International Journal of Mathematical Engineering and Science
ISSN : 2277-6982 Volume 1 Issue 2
https://sites.google.com/site/ijmesjournal/ http://www.ijmes.com/

A REMARKABLE SURVEY ON GENERALIZED MITTAG-LEFFLER FUNCTION AND APPLICATIONS

J. C. PRAJAPATI[1] and A. K. SHUKLA[2]

[1]*Department of Mathematical Sciences, Faculty of Applied Sciences, Charotar University of Science and Technology, Changa, Anand-388 421, India.*

[2]*Department of Applied Mathematics and Humanities, S.V.National Institute of Technology, Surat-395 007, India*

E-mails: jyotindra18@rediffmail.com
jyotindraprajapati.maths@charusat.ac.in
ajayshukla2@rediffmail.com

Abstract. The principal aim of the paper is to introduce the various generalizations of Shukla-Prajapati functions and polynomials. For these new functions and polynomials their various properties including usual differentiation and integration, Integral transforms, Generalised hypergeometric series form, Mellin – Barnes integral representation, Recurrence relations, Integral representation, Decomposition, Fractional calculus operators properties, generating relations, bilateral generating relation and finite summation formulae of new class of polynomials also established.

Key Words: Mittag–Leffler function, Integral Transforms, Fractional integral and differential operators.

***AMS classification* (2000)**: 33E12, 33C20, 44A20, 30E20.

1 INTRODUCTION AND PRELIMINARIES

Recently, Shukla and Prajapati (2007), investigated and studied the function $E_{\alpha,\beta}^{\gamma,q}(z)$ which is defined for $\alpha,\beta,\gamma \in \mathbb{C}$; $\operatorname{Re}(\alpha)>0, \operatorname{Re}(\beta)>0, \operatorname{Re}(\gamma)>0$ and $q \in (0,1) \cup \mathbb{N}$ as:

$$E_{\alpha,\beta}^{\gamma,q}(z)=\sum_{n=0}^{\infty}\frac{(\gamma)_{qn}}{\Gamma(\alpha n+\beta)}\frac{z^n}{n!} \quad . \tag{1.1}$$

where $(\gamma)_{qn}=\frac{\Gamma(\gamma+qn)}{\Gamma(\gamma)}$ denotes the generalized Pochhammer symbol Rainville(1960), which in particular reduces to $q^{qn}\prod_{r=1}^{q}\left(\frac{\gamma+r-1}{q}\right)_n$ if $q\in\mathbb{N}$.

In continuation of the study, the generalization of (1) can be written as $E_{\alpha,\beta}^{\gamma,\varsigma}(z)$ which is defined for $\operatorname{Re}(\alpha)>0, \operatorname{Re}(\beta)>0$, $\operatorname{Re}(\gamma)>0$ and $\varsigma>0$ as

$$E_{\alpha,\beta}^{\gamma,\varsigma}(z)=\sum_{n=0}^{\infty}\frac{(\gamma)_{\varsigma n}}{\Gamma(\alpha n+\beta)}\frac{z^n}{n!} \quad . \tag{1.2}$$

This is a generalization of the exponential function exp(z), the confluent hypergeometric function $\Phi(\gamma, \alpha ; z)$ Rainville (1960), the Mittag – Leffler function Mittag-Leffler (1903), the Wiman's function Wiman (1905) and the function $E_{\alpha,\beta}^{\gamma}(z)$ defined by Prabhakar (1971) as well as equation (1.1). Gorenflo *et al*. (1998), Gorenflo and Mainardi (2000), Kilbas *et al.* (1996, 2004), Saigo and Kilbas (1998), Srivastava and Tomovski (2009), Tomovski *et al* (2010) and many other researchers also studied the various properties of Mittag-Leffler functions and its generalizations with their applications. The applications of Integral Transforms discussed by Sneddon (1979).

The function $E_{\alpha,\beta}^{\gamma,\varsigma}(z)$ is an entire function of order $(\operatorname{Re}\alpha-\varsigma+1)^{-1}$ if $\operatorname{Re}\alpha>\varsigma-1$, and absolutely convergent in $\{|z|<R, R<1\}$ if $\operatorname{Re}\alpha=\varsigma-1$. The truncated power series of the function $E_{\alpha,\beta}^{\gamma,\varsigma}(z)$ can be defined as,

$$E_{\frac{1}{n},\beta}^{\gamma,\varsigma,nN}(z)=\sum_{m=0}^{N}\frac{(\gamma)_{\varsigma m}\ z^{nm}}{\Gamma(m+\beta)\ m!}+\sum_{m=0}^{N-1}\sum_{j=1}^{n-1}\frac{(\gamma)_{\varsigma m}\ z^{nm+j}}{\Gamma(m+\beta)\ \Gamma\left(\frac{nm+j}{n}+1\right)} \tag{1.3}$$

and a special case for the study of $E_{\alpha,\beta}^{\gamma,q}(z)$ for $\alpha=\frac{1}{n}$ as:

$$E_{\frac{1}{n},\beta}^{\gamma,\varsigma}(z)=\sum_{m=0}^{\infty}\frac{(\gamma)_{\varsigma m}\ z^{nm}}{\Gamma(m+\beta)\ m!}+\sum_{m=0}^{\infty}\sum_{j=1}^{n-1}\frac{(\gamma)_{\varsigma m}\ z^{nm+j}}{\Gamma(m+\beta)\ \Gamma\left(\frac{nm+j}{n}+1\right)}. \tag{1.4}$$

where $n\geq 2, N\geq 1$; $\operatorname{Re}(\beta)>0, \operatorname{Re}(\gamma)>0$ and $\varsigma>0$.

Authors investigated the operator $E^{\gamma,\varsigma}_{\alpha,\beta,w;a+}f$ as,

$$(E^{\gamma,\varsigma}_{\alpha,\beta,w;a+}f)(x)=\int_a^x (x-t)^{\beta-1}E^{\gamma,\varsigma}_{\alpha,\beta}[w(x-t)^{\alpha}]\,f(t)dt \quad (x>a). \quad (1.5)$$

where $w\in\mathbb{C}$; $\mathrm{Re}(\alpha)>0$, $\mathrm{Re}(\beta)>0$, $\mathrm{Re}(\gamma)>0$; $\varsigma>0$.
If $\varsigma=1$, then (1.5) reduces to the result Prabhakar (1971).

Shukla and Prajapati (2008 A) introduced a class of polynomials which are connected by Mittag-Leffler function $E_{\alpha}(x)$, in continuation of the study of class of polynomials, Authors introduced a general class of polynomial defined as,

$$A^{(\alpha,\beta,\gamma,\delta)}_{\varsigma n}(x;a,k,s)=\frac{x^{-\delta-an}}{n!}E^{\gamma,\varsigma}_{\alpha,\beta}\{p_k(x)\}\,\theta^{n}\left[x^{\delta}\,E^{\gamma,\varsigma}_{\alpha,\beta}\{-p_k(x)\}\right] \quad (1.6)$$

where $\alpha,\beta,\gamma,\delta$ are real or complex numbers; a,k,s are constants and generalized Mittag-Leffler function defined as (1.2).

