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Decomposition of J - closed sets in Bigeneralized Topological Spaces

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ABSTRACT

The aim of this paper is to introduce the concept of J- locally- closed sets in bigeneralized topological spaces and study some of their properties. Using these concepts, some of the generalizations of pairwise LC-continuous maps in bigeneralized topological spaces are defined and their characterizations are investigated.

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1. Introduction

The concept of generalized topological spaces was developed by Á.Császár [2] in 2002.He also introduced the concepts of neighborhood systems, continuous functions and associated interior and closure operators on generalized neighborhood systems and generalized topological spaces. In particular, he investigated characterizations for generalized continuous functions (= (g,g')- continuous functions). In [3], the notions of g- α - open sets, g - semi-open sets, g – pre open sets and g - β open sets in generalized topological spaces are analyzed. Recently, many topologist are working on GTS.

Kelly[8] initiated the notion of bitopological spaces. The notion of locally closed set in bitopological space was introduced by Kuratowski and Sierpinski[9].In this paper, we introduce the notion of $\lambda_{(m, n)}$- J – locally closed sets in bigeneralized topological spaces and study some of their properties. We also study the concept of generalizations of J-locally closed set in BGTS.

2 Preliminaries

A Generalized topology is a subset μ of P(X) that contains φ and any union of elements of μ belongs to μ. Every topology is a generalized topology. A set with generalized topology on it, is called generalized topological space and is denoted by (X, μ), a generalized topological space(briefly GTS)on X. The elements of λ are called λ-open sets and the complements are called λ- closed sets. The generalized closure of a subset S of X, denoted by $c_\lambda(S)$, is the intersection of generalized closed sets including S and the interior of S, denoted by $i_\lambda(S)$, is the union of generalized open sets contained in S.

Definitions 2.1.[3] Let (X, λ) be a generalized topological space and $A \subseteq X$, then A is said to be

(1) λ - semi open if $A \subseteq c_\lambda (i_\lambda (A))$

(2) λ - pre open if $A \subseteq i_\lambda(c_\lambda(A))$

(3) λ - α- open if $A \subseteq i_\lambda(c_\lambda (i_\lambda (A)))$

(4) λ - β open if $A \subseteq c_\lambda (i_\lambda (c_\lambda(A)))$

The complement of λ - semi open (resp. λ - pre open, λ - α- open, λ - β open) is said to be λ - semi closed (resp λ - pre closed , λ - α- closed, λ - β closed). The class of all λ - semi open sets on X is denoted by $\sigma(\lambda_x)$(briefly σ_x or σ). The class of λ - pre open (λ - α- open and λ - β open) sets on X as $[\pi(gx), \alpha(gx), \beta(gx)]$ or briefly $[[\pi, \alpha, \beta]$.

Definition 2.2. [1] Let X be non-empty set and let λ_1 and λ_2 be generalized topologies on X. A triple $(X, \lambda_1, \lambda_2)$ is said to be a bigeneralized topological space (briefly BGTS).

Let $(X, \lambda_1, \lambda_2)$ be a bigeneralized topological space and A be a subset of X. The closure of A and the interior of A with respect to λ_m are denoted by $c_{\lambda m}$ (A) and $i_{\lambda m}$ (A), respectively, for m = 1,2.

Definitions 2.3.[12] Let (X,λ) be a generalized topological space and $A \subseteq X$, then A is said to be λ - J- open if $A \subseteq i_\lambda (c_\pi (A))$. The complement of λ - J- open is called as λ - J- closed.

Definition 2.4. A subset A of a bigeneralized topological space $(X, \lambda_1, \lambda_2)$ is said to be $\lambda_{(m, n)}$ – Locally Closed set(briefly($\lambda_{(m, n)}$ - *LC*) if $A= S \cap F$, where S is a λ_m – open and F is λ_n – closed in $(X, \lambda_1, \lambda_2)$, where m, n = 1,2 and $m\neq n$.

3 $\lambda_{(m, n)}$ – J - Locally closed sets

Definition 3.1 : A subset A of a bigeneralized topological space $(X, \lambda_1, \lambda_2)$ is called $\lambda_{(m, n)}$ – J- Locally closed set (briefly $\lambda_{(m, n)}$ –Jlc) if $A = S \cap F$,where S is λ_m – J-open and F is λ_n – J- closed in $(X, \lambda_1, \lambda_2)$,where m,n=1,2 and $m\neq n$.

