International Journal of Mathematical Engineering and Science
ISSN : 2277-6982 Volume 1 Issue 4 (April 2012)
http://www.ijmes.com/ https://sites.google.com/site/ijmesjournal/

WEAK FORMS OF STRONGLY CONTINUOUS FUNCTIONS

G.SHANMUGAM [1], N.RAJESH [2]

[1] Department of Mathematics, Jeppiaar Engineering College,
Chennai-600199, gsm.maths@gmail.com

[2] Department of Mathematics, Rajah Serfoji Govt. College, Thanjavur
Pin - 613005, Tamilnadu, India, nrajesh_topology@yahoo.co.in

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Abstract. The purpose of this paper is to give two new types of irresolute functions called, completely $sg\alpha$- irresolute functions and weakly $sg\alpha$-irresolute functions. We obtain theirs characterizations and theirs basic properties.

Keywords and phrases: Topological spaces, $sg\alpha$-open sets, $sg\alpha$-irresolute functions.

1 INTRODUCTION AND PRELIMINARIES

Functions and of course irresolute functions stand among the most important and most researched points in the whole of mathematical science. In 1972, Crossley and Hildebrand [3] introduced the notion of irresoluteness. Many different forms of irresolute functions have been introduced over the years. Various interesting problems arise when one considers irresoluteness. Its importance is significant in various areas of mathematics and related sciences. Recently, as generalization of closed sets, the notion of $sg\alpha$-closed sets were introduced and studied by Rajesh and Krsteska [19]. In this paper, we will continue the study of related irresolute functions with $sg\alpha$-open sets. We introduced and characterize the concepts of completely $sg\alpha$-irresolute and weakly $sg\alpha$ irresolute functions. Throughout this paper, spaces means topological spaces on which no separation axioms are assumed unless otherwise mentioned and $f:(X,\tau)\to(Y,\sigma)$ (or simply $f:X\to Y$) denotes a function f of a space (X,τ) into a space (Y,σ). Let A be a subset of a space X. The closure and the interior of A are denoted by $cl(A)$ and $\text{int}(A)$, respectively.

Definition 1.1 A subset A of a space (X,τ) *is called a semi – open* [9] ($resp.\ \alpha - open$) if $A \subset cl(\text{int}(A))$ ($resp.\ A \subset \text{int}(cl(\text{int}(A)))$). The complement of semi- open ($resp.$ α - open) set is called semi- closed (resp. α - closed).

The semi – closure [4] of a subset A of *X,* denoted by $scl(A)$ is defined to be the intersection of all semi-closed sets containing A in X. The α – closure and α – interior of a set A are similarly defined.

Definition 1.2 A subset A of a space X *is called semi – generalized* closed ($briefly\ sg\alpha - closed$) [19] if $\alpha cl(A) \subset U$ whenever $A \subset U$ and U is semi-open in X. The complement of $sg\alpha - closed$ set is called $sg\alpha - open$.

The union (resp. intersection) of all $sg\alpha - open$ (resp. $sg\alpha - closed$) sets, each contained in (resp. containing) a set A in a space X is called the $sg\alpha$ -interior (resp. $sg\alpha - closure$) of A and is denoted by $sg\alpha - \text{int}(A)$ (resp. $sg\alpha$ - $cl(A)$) [19].

Note that the family of $sg\alpha$ - open subsets of (X,τ) forms a topology [19].

Definition 1.3 A function $f;(X,\tau) \to (Y,\sigma)$ is called:

(i) strongly continuous [10] (resp. *strongly* $sg\alpha - continuous$ [17] if $f^{-1}(V)$ is both open ($resp.$ $sg\alpha$ - open) and closed ($resp.$ $sg\alpha$ - closed) in X for each subset V of Y;

(ii) completely continuous [2] if $f^{-1}(V)$ is regular open in X for every open set V of X;

(iii) $sg\alpha$ -*irresolute* [18] if $f^{-1}(V)$ is $sg\alpha$ - closed (resp. $sg\alpha$ - open) in X for every $sg\alpha$ -closed (resp. $sg\alpha$ -open) subset V of *Y*.