The proofs of all results established in this paper are parallel to Shukla and Prajapati (2007 A, 2007 B, 2007 C, 2008 A, 2008 B, 2009 A, 2009 B, 2010).

2 BASIC PROPORTIES OF THE FUNCTION $E^{\gamma,\varsigma}_{\alpha,\beta}(z)$

As a consequence of the definitions (1.2) the following results hold:

THEOREM (2.1). If $\alpha,\beta,\gamma\in\mathbb{C}$; $\mathrm{Re}(\alpha)>0$, $\mathrm{Re}(\beta)>0$, $\mathrm{Re}(\gamma)>0$ and $\varsigma>0$ then

$$E^{\gamma,\varsigma}_{\alpha,\beta+1}(z)=\beta\,E^{\gamma,\varsigma}_{\alpha,\beta+1}(z)+\alpha\,z\,\frac{d}{dz}\,E^{\gamma,\varsigma}_{\alpha,\beta+1}(z) \quad (2.1)$$

$$E^{\gamma,\varsigma}_{\alpha,\beta-\alpha}(z)-E^{\gamma-1,\varsigma}_{\alpha,\beta-\alpha}(z)=\varsigma\,z\sum_{n=0}^{\infty}\frac{(\gamma)_{\varsigma n+\varsigma-1}\,z^{n}}{\Gamma(\alpha n+\beta)\,n!} \quad (2.2)$$

in particular,

$$E^{\gamma}_{\alpha,\beta-\alpha}(z)-E^{\gamma-1}_{\alpha,\beta-\alpha}(z)=zE^{\gamma}_{\alpha,\beta}(z). \quad (2.3)$$

THEOREM (2.2). If $\alpha,\beta,\gamma,w\in\mathbb{C}$; $\mathrm{Re}(\alpha)>0$, $\mathrm{Re}(\beta)>0$, $\mathrm{Re}(\gamma)>0$ and $\varsigma>0$ then for $m\in\mathbb{N}$,

$$\left(\frac{d}{dz}\right)^m E_{\alpha,\beta}^{\gamma,\varsigma}(z) = (\gamma)_{\varsigma m}\; E_{\alpha,\beta+m\alpha}^{\gamma+\varsigma m,\varsigma}(z) \tag{2.4}$$

$$\left(\frac{d}{dz}\right)^m \left[z^{\beta-1} E_{\alpha,\beta}^{\gamma,\varsigma}\left(wz^{\alpha}\right)\right] = z^{\beta-m-1} E_{\alpha,\beta-m}^{\gamma,\varsigma}(z),\quad \mathrm{Re}(\beta-m)>0 \tag{2.5}$$

in particular,

$$\left(\frac{d}{dz}\right)^m \left[z^{\beta-1} E_{\alpha,\beta}\left(wz^{\alpha}\right)\right] = z^{\beta-m-1} E_{\alpha,\beta-m}(z) \tag{2.6}$$

and

$$\left(\frac{d}{dz}\right)^m \left[z^{\beta-1}\Phi(\gamma,\,\beta;wz)\right] = \frac{\Gamma(\beta)}{\Gamma(\beta-m)}\, z^{\beta-m-1}\,\Phi(\gamma,\,\beta-m;wz) \tag{2.7}$$

THEOREM (2.3). If $\alpha = \dfrac{p}{r}$ with $p, r \in \mathbb{N}$ relatively prime; $\beta,\gamma \in \mathbb{C}$ and $\varsigma > 0$ then

$$\frac{d^p}{dz^p} E_{\frac{p}{r},\beta}^{\gamma,\varsigma}(z^{\frac{p}{r}}) = \sum_{n=1}^{\infty} \frac{(\gamma)_{\varsigma n}\, \Gamma\left(\frac{np}{r}+1\right)}{\Gamma\left(\frac{np}{r}+\beta\right)\Gamma\left(\frac{np}{r}-p+1\right)} \frac{z^{\left(\frac{n}{r}-1\right)p}}{n!}, \tag{2.8}$$

in particular,

$$E_{\frac{1}{r}}(z^{\frac{1}{r}}) = e^z \sum_{n=1}^{r-1} \frac{\gamma\left(1-\frac{n}{r},\,z\right)}{\Gamma\left(1-\frac{n}{r}\right)}, \qquad (\mathrm{r} = 2, 3\ldots). \tag{2.9}$$

3 GENERALIZED HYPERGEOMETRIC FUNCTION REPRESENTATION OF $E_{\alpha,\beta}^{\gamma,q}(z)$

Using (1.2), taking $\alpha = k \in \mathbb{N}$ and $\varsigma \in \mathbb{N}$ then we have

$$E_{\alpha,\beta}^{\gamma,\varsigma}(z) = \frac{1}{\Gamma(\beta)}\; {}_{\varsigma}F_{k}\left[\begin{matrix}\Delta(\varsigma;\gamma);\\ \Delta(\varsigma;\beta);\end{matrix}\;\frac{z}{\varsigma^{\varsigma}}\right]. \tag{3.1}$$

Convergence criteria for generalized hypergeometric function ${}_{\varsigma}F_k$:

(i) If $\varsigma \le k$, the function ${}_{\varsigma}F_k$ converges for all finite z.

(ii) If $\varsigma = k+1$, the function ${}_{\varsigma}F_k$ converges for $|z|<1$ and diverges for $|z|>1$.

(iii) If $\varsigma > k+1$, the function ${}_{\varsigma}F_k$ divergent for $z \ne 0$.

(iv) If $\varsigma = k+1$, the function ${}_{\varsigma}F_k$ is absolutely convergent on the circle $|z|=1$ if

$$\operatorname{Re}\left(\sum_{j=1}^{k}\frac{\beta+j-1}{k}-\sum_{i=1}^{\varsigma}\frac{\gamma+i-1}{\varsigma}\right)>0.$$

where $\Delta(\varsigma;\gamma)$ is a ς-tuples $\frac{\gamma}{\varsigma}, \frac{\gamma+1}{\varsigma}, \ldots, \frac{\gamma+\varsigma-1}{\varsigma}$;

$\Delta(\varsigma;\beta)$ is a ς-tuples $\frac{\beta}{\varsigma}, \frac{\beta+1}{\varsigma}, \ldots, \frac{\beta+\varsigma-1}{\varsigma}$.

4 MELLIN-BARNES INTEGRAL REPRESENTATION OF $E_{\alpha,\beta}^{\gamma,\varsigma}(z)$

THEOREM (4.1). *Let $\alpha \in R_+$; $\gamma, \delta \in \mathbb{C}$ $(\gamma \ne 0)$ and $\varsigma \in \mathbb{N}$. The function $E_{\alpha,\beta}^{\gamma,\varsigma}(z)$ is represented by the Mellin – Barnes integral as:*

$$E_{\alpha,\beta}^{\gamma,\varsigma}(z)=\frac{1}{2\pi i\,\Gamma(\gamma)}\int_L \frac{\Gamma(s)\Gamma(\gamma-qs)}{\Gamma(\beta-\alpha s)}(-z)^{-s}\,ds, \tag{4.1}$$

where $|\arg(z)|<\pi$; the contour of integration beginning at $-i\infty$ and ending at $+i\infty$, and indented to separate the poles of the integrand at $s=-n$ for all $n\in\mathbb{N}_0$ (to the left) from those at $s=\frac{\gamma+n}{q}$ for all $n\in\mathbb{N}_0$ (to the right).