Definition 3.2 : A subset A of a bigeneralized topological space $(X, \lambda_1, \lambda_2)$ is said to be $\lambda_{(m, n)}$ –Jlc* if $A = S \cap F$,where S is λ_m – J-open and F is λ_n – closed in (X, λ_1,λ_2),where m,n=1,2 and $m\neq n$.

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Definition 3.3 : A subset A of a bigeneralized topological space $(X, \lambda_1, \lambda_2)$ is said to be $\lambda_{(m, n)}$ –Jlc** if $A = S \cap F$,where S is λ_m – open and F is λ_n – J- closed in $(X, \lambda_1, \lambda_2)$,where m,n=1,2 and m≠n.

The collection of all $\lambda_{(m, n)})$ - Jlc (resp. $\lambda_{(m, n)})$ - Jlc*, $\lambda_{(m, n)})$ - Jlc**) sets of $(X, \lambda_1, \lambda_2)$ will be denoted by $\lambda_{(m, n)}$ -JLC(X) (resp. $\lambda_{(m, n)})$ - JLC*(X), $\lambda_{(m, n)})$ - JLC**(X)).

Proposition 3.4: Let A be subset of a space $(X, \lambda_1, \lambda_2)$.

(i)If $A \in \lambda_{(m, n)}$ - LC(X), then $A \in \lambda_{(m, n)}$ - JLC(X), $\lambda_{(m, n)}$ - JLC*(X) and $\lambda_{(m, n)}$ - JLC**(X).

(ii)If $A \in \lambda_{(m, n)}$ - JLC*(X), then $A \in \lambda_{(m, n)})$ - JLC(X).

(iii) If $A \in \lambda_{(m, n)}$ - JLC**(X), then $A \in \lambda_{(m, n)})$ - JLC(X).

Proof: Every closed set is λ-J-closed set, therefore the proof.

Remark 3.5. The concept of (λ_1, λ_2) – JLC*(X) and (λ_1, λ_2) – JLC**(X) are independent of each other.

Proposition 3.6. Let A be a subset of a space $(X, \lambda_1, \lambda_2)$.

(i) If $A \in (\lambda_1, \lambda_2)$ - JLC*(X).

(ii) $A = G \cap \lambda_2$- $c_\lambda(A)$ for some λ_1 – J- open set G.

(iii) λ_2- $c_\lambda(A)$ –A is λ_1 – J- closed.

(iv) $A \cup (X$- λ_2- $c_\lambda(A))$ is λ_1 – J- open.

Proof: (i)⇒(ii): Let $A \in (\lambda_1, \lambda_2)$ - JLC*(X). Then there exist λ_1 – J- open set G and a λ_2 – closed set F in $(X, \lambda_1, \lambda_2)$such that $A = G \cap F$. Since $A \subseteq G$ and $A \subseteq \lambda_2$- $c_\lambda(A)$, we have $A \subseteq G \cap \lambda_2$- $c_\lambda(A)$. Also , since λ_2- $c_\lambda(A) \subseteq F$, $G \cap \lambda_2$- $c_\lambda(A) \subseteq G \cap F = A$. Therefore $A = G \cap \lambda_2$- $c_\lambda(A)$.

(ii)⇒(i): Since G is λ_1 – J- open set and λ_2- $c_\lambda(A)$ is a λ_2 – closed set, we have $A = G \cap \lambda_2$- $c_\lambda(A) \in (\lambda_1, \lambda_2)$ - JLC*(X).

(iii)⇒(iv): Let $F = \lambda_2$- $c_\lambda(A)$ –A. Then by (iii), F is λ_1 – J- closed. Now $X – F = A \cup (X$ - λ_2- $c_\lambda(A))$. Since (X - F) is λ_1 – J- open , we get $A \cup (X$ - λ_2- $c_\lambda(A))$ is λ_1 – J- open.

(iv)⇒(iii): Let $G = A \cup (X$ -λ_2- $c_\lambda(A))$. This implies that X- G is λ_1 – J- closed and X- G = λ_2- $c_\lambda(A)$ –A . Hence λ_2-$c_\lambda(A)$ – A is λ_1 – J- closed.

