{\lambda_3 \rho g_x y}{B_y} + \frac{\psi_y(B_y)}{B_y}$$
$$B_y \neq 0$$

22.

$$V_y \frac{\partial B_x}{\partial y} = \lambda_4 \rho g_x$$
$$V_y \frac{dB_x}{dy} = \lambda_4 \rho g_x$$
$$V_y dB_x = \lambda_4 \rho g_x dy$$
$$V_y B_x = \lambda_4 \rho g_x y + \psi_y(V_y)$$
$$B_x = \frac{\lambda_4 \rho g_x y}{V_y} + \frac{\psi_y(V_y)}{V_y}$$
$$V_y \neq 0$$

23.

$$B_x \frac{\partial V_y}{\partial y} = \lambda_5 \rho g_x$$
$$B_x \frac{dV_y}{dy} = \lambda_5 \rho g_x$$
$$B_x dV_y = \lambda_5 \rho g_x dy$$
$$B_x V_y = \lambda_5 \rho g_x y + \psi_x(B_x)$$
$$V_y = \frac{\lambda_5 \rho g_x y}{B_x} + \frac{\psi_x(B_x)}{B_x}$$
$$B_x \neq 0$$

24.

$$V_z \frac{\partial B_x}{\partial z} = \lambda_6 \rho g_x$$
$$V_z \frac{dB_x}{dz} = \lambda_6 \rho g_x$$
$$V_z dB_x = \lambda_6 \rho g_x dz$$
$$V_z B_x = \lambda_6 \rho g_x z + \psi_z(V_z)$$
$$B_x = \frac{\lambda_6 \rho g_x z}{V_z} + \frac{\psi_z(V_z)}{V_z}$$
$$V_z \neq 0$$

25.

$$B_x \frac{\partial V_z}{\partial z} = \lambda_7 \rho g_x$$
$$B_x \frac{dV_z}{dz} = \lambda_7 \rho g_x$$
$$B_x dV_z = \lambda_7 \rho g_x dz$$
$$B_x V_z = \lambda_7 \rho g_x z + \psi_x(B_x)$$
$$V_z = \frac{\lambda_7 \rho g_x z}{B_x} + \frac{\psi_x(B_x)}{B_x}$$
$$B_x \neq 0$$

26

$$-V_x \frac{\partial B_z}{\partial z} = \lambda_8 \rho g_x$$
$$V_x \frac{dB_z}{dz} = -\lambda_8 \rho g_x$$
$$V_x dB_z = -\lambda_8 \rho g_x dz$$
$$V_x B_z = -\lambda_8 \rho g_x z + \psi_x(V_x)$$
$$B_z = -\frac{\lambda_8 \rho g_x z}{V_x} + \frac{\psi_x(V_x)}{V_x}$$
$$V_x \neq 0$$

27.

$$-B_z \frac{\partial V_x}{\partial z} = \lambda_9 \rho g_x$$
$$B_z \frac{dV_x}{dz} = -\lambda_9 \rho g_x$$
$$B_z dV_x = -\lambda_9 \rho g_x dz$$
$$B_z V_x = -\lambda_9 \rho g_x z + \psi_z(B_z)$$
$$V_x = -\frac{\lambda_9 \rho g_x z}{B_z} + \frac{\psi_z(B_z)}{B_z}$$
$$B_z \neq 0$$

Step 6: One collects the integrals of the sub-equations, above, for V_x, $V_{y,}$ $V_{z,}$ B_x, $B_{y,}$ $B_{z,}$ $P(x)$

$V_x(x,y,z,t) =$ (sum of integrals from sub-equations #1, #4,#13,#14,#15,#21,#27)

$$\beta_1\rho g_x x + a g_x t \pm \sqrt{2\omega_1 g_x x} + \frac{\omega_2 g_x y}{V_y} - \frac{\lambda_3\rho g_x y}{B_y} + \frac{\omega_3 g_x z}{V_z} - \frac{\lambda_9\rho g_x z}{B_z} + \underbrace{\frac{\psi_z(V_z)}{V_z} + \frac{\psi_y(B_y)}{B_y} + \frac{\psi_y(V_y)}{V_y} + \frac{\psi_z(B_z)}{B_z}}_{\text{arbitrary functions}} + C_1;$$

(integral from sub–equation #5)

$$P(x) = b\rho g_x x + C_2$$

(sum of integrals from sub–equations #2,#23)

$$V_y(y) = \beta_2\rho g_x y + \frac{\lambda_5\rho g_x y}{B_x} + \underbrace{\frac{\psi_x(B_x)}{B_x}}_{\text{arbitrary function}} + C_3$$

(sum of integrals from sub–equations #3, #25)

$$V_z(z) = \beta_3\rho g_x z + \frac{\lambda_7\rho g_x z}{B_x} + \underbrace{\frac{\psi_x(B_x)}{B_x}}_{\text{arbitrary function}} + C_4$$

(sum of integrals from sub-equations #6, #7, #8, #9, #10, #16,#19, #22, #24)

$B_x(x,y,z,t) =$

$$B_x = -\frac{\rho g_x}{2\eta}(dx^2 + fy^2 + mz^2) + q\rho g_x x + C_2 x + C_4 y + C_6 z + + c g_x t - \frac{\lambda_1\mu\rho g_x y}{B_y} + \frac{\lambda_4\rho g_x y}{V_y} - \frac{\omega_4\mu\rho g_x z}{B_z} +$$

$$\frac{\lambda_6\rho g_x z}{V_z} + \underbrace{\frac{\psi_z(B_z)}{B_z} + \frac{\psi_y(B_y)}{B_y} + \frac{\psi_y(V_y)}{V_y} + \frac{\psi_z(V_z)}{V_z}}_{\text{arbitrary functions}} + C_7$$

(sum of integrals from sub–equations #11,#18,#20)

$$B_y = r\rho g_x y \pm \sqrt{2\omega_6\mu\rho g_x x} - \frac{\lambda_2\rho g_x y}{V_x} + \underbrace{\frac{\psi_x(V_x)}{V_x}}_{\substack{\text{arbitrary}\\ \text{function}}} + C_8$$

(sum of integrals from sub–equations #12,#17,#26)

$$B_z = s\rho g_x z \pm \sqrt{2\omega_5\mu\rho g_x x} - \frac{\lambda_8\rho g_x z}{V_x} + \underbrace{\frac{\psi_x(V_x)}{V_x}}_{\text{arbitrary function}} + C_{21}$$

Step 7: Find the test derivatives for the linear part

1.	2.	3.	4.	5.	6.
$\frac{\partial V_x}{\partial x}=(\beta_1\rho g_x)$	$\frac{\partial V_y}{\partial y}=(\beta_2\rho g_x)$	$\frac{\partial V_z}{\partial z}=(\beta_3\rho g_x)$	$\frac{\partial V_x}{\partial t}=(ag_x)$	$\frac{\partial p}{\partial x}=(b\rho g_x)$	$\frac{dB_x}{dt}=(cg_x)$

7.	8.	9.	10.	11.	12.
$\frac{\partial^2 B_x}{\partial x^2}=-\frac{d\rho g_x}{\eta}$	$\frac{\partial^2 B_x}{\partial y^2}=-\frac{f\rho g_x}{\eta}$	$\frac{\partial^2 B_x}{\partial z^2}=-\frac{m\rho g_x}{\eta}$	$\frac{\partial B_x}{\partial x}=q\rho g_x$	$\frac{\partial B_y}{\partial y}=r\rho g_x$	$\frac{\partial B_z}{\partial z}=s\rho g_x$

Test derivatives for the nonlinear part

13.	14.	15.	16	17.
$\frac{\partial V_x}{\partial x}=\frac{\omega_1 g_x}{V_x}$	$\frac{\partial V_x}{\partial y}=\frac{\omega_2 g_x}{V_y}$	$\frac{\partial V_x}{\partial z}=\frac{\omega_3 g_x}{V_z}$	$\frac{\partial B_x}{\partial z}=-\frac{\omega_4\mu\rho g_x}{B_z}$	$\frac{\partial B_z}{\partial x}=\frac{\omega_5\mu\rho g_x}{B_z}$

18.	19.	20.	21.	22.
$\frac{\partial B_y}{\partial x}=\frac{\omega_6\mu\rho g_x}{B_y}$	$\frac{\partial B_x}{\partial y}=-\frac{\lambda_1\mu\rho g_x}{B_y}$	$\frac{\partial B_y}{\partial y}=-\frac{\lambda_2\rho g_x}{V_x}$	$\frac{\partial V_x}{\partial y}=-\frac{\lambda_3\rho g_x}{B_y}$	$\frac{\partial B_x}{\partial y}=\frac{\lambda_4\rho g_x}{V_y}$