5 INTEGRAL TRANSFORMS OF $E_{\alpha,\beta}^{\gamma,\varsigma}(z)$

In this section, some useful integral transforms like Euler transforms, Laplace transforms, Mellin transforms and Whittaker transforms also discussed.

THEOREM (5.1). *Euler (Beta) transforms*

$$\int_0^1 z^{a-1}(1-z)^{b-1} E_{\alpha,\beta}^{\gamma,\varsigma}\left(xz^{\sigma}\right)dz = \frac{\Gamma(b)}{\Gamma(\gamma)}\ {}_2\psi_2\left[\begin{matrix}(\gamma,\varsigma),(a,\sigma)\ ; & x\\ (\beta,\alpha),(a+b,\sigma)\ ; & \end{matrix}\right], \tag{5.1}$$

where $\mathrm{Re}(a)>0, \mathrm{Re}(b)>0, \mathrm{Re}(\alpha)>0, \mathrm{Re}(\beta)>0, \mathrm{Re}(\gamma)>0$, $\mathrm{Re}(\sigma)>0$ and $\varsigma>0$.

THEOREM (5.2). *Laplace transforms*.

$$\int_0^\infty z^{a-1} e^{-sz} E_{\alpha,\beta}^{\gamma,\varsigma}\left(xz^{\sigma}\right)dz = \frac{s^{-a}}{\Gamma(\gamma)}\ {}_2\psi_1\left[\begin{matrix}(\gamma,\varsigma),(a,\sigma)\ ; & \dfrac{x}{s^{\sigma}}\\ (\beta,\alpha)\ ; & \end{matrix}\right], \tag{5.2}$$

Where, $\mathrm{Re}(a)>0$, $\mathrm{Re}(\alpha)>0, \mathrm{Re}(\beta)>0, \mathrm{Re}(\gamma)>0, \mathrm{Re}(\sigma)>0$, $\varsigma>0$ and $\left|\dfrac{x}{s^{\sigma}}\right|<1$.

THEOREM (5.3). *Mellin transforms*.

$$\int_0^\infty t^{s-1} E_{\alpha,\beta}^{\gamma,\varsigma}(-wt)\,dt = \frac{\Gamma(s)\,\Gamma(\gamma-\varsigma s)}{w^s\,\Gamma(\gamma)\Gamma(\beta-\alpha s)}, \tag{5.3}$$

where $\mathrm{Re}(\alpha)>0, \mathrm{Re}(\beta)>0, \mathrm{Re}(\gamma)>0, \mathrm{Re}(s)>0$ and $\varsigma>0$.

To obtain Whittaker transform, we use the following integral,

$$\int_0^\infty e^{-t/2}\, t^{v-1}\, W_{\lambda,\mu}(t)dt = \frac{\Gamma\left(\frac{1}{2}+\mu+v\right)\Gamma\left(\frac{1}{2}-\mu+v\right)}{\Gamma(1-\lambda+v)},$$

where $\mathrm{Re}(v\pm\mu)>-\dfrac{1}{2}$. (5.4)

THEOREM (5.4). *Whittaker Transforms*.

$$\int_0^\infty t^{\rho-1} e^{-\frac{1}{2}pt} W_{\lambda,\mu}(pt) E_{\alpha,\beta}^{\gamma,\varsigma}\left(wt^\delta\right)dt$$

$$= \frac{p^{-\rho}}{\Gamma(\gamma)} {}_3\psi_2 \left[\begin{matrix} (\gamma,q), \left(\frac{1}{2} \pm \mu + \rho,\, \delta\right); & \frac{w}{p^\delta} \\ (\beta,\, \alpha), (1-\lambda+\rho, \delta); & \end{matrix} \right], \tag{5.5}$$

where, $\operatorname{Re}(\alpha) > 0, \operatorname{Re}(\beta) > 0, \operatorname{Re}(\gamma) > 0, \operatorname{Re}(\rho) > 0, \operatorname{Re}(\delta) > 0$ and $\varsigma > 0$.

6 RECURRENCE RELATIONS

THEOREM (6.1). For any $\operatorname{Re}(\alpha + p) > 0$, $\operatorname{Re}(\beta + s) > 0$ and $\operatorname{Re}(\gamma) > 0$, $\varsigma > 0$ we get

$$E_{\alpha+p,\beta+s+1}^{\gamma,\varsigma}(z) - E_{\alpha+p,\beta+s+2}^{\gamma,\varsigma}(z)$$
$$= (\beta+s)(\beta+s+2) E_{\alpha+p,\beta+s+3}^{\gamma,\varsigma}(z) + (\alpha+p)^2 z^2 \ddot{E}_{\alpha+p,\beta+s+3}^{\gamma,\varsigma}(z) +$$
$$(\alpha+p)\{\alpha+p+2(\beta+s+1)\}\, z\, \dot{E}_{\alpha+p,\beta+s+3}^{\gamma,\varsigma}(z) \tag{6.1}$$

where $\dot{E}_{\alpha,\beta}^{\gamma,q}(z) = \frac{d}{dz}[E_{\alpha,\beta}^{\gamma,q}(z)]$ and $\ddot{E}_{\alpha,\beta}^{\gamma,q}(z) = \frac{d^2}{dz^2}[E_{\alpha,\beta}^{\gamma,q}(z)]$.

THEOREM (6.2). For k and $m \in \mathbb{N}$

$$E_{k,m+1}^{\gamma,\varsigma}(z) = k^2 z^2 \ddot{E}_{k,m+3}^{\gamma,\varsigma}(z) + kz[k + 2(m+1)] \dot{E}_{k,m+3}^{\gamma,\varsigma}(z)$$
$$+ m(m+2) E_{k,m+3}^{\gamma,\varsigma}(z) + E_{k,m+2}^{\gamma,\varsigma}(z). \tag{6.2}$$

7 INTEGRAL REPRESENTATIONS

THEOREM (7.1). For any $\operatorname{Re}(\alpha + p) > 0$, $\operatorname{Re}(\beta + s) > 0$ and $\operatorname{Re}(\gamma) > 0$, $\varsigma > 0$ and setting $\alpha + p = k$, $\beta + s = m$, where k and $m \in \mathbb{N}$ then

$$\int_0^1 t^m E_{k,m}^{\gamma,\varsigma}(t^k)\, dt = E_{k,m+1}^{\gamma,\varsigma}(1) - E_{k,m+2}^{\gamma,\varsigma}(1) \tag{7.1}$$

