An observation about the digital root of the twin primes, few conjectures and an open problem on primes

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Abstract. Few interesting properties which distinquish twin primes from the general set of primes there are already known. I wrote myself an article regarding an interesting property of a set of (pairs of) twin primes based on the sum of the digits of the lesser (implicitly greater) prime from a pair of twin primes. This paper notes a property regarding twin primes based on their digital root.

Observation:

The digital root of the lesser prime p from a pair of twin primes [p, q], under the condition that p > 3, is, for the first such 100 primes (for a list of lesser of twin primes see the sequence A001359 in OEIS):

: 5, 2, 8, 2, 5, 5, 8, 2, 8, 2, 5, 8, 2, 8, 2, 5, 8, 2, 5, 5, 8, 2, 8, 2, 5, 5, 2, 2, 8, 2, 8, 2, 8, 2, 5, 5, 8, 2, 8, 5, 8, 2, 5, 5, 5, 2, 5, 2, 5, 8, 2, 5, 2, 5, 8, 5, 5, 5, 8, 2, 2, 8, 5, 5, 8, 5, 8, 5, 8, 5, 2, 8, 2, 5, 2, 2, 8, 2, 8, 2, 5, 8, 2, 8, 5, 8, 2, 5, 5, 5, 2, 8, 2, 2, 8, 8, 5, 5.

Note:

Obviously the digital root of a lesser from a pair of twin primes can never be equal to 3, 6 or 9 (it would be then a number divisible by 3 not a prime) or 1, 4 or 7 (the greater from the pair of twin primes would be in this case divisible by 3).

Conjecture 1:

There is an infinity of pairs of twin primes for which the lesser term of the pair has the property that its digital root is equal to 2.

Conjecture 2:

There is an infinity of pairs of twin primes for which the lesser term of the pair has the property that its digital root is equal to 5.

Conjecture 3:

There is an infinity of pairs of twin primes for which the lesser term of the pair has the property that its digital root is equal to 8.

Note:

Is remarkable that the three subsets of the set of (lesser of) twin primes seem to have (of course, for a given term great enough) an approximately equal number of terms; for instance, from the first 100 from the lesser of twin primes, 33 of them have the digital root equal to 2, 35 have the digital root equal to 5 and 32 have the digital root equal to 7.

Conjecture 4:

Let a_i be the sequence of the lesser of twin primes whose digital root is equal to 2, b_i be the sequence of the lesser of twin primes whose digital root is equal to 5 and c_i be the sequence of the lesser of twin primes whose digital root is equal to 8. Than:

: there exist an infinity of terms n of a_i for which the number of the terms of b_i smaller than n is equal to the number of the terms of c_i smaller than n;

: there exist an infinity of terms n of b_i for which the number of the terms of a_i smaller than n is equal to the number of the terms of c_i smaller than n;

: there exist an infinity of terms n of c_i for which the number of the terms of a_i smaller than n is equal to the number of the terms of b_i smaller than n.

Conjecture 5:

There is an infinity of integers n for which the set of the lesser (greater) of the twin primes smaller than n is divided in three subsets whith an equal number of terms, a_i with the property that the digital root of its terms is equal to 2 (4), b_i with the property that the digital root of its terms is equal to 5 (7) and c_i with the property that the digital root of its terms is equal to 8 (1).

Open problem:

Is there any other prime p beside p = 23 with the property that the following six subsets of odd primes have an equal number of terms smaller than p? The terms of the six subsets are the primes whose digital root is equal to 1, 2, 4, 5, 7 respectively 8 (it can be seen that, for p = 23, we have the following odd primes smaller than 23 that belong to the six subsets: 5, 7, 11, 13, 17, 19, whose digital root is 5, 7, 2, 4, 8, 1).