23.	24.	25.	26.	27.
$\frac{\partial V_y}{\partial y}=\frac{\lambda_5\rho g_x}{B_x}$	$\frac{\partial B_x}{\partial z}=\frac{\lambda_6\rho g_x}{V_z}$	$\frac{\partial V_z}{\partial z}=\frac{\lambda_7\rho g_x}{B_x}$	$\frac{\partial B_z}{\partial z}=-\frac{\lambda_8\rho g_x}{V_x}$	$\frac{\partial V_x}{\partial z}=-\frac{\lambda_9\rho g_x}{B_z}$

Step 8: Substitute the above test derivatives respectively in the following 28-term equation

$$\begin{cases}\frac{\partial V_x}{\partial x}+\frac{\partial V_y}{\partial y}+\frac{\partial V_z}{\partial z}+\rho\frac{\partial V_x}{\partial t}+\frac{\partial p}{\partial x}+\frac{\rho\partial B_x}{\partial t}-\frac{\eta\partial^2 B_x}{\partial x^2}-\frac{\eta\partial^2 B_x}{\eta\partial y^2}-\frac{\eta\partial^2 B_x}{\eta\partial z^2}+\frac{\partial B_x}{\partial x}+\frac{\partial B_y}{\partial y}+\frac{\partial B_z}{\partial z}\\ +\rho V_x\frac{\partial V_x}{\partial x}+\rho V_y\frac{\partial V_x}{\partial y}+\rho V_z\frac{\partial V_x}{\partial z}-\frac{1}{\mu}B_z\frac{\partial B_x}{\partial z}+\frac{1}{\mu}B_z\frac{\partial B_z}{\partial x}+\frac{1}{\mu}B_y\frac{\partial B_y}{\partial x}-\frac{1}{\mu}B_y\frac{\partial B_x}{\partial y}-V_x\frac{\partial B_y}{\partial y}-B_y\frac{\partial V_x}{\partial y}\\ +V_y\frac{\partial B_x}{\partial y}+B_x\frac{\partial V_y}{\partial y}+V_z\frac{\partial B_x}{\partial z}+B_x\frac{\partial V_z}{\partial z}-V_x\frac{\partial B_z}{\partial z}-B_z\frac{\partial V_x}{\partial z}=\rho g_x \qquad \text{(Three lines per equation)}\end{cases}$$

$$\begin{cases}(\beta_1\rho g_x)+(\beta_2\rho g_x)+(\beta_3\rho g_x)+\rho(ag_x)+(b\rho g_x)+\rho(cg_x)-\eta(-\frac{d\rho g_x}{\eta})-\eta(-\frac{f\rho g_x}{\eta})-\eta(-\frac{m\rho g_x}{\eta})+\\ (q\rho g_x)+(r\rho g_x)+(s\rho g_x)+\rho V_x(\frac{\omega_1 g_x}{V_x})+\rho V_y(\frac{\omega_2 g_x}{V_y})+\rho V_z(\frac{\omega_3 g_x}{V_z})-\frac{1}{\mu}B_z(-\frac{\omega_4\mu\rho g_x}{B_z})+\\ \frac{1}{\mu}B_z(\frac{\omega_5\mu\rho g_x}{B_z})+\frac{1}{\mu}B_y(\frac{\omega_6\mu\rho g_x}{B_y})-\frac{1}{\mu}B_y(-\frac{\lambda_1\mu\rho g_x}{B_y})-V_x(-\frac{\lambda_2\rho g_x}{V_x})-B_y(-\frac{\lambda_3\rho g_x}{B_y})+V_y(\frac{\lambda_4\rho g_x}{V_y})+\\ B_x(\frac{\lambda_5\rho g_x}{B_x})+V_z(\frac{\lambda_6\rho g_x}{V_z})+B_x(\frac{\lambda_7\rho g_x}{B_x})-V_x(-\frac{\lambda_8\rho g_x}{V_x})-B_z(-\frac{\lambda_9\rho g_x}{B_z})\stackrel{?}{=}\rho g_x \qquad \text{(Four lines per equation)}\end{cases}$$

$$\begin{cases}\beta_1\rho g_x+\beta_2\rho g_x+\beta_3\rho g_x+a\rho g_x+b\rho g_x+c\rho g_x+d\rho g_x+f\rho g_x+m\rho g_x q\rho g_x+r\rho g_x+s\rho g_x+\omega_1\rho g_x\\ +\omega_3\rho g_x+\omega_5\rho g_x+\omega_6\rho g_x+\lambda_1\mu\rho g_x+\lambda_2\rho g_x+\lambda_3\rho g_x+\lambda_4\rho g_x+\lambda_5\rho g_x+\omega_2\rho g_x+\omega_3\rho g_x\\ +\lambda_6\rho g_x+\lambda_7\rho g_x+\lambda_8\rho g_x+\lambda_9\rho g_x\stackrel{?}{=}\rho g_x \qquad \text{(Three lines per equation)}\end{cases}$$

$$\begin{cases}\beta_1 g_x + \beta_2 g_x + \beta_3 g_x + a g_x + b g_x + c g_x + d g_x + f g_x + m g_x q g_x + r g_x + s g_x + \omega_1 g_x + \omega_3 g_x + \omega_5 g_x \\ +\omega_6 g_x + \lambda_1 g_x + \lambda_2 g_x + \lambda_3 g_x + \lambda_4 g_x + \lambda_5 g_x + \omega_2 g_x + \omega_3 g_x + \lambda_6 g_x + \lambda_7 g_x + \lambda_8 g_x + \lambda_9 g_x \stackrel{?}{=} g_x \quad \text{(2 lines)}\end{cases}$$

$$\begin{cases} g_x(\beta_1 + \beta_2 + \beta_3 + a + b + c + d + f + m + q + r + s + \omega_1 + \omega_3 + \omega_5 + \lambda_3 + \lambda_4 + \lambda_5 + \omega_2 + \omega_3 + \lambda_6 + \lambda_7 \\ +\omega_6 + \lambda_1 + \lambda_2 + \lambda_8 + \lambda_9) \stackrel{?}{=} g_x \qquad \text{(Two lines per equation)}\end{cases}$$

$$g_x(1) \stackrel{?}{=} g_x \qquad \text{(Sum of the ratio terms = 1)}$$

$$g_x \stackrel{?}{=} g_x \quad \text{Yes}$$

Since an identity is obtained, the solutions to the 28-term equation are as follows

$$\boxed{\begin{array}{l} V_x(x,y,z,t) = \qquad \text{(sum of integrals from sub - equations \#1, \#4,\#13,\#14,\#15,\#21,\#27)} \\ \beta_1 \rho g_x x + a g_x t \pm \sqrt{2\omega_1 g_x x} + \dfrac{\omega_2 g_x y}{V_y} - \dfrac{\lambda_3 \rho g_x y}{B_y} + \dfrac{\omega_3 g_x z}{V_z} - \dfrac{\lambda_9 \rho g_x z}{B_z} + \underbrace{\dfrac{\psi_z(V_z)}{V_z} + \dfrac{\psi_y(B_y)}{B_y} + \dfrac{\psi_y(V_y)}{V_y} + \dfrac{\psi_z(B_z)}{B_z}}_{\text{arbitrary functions}} + C_1 \end{array}};$$

$$\boxed{\begin{array}{l} \qquad \text{(integral from sub–equation \#5)} \\ P(x) = b\rho g_x x + C_2 \end{array}}$$

$$\boxed{\begin{array}{l} \text{(sum of integrals from sub–equations \#2,\#23)} \\ V_y = \beta_2 \rho g_x y + \dfrac{\lambda_5 \rho g_x y}{B_x} + \underbrace{\dfrac{\psi_x(B_x)}{B_x}}_{\text{arbitrary function}} + C_3 \end{array}}$$

$$\boxed{\begin{array}{l} \text{(sum of integrals from sub–equations \#3, \#25)} \\ V_z = \beta_3 \rho g_x z + \dfrac{\lambda_7 \rho g_x z}{B_x} + \underbrace{\dfrac{\psi_x(B_x)}{B_x}}_{\text{arbitrary function}} + C_4 \end{array}}$$

$$\boxed{\begin{array}{l} \qquad \text{(sum of integrals from sub - equations \#6, \#7, \#8, \#9, \#10, \#16,\#19, \#22, \#24)} \\ B_x(x,y,z,t) = \\ B_x = -\dfrac{\rho g_x}{2\eta}(dx^2 + fy^2 + mz^2) + q\rho g_x x + C_2 x + C_4 y + C_6 z + + c g_x t - \dfrac{\lambda_1 \mu \rho g_x y}{B_y} + \dfrac{\lambda_4 \rho g_x y}{V_y} - \dfrac{\omega_4 \mu \rho g_x z}{B_z} + \\ \qquad \dfrac{\lambda_6 \rho g_x z}{V_z} + \underbrace{\dfrac{\psi_z(B_z)}{B_z} + \dfrac{\psi_y(B_y)}{B_y} + \dfrac{\psi_y(V_y)}{V_y} + \dfrac{\psi_z(V_z)}{V_z}}_{\text{arbitrary functions}} + C_7 \end{array}}$$

$$\boxed{\begin{array}{l} \qquad \text{(sum of integrals from sub–equations \#11,\#18,\#20)} \\ B_y = r\rho g_x y \pm \sqrt{2\omega_6 \mu \rho g_x x} - \dfrac{\lambda_2 \rho g_x y}{V_x} + \underbrace{\dfrac{\psi_x(V_x)}{V_x}}_{\substack{\text{arbitrary} \\ \text{function}}} + C_8 \end{array}}$$

$$\boxed{\begin{array}{l} \qquad \text{(sum of integrals from sub–equations \#12,\#17,\#26)} \\ B_z = s\rho g_x z \pm \sqrt{2\omega_5 \mu \rho g_x x} - \dfrac{\lambda_8 \rho g_x z}{V_x} + \underbrace{\dfrac{\psi_x(V_x)}{V_x}}_{\text{arbitrary function}} + C_{21} \end{array}}$$