(iv)⇒(ii): Let $G = A \cup (X$ -λ_2- $c_\lambda(A))$. So $G \cap \lambda_2$- $c_\lambda(A) = A$. Hence $A = G \cap \lambda_2$- $c_\lambda(A)$ for some λ_1 – J- open set G. (ii)⇒(iv): Let $A = G \cap \lambda_2$- $c_\lambda(A)$ for some λ_1 – J- open set G. Then $A \cup (X$ -λ_2- $c_\lambda(A)) = G$ which is λ_1 – J- open.

Proposition 3.7. Let A , B be any two subsets of a space $(X, \lambda_1, \lambda_2)$.If $A \in (\lambda_1, \lambda_2)$ – JLC(X) and B is λ_1 – J- open or λ_2 – J- closed, then $A \cap B \in (\lambda_1, \lambda_2)$ – JLC(X).

Proof: Let $A \in (\lambda_1, \lambda_2)$ - JLC(X). This implies $A = G \cap F$, where G is λ_1 – J- open and F is λ_2 – J- closed in $(X, \lambda_1, \lambda_2)$. Now $A \cap B = (G \cap B) \cap F$.

case(i) If B is λ_1 – J- open, then $G \cap B$ is also λ_1 – J- open and F is λ_2 – J- closed in $(X, \lambda_1, \lambda_2)$. Hence $A \cap B \in (\lambda_1, \lambda_2)$ - JLC(X).

case(ii) If B is λ_2 – J- closed, then $A \cap B = G \cap (B \cap F)$, where G is λ_1 – J- open and $B \cap F$ is λ_2 – J- closed in $(X, \lambda_1, \lambda_2)$. Hence $A \cap B \in (\lambda_1, \lambda_2)$ - JLC(X).

Proposition 3.8. Let A and $B \in (\lambda_1, \lambda_2)$ – JLC*(X) , then $A \cap B \in (\lambda_1, \lambda_2)$ – JLC*(X).

Proof: Let A, $B \in (\lambda_1, \lambda_2)$ – JLC*(X). then there exist λ_1 – J- open sets G and H such that $A = G \cap \lambda_2$- $c_\lambda(A)$ and $B = H \cap (\lambda_2$- $c_\lambda(A)$ (by theorem 3.11) Since $G \cap H$ is λ_1 – J- open and $A \cap B = (G \cap H) \cap (\lambda_2$- $c_\lambda(A) \cap \lambda_2$- $c_\lambda(B))$, then $A \cap B \in (\lambda_1, \lambda_2)$ – JLC*(X).

Remark 3.9. The union of any two (λ_1, λ_2) – JLC*(X) sets need not be a (λ_1, λ_2) – JLC*(X) set as can be seen from the following example.

Example 3.10. Let X={a, b, c}, λ_1= {X, ϕ, {a, b}} and λ_2= {X, ϕ, {a},{a, b}}. Then the subsets {a}, {c} $\in (\lambda_1, \lambda_2)$ - JLC*(X) but their union {a, c} $\notin (\lambda_1, \lambda_2)$ – JLC*(X).

Proposition 3.11. Let A , $B \in (\lambda_1, \lambda_2)$ – JLC**(X) , then $A \cap B \in (\lambda_1, \lambda_2)$ – JLC**(X).

Proof: Let A, $B \in (\lambda_1, \lambda_2)$ – JLC**(X). then there exist λ_1 – J- open sets G and H such that $A = G \cap \lambda_2$- $c_\lambda(A)$ and $B = H \cap (\lambda_2$- $c_\lambda(A)$ (by theorem 3.12) Since $G \cap H$ is λ_1 – J- open and $A \cap B = (G \cap H) \cap (\lambda_2$- $c_\lambda(A) \cap \lambda_2$- $c_\lambda(B))$, then $A \cap B \in (\lambda_1, \lambda_2)$ – JLC**(X).

Proposition 3.12. Let $A \in (\lambda_1, \lambda_2)$ – JLC**(X) and B is either λ_2 - closed or λ_1 –open subset of $(X, \lambda_1, \lambda_2)$, then $A \cap B \in (\lambda_1, \lambda_2)$ – JLC**(X).

Proof: Let $A \in (\lambda_1, \lambda_2)$ – JLC**(X). This implies $A = G \cap F$, where G is λ_1 – J- open and F is λ_2 – J- closed in (X,λ_1, λ_2). Now $A \cap B = (G \cap F) \cap B$. If B is λ_1 - open, then $B \cap G$ is also λ_1 – open. Hence $A \cap B \in (\lambda_1, \lambda_2)$ – JLC**(X). If B is λ_2 - closed, then $B \cap F$ is λ_2 – J- closed. Therefore $A \cap B \in (\lambda_1 , \lambda_2)$ – JLC**(X).