2. COMPLETELY $sg\alpha$ -IRRESOLUTE FUNCTIONS

Definition 2.1 A *function* f*:* $X \to Y$ *is called completely* $sg\alpha$ *- irresolute if the inverse image of each* $sg\alpha$ *-open subset of Y is regular open in X.*

Clearly, every strongly continuous function is completely $sg\alpha$ - irresolute and every completely $sg\alpha$ -irresolute function is $sg\alpha$ -irresolute.

Remark 2.2 The *converses of the above implications are not true in general as seen from the following examples.*

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Example 2.3 *Let* $X=\{a,b,c\}=Y, \tau=\{\phi,\{a\},\{b,c\},X\}$ *and* $\sigma=\{\phi,\{a\},Y\}$. *Clearly, the identity function* $f:(X,\tau)\to(Y,\sigma)$ *is completely* $sg\alpha$*-irresolute but not strongly continuous.*

Example 2.4 *Let* $X=\{a,b,c\}=Y, \tau=\{\phi,\{a,b\},X\}$ *and* $\sigma=\{\phi,\{a\},\{a,b\},Y\}$. *Then the identity function* $f:(X,\tau)\to(Y,\sigma)$ *is* $sg\alpha$*-irresolute but not completely* $sg\alpha$*-irresolute*.

Theorem 2.5 *The following statements are equivalent for a function* $f:X\to Y$

(i) *f is completely* $sg\alpha$*-irresolute;*

(ii) $f:(X,\tau)\to(Y,sg\alpha O(X))$ *is completely continuous;*

(iii) $f^{-1}(F)$*.is regular closed in X for every sga-closed set F of Y.*

Proof. (i) $\Leftrightarrow$ (ii): Follows from the definitions. (i) $\Leftrightarrow$ (iii): Let F be any $sg\alpha$ - closed set of Y. Then $Y\setminus F\in sg\alpha\, O(Y)$. By (i), $f^{-1}(Y\setminus F)=X\setminus f^{-1}(F)\in RO(X)$ .We have $f^{-1}(F)\in RC(X)$.Converse is similar.

Lemma 2.6 [11] *Let S be an open subset of a space* (X,τ). *Then the following hold:*

(i) *If* U *is regular open in X, then so is* $U\cap S$ *in the subspace* (S,τ_s).

(ii) *If* $B\subset S$ *is regular open in* (S,τ_s), *then there exists a regular open set U in* (X,τ) *such that B=U*$\cap$*S.*

Theorem 2.7 *If* $f:(X,\tau)\to(Y,\sigma)$ *is a completely* $sg\alpha$*-irresolute function and A is any open subset of X, then the restriction* $f|_A: A\to Y$ *is completely* $sg\alpha$*-irresolute.*

Proof. Let F be a $sg\alpha$-open subset of Y. *By hypothesis* $f^{-1}(F)$ is regular open in *X*. Since *A* is open in *X*, it follows from the previous Lemma that $(f|_A)^{-1}(F)=A\cap f^{-1}(F)$, which is regular open in A. Therefore, $f|_A$ *is* completely $sg\alpha$-irresolute.

Theorem 2.8 *The following hold for functions* $f:(X,\tau)\to(Y,\sigma)$ *and* $g:(Y,\sigma)\to(Z,\eta)$*:*

(i) *If f is completely* $sg\alpha$*-irresolute and g is strongly* $sg\alpha$*-continuous, then* $g\circ f$ *is completely continuous;*

(ii) *If f is completely continuous and g is strongly* $sg\alpha$*-continuous, then* $g\circ f$ *is completely* $sg\alpha$*-irresolute;*

(iii) *If f is completely* $sg\alpha$*-irresolute and g is* $sg\alpha$*-irresolute, then* $g\circ f$ *is completely* $sg\alpha$*-irresolute*

(iv) *If f is completely continuous and g is completely* $sg\alpha$*-irresolute functions,* *then* $g \circ f$ *is completely* $sg\alpha$*-irresolute function*

Proof. The proof of the theorem is easy and hence omitted.

Definition 2.9 *A space X is said to be almost connected [5] (resp.* $sg\alpha$*-connected [20] if there does not exist disjoint regular open (resp.* $sg\alpha$*-open) sets A and B such that* $A \cup B = X$.