Supporter Equation Contributions

Note above that there are 28 terms in the driver equation, and 27 supporter equations, Each supporter equation provides useful information about the driver equation. The more of these supporter equations that are integrated, the more the information one obtains about the driver equation. However, without solving a supporter equation, one can sometimes write down some characteristics of the integration relation of the supporter equation by referring to the subjects of the supporter equations of the Navier-Stokes equations. For example, if one uses $(\eta\partial^2 B_x/\partial x^2)$ as the subject of a supporter equation here, the curve for the integration relation obtained would be parabolic, periodic, and decreasingly exponential. Using $\rho(\partial V/\partial t)$ as the subject of the supporter equation, the curve would be periodic and decreasingly exponential. Using $(\partial p/\partial x)$, the curve would be parabolic.

Comparison of Solutions of Navier-Stokes Equations
and
Solutions of Magnetohydrodynamic Equations

Navier-Stokes x–direction **solution**

$$V_x(x,y,z,t) = -\frac{\rho g_x}{2\mu}(ax^2+by^2+cz^2) + C_1x + C_3y + C_5z + fg \pm \sqrt{2hgx} + \frac{ngy}{V_y} + \frac{qgz}{V_z} + \underbrace{\frac{\psi_y(V_y)}{V_y} + \frac{\psi_z(V_z)}{V_z}}_{\text{arbitrary functions}}$$

$$P(x) = d\rho g_x x \qquad (V_y \neq 0,\ V_z \neq 0)$$

For magnetohydrodynamic solutions, see previous page

1. V_x for MHD system looks like the V_x for the Euler solution.
2. $P(x)$) for N-S and MHD equations are the same.
3. V_y and V_z for MHD are different from those of N-S equations.
4. B_x is parabolic and resembles V_x for N-S, except for the absence of the square root function.
5. B_y and B_z resemble the Euler solution.

Conclusion for Magnetohydrodynamics

The author proposes the law of definite ratio for magnetohyrodynamics.
This law states that in magnetohydrodynamics, all the other terms in the system of equations divide the gravity term in a definite ratio and each term utilizes gravity to function. As in the case of incompressible fluid flow, one can add that, without gravity forces, there would be no magnetohydrodynamics on earth as is known, according to the solutions of the magnetohydrodynamic equations.

Encouraged by the solution method for the magnetohydrodynamic equations, one will next solve the Navier-Stokes equations again by a second method in which the three equations in the system are added and a single equation integrated

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Option 7
Solutions of 3-D Navier-Stokes Equations
(Method 2)

Here, the three equations below, will be added together; and a single equation will be integrated

$$\begin{cases} -\mu(\frac{\partial^2 V_x}{\partial x^2}+\frac{\partial^2 V_x}{\partial y^2}+\frac{\partial^2 V_x}{\partial z^2})+\frac{\partial p}{\partial x}+\rho(\frac{\partial V_x}{\partial t}+V_x\frac{\partial V_x}{\partial x}+V_y\frac{\partial V_x}{\partial y}+V_z\frac{\partial V_x}{\partial z})=\rho g_x & (1)\\ -\mu(\frac{\partial^2 V_y}{\partial x^2}+\frac{\partial^2 V_y}{\partial y^2}+\frac{\partial^2 V_y}{\partial z^2})+\frac{\partial p}{\partial y}+\rho(\frac{\partial V_y}{\partial t}+V_x\frac{\partial V_y}{\partial x}+V_y\frac{\partial V_y}{\partial y}+V_z\frac{\partial V_y}{\partial z})=\rho g_y & (2)\\ -\mu(\frac{\partial^2 V_z}{\partial x^2}+\frac{\partial^2 V_z}{\partial y^2}+\frac{\partial^2 V_z}{\partial z^2})+\frac{\partial p}{\partial z}+\rho(\frac{\partial V_z}{\partial t}+V_x\frac{\partial V_z}{\partial x}+V_y\frac{\partial V_z}{\partial y}+V_z\frac{\partial V_z}{\partial z})=\rho g_z & (3) \end{cases}$$

Step 1: Apply the axiom, if $a=b$ and $c=d$, then $a+c=b+d$; and therefore, add the left sides and add the right sides of the above equations . That is, $(1)+(2)+(3)=\rho g_x+\rho g_y+\rho g_z$

$$-\mu\frac{\partial^2 V_x}{\partial x^2}-\mu\frac{\partial^2 V_x}{\partial y^2}-\mu\frac{\partial^2 V_x}{\partial z^2}-\mu\frac{\partial^2 V_y}{\partial x^2}-\mu\frac{\partial^2 V_y}{\partial y^2}-\mu\frac{\partial^2 V_y}{\partial z^2}-\mu\frac{\partial^2 V_z}{\partial x^2}-\mu\frac{\partial^2 V_z}{\partial y^2}-\mu\frac{\partial^2 V_z}{\partial z^2}+\frac{\partial p}{\partial x}+\frac{\partial p}{\partial y}$$
$$+\frac{\partial p}{\partial z}++\rho\frac{\partial V_x}{\partial t}+\rho\frac{\partial V_y}{\partial t}+\rho\frac{\partial V_z}{\partial t}+\rho V_x\frac{\partial V_x}{\partial x}+\rho V_y\frac{\partial V_x}{\partial y}+\rho V_z\frac{\partial V_x}{\partial z}+\rho V_x\frac{\partial V_y}{\partial x}+\rho V_y\frac{\partial V_y}{\partial y}+\rho V_z\frac{\partial V_y}{\partial z}$$
$$+\rho V_x\frac{\partial V_z}{\partial x}+\rho V_y\frac{\partial V_z}{\partial y}+\rho V_z\frac{\partial V_z}{\partial z}=(\rho g_x+\rho g_y+\rho g_z)$$

(Three lines per equation)

Let $\rho g_x+\rho g_y+\rho g_z=\rho G$, where $G=|g_x+g_y+g_z|$ to obtain

$$-\mu\frac{\partial^2 V_x}{\partial x^2}-\mu\frac{\partial^2 V_x}{\partial y^2}-\mu\frac{\partial^2 V_x}{\partial z^2}-\mu\frac{\partial^2 V_y}{\partial x^2}-\mu\frac{\partial^2 V_y}{\partial y^2}-\mu\frac{\partial^2 V_y}{\partial z^2}-\mu\frac{\partial^2 V_z}{\partial x^2}-\mu\frac{\partial^2 V_z}{\partial y^2}-\mu\frac{\partial^2 V_z}{\partial z^2}$$
$$+\frac{\partial p}{\partial x}+\frac{\partial p}{\partial y}+\frac{\partial p}{\partial z}++\rho\frac{\partial V_x}{\partial t}+\rho\frac{\partial V_y}{\partial t}+\rho\frac{\partial V_z}{\partial t}+\rho V_x\frac{\partial V_x}{\partial x}+\rho V_y\frac{\partial V_x}{\partial y}+\rho V_z\frac{\partial V_x}{\partial z}$$
$$+\rho V_x\frac{\partial V_y}{\partial x}+\rho V_y\frac{\partial V_y}{\partial y}+\rho V_z\frac{\partial V_y}{\partial z}+\rho V_x\frac{\partial V_z}{\partial x}+\rho V_y\frac{\partial V_z}{\partial y}+\rho V_z\frac{\partial V_z}{\partial z}=\rho G$$

Step 2: Solve the above 25-term equation using the ratio method. (24 ratio terms)

The ratio terms to be used are respectively the following: (Sum of the ratio terms = 1)

a ,b, c, d, f, m, n, q, r, β_1, β_2, β_3, β_4, β_5, β_6, λ_1, λ_2, λ_3, λ_4, λ_5, λ_6, λ_7, λ_8, λ_9

$-\mu\frac{\partial^2 V_x}{\partial x^2}=a\rho G$; $-\mu\frac{\partial^2 V_x}{\partial y^2}=b\rho G$; $-\mu\frac{\partial^2 V_x}{\partial z^2}=c\rho G$; $-\mu\frac{\partial^2 V_y}{\partial x^2}=d\rho G$;