THEOREM (7.2). *If* $\operatorname{Re}(\alpha) > 0$, $\operatorname{Re}(\beta) > 0, \operatorname{Re}(\gamma) > 0, \operatorname{Re}(\nu) > 0$ *and* $\varsigma > 0$ *then*

$$\frac{1}{\Gamma(\nu)}\int_0^z t^{\beta-1}(z-t)^{\nu-1}\,E_{\alpha,\beta}^{\gamma,\varsigma}(\lambda t^{\alpha})\,dt = z^{\beta+\nu-1}\,E_{\alpha,\beta+\nu}^{\gamma,\varsigma}(\lambda z^{\alpha})\,. \quad (7.2)$$

Special cases of Theorem (7.2): For $\mathrm{Re}(\nu)>0$, from (7.2), the particular cases listed as below:

$$\frac{1}{\Gamma(\nu)}\int_0^z (z-t)^{\nu-1}\,e^{t}\,dt = z^{\nu}\,E_{1,\nu+1}^{1,1}(z), \quad (7.3)$$

$$\frac{1}{\Gamma(\nu)}\int_0^z (z-t)^{\nu-1}\,\cosh(\sqrt{t})\,dt = z^{\nu}\,E_{2,\nu+1}^{1,1}(z^2), \quad (7.4)$$

$$\frac{1}{\Gamma(\nu)}\int_0^z (z-t)^{\nu-1}\,\frac{\sinh(\sqrt{t})}{\sqrt{t}}\,dt = z^{\nu}\,E_{2,\nu+2}^{1,1}(z^2), \quad (7.5)$$

$$\frac{1}{\Gamma(\nu)}\int_0^z (z-t)^{\nu-1}\exp\left(z^2\right)erf_c(-z)\,dt = z^{\nu}\,E_{\frac{1}{2},\nu+1}^{1,1}(z)\,. \quad (7.6)$$

THEOREM (7.3). *If* $\mathrm{Re}(\alpha)>0,\ \mathrm{Re}(\beta)>0, \mathrm{Re}(\gamma)>0$ *and* $\varsigma>0$ *then*

$$\frac{1}{\Gamma(\alpha)}\int_0^z t^{\beta-1}(z-t)^{\alpha-1}\,E_{2\alpha,\beta}^{\gamma,\varsigma}(t^{2\alpha})\,dt = z^{\beta-1}[E_{\alpha,\beta}^{\gamma,\varsigma}(z^{\alpha}) - E_{2\alpha,\beta}^{\gamma,\varsigma}(z^{2\alpha})]. \quad (7.7)$$

THEOREM (7.4). *If* $\mathrm{Re}(\alpha)>0,\ \mathrm{Re}(\beta)>0, \mathrm{Re}(\gamma)>0$ *and* $\varsigma>0$ *then*

$$\int_0^\infty e^{-\frac{x^2}{4t}}\,E_{\alpha,\beta}^{\gamma,\varsigma}(x^{\alpha})\,x^{\beta-1}\,dx = \sqrt{\pi}\;t^{\frac{\beta}{2}}\,E_{\frac{\alpha}{2},\frac{\beta+1}{2}}^{\gamma,\varsigma}\left(t^{\frac{\alpha}{2}}\right). \quad (7.8)$$

THEOREM (7.5). *If* $\mathrm{Re}(\alpha)>0,\ \mathrm{Re}(\beta)>0, \mathrm{Re}(\gamma)>0$ *and* $\varsigma>0$ *then*

$$\frac{d}{dx}\left[x^{\gamma-1}\,E_{\alpha,\beta}^{\gamma,\varsigma}(ax^{\alpha})\right] = x^{\gamma-2}\left[E_{\alpha,\beta-1}^{\gamma,\varsigma}(ax^{\alpha}) + (\gamma-\beta)\,E_{\alpha,\beta}^{\gamma,\varsigma}(ax^{\alpha})\right]. \quad (7.9)$$

THEOREM (7.6). *If* $\mathrm{Re}(\alpha)>0,\ \mathrm{Re}(\beta)>0, \mathrm{Re}(\gamma)>0$ *and* $\varsigma>0$ *then*

$$L^{-1}\left\{s^{-\beta}\left(1-\frac{z}{s^{\alpha}}\right)^{-\gamma,\varsigma}\right\}=t^{\beta-1}\,E_{\alpha,\beta}^{\gamma,\varsigma}(zt^{\alpha})\,. \tag{7.10}$$

THEOREM (7.7). If $\mathrm{Re}(\alpha)>0$, $\mathrm{Re}(\beta)>0, \mathrm{Re}(\gamma)>0$, $\beta>\alpha>0$ and $\varsigma>0$ then

$$E_{\alpha,\beta}^{\gamma,\varsigma}(z)=k\,z^{\alpha-\beta}\int_0^{\infty}\exp\left(-\frac{t^k}{z^k}\right)t^{\beta-\alpha-1}\sum_{n=0}^{\infty}\frac{(\gamma)_{\varsigma n}\,t^n}{\Gamma(\alpha n+\beta)\,n!\;\Gamma\left(\frac{\beta-\alpha+n}{k}\right)}\,dt\,. \tag{7.11}$$

$$E_{\alpha,\beta}^{\gamma,q}(z)=\frac{1}{\alpha\,\Gamma(\beta-\alpha)}\int_0^1(1-t^{\frac{1}{\alpha}})^{\beta-\alpha-1}\;E_{\alpha,\alpha}^{\gamma,q}(t\,z)\,dt\,. \tag{7.12}$$

$$E_{\alpha,\beta}^{\gamma,q}(z)=\frac{1}{\Gamma(\alpha)}\int_0^1 t^{\alpha-1}(1-t)^{\beta-\alpha-1}\;E_{\alpha,\beta-\alpha}^{\gamma,q}\left(z(1-t)^{\alpha}\right)dt\,. \tag{7.13}$$

THEOREM (7.8). If $\mathrm{Re}(\alpha)>0$, $\mathrm{Re}(\beta)>0, \mathrm{Re}(\gamma)>0, \mathrm{Re}(\delta)>0$, $\mathrm{Re}(\mu)>0$, $\mathrm{Re}(\nu)>0$, $\lambda\in\mathbb{C}$ and $\varsigma>0$ then

$$\frac{1}{\Gamma(\delta)}\int_0^1 u^{\beta-1}(1-u)^{\delta-1}E_{\alpha,\beta}^{\gamma,\varsigma}(zu^{\alpha})du=E_{\alpha,\beta+\delta}^{\gamma,\varsigma}(z)\,. \tag{7.14}$$

$$\frac{1}{\Gamma(\delta)}\int_t^x(x-s)^{\delta-1}(s-t)^{\beta-1}E_{\alpha,\beta}^{\gamma,\varsigma}\left[\lambda(s-t)^{\alpha}\right]ds=$$
$$(x-t)^{\delta+\beta-1}E_{\alpha,\beta+\delta}^{\gamma,\varsigma}\left[\lambda(x-t)^{\alpha}\right]. \tag{7.15}$$