Theorem 2.10 *If* $f:(X,\tau)\to(Y,\sigma)$ *is completely* $sg\alpha$*-irresolute surjective function and X is almost connected, then Y is* $sg\alpha$*-connected.*

Proof. Suppose that *Y* is not $sg\alpha$- connected. Then there exist disjoint $sg\alpha$-open sets A and B of Y such that A $\cup$ B = X. Since *f* is completely $sg\alpha$-irresolute surjective, $f^{-1}(A)$ and $f^{-1}(B)$ are regular open sets in *X*. Moreover, $f^{-1}(A) \cup f^{-1}(B) = X$, $f^{-1}(A) \neq \emptyset$ and $f^{-1}(B) \neq \emptyset$. This shows that X is not almost connected, which is a contradiction to the assumption that X is almost connected. By contradiction, Y is $sg\alpha$*- connected.*

Definition 2.11 *A space X is said to be*

(i) *nearly compact* [13, 22] *if every regular open cover of X has a finite subcover:*
(ii) nearly countable compact [6] *if every cover by regular open sets has a countable subcover;*
(iii) nearly Lindelof [5] *if every cover of X by regular open sets has a countable subcover;*
(iv) $sg\alpha$*-compact* [20] *if every* $sg\alpha$*- open cover of X has a finite subcover;*
(v) Countably $sg\alpha$*-compact* [21] *if every* $sg\alpha$*-open countable cover of X has a finite subcover;*
(vi) $sg\alpha$*- Lindelof* [21] *if every cover of X by* $sg\alpha$*-open sets has a countable subcover;*

Theorem 2.12 *Let* $f:(X,\tau)\to(Y,\sigma)$ *be a completely* $sg\alpha$ *irresolute surjective function. Then the following statements hold:*

(i) *If X is nearly compact, then Y is* $sg\alpha$*- compact.*
(ii) *If X is nearly Lindelof, then Y is* $sg\alpha$*- Lindelof.*
(iii) *If X is nearly countably compact, then Y is countably* $sg\alpha$*- compact.*

Proof. (i) Let $f: X \to Y$ be a completely $sg\alpha$-irresolute function of nearly compact space X onto a space Y. Let $\{U_\alpha : \alpha \in \Delta\}$ be any $sg\alpha$-open cover of Y.
Then $\{f^{-1}(U_\alpha) : \alpha \in \Delta\}$ is a regular open cover of X. Since X is nearly compact, there exists a finite subfamily, $\{f^{-1}(U_{\alpha_i}) \mid i = 1,2,\ldots n\}$ of $\{f^{-1}(U_\alpha) : \alpha \in \Delta\}$ which cover X.

It follows then that $\{U_{\alpha_i} : i = 1,2,\ldots n\}$ is a finite subfamily of $\{U_\alpha : \alpha \in \Delta\}$ which cover Y. Hence, space Y is a $sg\alpha$-compact space. The proof of other cases are similar.

Definition 2.13 *A space* (X,τ) *is said to be:*

(i) *S- closed* [24] *(resp.* $sg\alpha$*-closed compact* [21]*) if every regular closed (resp.* $sg\alpha$*- closed) cover of X has a finite subcover;*
(ii) *countably S- closed - compact [1] (resp. countably* $sg\alpha$*-closed compact* [21] *) if every countable cover of X by regular closed (resp.* $sg\alpha$*-closed) sets has a finite subcover;*
(iii) *S – Lindelof* [12] *(resp.* $sg\alpha$*-Lindelof [21]) if every cover of X by regular closed (resp.* $sg\alpha$*-closed) sets has a countable subcover.*

Theorem 2.14 *Let* $f:(X,\tau)\to(Y,\sigma)$ *be a completely* $sg\alpha$ *irresolute surjective function. Then the following statements hold:*

(i) *If X is S-closed, then Y is* $sg\alpha$*- closed compact.*
(ii) *If X is S- Lindelof , then Y is* $sg\alpha$*-closed Lindelof.*
(iii) *If X is countably S-closed, then Y is countably* $sg\alpha$*- closed compact.*

Proof. It can be obtained similarly as the previous Theorem.

Definition 2.15 *A space X is said to be strongly* $sg\alpha$*-normal (resp. mildly* $sg\alpha$*-normal) if for each pair of distinct* $sg\alpha$*-closed (resp. regular closed) sets A and B of X, there exist disjoint* $sg\alpha$*-open sets U and V such that* $A \subset U$ *and* $B \subset$ V.