$-\mu\frac{\partial^2 V_y}{\partial y^2}=f\rho G$; $-\mu\frac{\partial^2 V_y}{\partial z^2}=h\rho G$; $-\mu\frac{\partial^2 V_z}{\partial x^2}=m\rho G$; $-\mu\frac{\partial^2 V_z}{\partial y^2}=n\rho G$;

$-\mu\frac{\partial^2 V_z}{\partial z^2}=r\rho G$; $\frac{\partial p}{\partial x}=\beta_1\rho G$; $\frac{\partial p}{\partial y}=\beta_2\rho G$; $\frac{\partial p}{\partial z}=\beta_3\rho G$;

$\rho\frac{\partial V_x}{\partial t}=\beta_4\rho G$; $\rho\frac{\partial V_y}{\partial t}=\beta_5\rho G$; $\rho\frac{\partial V_z}{\partial t}=\beta_6\rho G$; $\rho V_x\frac{\partial V_x}{\partial x}=\lambda_1\rho G$;

$\rho V_y\frac{\partial V_x}{\partial y}=\lambda_2\rho G$; $\rho V_z\frac{\partial V_x}{\partial z}=\lambda_3\rho G$; $\rho V_x\frac{\partial V_y}{\partial x}=\lambda_4\rho G$; $\rho V_y\frac{\partial V_y}{\partial y}=\lambda_5\rho G$;

$\rho V_z\frac{\partial V_y}{\partial z}=\lambda_6\rho G$; $\rho V_x\frac{\partial V_z}{\partial x}=\lambda_7\rho G$; $\rho V_y\frac{\partial V_z}{\partial y}=\lambda_8\rho G$; $\rho V_z\frac{\partial V_z}{\partial z}=\lambda_9\rho G$

1 $\frac{\partial^2 V_x}{\partial x^2} = -\frac{a}{\mu}\rho G$ $\frac{\partial V_x}{\partial x} = -\frac{a}{\mu}\rho G x + C_1$ $V_x = -\frac{a}{\mu}\rho G\frac{x^2}{2} + C_1 x + C_2$	**2** $-\mu\frac{\partial^2 V_x}{\partial y^2} = b\rho G$ $\frac{\partial^2 V_x}{\partial y^2} = -\frac{b}{\mu}\rho G$ $\frac{\partial V_x}{\partial y} = -\frac{b}{\mu}\rho G y + C_3$ $V_x = -\frac{b}{\mu}\rho G\frac{y^2}{2} + C_3 y + C_4$	**3** $-\mu\frac{\partial^2 V_x}{\partial z^2} = c\rho G$ $-\mu\frac{\partial^2 V_x}{\partial z^2} = c\rho G$ $\frac{\partial^2 V_x}{\partial z^2} = -\frac{c}{\mu}\rho G$ $\frac{\partial V_x}{\partial z} = -\frac{c}{\mu}\rho G z + C_5$ $V_x = -\frac{c}{\mu}\rho G\frac{z^2}{2} + C_5 z + C_6$
4 $-\mu\frac{\partial^2 V_y}{\partial x^2} = d\rho G$ $-\mu\frac{\partial^2 V_y}{\partial x^2} = d\rho G$ $\frac{\partial^2 V_y}{\partial x^2} = -\frac{d}{\mu}\rho G$ $\frac{\partial V_y}{\partial x} = -\frac{d}{\mu}\rho G x + C_7$ $V_y = -\frac{d}{\mu}\rho G\frac{x^2}{2} + C_7 x + C_8$	**5** $-\mu\frac{\partial^2 V_y}{\partial y^2} = f\rho G$ $\frac{\partial^2 V_y}{\partial y^2} = -\frac{f}{\mu}\rho G$ $\frac{\partial V_y}{\partial y} = -\frac{f}{\mu}\rho G y + C_9$ $V_y = -\frac{f}{\mu}\rho G\frac{y^2}{2} + C_9 y + C_{10}$	**6** $-\mu\frac{\partial^2 V_y}{\partial z^2} = h\rho G$ $\frac{\partial^2 V_y}{\partial z^2} = -\frac{h}{\mu}\rho G$ $\frac{\partial V_y}{\partial z} = -\frac{h}{\mu}\rho G z + C_{11}$ $V_y = -\frac{h}{\mu}\rho G\frac{z^2}{2} + C_{11} z + C_{12}$
7 $-\mu\frac{\partial^2 V_z}{\partial x^2} = m\rho G$ $\frac{\partial^2 V_z}{\partial x^2} = -\frac{m}{\mu}\rho G$ $\frac{\partial V_z}{\partial x} = -\frac{m}{\mu}\rho G x + C_{13}$ $V_z = -\frac{m}{\mu}\rho G\frac{x^2}{2} + C_{13} x + C_{14}$	**8** $-\mu\frac{\partial^2 V_z}{\partial y^2} = n\rho G$ $\frac{\partial^2 V_z}{\partial y^2} = -\frac{n}{\mu}\rho G$ $\frac{\partial V_z}{\partial y} = -\frac{n}{\mu}\rho G y + C_{15}$ $V_z = -\frac{m}{\mu}\rho G\frac{y^2}{2} + C_{15} y + C_{16}$	**9** $-\mu\frac{\partial^2 V_z}{\partial z^2} = r\rho G$ $\frac{\partial^2 V_z}{\partial z^2} = -\frac{r}{\mu}\rho G$ $\frac{\partial V_z}{\partial z} = -\frac{r}{\mu}\rho G z + C_{17}$ $V_z = -\frac{r}{\mu}\rho G\frac{z^2}{2} + C_{17} z + C_{18}$

10 $\frac{\partial p}{\partial x} = \beta_1\rho G$ $\frac{dp}{dx} = \beta_1\rho G$ $P(x) = \beta_1\rho G x + C_{19}$	**11** $\frac{\partial p}{\partial y} = \beta_2\rho G$ $\frac{dp}{dy} = \beta_2\rho G$ $P(y) = \beta_2\rho G y + C_{20}$	**12** $\frac{\partial p}{\partial z} = \beta_3\rho G$ $\frac{dp}{dz} = \beta_3\rho G$ $P(z) = \beta_3\rho G z + C_{21}$

13 $\rho \frac{\partial V_x}{\partial t} = \beta_4 \rho G$ $\frac{dV_x}{dt} = \beta_4 G$ $V_x = \beta_4 Gt + C_{22}$	**14** $\rho \frac{\partial V_y}{\partial t} = \beta_5 \rho G$ $\frac{dV_y}{dt} = \beta_5 G$ $V_y = \beta_5 Gt + C_{23}$	**15** $\rho \frac{\partial V_z}{\partial t} = \beta_6 \rho G$ $\frac{dV_z}{dt} = \beta_6 G$ $V_z = \beta_6 Gt + C_{24}$	**16** $\rho V_x \frac{\partial V_x}{\partial x} = \lambda_1 \rho G$ $V_x \frac{\partial V_x}{\partial x} = \lambda_1 G$ $V_x \frac{dV_x}{dx} = \lambda_1 G$ $V_x dV_x = \lambda_1 G\, dx$ $\frac{V^2{}_x}{2} = \lambda_1 G\, x$ $V^2{}_x = 2\lambda_1 G\, x$ $V_x = \pm\sqrt{2\lambda_1 G\, x} + C_{25}$
17 $\rho V_y \frac{\partial V_x}{\partial y} = \lambda_2 \rho G$ $V_y \frac{dV_x}{dy} = \lambda_2 G$ $V_y dV_x = \lambda_2 G\, dy$ $V_y V_x = \lambda_2 G\, y + \psi_y(V_y)$ $V_x = \frac{\lambda_2 G\, y}{V_y} + \frac{\psi_y(V_y)}{V_y}$	**18** $\rho V_z \frac{\partial V_x}{\partial z} = \lambda_3 \rho G$ $V_z \frac{dV_x}{dz} = \lambda_3 G$ $V_z dV_x = \lambda_3 G\, dz$ $V_z V_x = \lambda_3 G\, z + \psi_z(V_z)$ $V_x = \frac{\lambda_3 G\, z}{V_z} + \frac{\psi_z(V_z)}{V_z}$	**19** $\rho V_x \frac{\partial V_y}{\partial x} = \lambda_4 \rho G$ $V_x \frac{dV_y}{dx} = \lambda_4 G$ $V_x dV_y = \lambda_4 G\, dx$ $V_x V_y = \lambda_4 G\, x + \psi_x(V_x)$ $V_y = \frac{\lambda_4 G\, x}{V_x} + \frac{\psi_x(V_x)}{V_x}$	**20** $\rho V_y \frac{\partial V_y}{\partial y} = \lambda_5 \rho G$ $V_y \frac{dV_y}{dy} = \lambda_5 G$ $V_y dV_y = \lambda_5 G\, dy$ $\frac{V_y^{\ 2}}{2} = \lambda_5 G\, y$ $V_y^{\ 2} = 2\lambda_5 G\, y$ $V_y = \pm\sqrt{2\lambda_5 G\, y} + C_{26}$
21 $\rho V_z \frac{\partial V_y}{\partial z} = \lambda_6 \rho G$ $V_z \frac{dV_y}{dz} = \lambda_6 G$ $V_z dV_y = \lambda_6 G\, dz$ $V_z V_y = \lambda_6 G\, z + \psi_z(V_z)$ $V_y = \frac{\lambda_6 G\, z}{V_z} + \frac{\psi_z(V_z)}{V_z}$	**22** $\rho V_x \frac{\partial V_z}{\partial x} = \lambda_7 \rho G$ $V_x \frac{dV_z}{dx} = \lambda_7 G$ $V_x dV_z = \lambda_7 G\, dx$ $V_x V_z = \lambda_7 G\, x + \psi_x(V_x)$ $V_z = \frac{\lambda_7 G\, x}{V_x} + \frac{\psi_x(V_x)}{V_x}$	**23** $\rho V_y \frac{\partial V_z}{\partial y} = \lambda_8 \rho G$ $V_y \frac{dV_z}{dy} = \lambda_8 G$ $V_y dV_z = \lambda_8 G\, dy$ $V_y V_z = \lambda_8 G\, y + \psi_y(V_y)$ $V_z = \frac{\lambda_8 G\, y}{V_y} + \frac{\psi_y(V_y)}{V_y}$	**24** $\rho V_z \frac{\partial V_z}{\partial z} = \lambda_9 \rho G$ $V_z \frac{dV_z}{dz} = \lambda_9 G$ $V_z dV_z = \lambda_9 G\, dz$ $\frac{V_z^{\ 2}}{2} = \lambda_9 G\, z$ $V_z^{\ 2} = 2\lambda_9 G\, z$ $V_z = \pm\sqrt{2\lambda_9 G\, z} + C_{27}$