If $\varsigma=1$ then

$$\int_0^x t^{\nu-1}(x-t)^{\mu-1}E_{\alpha,\mu}^{\gamma,1}\left[w(x-t)^{\alpha}\right]E_{\alpha,\nu}^{\delta,1}(wt^{\alpha})\,dt=x^{\mu+\nu-1}E_{\alpha,\mu+\nu}^{\gamma+\delta,1}\left(wx^{\alpha}\right). \tag{7.16}$$

$$\int_0^z t^{\beta-1}E_{\alpha,\beta}^{\gamma,\varsigma}\left(wt^{\alpha}\right)dt=z^{\beta}E_{\alpha,\beta+1}^{\gamma,\varsigma}\left(wz^{\alpha}\right). \tag{7.17}$$

in particular,

$$\int_0^z t^{\beta-1} E_{\alpha,\beta}\left(wt^{\alpha}\right)dt = z^{\beta} E_{\alpha,\beta+1}\left(wz^{\alpha}\right) \tag{7.18}$$

and
$$\int_0^z t^{\beta-1}\,\Phi(\gamma,\beta;wt)\,dt = \frac{z^{\beta}}{\beta}\,\Phi(\gamma,\beta+1;wz)\,. \tag{7.19}$$

8 DECOMPOSITION OF MITTAG-LEFFLER FUNCTION

THEOREM (8.1). *Integral Representations of the function* $E^{\gamma,\varsigma}_{\frac{1}{n},\beta}(z)$

$$E^{\gamma,\varsigma}_{\frac{1}{n},\beta}(z) = E^{\gamma,\varsigma}_{1,\beta}(z^n) + n\sum_{k=1}^{n-1}\frac{1}{\Gamma\left(1-\frac{k}{n}\right)}\int_0^z E^{\gamma,\varsigma}_{1,\beta}(z^n - u^n)\,u^{n-k-1}\,du\,, \tag{8.1}$$

where $n \geq 2$; $\operatorname{Re}(\beta) > 0$, $\operatorname{Re}(\gamma) > 0$ *and* $\varsigma > 0$.

THEOREM (8.2). *If* $n \geq 2$, $\operatorname{Re}(\beta) > 0$, $\operatorname{Re}(\gamma) > 0$ *and* $\varsigma > 0$ *then*

$$\frac{d}{dz}\left[z^{\beta-1}E^{\gamma,\varsigma}_{\frac{1}{n},\beta}\left(z^{\frac{1}{n}}\right)\right]$$
$$= z^{\beta-1}\left[\sum_{k=1}^{n-1}\frac{(\gamma)_{\varsigma(n-k)}\,z^{-\frac{k}{n}}}{\Gamma\left(-\frac{k}{n}+\beta\right)(n-k)!} + \sum_{k=1}^{n-1}\frac{(\gamma)_{\varsigma(m+n)}\,z^{\frac{m}{n}}}{\Gamma\left(\frac{m}{n}+\beta\right)(m+n)!}\right] \tag{8.2}$$

THEOREM (8.3).
$$\left|E^{\gamma,\varsigma}_{\frac{1}{n},\beta}(z)\right| \leq \sum_{k=0}^{n-1}\frac{|z|^k}{\Gamma\left(1+\frac{k}{n}\right)}E^{\gamma,\varsigma}_{1,\beta}\left(\left|\operatorname{Re} z^n\right|\right), \tag{8.3}$$

where $n \geq 2$; $\operatorname{Re}(\beta) > 0$, $\operatorname{Re}(\gamma) > 0$ *and* $\varsigma > 0$.

Remark of Theorem 8.3: It is easy to verify that,

$$\left|E^{\gamma,\varsigma,nN}_{\frac{1}{n},\beta}(z)\right| \leq E^{\gamma,\varsigma,nN}_{\frac{1}{n},\beta}\left(|z|\right) \leq E^{\gamma,\varsigma}_{\frac{1}{n},\beta}\left(|z|\right) \leq \sum_{k=0}^{n-1}\frac{|z|^k}{\Gamma\left(1+\frac{k}{n}\right)}E^{\gamma,\varsigma}_{1,\beta}\left(|z|^n\right). \tag{8.4}$$

THEOREM (8.4). *If* $n \geq 2$; $\mathrm{Re}(\beta) > 0$, $\mathrm{Re}(\gamma) > 0$ *and* $\varsigma > 0$ *then*

$$\left| E_{\frac{1}{n},\beta}^{\gamma,\varsigma}(z) - E_{\frac{1}{n},\beta}^{\gamma,\varsigma.nN}(z) \right| \leq T \tag{8.5}$$

where

$$T = \left[\frac{(\gamma)_{\varsigma(N+1)} \left| z^n \right|^{N+1}}{\Gamma(\beta+N+1)\,(N+1)!} + \frac{(\gamma)_{\varsigma N}}{\Gamma(\beta+N)} \sum_{k=1}^{n-1} \frac{|z|^{k+nN}}{\Gamma\left(\frac{k+nN}{n}+1\right)} \right] E_{1,\beta}^{\gamma,\varsigma}\left(\left|\mathrm{Re}\, z^n\right|\right). \tag{8.6}$$

9 FRACTIONAL INTEGRAL AND DIFFERENTIAL OPERATORS ASSOCIATEDWITH THE FUNCTION $E_{\alpha,\beta}^{\gamma,\varsigma}(z)$

The following well-known facts are prepared for studying properties of the Riemann-Liouville fractional integrals and differential operators associated with the function $E_{\alpha,\beta}^{\gamma,q}(z)$ and also the properties of operator $E_{\alpha,\beta,w;a+}^{\gamma,\varsigma} f$.

• $L(a,b)$ *Space of Lebesgue measurable real or complex valued functions* Kilbas *et al.*(2004)**:**
$L(a,b)$ Consists of Lebesgue measurable real or complex valued functions $f(x)$ on $[a,b]$

$$\text{i.e.} \quad L(a,b) = \left\{ f : \|f\|_1 \equiv \int_a^b |f(t)|\,dt < \infty \right\}. \tag{9.1}$$

Kilbas et al. (2004) studied the several properties of fractional integral and differential integral operators.

• *Confluent hypergeometric functions* Rainville (1960):
This is also known as the Pochhammer – Barnes confluent hypergeometric function defined as,

$$\Phi(a,b;z) = {}_1F_1(a;b;z) = \sum_{n=0}^{\infty} \frac{(a)_n\, z^n}{(b)_n\, n!}, \tag{9.2}$$

where $b \neq 0$ or a negative integer is convergent for all finite z.