It is evident that every mildly $sg\alpha$-normal space is strongly $sg\alpha$- normal.

Recall that a function $f: X \to Y$ is called $sg\alpha^*$-closed [15] if the image of each $sg\alpha$-closed set of X is a $sg\alpha$-closed set in Y.

In [15], the following theorem is proved.

Theorem 2.16 If *a mapping* $f: X \to Y$ *is* $sg\alpha^*$*-closed, then for each subset B of Y and a* $sg\alpha$*-open set U of X containing* $f^{-1}(B)$*, there exists a* $sg\alpha$*-open set V in Y containing B such that* $f^{-1}(V) \subset U$.

Theorem 2.17 If $f: X \to Y$ *is completely* $sg\alpha$*-irresolute,* $sg\alpha^*$ *closed function from a mildly* $sg\alpha$*-normal space X onto a space Y, then Y is strongly* $sg\alpha$*- normal.*

Proof. Let A and B be two disjoint $sg\alpha$-closed subsets of Y. then $f^{-1}(A)$ and $f^{-1}(B)$ are disjoint regular closed subsets of X. Since X is mildly $sg\alpha$-normal space, there exist disjoint $sg\alpha$-open sets in X such that $f^{-1}(A) \subset$ U and $f^{-1}(B) \subset$ V. Then by Theorem 2.16, there exist $sg\alpha$-open sets $G = Y \setminus f(X \setminus U)$ and $H = Y \setminus f(X \setminus V)$ such that A$\subset$G, $f^{-1}(G) \subset U$; B$\subset$H, $f^{-1}(H) \subset V$. Clearly G and H are disjoint $sg\alpha$*-open* subsets of Y. Hence, Y is strongly $sg\alpha$*-normal.*

Definition 2.18 A *space X is said to be strongly* $sg\alpha$*-regular if for each* $sg\alpha$*-closed set F and each point x* $\notin$ *F, there exists disjoint* $sg\alpha$*-open sets U and V in X such that x* $\in$ *U and F* $\subset$ *V.*

Definition 2.19 A *space X is called almost* $sg\alpha$*-normal if for each regular closed subset F and every point x* $\notin$ *F, there exist disjoint* $sg\alpha$*-open sets U and V such that x* $\in$ *U and F* $\subset$ *V.*

Theorem 2.20 *If f is a completely* $sg\alpha$*-irresolute,* $sg\alpha^*$*-closed injection of an almost* $sg\alpha$*-regular space X onto a space Y, then Y is strongly* $sg\alpha$*-regular space.*

Proof. Let F be a $sg\alpha$-closed subset of Y and let $y \notin$ F. Then, $f^{-1}(F)$ is regular closed subset of X such that $f^{-1}(y) = x \notin f^{-1}(F)$. Since X being almost $sg\alpha$-regular space, there exists disjoint $sg\alpha$-open sets U and V in X such that $f^{-1}(y) \in U$ and $f^{-1}(F) \subset V$. By Theorem 2.17, there exist $sg\alpha$-open sets $G = Y \setminus f(X \setminus U)$ such that $f^{-1}(G) \subset U$, $y \in G$ and $H = Y \setminus f(X \setminus V)$ such that $f^{-1}(H) \subset V, F \subset H$. Clearly, G and H are disjoint $sg\alpha$-open subsets of Y. Hence, Y is strongly $sg\alpha$-regular space.

Definition 2.21 *A space* (X, τ) *is said to be* $sg\alpha - T_1$ [14] (resp. $r - T_1$ *[5]) if for each pair of distinct points x and y of X, there exist* $sg\alpha$*- open (resp. regular open) sets* U_1 *and* U_2 *such that* $x \in U_1$ *and* $y \in U_2$, $x \notin U_2$ *and* $y \notin U_1$.

Theorem 2.22 *If* $f : (X, \tau) \to (Y, \sigma)$ *is completely* $sg\alpha$*-irresolute injective function and Y is* $sg\alpha$*-*T_1*, then X is* $r - T_1$.

Proof. Suppose that Y is $sg\alpha - T_1$. For any two distinct points x and y of X, there exist $sg\alpha$-open sets F_1 and F_2 in Y such that $f(x) \in F_1$, $f(y) \in F_2$, $f(x) \notin F_2$ and $f(y) \notin F_1$. Since f is injective completely $sg\alpha$-irresolute function, we have X is $r - T_1$.