Step 3 : One Collects the integrals of the sub-equations, above, for $V_x, V_y, V_z, P(x), P(y), P(z)$

For V_x, $P(x)$	For V_y $P(y)$	For V_z $P(z)$
Sum of integrals from sub-equations #1, #2, #3, #13, #16, #17, #18, #10	**S**um of integrals from sub-equations #4, #5, #6, #14, #19, #20, #21,#11	**S**um of integrals from sub-equations #7, #8, #9, #15, #22, #23, #24, #12,
$V_x = -\frac{a}{\mu}\rho G\frac{x^2}{2} + C_1x + C_2$	$V_y = -\frac{d}{\mu}\rho G\frac{x^2}{2} + C_7x + C_8$	$V_z = -\frac{m}{\mu}\rho G\frac{x^2}{2} + C_{13}x + C_{14}$
$V_x = -\frac{b}{\mu}\rho G\frac{y^2}{2} + C_3y + C_4$	$V_y = -\frac{f}{\mu}\rho G\frac{y^2}{2} + C_9y + C_{10}$	$V_z = -\frac{m}{\mu}\rho G\frac{y^2}{2} + C_{15}y + C_{16}$
$V_x = -\frac{c}{\mu}\rho G\frac{z^2}{2} + C_5z + C_6$	$V_y = -\frac{h}{\mu}\rho G\frac{z^2}{2} + C_{11}z + C_{12}$	$V_z = -\frac{r}{\mu}\rho G\frac{z^2}{2} + C_{17}z + C_{18}$
$V_x = \beta_4 Gt + C_{22}$	$V_y = \beta_5 Gt + C_{21}$	$V_z = \beta_6 Gt + C_{24}$
$V_x = \pm\sqrt{2\lambda_1 G\,x} + C_{25}$	$V_y = \frac{\lambda_4 G\,x}{V_x} + \frac{\psi_x(V_x)}{V_x}$	$V_z = \frac{\lambda_7 G\,x}{V_x} + \frac{\psi_x(V_x)}{V_x}$
$V_x = \frac{\lambda_2 G\,y}{V_y} + \frac{\psi_y(V_y)}{V_y}$	$V_y = \pm\sqrt{2\lambda_5 G\,y} + C_{26}$	# $V_z = \frac{\lambda_8 G\,y}{V_y} + \frac{\psi_y(V_y)}{V_y}$
$V_x = \frac{\lambda_3 G\,z}{V_z} + \frac{\psi_z(V_z)}{V_z}$	$V_y = \frac{\lambda_6 G\,z}{V_z} + \frac{\psi_z(V_z)}{V_z}$	$V_z = \pm\sqrt{2\lambda_9 G\,z} + C_{27}$
$P(x) = \beta_1\rho Gx + C_{19}$	$P(y) = \beta_2\rho Gy + C_{20}$	$P(z) = \beta_3\rho Gz + C_{21}$

From above**,**

For V_x, Sum of integrals from sub-equations #1, #2, #3, #13, #16, #17, #18, #10

$V_x(x,y,z,t)$

$$= -\frac{a}{\mu}\rho G\frac{x^2}{2} + C_1x - \frac{b}{\mu}\rho G\frac{y^2}{2} + C_3y - \frac{c}{\mu}\rho G\frac{z^2}{2} + C_5z + \beta_4 Gt \pm \sqrt{2\lambda_1 G\,x} + \frac{\lambda_2 G\,y}{V_y} + \frac{\lambda_3 G\,z}{V_z} + \underbrace{\frac{\psi_y(V_y)}{V_y} + \frac{\psi_z(V_z)}{V_z}}_{\text{arbitrary functions}}$$

$$P(x) = \beta_1\rho Gx + C_{19}$$

For V_y : Sum of integrals from sub-equations #4, #5, #6,#14, #19, #20, #21,#11

$V_y(x,y,z,t)$

$$= -\frac{d}{\mu}\rho G\frac{x^2}{2} + C_7x - \frac{f}{\mu}\rho G\frac{y^2}{2} + C_9y - \frac{h}{\mu}\rho G\frac{z^2}{2} + C_{11}z + \beta_5 Gt + \pm\sqrt{2\lambda_5 G\,y} + \frac{\lambda_4 G\,x}{V_x} + \frac{\lambda_6 G\,z}{V_z} + \underbrace{\frac{\psi_x(V_x)}{V_x}\frac{\psi_z(V_z)}{V_z}}_{\text{arbitrary functions}}$$

$$P(y) = \beta_2\rho Gy + C_{20}$$

For V_z: Sum of integrals from sub-equations #7, #8, #9,#15, #22, #23, #24, #12,

$$V_z = -\frac{m}{\mu}\rho G\frac{x^2}{2} + C_{13}x - \frac{n}{\mu}\rho G\frac{y^2}{2} + C_{15}y - \frac{r}{\mu}\rho G\frac{z^2}{2} + C_{17}z + \beta_6 Gt \pm \sqrt{2\lambda_9 G\,z} + \frac{\lambda_7 G\,x}{V_x} + \frac{\lambda_8 G\,y}{V_y} + \underbrace{\frac{\psi_x(V_x)}{V_x} + \frac{\psi_y(V_y)}{V_y}}_{\text{arbitrary functions}}$$

$$P(z) = \beta_3\rho Gz + C_{21}$$

Step 4: Simplify the sums of the integrals from above..(**Method 2 solutions of N-S equations**

$$V_x(x,y,z,t) = -\frac{\rho G}{2\mu}(ax^2 - by^2 - cz^2) + C_1x + C_3y + C_5z + \beta_4 Gt \pm \sqrt{2\lambda_1 G\,x} + \frac{\lambda_2 G\,y}{V_y} + \frac{\lambda_3 G\,z}{V_z}$$

$$P(x) = \beta_1 \rho G x + C_{19} \qquad (V_y \neq 0,\ V_z \neq 0) \qquad + \underbrace{\frac{\psi_y(V_y)}{V_y} + \frac{\psi_z(V_z)}{V_z}}_{\text{arbitrary functions}}$$

$$V_y(x,y,z,t) = -\frac{\rho G}{2\mu}(dx^2 - fy^2 - hz^2) + C_7x + C_9y + C_{11}z + C_{10}\beta_5 Gt \pm \sqrt{2\lambda_5 G\,y} + \frac{\lambda_4 G\,x}{V_x} + \frac{\lambda_6 G\,z}{V_z}$$

$$P(y) = \beta_2 \rho G y + C_{20} \qquad (V_x \neq 0,\ V_z \neq 0) \qquad \underbrace{+\frac{\psi_x(V_x)}{V_x} + \frac{\psi_z(V_z)}{V_z}}_{\text{arbitrary functions}}$$

$$V_z(x,y,z,t) = -\frac{\rho G}{2\mu}(mx^2 - ny^2 - rz^2) + C_{13}x + C_{15}y + C_{17}z + \beta_6 Gt \pm \sqrt{2\lambda_9 G\,z} + \frac{\lambda_7 G\,x}{V_x} + \frac{\lambda_8 G\,y}{V_y}$$

$$P(z) = \beta_3 \rho G z + C_{21} \qquad (V_y \neq 0,\ V_y \neq 0) \qquad + \underbrace{\frac{\psi_x(V_x)}{V_x} + \frac{\psi_y(V_y)}{V_y}}_{\text{arbitrary functions}}$$