4. Pairwise JLC-continuous maps

We introduce new class of LC-continuous maps namely, pair-wise JLC-continuous maps, JLC*-continuous maps, JLC**- continuous maps, JLC-irresolute maps, JLC*-irresolute maps, JLC**-irresolute maps and investigate some of their properties.

Definition 4.1. A map $f:((X, \lambda_1, \lambda_2) \to (Y, \sigma_1,\sigma_2)$ is called pair-wise JLC-continuous(resp. pair-wise JLC*-continuous, pair-wise JLC**-continuous) if $f^{-1}(V) \in(\lambda_1, \lambda_2)$ – JLC(X)(resp. $f^{-1}(V) \in(\lambda_1, \lambda_2)$ – JLC*(X), $f^{-1}(V) \in(\lambda_1,\lambda_2)$ – JLC**(X)) for every σ_1 – closed set V in (Y, σ_1,σ_2).

Definition 4.2. A map $f:((X, \lambda_1, \lambda_2) \to (Y, \sigma_1,\sigma_2)$ is called pair-wise JLC-irresolute(resp. pair-wise JLC*-irresolute, pair-wise JLC**-irresolute) if $f^{-1}(V) \in(\lambda_1, \lambda_2)$ – JLC(X)(resp. $f^{-1}(V) \in(\lambda_1, \lambda_2)$ – JLC*(X), $f^{-1}(V) \in(\lambda_1, \lambda_2)$ – JLC**(X)) for every $V \in (\sigma_1,\sigma_2)$ - JLC(Y) (resp, . $V \in (\sigma_1,\sigma_2)$ - JLC*(Y) , $V \in (\sigma_1,\sigma_2)$ - JLC**(Y)).

Proposition 4.3. Let $f : ((X, \lambda_1, \lambda_2) \to (Y, \sigma_1,\sigma_2))$ be a map . Then the following hold.

(i) If f is pair-wise LC-continuous, then it is pair-wise JLC- continuous (pair-wise JLC*-continuous, pair-wise JLC**-continuous).

(ii) If f is pair-wise JLC*-continuous, then it is pair-wise JLC-continuous.

(iii) If f is pair-wise JLC**-continuous, then it is pair-wise JLC-continuous.

(iv) If f is pair-wise JLC - irresolute, then it is pair-wise JLC-continuous.

(v) If f is pair-wise JLC* - irresolute, then it is pair-wise JLC*-continuous.

Proof: Obvious

Proposition 4.4. Let $f : ((X, \lambda_1, \lambda_2) \to (Y, \sigma_1,\sigma_2))$ and $g: (Y, \sigma_1,\sigma_2) \to (Z, \eta_1,\eta_2))$ are functions . Then the following statements hold.

(i) If f is pair-wise JLC- irresolute and g is pair-wise JLC-continuous, then $g \circ f$ is pair-wise JLC-continuous.

(ii) If f and g are pair-wise JLC- irresolute , then $g \circ f$ is pair-wise JLC-irresolute.

(iii) If f is pair-wise JLC*- irresolute and g is pair-wise JLC*-continuous, then $g \circ f$ is pair-wise JLC*-continuous.

(iv) If f is pair-wise JLC**- irresolute and g is pair-wise JLC**-continuous, then $g \circ f$ is pair-wise JLC**-continuous.

Proof: Obvious.

Remark 4.5. The composition of pairwise JLC- continuous maps need not be pairwise JLC- continuous.

5 $\lambda_{(m, n)}$ – Jg - Locally closed sets

Definition:5.1. A subset A of a space (X, λ) is called J – generalized closed (briefly Jg-closed) if $c_J(A) \subset U$ whenever $A \subset U$, U is open in (X, λ).

Definition:5.2. A subset A of a space (X, λ) is called generalized J – closed (briefly gJ-closed) if $c_J(A) \subset U$ whenever $A \subset U$, U is J - open in (X, λ).