Definition 2.23 *A space* (X, τ) *is said to be* $sg\alpha - T_2$ [14] (resp. $r - T_2$) *for each pair of distinct points x and y in X, there exist disjoint* $sg\alpha$*-open (resp. regular open) sets A and B in X such that* $x \in A$ *and* $y \in B$.

Theorem 2.24 If $f:(X,\tau)\to(Y,\sigma)$ *is completely* $sg\alpha$ *-irresolute injective function and Y is* $sg\alpha - T_2$, *then X is* $r - T_2$.

Proof. Similar to the proof of Theorem 2.22

3. WEAKLY $sg\alpha$ -IRRESOLUTE FUNCTIONS

Definition 3.1 *A function* $f: X \to Y$ *is said to be weakly* $sg\alpha$ *-irresolute if for each point* $x \in X$ *and each* $V \in sg\alpha\, O(Y, f(x))$, *there exists* $aU \in sg\alpha O(X,x)$ *such that* $f(U) \subset sg\alpha\text{-}cl(V)$.

It is evident that every $sg\alpha$-irresolute function is weakly $sg\alpha$- irresolute but the converse is not true.

Example 3.2 Let $X = Y = \{a,b,c\}$, with topologies $\tau = \{\phi, \{a\}, \{b,c\}, X\}$ *and* $\sigma = \{\phi, \{a\}, Y\}$. *Define a function* $f:(X,\tau)\to(Y,\sigma)$ *by* $f(a) = b$, $f(b) = a$ *and* $f(c) = c$. *Clearly f is weakly* $sg\alpha$ *-irresolute but not* $sg\alpha$ *-irresolute.*

Theorem 3.3 For a function $f: X \to Y$, *the following statements are equivalent:*

(i) *f is weakly* $sg\alpha$ *-irresolute;*

(ii) $f^{-1}(V) \subset sg\alpha\text{-}\mathrm{int}\left(f^{-1}(sg\alpha\text{-}cl(V))\right)$ *for every* $V \in sg\alpha O(Y)$;

(iii) $sg\alpha\text{-}cl\left(f^{-1}(V)\right) \subset f^{-1}(sg\alpha\text{-}cl(V))$ *for every* $V \in sg\alpha O(Y)$.

Proof. (i) $\Rightarrow$ (ii): Suppose that $V \in sg\alpha O(Y)$ and let $x \in f^{-1}(V)$. It follows form (i) that $f(U) \subset sg\alpha\text{-}cl(V)$ for some $U \in sg\alpha O(X,x)$. Therefore, we have $U \subset f^{-1}(sg\alpha\text{-}cl(V))$ and $x \in U \subset sg\alpha\text{-}\mathrm{int}\left(f^{-1}(sg\alpha\text{-}cl(V))\right)$. This shows that $f^{-1}(V) \subset sg\alpha\text{-}\mathrm{int}\left(f^{-1}(sg\alpha\text{-}cl(V))\right)$.

(ii) $\Rightarrow$ (iii): Suppose that $V \in sg\alpha O(Y)$ and $x \notin f^{-1}(sg\alpha\text{-}cl(V))$.Then $f(x) \notin sg\alpha\text{-}cl(V)$. There exists $G \in sg\alpha O(Y, f(x))$ such that $G \cap V = \emptyset$ Since $V \in sg\alpha O(Y)$ we have $sg\alpha$-cl(G) $\cap$ V= $\emptyset$ and hence $sg\alpha\text{-}\mathrm{int}\left(f^{-1}(sg\alpha\text{-}cl(G))\right) \cap f^{-1}(V) = \emptyset$
By (ii), we have $x \in f^{-1}(G) \subset sg\alpha\text{-}\mathrm{int}\left(f^{-1}(sg\alpha\text{-}cl(G))\right) \in sg\alpha O(X)$ Therefore we obtain $x \notin sg\alpha\text{-}cl\left(f^{-1}(V)\right)$. This shows that $sg\alpha\text{-}cl\left(f^{-1}(V)\right) \subset f^{-1}(sg\alpha\text{-}cl(V))$.