• *Gauss multiplication theorem* Rainville (1960):
If *m* is a positive integer and $z \in \mathbb{C}$ then,

$$\prod_{k=1}^{m}\Gamma\left(z+\frac{k-1}{m}\right)=(2\pi)^{\frac{m-1}{2}}\, m^{\frac{1}{2}-mz}\,\Gamma(mz)\ . \tag{9.3}$$

• *Riemann-Liouville fractional integrals of order* μ Khan and Abukhammash GS (2003)

Let $f(x)\in L(a,b)$, $\mu\in\mathbb{C}$ $(\mathrm{Re}(\mu)>0)$ then

$$ {}_aI_x^{\mu}\, f(x)=I_{a+}^{\mu}\, f(x)=\frac{1}{\Gamma(\mu)}\int_a^x \frac{f(t)}{(x-t)^{1-\mu}}dt \quad (x>a) \tag{9.4}$$

is called R-L left-sided fractional integral of order μ.

Let $f(x)\in L(a,b)$, $\mu\in\mathbb{C}$ $(\mathrm{Re}(\mu)>0)$ then

$$ {}_xI_b^{\mu}\, f(x)=I_{b-}^{\mu}\, f(x)=\frac{1}{\Gamma(\mu)}\int_x^b \frac{f(t)}{(t-x)^{1-\mu}}dt \quad (x<b) \tag{9.5}$$

is called R-L right-sided fractional integral of order μ.

THEOREM (9.1). *Let* $a\in R_+=[0,\infty)$ *and let* $w\in\mathbb{C}$; $\mathrm{Re}(\alpha),\mathrm{Re}(\beta),\mathrm{Re}(\gamma)$, $\mathrm{Re}(\mu)>0$, $\varsigma>0$ *for* $x>a$, *then*

$$\left(I_{a+}^{\mu}[(t-a)^{\beta-1}\, E_{\alpha,\beta}^{\gamma,\varsigma}\{w(t-a)^{\alpha}\}\right)(x)= (x-a)^{\mu+\beta-1}\, E_{\alpha,\beta+\mu}^{\gamma,\varsigma}[w(x-a)^{\alpha}] \tag{9.6}$$

and

$$\left(D_{a+}^{\mu}[(t-a)^{\beta-1}\, E_{\alpha,\beta}^{\gamma,\varsigma}\{w(t-a)^{\alpha}\}\right)(x)=(x-a)^{\beta-\mu-1}\, E_{\alpha,\beta-\mu}^{\gamma,\varsigma}[w(x-a)^{\alpha}]. \tag{9.7}$$

THEOREM (9.2). *Let* $\mu,\alpha,\lambda\in\mathbb{C}$ *and* $\varsigma>0$ *then*

$$ {}_0I_x^{\mu}[\lambda\, E_{\alpha,1}^{1,\varsigma}(\lambda\, x^{\alpha})]=\lambda\, x^{\mu}\, E_{\alpha,\mu+1}^{1,\varsigma}(\lambda\, x^{\alpha}) \tag{9.8}$$

in particular,

$$ {}_0I_x^{\mu}(e^{\lambda x})=x^{\mu}\, E_{1,\mu+1}(\lambda\, x)\,. \tag{9.9}$$

THEOREM (9.3). *Let* $a\in R_+=[0,\infty)$ *and let* $w\in\mathbb{C}$; $\mathrm{Re}(\alpha),\mathrm{Re}(\beta),\mathrm{Re}(\gamma)$, $\mathrm{Re}(\mu)>0$, $\varsigma>0$ *for* $x>a$, *then*

$$\left(E^{\gamma,q}_{\alpha,\beta,w;a+}(t-a)^{\mu-1}\right)(x)=(x-a)^{\beta+\mu-1}\,\Gamma(\mu)\;E^{\gamma,q}_{\alpha,\beta+\mu}\left(w(x-a)^{\alpha}\right). \quad (9.10)$$

THEOREM (9.4). *Let* $a\in R_{+}=[0,\infty)$ *and let* $w\in\mathbb{C}$; $\mathrm{Re}(\alpha),\mathrm{Re}(\beta),\mathrm{Re}(\gamma),\mathrm{Re}(\mu)>0$, $\varsigma>0$ *for* $b>a$ *then the operator* $E^{\gamma,q}_{\alpha,\beta,w;a+}$ *is bounded on* $L(a,b)$ *and*

$$\left\|E^{\gamma,q}_{\alpha,\beta,w;a+}f\right\|_1\le B\|f\|_1 \quad (9.11)$$

Where

$$B=(b-a)^{\mathrm{Re}(\beta)}\sum_{k=0}^{\infty}\frac{|(\gamma)_{qk}|}{|\Gamma(\alpha k+\beta)|[\mathrm{Re}(\alpha)k+\mathrm{Re}(\beta)]}\frac{\left|w(b-a)^{\mathrm{Re}(\alpha)}\right|^{k}}{k!} \quad (9.12)$$

the relation

$$I^{\mu}_{a+}\,E^{\gamma,q}_{\alpha,\beta,w;a+}f=E^{\gamma,q}_{\alpha,\beta+\mu,w;a+}f \quad (9.13)$$

hold for any summable function $f\in L(a,b)$.

We can express the function $E^{\gamma,\varsigma}_{m,\beta}(z)$, $m\in\mathbb{N}$ as:

$$E^{\gamma,\varsigma}_{m,\beta}(z)=\frac{(2\pi)^{\frac{m-1}{2}}}{m^{\beta-\frac{1}{2}}}\prod_{k=0}^{m-1}\frac{1}{\Gamma\left(\frac{\beta+k}{m}\right)}\sum_{n=0}^{\infty}\frac{(\gamma)_{\varsigma n}}{\left(\frac{\beta+k}{m}\right)_n n!}\frac{z^n}{m^{nm}}. \quad (9.14)$$

On substituting $\varsigma=1$ in (9.14) and then the result Kilbas *et al.* (2004) becomes a special case of (9.14) as:

$$E^{\gamma,1}_{m,\beta}(z)=E^{\gamma}_{m,\beta}(z)=\frac{(2\pi)^{\frac{m-1}{2}}}{m^{\beta-\frac{1}{2}}}\prod_{k=0}^{m-1}\frac{1}{\Gamma\left(\frac{\beta+k}{m}\right)}\Phi\left(\gamma,\frac{\beta+k}{m};\frac{z}{m^m}\right) \quad (9.15)$$

10 FRACTIONAL OPERATORS AND FUNCTION $E^{\gamma,\varsigma}_{\alpha,\beta}(z)$

Consider the function $f(t)=\sum_{n=0}^{\infty}\frac{(\gamma)_{\varsigma n}(ct)^n}{(n!)^2}$, where $\mathrm{Re}(\gamma)>0$, $\varsigma>0$ and c is an arbitrary constant then the fractional integral operator of order ν can be written as,

$$I^{\nu} f(t) = t^{\nu} E_{1,\nu+1}^{\gamma,\varsigma}(ct). \tag{10.1}$$

We denote the function (10.1) as $E_t(c,\nu,\gamma,q)$, i.e.

$$E_t(c,\nu,\gamma,\varsigma) = t^{\nu} E_{1,\nu+1}^{\gamma,\varsigma}(ct). \tag{10.2}$$

The fractional differential operator of order μ can be written as

$$D^{\mu} f(t) = D^{n}\left[I^{k-\mu} \sum_{n=0}^{\infty} \frac{(\gamma)_{\varsigma n} (ct)^{n}}{(n!)^{2}} \right] = t^{-\mu} E_{1,1-\mu}^{\gamma,\varsigma}(ct). \tag{10.3}$$