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Definition 5.3. A subset A of a bigeneralized topological space $(X, \lambda_1, \lambda_2)$ is said to be $\lambda_{(m, n)}$ – Jg- Locally Closed *set* (briefly($\lambda_{(m, n)}$) - Jglc) if A= S $\cap$ F, where S is a λ_m – Jg- open and F is λ_n – Jg- closed in $(X, \lambda_1, \lambda_2)$, where m, n = 1,2 and m$\neq$n.

Definition 5.4 A subset A of a bigeneralized topological space $(X, \lambda_1, \lambda_2)$ is said to be $\lambda_{(m, n)}$ – Jglc*) if there exists a λ_m – Jg- open set S and λ_n – closed set F of $(X, \lambda_1, \lambda_2)$ such that A= S $\cap$ F, where m, n = 1,2 and m$\neq$n.

Definition 5.5 A subset A of a bigeneralized topological space $(X, \lambda_1, \lambda_2)$ is said to be $\lambda_{(m, n)}$ – *Jglc***) if exists a λ_m – open set S and λ_n –Jg- closed set F of $(X, \lambda_1, \lambda_2)$ such that A= S $\cap$ F, where m, n = 1,2 and m$\neq$n.

The collection of all $\lambda_{(m, n)})$ - Jglc (resp. $\lambda_{(m, n)})$ - Jglc*, $\lambda_{(m, n)})$ - Jglc**) sets of $(X, \lambda_1, \lambda_2)$ will be denoted by- JGLC(X) (resp. $\lambda_{(m, n)})$ - JGLC*(X), $\lambda_{(m, n)})$ - JGLC**(X)).

Proposition 5.6: Let A be subset of a space $(X, \lambda_1, \lambda_2)$,

(i)If A$\in$ $\lambda_{(m, n)}$ - *JG*LC(X), then A$\in$ $\lambda_{(m, n)}$ - JGLC*(X) and $\lambda_{(m, n)}$ - JGLC**(X).

(ii) If A$\in$ $\lambda_{(m, n)}$ - *G*LC(X), then A$\in$ $\lambda_{(m, n)}$ - JGLC(X)

(iii)(ii)If A$\in\lambda_{(m,n)}$ - JGLC*(X), then A$\in\lambda_{(m, n)})$ - JGLC(X).

(iv) If A$\in\lambda_{(m, n)}$ - JGLC**(X), then A$\in\lambda_{(m, n)})$ - JGLC(X).

Proof: By definition of locally closed set , it is the intersection of open and closed set, Every open set is J – open set and therefore it is Jglc – set.

Proposition 5.7. Let A be a subset of a space $(X, \lambda_1, \lambda_2)$, then the following are equivalent.

(i) If A $\in$ (λ_1, λ_2) - JGLC*(X).

(ii) A = G $\cap$ λ_2- c_λ(A) for some λ_1 – Jg- open set G.

(iii) λ_2- c_λ (A) –A is λ_1 – Jg- closed.

(iv) A $\cup$ (X- λ_2- c_λ(A)) is λ_1 – Jg- open.

Proof: (i)$\Rightarrow$(ii): Let A $\in(\lambda_1, \lambda_2)$ - JGLC*(X). Then there exist λ_1 – Jg- open set G and a λ_2 – closed set F in $(X, \lambda_1, \lambda_2)$ such that A = G $\cap$ F. Since A $\subseteq$ G and A $\subseteq$ λ_2- c_λ(A), we have A $\subseteq$ G$\cap$ λ_2- c_λ(A). Conversely, since λ_2- c_λ(A) $\subseteq$ F, G$\cap$ λ_2- c_λ(A) $\subseteq$ G $\cap$ F = A. Therefore A = G $\cap$ λ_2- c_λ(A).

(ii)$\Rightarrow$(i): Since G is λ_1 – Jg- open set and λ_2- c_λ(A) is a λ_2 – closed set, we have A = G

$\cap \lambda_2$- $c_\lambda(A) \in (\lambda_1, \lambda_2)$ - JGLC*(X).

(iii)⇒(iv): Let F = λ_2- $c_\lambda(A)$ –A. Then by (iii), F is λ_1 – Jg- closed. Now X – F = A ∪ (X -λ_2- $c_\lambda(A)$) . Since (X - F) is λ_1 – Jg- open ,we get A ∪ (X - λ_2- $c_\lambda(A)$) is λ_1 – Jg- open.