The above are solutions for $V_x\ V_y,\ V_z\quad P(x),\ P(y),\ P(z)$ .of the Navier-Stokes Equations

Comparison of Method 1 (Option 4) and Method 2 (Option 7) of Solutions of Navier-Stokes Equations

Method 1: x–direction solution of Navier-Stokes equation

$$V_x(x,y,z,t) = -\frac{\rho g_x}{2\mu}(ax^2 + by^2 + cz^2) + C_1 x + C_3 y + C_5 z + f g_x t \pm \sqrt{2hg_x x} + \frac{ng_x y}{V_y} + \frac{qg_x z}{V_z} + \underbrace{\frac{\psi_y(V_y)}{V_y} + \frac{\psi_z(V_z)}{V_z}}_{\text{arbitrary functions}} + C_9$$

$$P(x) = d\rho g_x x;\ (a+b+c+d+h+n+q=1) \quad (V_y \neq 0,\ V_z \neq 0) \qquad \text{(A)}$$

Method 2: x–direction solution of Navier-Stokes equation

$$V_x(x,y,z,t) = -\frac{\rho G}{2\mu}(ax^2 - by^2 - cz^2) + C_1 x + C_3 y + C_5 z + \beta_4 G t \pm \sqrt{2\lambda_1 G\, x} + \frac{\lambda_2 G\, y}{V_y} + \frac{\lambda_3 G\, z}{V_z} + \underbrace{\frac{\psi_y(V_y)}{V_y} + \frac{\psi_z(V_z)}{V_z}}_{\text{arbitrary functions}}$$

$$P(x) = \beta_1 \rho G x + C_{19} \qquad (V_y \neq 0,\ V_z \neq 0) \qquad \textbf{(B)}$$

It is pleasantly surprising that the above solutions (A) and (B) are almost identical (except for the constants) even though they were obtained by different approaches as in Option 4 and Option 8. Such an agreement confirms the validity of the solution method for the system of magnetohydrodynamic equations (Option 7). For the system of magnetohydrodynamic equations, there is only a single "driver" equation. For the system of N-S equations, there are three driver equations, since each equation contains the gravity term. Therefore, one was able to solve each of the three simultaneous equations separately (as in Method 1); but in addition, one obtained an identical solution (except for the constants) in solving the simultaneous N-S system by adding the three equations and integrating a single driver equation. In Method 1, the gravity term was ρg. In Method 2, the gravity term was ρG, where G is the magnitude of the vector sum of the gravity terms. Note that in Method 1, the sum of the ratio terms (8 ratio terms for each equation) equals unity, but in Method 2, the sum of the ratio terms (24 ratio terms) for the single driver equation solved equals unity. Note that in Method 2, only a single "driver" equation was solved, but in Method 1, three "driver" equations were solved. In Method 2, one could say that the system of N-S equations was "more simultaneously" solved than in Method 1.

To summarize, solving the Navier-Stokes equations by the first method helped one to solve the magnetohydrodynamic equations; and solving the magnetohydrodynamic equations encouraged one to solve the Navier-Stokes equations by the second method.
(" Navier-Stokes equations "scratched" the back of magnetohydrodynamic equations; and in return, magnetohydrodynamic equations "scratched" the back of Navier-Stokes equations")

About integrating only a single equation
If one asked for help in solving the N-S equations, and one was told to add the three equations together and then solve them, one would think that one was being given a nonsensical advice; but now, after studying the above Option 7 method, one would appreciate such a suggestion.

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Option 8
Solutions of 3-D Linearized Navier-Stokes Equations
Method 2

Here, the three equations below, will be added together; and a single equation will be integrated.

$$\begin{cases} -\mu\frac{\partial^2 V_x}{\partial x^2}-\mu\frac{\partial^2 V_x}{\partial y^2}-\mu\frac{\partial^2 V_x}{\partial z^2}+\frac{\partial p}{\partial x}+4\rho\frac{\partial V_x}{\partial t}=\rho g_x & (1)\\ -\mu\frac{\partial^2 V_y}{\partial x^2}-\mu\frac{\partial^2 V_y}{\partial y^2}-\mu\frac{\partial^2 V_y}{\partial z^2}+\frac{\partial p}{\partial y}+4\rho\frac{\partial V_y}{\partial t}=\rho g_y & (2)\\ -\mu\frac{\partial^2 V_z}{\partial x^2}-\mu\frac{\partial^2 V_z}{\partial y^2}-\mu\frac{\partial^2 V_z}{\partial z^2}+\frac{\partial p}{\partial z}+4\rho\frac{\partial V_z}{\partial t}=\rho g_z & (3)\end{cases}$$

Step 1: Apply the axiom, if $a=b$ and $c=d$, then $a+c=b+d$; and therefore, add the left sides and add the right sides of the above equations . That is, $(1)+(2)+(3)=\rho g_x+\rho g_y+\rho g_z$

$$-\mu\frac{\partial^2 V_x}{\partial x^2}-\mu\frac{\partial^2 V_x}{\partial y^2}-\mu\frac{\partial^2 V_x}{\partial z^2}+\frac{\partial p}{\partial x}+4\rho\frac{\partial V_x}{\partial t}-\mu\frac{\partial^2 V_y}{\partial x^2}-\mu\frac{\partial^2 V_y}{\partial y^2}-\mu\frac{\partial^2 V_y}{\partial z^2}+\frac{\partial p}{\partial y}+4\rho\frac{\partial V_y}{\partial t}$$
$$-\mu\frac{\partial^2 V_z}{\partial x^2}-\mu\frac{\partial^2 V_z}{\partial y^2}-\mu\frac{\partial^2 V_z}{\partial z^2}+\frac{\partial p}{\partial z}4\rho\frac{\partial V_z}{\partial t}=\rho g_x+\rho g_y+\rho g_z$$ (Two lines per equation)

Let $\rho g_x+\rho g_y+\rho g_z=\rho G$ **,** where $G=\left|g_x+g_y+g_z\right|$ to obtain

$$-\mu\frac{\partial^2 V_x}{\partial x^2}-\mu\frac{\partial^2 V_x}{\partial y^2}-\mu\frac{\partial^2 V_x}{\partial z^2}+\frac{\partial p}{\partial x}+4\rho\frac{\partial V_x}{\partial t}-\mu\frac{\partial^2 V_y}{\partial x^2}-\mu\frac{\partial^2 V_y}{\partial y^2}-\mu\frac{\partial^2 V_y}{\partial z^2}+\frac{\partial p}{\partial y}+4\rho\frac{\partial V_y}{\partial t}$$
$$-\mu\frac{\partial^2 V_z}{\partial x^2}-\mu\frac{\partial^2 V_z}{\partial y^2}-\mu\frac{\partial^2 V_z}{\partial z^2}+\frac{\partial p}{\partial z}+4\rho\frac{\partial V_z}{\partial t}=\rho G$$ (Two lines per equation)

Step 2: Solve the above 25-term equation using the ratio method. (24 ratio terms)

The ratio terms to be used are respectively the following: (Sum of the ratio terms = 1)

$a,\ b,\ c,\ d,\ f,\ h,\ j,\ m,\ n,\ q,\ r,\ s,\ u,\ v,\ w.$ (Sum of the ratio terms = 1)

$-\mu\frac{\partial^2 V_x}{\partial x^2}=a\rho G;\quad -\mu\frac{\partial^2 V_x}{\partial y^2}=b\rho G;\quad -\mu\frac{\partial^2 V_x}{\partial z^2}=c\rho G;\quad \frac{\partial p}{\partial x}=d\rho G$

$4\rho\frac{\partial V_x}{\partial t}=f\rho G;\quad -\mu\frac{\partial^2 V_y}{\partial x^2}=h\rho G;\quad -\mu\frac{\partial^2 V_y}{\partial y^2}=j\rho G;\quad -\mu\frac{\partial^2 V_y}{\partial z^2}=m\rho G;$

$\frac{\partial p}{\partial y}=n\rho G;\quad 4\rho\frac{\partial V_y}{\partial t}=q\rho G;\quad -\mu\frac{\partial^2 V_z}{\partial x^2}=r\rho G;\quad -\mu\frac{\partial^2 V_z}{\partial y^2}=s\rho G;$

$-\mu\frac{\partial^2 V_z}{\partial z^2}=u\rho G;\quad \frac{\partial p}{\partial z}=v\rho G;\quad 4\rho\frac{\partial V_z}{\partial t}=w\rho G$