We denote the function (10.3) as $E_t(c,-\mu,\gamma,q)$, i.e.

$$E_t(c,-\mu,\gamma,\varsigma) = t^{-\mu} E_{1,1-\mu}^{\gamma,\varsigma}(ct) \tag{10.4}$$

THEOREM (10.1). If $\mathrm{Re}(\gamma) > 0$, $\varsigma > 0$, c is an arbitrary constant and fractional integral operator of order ν then

$$I^{\lambda} E_t(c,\nu,\gamma,\varsigma) = E_t(c,\lambda+\nu,\gamma,\varsigma). \tag{10.5}$$

$$D^{\lambda} E_t(c,\nu,\gamma,\varsigma) = E_t(c,\nu-\lambda,\gamma,\varsigma). \tag{10.6}$$

The Laplace transforms of $E_t(c,\nu,\gamma,\varsigma)$ is given as

$$L\{E_t(c,\nu,\gamma,\varsigma)\} = \frac{1}{s^{\nu+1}}\left(1-\frac{c}{s}\right)^{-\gamma,\varsigma}, \tag{10.7}$$

In the light of Theorem (10.1), we can prove following Theorem (10.2).

THEOREM (10.2). If $\mathrm{Re}(\gamma) > 0$, $\varsigma > 0$, c is an arbitrary constant and fractional integral operator of order μ then

$$I^{\lambda} E_t(c,-\mu,\gamma,q) = E_t(c,\lambda-\mu,\gamma,\varsigma). \tag{10.8}$$

$$D^{\lambda} E_t(c,-\mu,\gamma,q) = E_t(c,-\lambda-\mu,\gamma,\varsigma). \tag{10.9}$$

$$L\{E_t(c,-\mu,\gamma,\varsigma)\} = \frac{1}{s^{1-\mu}}\left(1-\frac{c}{s}\right)^{-\gamma,\varsigma}. \tag{10.10}$$

11 GENERATING RELATIONS AND FINITE SUMMATION FORMULAE OF (6)

A considerably large number of special functions (including all of the classical orthogonal polynomials) are known to possess generating relations.

We used operational technique by employing θ Mittal (1977), Patil and Thakare (1975) as a differential operator, where $\theta \equiv x^a(s+xD)$ and $\theta_1 \equiv x^a(1+xD)$, $D \equiv \dfrac{d}{dx}$, for obtaining following generating relations and finite summation formulae of (1.6).

$$\sum_{n=0}^{\infty} A_{\varsigma n}^{(\alpha,\beta,\gamma,\delta)}(x;a,k,s)t^n = (1-at)^{-\left(\frac{\delta+s}{a}\right)} E_{\alpha,\beta}^{\gamma,\varsigma}\{p_k(x)\} E_{\alpha,\beta}^{\gamma,\varsigma}[-p_k\{x(1-at)^{-\frac{1}{a}}\}]. \quad (11.1)$$

$$\sum_{n=0}^{\infty}\left(\frac{u}{a}\right)^n \sum_{m=0}^{\infty} A_n^{(1,1,1,\delta+km)}(x;a,k,s)\frac{t^m}{m!} = e^t \sum_{n=0}^{\infty} A_n^{(1,1,1,\delta)}\left(\left[x^k - \frac{t}{\rho_k}\right]^{\frac{1}{k}};a,k,s\right)\left(\frac{u}{a}\right)^n \quad (11.2)$$

$$\sum_{n=0}^{\infty} A_{\varsigma n}^{(\alpha,\beta,\gamma,\delta-an)}(x;a,k,s)t^n = (1+at)^{\frac{\delta+s}{a}-1} E_{\alpha,\beta}^{\gamma,\varsigma}\{p_k(x)\} E_{\alpha,\beta}^{\gamma,\varsigma}[-p_k\{x(1+at)^{\frac{1}{a}}\}]. \quad (11.3)$$

$$\sum_{m=0}^{\infty}\binom{n+m}{m} A_{\varsigma(m+n)}^{(\alpha,\beta,\gamma,\delta)}(x;a,k,s)t^m = (1-at)^{-n-\left(\frac{\delta+s}{a}\right)} \frac{E_{\alpha,\beta}^{\gamma,\varsigma}\{p_k(x)\}}{E_{\alpha,\beta}^{\gamma,\varsigma}[p_k\{x(1-at)^{-\frac{1}{a}}\}]} A_{\varsigma n}^{(\alpha,\beta,\gamma,\delta)}\{x(1-at)^{-\frac{1}{a}};a,k,s\}. \quad (11.4)$$

$$\sum_{n=0}^{\infty}\binom{n+m}{m} A_{\varsigma(m+n)}^{(\alpha,\beta,\gamma,\delta-an)}(x;a,k,s)t^{n} =$$

$$(1+at)^{-1+\left(\frac{\delta+s}{a}\right)} \frac{E_{\alpha,\beta}^{\gamma,\varsigma}\{p_k(x)\}}{E_{\alpha,\beta}^{\gamma,\varsigma}[p_k\{x(1+at)^{\frac{1}{a}}\}]} A_{\varsigma m}^{(\alpha,\beta,\gamma,\delta)}\{x(1+at)^{\frac{1}{a}};a,k,s\}. \tag{11.5}$$

Using (11.1) to (11.5), we obtained following generating relations

$$\sum_{m=0}^{\infty} m^{n} A_{\varsigma(m+n)}^{(\alpha,\beta,\gamma,\delta)}(x;a,k,s)z^{m} =$$

$$(1-az)^{-\frac{\delta+s}{a}} \frac{E_{\alpha,\beta}^{\gamma,\varsigma}\{p_k(x)\}}{E_{\alpha,\beta}^{\gamma,\varsigma}[p_k\{x(1-at)^{-\frac{1}{a}}\}]} \times$$

$$\sum_{m=0}^{n} m!\, S(n,m)\, A_{\varsigma m}^{(\alpha,\beta,\gamma,\delta)}[x(1-az)^{-\frac{1}{a}};a,k,s]\left(\frac{z}{1-az}\right)^{m}. \tag{11.6}$$

$(n \in N_0;\, |z| < |a|^{-1}; a \neq 0)$

$$\sum_{m=0}^{\infty} m^{n} A_{\varsigma(m+n)}^{(\alpha,\beta,\gamma,\delta-am)}(x;a,k,s)z^{m} =$$

$$(1-az)^{-1+\frac{\delta+s}{a}} \frac{E_{\alpha,\beta}^{\gamma,\varsigma}\{p_k(x)\}}{E_{\alpha,\beta}^{\gamma,\varsigma}[p_k\{x(1-at)^{-\frac{1}{a}}\}]} \times$$

$$\sum_{m=0}^{n} m!\, S(n,m)\, A_{\varsigma m}^{(\alpha,\beta,\gamma,\delta-am)}[x(1+az)^{\frac{1}{a}};a,k,s]\left(\frac{z}{1+az}\right)^{m}. \tag{11.7}$$