(iv)⇒(iii): Let G = A ∪ (X -λ_2- $c_\lambda(A)$). This implies that X- G is λ_1 – Jg- closed and X- G = λ_2- $c_\lambda(A)$ –A . Hence λ_2- $c_\lambda(A)$ – A is λ_1 – Jg- closed.

(iv)⇒(ii): Let G = A ∪ (X -λ_2- $c_\lambda(A)$). Then G is Jg-open. Hence we prove that A = G ∩ λ_2- $c_\lambda(A)$ for some λ_1 – Jg-open set G. G ∩ λ_2- $c_\lambda(A)$ = A ∪ (X -λ_2- $c_\lambda(A)$) ∩ λ_2- $c_\lambda(A)$) = A ∩ λ_2- $c_\lambda(A)$) ∪ [X- λ_2- $c_\lambda(A)$) ∩ λ_2- $c_\lambda(A)$] = A ∩ φ = A. Hence A = G ∩ λ_2- $c_\lambda(A)$.

(ii)⇒(iv): Let A = G ∩ λ_2- $c_\lambda(A)$ for some λ_1 – J- open set G. Then A ∪ (X -λ_2- $c_\lambda(A)$) = G ∩ A = G ∩ λ_2- $c_\lambda(A)$ ∪ (X - A = G ∩ λ_2- $c_\lambda(A)$) = G ∩ X = G which is λ_1 – J- open. Thus A ∪ (X- λ_2- $c_\lambda(A)$) is λ_1 – Jg- open.

6. J - Submaximal Spaces

Definition 6.1. A bigeneralized topological space (X, λ_1, λ_2) is

(i) λ_1, λ_2 – submaximal space if every λ_1 – dense subset of X is λ_2 – open in X.

(ii) λ_2, λ_1 – submaximal space if every λ_2 – dense subset of X is λ_1 – open in X.

(iii) λ_1, λ_2 – J - submaximal space if every λ_1 – dense subset of X is λ_2 – J - open in X.

(iv) λ_2, λ_1 – J - submaximal space if every λ_2 – dense subset of X is λ_1 – J - open in X.

Proposition 6.2. If (X,λ_1, λ_2) is λ_1, λ_2 – submaximal space then it is λ_1, λ_2 – J - sub-maximal space.

Proof: Since (X,λ_1, λ_2) is λ_1, λ_2 – submaximal space, we have every λ_1 – dense subset of X is λ_2 – open in X. Since every λ_2 – open in X is λ_2 – J - open in X, we have λ_1 – dense subset of X is λ_2 – open in X. Therefore (X,λ_1, λ_2) is λ_1, λ_2 –J- submaximal space.

Proposition 6.3. A bigeneralized topological space (X, λ_1, λ_2) is λ_1, λ_2 – J - submaximal space if and only if λ_2, λ_1 – JLC*(X, λ_1, λ_2) = P(X).

Proof: Suppose that (X,λ_1, λ_2) is λ_1, λ_2 – J- submaximal space. Obviously λ_2, λ_1 – JLC*(X, λ_1, λ_2) ⊂ P(X). Let A ∈ P(X) and U = A ∪ {X – [λ_1- $c_\lambda(A)$]}. Since λ_1- $c_\lambda(U)$ = X, we have U is λ_1 – dense subset of X. Since (X, λ_1, λ_2) is λ_1, λ_2 – J - submaximal space, we have U is λ_2 – J – open in X. Since every λ_2 – J – open in X is λ_2, λ_1 – JLC* set in (X, λ_1, λ_2), we have

$U \in \lambda_2, \lambda_1 - JLC^*(X, \lambda_1, \lambda_2)$. Therefore $P(X) \subset \lambda_2, \lambda_1 - JLC^*(X, \lambda_1, \lambda_2)$. Hence $\lambda_2, \lambda_1 - JLC^*(X, \lambda_1, \lambda_2) = P(X)$.

Conversely, suppose that,(λ_2, λ_1)– $JLC^*(X, \lambda_1, \lambda_2) = P(X)$. Let A be the λ_1 – dense subset of $(X, \lambda_1, \lambda_2)$. Then $A \cup \{X - [\lambda_1\text{- } c_\lambda(A)]\} = A \cup [\lambda_1\text{- } c_\lambda(A)]^c = A$. Therefore $A \in \lambda_2, \lambda_1 - JLC^*(X, \lambda_1, \lambda_2)$ implies that A is λ_2 – J – open in X. Hence X is a λ_1, λ_2 – J - submaximal space.

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