1. $-\mu\frac{\partial^2 V_x}{\partial x^2}=a\rho G$ $\frac{\partial^2 V_x}{\partial x^2}=-\frac{a}{\mu}\rho G$ $\frac{\partial V_x}{\partial x}=-\frac{a}{\mu}\rho Gx+C_1$ $V_x=-\frac{\rho Ga}{2\mu}x^2+C_1x+C_2$	**2.** $-\mu\frac{\partial^2 V_x}{\partial y^2}=b\rho G$ $\frac{\partial^2 V_x}{\partial y^2}=-\frac{b}{\mu}\rho G$ $\frac{\partial V_x}{\partial y}=-\frac{b}{\mu}\rho Gy+C_3$ $V_x=-\frac{\rho Gb}{2\mu}y^2+C_3y+C_4$	**3** $-\mu\frac{\partial^2 V_x}{\partial z^2}=c\rho G$ $\frac{\partial^2 V_x}{\partial z^2}=-\frac{c}{\mu}\rho G$ $\frac{\partial V_x}{\partial z}=-\frac{c}{\mu}\rho Gz+C_5$ $V_x=-\frac{\rho Gc}{2\mu}z^2+C_5z+C_6$

4 $\frac{\partial p}{\partial x} = d\rho G$ $P(x) = d\rho Gx + C_7$	**5** $4\rho \frac{\partial V_x}{\partial t} = f\rho G$ $\frac{\partial V_x}{\partial t} = \frac{fG}{4}$ $V_x = \frac{fG}{4} t + C_8$	**6** $-\mu \frac{\partial^2 V_y}{\partial x^2} = h\rho G$ $\frac{\partial^2 V_y}{\partial x^2} = -\frac{h}{\mu}\rho G$ $\frac{\partial V_y}{\partial x} = -\frac{h}{\mu}\rho Gx + C_9$ $V_y = -\frac{\rho Gh}{2\mu} x^2 + C_9 x + C_{10}$
7 $-\mu \frac{\partial^2 V_y}{\partial y^2} = j\rho G$ $\frac{\partial^2 V_y}{\partial y^2} = -\frac{j}{\mu}\rho G$ $\frac{\partial V_y}{\partial y} = -\frac{j}{\mu}\rho Gy + C_{11}$ $V_y = -\frac{\rho Gj}{2\mu} y^2 + C_{11} y + C_{12}$	**8** $-\mu \frac{\partial^2 V_y}{\partial z^2} = m\rho G$ $\frac{\partial^2 V_y}{\partial z^2} = -\frac{m}{\mu}\rho G$ $\frac{\partial V_y}{\partial z} = -\frac{m}{\mu}\rho Gz + C_{13}$ $V_y = -\frac{\rho Gm}{2\mu} z^2 + C_{13} z + C_{14}$	**9** $\frac{\partial p}{\partial y} = n\rho G$ $P(y) = n\rho Gx + C_{15}$
10 $4\rho \frac{\partial V_y}{\partial t} = q\rho G$ $\frac{\partial V_y}{\partial t} = \frac{qG}{4}$ $V_y = \frac{qG}{4} t + C_{16}$	**11** $-\mu \frac{\partial^2 V_z}{\partial x^2} = r\rho G$ $\frac{\partial^2 V_z}{\partial x^2} = -\frac{r}{\mu}\rho G$ $\frac{\partial V_z}{\partial x} = -\frac{r}{\mu}\rho Gx + C_{17}$ $V_z = -\frac{\rho Gr}{2\mu} x^2 + C_{17} x + C_{18}$	**12** $-\mu \frac{\partial^2 V_z}{\partial y^2} = s\rho G$ $\frac{\partial^2 V_z}{\partial y^2} = -\frac{s}{\mu}\rho G$ $\frac{\partial V_z}{\partial y} = -\frac{s}{\mu}\rho Gy + C_{19}$ $V_z = -\frac{\rho Gs}{2\mu} y^2 + C_{19} y + C_{20}$

13 $-\mu \frac{\partial^2 V_z}{\partial z^2} = u\rho G$ $\frac{\partial^2 V_z}{\partial z^2} = -\frac{u}{\mu}\rho G$ $\frac{\partial V_z}{\partial z} = -\frac{u}{\mu}\rho Gz + C_{21}$ $V_z = -\frac{\rho Gu}{2\mu} z^2 + C_{21} z + C_{22}$	**14** $\frac{\partial p}{\partial z} = v\rho G$ $P(z) = v\rho Gx + C_{23}$	**15** $4\rho \frac{\partial V_z}{\partial t} = w\rho G$ $\frac{\partial V_z}{\partial t} = \frac{wG}{4}$ $V_z = \frac{wG}{4} t + C_{24}$

Step 3: One collect the solutions from Step 2 for (V_x, V_y, V_z, $P(x)$, $P(y)$, $P(z)$)

For V_x, **S**um of integrals from sub-equations #1, #2, #3, #5, and for $P(x)$, from #4

$$V_x(x,y,z,t) = -\frac{\rho G}{2\mu}(ax^2+by^2+cz^2)+C_1x+C_3y+C_5z+\frac{fG}{4}t+C_8;\quad P(x)=d\rho Gx+C_7$$

For V_y **S**um of integrals from sub-equations #6, #7, #8, #10, and for $P(y)$, from #9.

$$V_y(x,y,z,t) = -\frac{\rho G}{2\mu}(hx^2+jy^2+mz^2)+C_9x+C_{11}y+C_{13}z+\frac{qG}{4}t+C_{16};\quad P(y)=n\rho Gx+C_{15}$$

For V_z: **S**um of integrals from sub-equations #11, #12, #13, and for $P(z)$, from #14

$$V_z(x,y,z,t) = -\frac{\rho G}{2\mu}(rx^2+sy^2+uz^2)+C_{17}x+C_{19}y+C_{21}z+\frac{wG}{4}t+C_{24};\quad P(z)=v\rho Gx+C_{23}$$

Comparison of the above methods for the solutions of Linearized Navier-Stokes Equations

Note below that the solutions by the two different methods are the same except for the constants involved. Now, one has two different methods for solving the system of Navier-Stokes equations. Such an agreement and consistency confirms the validity of the method used in solving the magnetohydrodynamic equations.

Solutions by Method 1

$$V_y(x,y,z,t) = -\frac{\rho g_x}{2\mu}(ax^2+by^2+cz^2)+C_1x+C_3y+C_5z+\frac{fg_x}{4}t+C_9;\quad P(x)=d\rho gx$$

$$V_y(x,y,z,t) = -\frac{\rho g_y}{2\mu}(hx^2+jy^2+mz^2)+C_1x+C_3y+C_5z+\frac{qg_y}{4}t+C;\quad P(y)=n\rho g_y y$$

$$V_z(x,y,z,t) = -\frac{\rho g_z}{2\mu}(rx^2+sy^2+uz^2)+C_1x+C_3y+C_5z+\frac{wg_z}{4}t+C;\quad P(z)=v\rho g_z z$$

Solutions by Method 2

$$V_x(x,y,z,t) = -\frac{\rho G}{2\mu}(ax^2+by^2+cz^2)+C_1x+C_3y+C_5z+\frac{fG}{4}t+C_8;\quad P(x)=d\rho Gx+C_7$$

$$V_y(x,y,z,t) = -\frac{\rho G}{2\mu}(hx^2+jy^2+mz^2)+C_9x+C_{11}y+C_{13}z+\frac{qG}{4}t+C_{16};\quad P(y)=n\rho Gx+C_{15}$$

$$V_z(x,y,z,t) = -\frac{\rho G}{2\mu}(rx^2+sy^2+uz^2)+C_{17}x+C_{19}y+C_{21}z+\frac{wG}{4}t+C_{24};\quad P(z)=v\rho Gx+C_{23}$$

Overall Conclusion

The Navier-Stokes (N-S) equations in 3-D and 4-D have been solved analytically for the first time by two different methods. In Method 1, the three equations were separately integrated.
In Method 2, the three equations were first added together and a single equation was integrated. The solutions from these two methods were the same, except for the constants involved. The system of magnetohydrodynamic (MHD) equations have also been solved analytically for the first time. The experience gained in solving the Navier-Stokes equations guided the author to solve MHD equations; and the experience from solving the MHD equations encouraged the second method of solving the N-S equations. The N-S solution is unique. After each term of the equation had been made subject of the equation to produce nine equations, and all nine equations had been integrated, only the equation with the gravity term as the subject of the equation produced the solution.