(iii) $\Rightarrow$ (i): Let $x \in X$ *and* $V \in sg\alpha O(Y, f(x))$. Then $x \notin f^{-1}(sg\alpha\text{-}cl(Y - sg\alpha\text{-}cl(V)))$ Since $Y - sg\alpha\text{-}cl(V) \in sg\alpha O(Y)$, by (iii), we have $x \notin sg\alpha\text{-}cl\left(f^{-1}(Y - sg\alpha\text{-}cl(V))\right)$- Hence there exists $U \in sg\alpha O(X,x)$ such that $U \cap f^{-1}(Y - sg\alpha\text{-}\mathrm{cl(V)}) = \emptyset$.

Therefore, we obtain $f(U)\cap(Y - sg\alpha\text{-cl}(V)) = \emptyset$ and hence $f(U) \subset sg\alpha\text{-cl}(V)$. This shows that f is weakly $sg\alpha$-irresolute.

Definition 3.4 *A space X is said to be* $sg\alpha$-regular [16] *if for closed set F and each point* $x \in X \setminus F$, *there exist disjoint* $sg\alpha$*-open sets U and V such that* $x \in U$ *and* $F \subset V$.

Lemma 3.5 [16] *For a space X the following are equivalent:*

(i) *X is* $sg\alpha$*-regular;*

(ii) *for each point* $x \in X$ *and for each open set U of X containing x, there exists* $V \in sg\alpha O(X)$ *such that* $x \in V \subset sg\alpha\text{-}cl(V) \subset U$.

Theorem 3.6 Let *Y be a* $sg\alpha$*-regular space. Then a function* $f : X \to Y$ *is weakly* $sg\alpha$*-irresolute if and only if it is* $sg\alpha$*-irresolute.*

Proof. Suppose that $f : X \to Y$ is weakly $sg\alpha$-irresolute. Let V be any $sg\alpha$-open set of Y and $x \in f^{-1}(V)$. Then $f(x) \in V$. Since *Y is* $sg\alpha$-regular, by lemma 3.5, there exists $W \in sg\alpha O(Y)$ such that $f(x) \in W \subset sg\alpha\text{-}cl(W) \subset V$. Since f is weakly $sg\alpha$-irresolute, there exists $U \in sg\alpha O(X, x)$ such that $f(U) \subset sg\alpha\text{-}cl(W)$. Therefore, we have $x \in U \subset f^{-1}(V)$ and $f^{-1}(V) \in sg\alpha O(X)$. This shows that f is $sg\alpha$*-irresolute.* The converse is obvious.

Theorem 3.7 *A function* $f : X \to Y$ is weakly $sg\alpha$*-irresolute if the graph function, defined by g(x) =(x, f(x)) for each* $x \in X$, *is weakly* $sg\alpha$*-irresolute.*

Proof. *Let* $x \in X$ *and* $V \in sg\alpha O(Y, f(x))$. *Then* $X \times V$ *is a* $sg\alpha$*-open set of* $X \times Y$ *containing g(x). Since g is weakly* $sg\alpha$*-irresolute, there exists* $U \in sg\alpha O(X, x)$ *such that* $g(U) \subset sg\alpha\text{-}cl(X \times V) \subset X \times sg\alpha\text{-}cl(V)$. *Therefore, we have* $f(U) \subset sg\alpha\text{-}cl(V)$.

Definition 3.8 *A topological space X is called* $sg\alpha\text{-}T_2$ *[14] if for any distinct pair of points x and y in X, there exist* $sg\alpha$*-open sets U and V in X containing x and y, respectively, such that* $U \cap V = \emptyset$.

Theorem 3.9 [14] *A space X is* $sg\alpha\text{-}T_2$ *if and only if for any pair of distinct points x, y of X there exists* $sg\alpha$*-open sets U and V such that* $x \in U$, *and* $y \in V$ *and* $sg\alpha\text{-}cl(U) \cap sg\alpha\text{-}cl(V) = \emptyset$.

Theorem 3.10 *If Y is a* $sg\alpha\text{-}T_2$ *space and f: X* $\to$ *Y is a weakly* $sg\alpha$*-irresolute injection, then X is* $sg\alpha\text{-}T_2$.