$(n \in N_0;\, |z| < |a|^{-1}; a \neq 0)$

Two finite summation formulae for (1.6) also obtained as

$$A_{\varsigma n}^{(\alpha,\beta,\gamma,\delta)}(x;a,k,s)=\sum_{n=0}^{n}\frac{1}{m!}a^{m}\left(\frac{\delta}{a}\right)_{m}A_{\varsigma(n-m)}^{(\alpha,\beta,\gamma,0)}(x;a,k,s). \quad (11.8)$$

$$A_{\varsigma n}^{(\alpha,\beta,\gamma,\delta)}(x;a,k,s)=\sum_{n=0}^{n}\frac{1}{m!}a^{m}\left(\frac{\delta-\sigma}{a}\right)_{m}A_{\varsigma(n-m)}^{(\alpha,\beta,\gamma,\sigma)}(x;a,k,s). \quad (11.9)$$

we get following bilateral generating relation for $A_{qn}^{(\alpha,\beta,\gamma,\delta)}(x;a,k,s)$,

$$\sum_{m=0}^{\infty}A_{\varsigma\,(\nu+m)}^{(\alpha,\beta,\gamma,\delta)}(x;a,k,s)R_{m,\nu}^{b}(y)t^{m}=$$

$$(1-at)^{-n-\left(\frac{\delta+s}{a}\right)}\frac{E_{\alpha,\beta}^{\gamma,\varsigma}\{p_{k}(x)\}}{E_{\alpha,\beta}^{\gamma,\varsigma}[p_{k}\{x(1-at)^{-\frac{1}{a}}\}]}\Phi_{b,\nu}[x(1-at)^{-\frac{1}{a}},yt^{b}], \quad (11.10)$$

Wher $\Phi_{b,\nu}[x,t]=\sum_{m=0}^{\infty}\delta_{\nu,m}\,A_{\varsigma(\nu+bm)}^{(\alpha,\beta,\gamma,\delta)}(x;a,k,s)t^{m}$, $\delta_{\nu,m}\neq 0$ and $R_{m,\nu}^{b}(y)$ is a polynomial of degree $\left[\frac{m}{b}\right]$ in *y*, which is defined as

$$R_{m,\nu}^{b}(y)=\sum_{k=0}^{[m/b]}\binom{\nu+m}{\nu+bk}\delta_{\nu,k}\,y^{k};$$

b is a positive integer, ν is an arbitrary complex number.

REFERENCES

1. Carlitz L.: “Some expansion and convolution formulas related to Mac Mohan’s master theorems”, SIAM Journal of Mathematical Analysis, 8 (2), 320 – 336 (1977).
2. Gorenflo R, Kilbas A.A. and Rogosin S.V.: “On the generalised Mittag-Leffler type function”, Integral Transforms and Special Functions, 7, 215--224 (1998).
3. Gorenflo R. and Mainardi F.: “On Mittag – Leffler function in fractional evaluation processes”, Journal of Computational and Applied Mathematics, 118, 283 – 299 (2000).

4. Khan M.A. and Abukhammash G.S.: "A study on two variable analogues of certain fractional operators", Pro Mathematica, 17(33), 32--48 (2003).
5. Kilbas A.A. and Saigo M.: "On Mittag – Leffler type function, fractional calculus operators and solution of Integral equations", Integral Transforms and Special Functions, 4, 355 – 370 (1996).
6. Kilbas A.A., Saigo M. and Saxena R.K.: "Generalised Mittag-Leffler function and generalised Fractional calculus operators", Integral Transforms and Special Functions, 15, 31--49 (2004).
7. Mittag – Leffler G.M.: "Sur la nouvelle fonction $E_{\alpha}(x)$", C.R. Acad. Sci. Paris, 137, 554 – 558 (1903).
8. Mittal H.B.: "Bilinear and bilateral generating relations", American Journal of Mathematics, 99, 23--55 (1977).
9. Patil K.R. and Thakare N.K.: "Operational formulas for a function defined by a generalized Rodrigues formula-II", Science Journal of Shivaji University, 15, 1--10 (1975).
10. Prabhakar T.R.: "A singular integral equation with a generalized Mittag-Leffler function in the Kernel", Yokohama Mathematical Journal, 19, 7--15 (1971).
11. Prajapati J.C.: "Generalization of Mittag-Leffler function and its applications", Ph.D. thesis, Department of Mathematics, Sardar Vallabhbhai National Institute of Technology, Surat, India (2008).
12. Rainville E.D.: "Special Functions" The Macmillan Company, New York (1960).
13. Saigo M. and Kilbas A.A.: "On Mittag – Leffler type function and applications", Integral Transforms and Special Functions, 7, 97--112 (1998).
14. Sneddon I.N.: "The use of Integral Transforms", Tata McGraw – Hill Pub. Co. Ltd., New Delhi (1979).
15. Shukla A.K. and Prajapati J.C.: "On a generalization of Mittag-Leffler function and its properties", Journal of Mathematical Analysis and Applications, 337, pp. 797--811 (2007 A).
16. Shukla A.K. and Prajapati J.C.: "Some properties of a class of Polynomials suggested by Mittal", Proyecciones Journal of Mathematics, 26(2), pp. 145--156 (2007 B).
17. Shukla A.K. and Prajapati J.C.: "Generalization of a class of Polynomials", Demonstratio Mathematica, 40 (4), 819--826 (2007 C).
18. Shukla AK and Prajapati J.C.: "A general class of polynomials associated with generalized Mittag-Leffler function. Integral Transforms and Special Functions 19(1), pp. 23--34 (2008 A).
19. Shukla AK and Prajapati J.C.: "On Generalized Mittag-Leffler type function and generated integral operator", Mathematical Sciences Research Journal 12 (12), pp. 283--290 (2008 B).
20. Shukla A.K. and Prajapati J.C.: "Some remarks on generalized Mittag-Leffler function", Proyecciones Journal of Mathematics 28(1), 27--34 (2009 A).
21. Shukla A.K. and Prajapati J.C.: "On a Recurrence relation of generalized Mittag-Leffler function. Surveys in Mathematics and its Applications, 4, 133--138 (2009 B).

22. Shukla A.K. and Prajapati J.C.: "Decomposition and properties of generalized Mittag-Leffler function", Advances in Pure Mathematics, 2(1), 8--14 (2012).
23. Srivastava H.M. and Tomovski Z.: "Fractional calculus with an integral operator containing a generalized Mittag-Leffler function in the kernel", Applied Mathematics and Computation, 211, 198–210 (2009).
24. Tomovski Z., Hilfer R. and Srivastava H.M.: "Fractional and operational calculus with generalized fractional derivative operators and Mittag-Leffler type functions", Integral Transforms and Special Functions, 21(11), 797--814 (2010).
25. Wiman A.: Uber de Fundamental satz in der Theorie der Funktionen $E_\alpha(x)$", Acta Mathematica, 29, 191 – 201 (1905).