The experience gained in solving the linearized equation helped the author to propose a new law, the law of definite ratio for incompressible fluid flow. This law states that in incompressible fluid flow, the other terms of the fluid flow equation divide the gravity term in a definite ratio, and each term utilizes gravity to function. The sum of the terms of the ratio is always unity. The application of this law helped speed-up the solutions of the non-linearized N-S equations as well as solutions of the magnetohydrodynamic equations, since there was no more experimentation as to subject of the equation. The experience from the linearized solution is that for a solution of the N-S equation, the gravity term must be the subject of the equation.
It was also shown that without gravity forces on earth, there will be no incompressible fluid flow on earth as is known; and there will also be no magnetohydrodynamics.
After using ratios to split the equation with the gravity term as the subject of the equation, the integration was straightforward. The solutions revealed the role of each term of the Navier-Stokes equations in fluid flow. Most importantly, the gravity term is the indispensable term in fluid flow, and it is involved in the parabolic as well as the forward motion. The pressure gradient term is also involved in the parabolic motion. The viscosity terms are involved in parabolic, periodic and decreasingly exponential motion.. As the viscosity increases, periodicity increases. The variable acceleration term is also involved in the periodic and decreasingly exponential motion. The convective acceleration terms produce square root function behavior and behavior of fractional terms containing square root functions with variables in denominators and consequent turbulence behavior.

From Option 1 and Option 4 results, the following statements can be made:
1. The N-S equations have unique solutions.
2. The N-S equations have parabolic solutions.
3. The N-S equations have square root function solutions.
4. The N-S equations do not have periodic solutions but have periodic relations.
5.. The N-S equations do not have decreasingly exponential solutions but have decreasingly exponential relations.

Determining the ratio terms

In applications, the ratio terms a, b, c, d, f, h, n, q and others may perhaps be determined using information such as initial and boundary conditions or may have to be determined experimentally. The author came to the experimental determination conclusion after referring to Example 5, page 100. The question is how did the grandmother determine the terms of the ratio for her grandchildren? Note that so far as the general solutions of the N-S equations are concerned, one needs not find the specific values of the ratio terms.
Finally, for any fluid flow design, one should always maximize the role of gravity for cost-effectiveness, durability, and dependability. Perhaps, Newton's law for fluid flow should read "Sum of everything else equals ρg" ; and this would imply the proposed new law that the other terms divide the gravity term in a definite ratio, and each term utilizes gravity to function.

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Option 9

Back to Options

Spin-off: CMI Millennium Prize Problem Requirements

Proof 1

For the linearized Navier-Stokes equations

Proof of the existence of solutions of the Navier-Stokes equations

Since from page 11, it has been shown that the smooth equations given by

$\boxed{V_x(x,y,z,t) = -\frac{\rho g_x}{2\mu}(ax^2 + by^2 + cz^2) + C_1x + C_3y + C_5z + \frac{fg_x}{4}t + C_9\ ;\ P(x) = d\rho g_x x}$ are solutions

of the linearized equation, $-\mu(\frac{\partial^2 v_x}{\partial x^2} + \frac{\partial^2 v_x}{\partial y^2} + \frac{\partial^2 v_x}{\partial z^2}) + \frac{\partial p_x}{\partial x} + 4\rho\frac{\partial v_x}{\partial t} = \rho g_x$, it has been shown that smooth solutions to the above differential equation exist. and the proof is complete.

From, above, if $y = 0,\ z = 0$, $\boxed{V_x(x,t) = -\frac{\rho g_x}{2\mu}ax^2 + C_1x + \frac{fg_x}{4}t + C_9}$; $P(x) = d\rho gx + C_{10}$

Therefore, $V_x(x,0) = V_x^0(x) = -\frac{\rho g_x}{2\mu}ax^2 + C_{10}x + C_9$

Finding $P(x,t)$

1. $V_x(x,t) = -\frac{\rho g_x}{2\mu}(ax^2) + C_1x + \frac{fg_x}{4}t + C_9$; $P(x) = d\rho g_x x$ **2.** $\frac{\partial p}{\partial x} = d\rho g$;

Required: To find $P(x,t)$ (that is, find a formula for P in terms of x and t)

$$\frac{dp}{dt} = \frac{dp}{dx}\frac{dx}{dt}$$

$$\frac{dp}{dt} = \frac{dp}{dx}V_x \qquad (\frac{dx}{dt} = V_x)$$

$$\frac{dp}{dt} = d\rho g_x\left(-\frac{\rho g_x}{2\mu}(ax^2) + C_1x + \frac{fg_x}{4}t + C_9\right) \qquad (\frac{dp}{dx} = d\rho g_x)$$

$$\frac{dp}{dt} = -\frac{ad\rho^2 g_x^2}{2\mu}x^2 + C_1 d\rho g_x x + \frac{d\rho f g_x^2}{4}t + C_9 d\rho g_x$$

$$P(x,t) = \int\left(-\frac{ad\rho^2 g_x^2}{2\mu}x^2 + C_1 d\rho g_x x + \frac{d\rho f g_x^2}{4}t + C_9 d\rho g_x\right)dt$$

$$P(x,t) = -\frac{ad\rho^2 g_x^2}{2\mu}x^2 t + C_1 d\rho g_x xt + \frac{d\rho f g_x^2}{8}t^2 + C_9 d\rho g_x t + C_{10}$$

$$= -d\rho g_x\left(\frac{a\rho g_x}{2\mu}x^2 t + C_1 xt + \frac{fg_x}{8}t^2 + C_9 t\right) + C_{10}$$

For the corresponding coverage for the original Navier-Stokes equation, see the next page

Proof 2

For the Non-linearized Navier-Stokes equations (Original Equations)

Proof of the existence of solutions of the Navier-Stokes equations

From page 22, if $y=0,\ z=0$ in

Solution to Linear part

$$V_x(x,y,z,t) = \overbrace{-\frac{\rho g_x}{2\mu}(ax^2+by^2+cz^2)+C_1x+C_3y+C_5z+\underbrace{fg_xt}_{\text{continued}}}\ \underbrace{\pm\sqrt{2hg_xx}+\frac{ng_xy}{V_y}+\frac{qg_xz}{V_z}+\frac{\psi_y(V_y)}{V_y}+\frac{\psi_z(V_z)}{V_z}}_{\text{solution of Euler equation}}$$

$$P(x) = d\rho g_x x$$

one obtains

$$\boxed{V_x(x,t) = -\frac{\rho g_x}{2\mu}ax^2 + C_1x + fg_xt \pm \sqrt{2hg_xx} + C_9\ ; \quad P(x) = d\rho g_x x;}$$

$$V_x(x,0) = V_x^0(x) = -\frac{\rho g_x}{2\mu}ax^2 + C_1x \pm \sqrt{2hg_xx} + C_9\ ; \quad P(x) = d\rho g_x x;$$

Since previously, from p.113, it has been shown that the smooth equations given by

$V_x(x,t) = -\frac{\rho g_x}{2\mu}ax^2 + C_1x + fg_xt \pm \sqrt{2hg_xx} + C_9\ ;\ P(x) = d\rho g_x x;$ are solutions of

$-\mu\frac{\partial^2 V_x}{\partial x^2} + \frac{\partial p_x}{\partial x} + \rho\frac{\partial V_x}{\partial t} + \rho V_x\frac{\partial V_x}{\partial x} = \rho g_x$ (deleting the $x-$ and $y-$ terms of (A)), p.112, one has shown that smooth solutions to the above differential equation exist, and the proof is complete.

Finding $P(x,t)$:

1. $V_x(x,t) = -\frac{\rho g_x}{2\mu}ax^2 + C_1x + fg_xt \pm \sqrt{2hg_xx} + C_9;\ \ P(x) = d\rho g_x x;$ **2.** $\frac{\partial p}{\partial x} = d\rho g;$

$$\frac{dp}{dt} = \frac{dp}{dx}\frac{dx}{dt}$$

$$\frac{dp}{dt} = \frac{dp}{dx}V_x \qquad \left(\frac{dx}{dt} = V_x\right)$$

$$\frac{dp}{dt} = d\rho g_x\left(-\frac{\rho g_x}{2\mu}(ax^2) + C_1x \pm \sqrt{2hg_xx} + fg_xt + C_9\right) \qquad \left(\frac{dp}{dx} = d\rho g_x\right)$$

$$P(x,t) = \int d\rho g_x\left(-\frac{\rho g_x}{2\mu}(ax^2) + C_1x \pm \sqrt{2hg_xx} + fg_xt + C_9\right)dt$$

$$P(x,t) = -d\rho g_x\left(\frac{a\rho g_x}{2\mu}x^2t + C_1xt \pm \left(\sqrt{2hg_xx}\right)t + \frac{fg_x}{2}t^2 + C_9t\right) + C_{10}$$

References:

For paper edition of the above paper, see Chapter 11 of the book entitled "Power of Ratios" by A. A. Frempong, published by Yellowtextbooks.com.

Without using ratios or proportion, the author would never be able to split-up the Navier-Stokes equations into sub-equations which were readily integrable. The impediment to solving the Navier-Stokes equations for over 150 years (whether linearized or non-linearized) has been due to finding a way to split-up the equations. Since ratios were the key to splitting the Navier-Stokes equations, and also splitting the 28-term system of magnetohydrodynamic equations, and solving them, the solutions have also been published in the " Power of Ratios" book which covers definition of ratio and applications of ratio in mathematics, science, engineering, economics and business fields.