Proof. Let x, y be any two distinct points of X. Since f is injective, we have $f(x)\neq f(y)$. Since Y is $sg\alpha$-T_2, by theorem 3.9 there exists $V \in sg\alpha O(Y, f(x))$ and $W \in sg\alpha O(Y, f(x))$ such that $sg\alpha\text{-}cl(V) \cap sg\alpha\text{-}cl(W) = \emptyset$. Since f is weakly sgα-irresolute, there exists $G \in sg\alpha O(X, x)$ and $H \in sg\alpha O(X, y)$ such that $f(G) \subset sg\alpha\text{-}cl(V)$ and $f(H) \subset sg\alpha\text{-}cl(W)$. Hence we obtain $G \cap H = \emptyset$. This shows that X is $sg\alpha$- T_2.

Definition 3.11 *A function $f : X \rightarrow Y$ is said to have a strongly sgα-closed graph if for each $(x, y) \in (X \times Y) \setminus G(f)$, there exist $U \in sg\alpha O(X, x)$ and $V \in sg\alpha O(Y, y)$ such that $(sg\alpha\text{-}cl(U) \times sg\alpha\text{-}cl(V)) \cap G(f) = \emptyset$.*

Definition 3.12 *If Y is a $sg\alpha$-T_2 space and $f : X \rightarrow Y$ is weakly sgα-irresolute, then $G(f)$ is strongly $sg\alpha$-closed.*

Proof. Let $(x, y) \in (X \times Y) \setminus G(f)$. Then $y \neq f(x)$ and by Theorem 3.9 there exists $V \in sg\alpha O(Y, f(x))$ *and* $W \in sg\alpha O(Y, y)$ such that $sg\alpha\text{-}cl(V) \cap sg\alpha\text{-}cl(W)=\emptyset$. Since f is weakly $sg\alpha$-irresolute, there exists $U \in sg\alpha O(X, x)$ such that $f(sg\alpha\text{-}cl(U)) \subset sg\alpha\text{-}cl(V)$. Therefore we obtain $f(sg\alpha\text{-}cl(V)) \cap sg\alpha\text{-}cl(W) = \emptyset$ and hence $(sg\alpha\text{-}cl(U) \times sg\alpha\text{-}cl(W)) \cap G(f) = \emptyset$. This shows that $G(f)$ is strongly $sg\alpha$-closed in $X \times Y$.

Theorem 3.13 If *a function $f : X \rightarrow Y$ is weakly $sg\alpha$-irresolute, injective and $G(f)$ is strongly $sg\alpha$-closed, then X is $sg\alpha$- T_2.*

Proof. Let x, y be a pair of distinct points of X. Since f is injective, $f(x)\neq f(y)$ *and* $(x, f(y)) \notin G(f)$. Since $G(f)$ is strongly $sg\alpha$-closed there exists $G \in sg\alpha O(X, x)$ and $V \in sg\alpha O(Y, f(y))$ such that $f(sg\alpha\text{-}cl(G)) \cap sg\alpha\text{-}cl(V) = \emptyset$. Since f is weakly sgα-irresolute, there exists $H \in sg\alpha O(X, y)$ such that $f(H) \subset sg\alpha\text{-}cl(V)$. Hence we have $f(sg\alpha\text{-}cl(G)) \cap f(H) = \emptyset$; hence $G \cap H = \emptyset$. This shows that X is $sg\alpha$-T_2.

Recall that a topological space X is said to be $sg\alpha$-connected [20] if it cannot be written as the union of two non empty disjoint $sg\alpha$-open sets.

Theorem 3.14 *If a function $f : X \rightarrow Y$ is a weakly sgα-irresolute surjection and X is sgα-connected, then Y is $sg\alpha$-connected.*

Proof. Suppose that Y is not $sg\alpha$-connected. There exist nonempty sgα-open sets V and W of Y such that $V \cup W = Y$ and $V \cap W =\emptyset$. Then we have, $V, W \in sg\alpha O(Y)$. Since f is weakly $sg\alpha$-irresolute $f^{-1}(V), f^{-1}(W) \in sg\alpha O(X)$. Moreover, we have $f^{-1}(V) \cup f^{-1}(W) = X, f^{-1}(V) \cap f^{-1}(W) = \phi$, and $f^{-1}(V)$ and $f^{-1}(W)$ are nonempty. Therefore, X is not $sg\alpha$-connected.

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