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Almost *v*-continuity

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Abstract: The object of the paper is to study basic properties of Almost *v*-continuity.

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1. Introduction:

In 1963 M. K. Singhal and A. R. Singhal introduced Almost continuous mappings. In 1980, Joseph and Kwack introduced the notion of (θ, s)-continuous functions. In 1982, Jankovic introduced the notion of almost weakly continuous functions. Dontchev, Ganster and Reilly introduced a new class of functions called regular set-connected functions in 1999. Jafari introduced the notion of (p, s)-continuous functions in 1999. T. Noiri and V. Popa studied some properties of almost-precontinuity in 2005 and unified theory of almost-continuity in 2008. E. Ekici introduced almost-precontinuous functions in 2004 and recently have been investigated further by Noiri and Popa. Ekici E., introduced almost-precontinuous functions in 2006. Ahmad Al-Omari and Mohd. Salmi Md. Noorani studied Some Properties of almost-b-Continuous Functions in 2009. Recently S. Balasubramanian, C. Sandhya and P.A.S.Vyjayanthi introduced *v*-continuous functions in 2010. Inspired with these developments, we introduce almost-*v*-continuous functions, obtain basic properties, preservation Theorems.

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2. Preliminaries:

Definition 2.1: $A\subset X$ is said to be

(i) regular open[pre-open; semi-open; α-open; β-open] if $A = \text{int}(cl(A))[A\subseteq \text{int}(cl(A); A\subseteq cl(\text{int}(A)); A\subseteq \text{int}(cl(\text{int}(A))); A\subseteq cl(\text{int}(cl(A))]$.

(ii) v-open[rα-open] if $\exists$ a regular open set O such that $O\subset A\subset cl(O)[O\subset A\subset \alpha cl(O)]$

(iii) θ-closed[θ-semi-closed] if $A = Cl_\theta(A) = \{x\in X: cl(V)\cap A \neq \phi;$ for every $V\in\tau\}[A = sCl_\theta(A) = \{x\in X: cl(V)\cap A \neq \phi;$ for every $V\in SO(X, x)\}]$. The complement of a θ-closed[θ-semi-closed] set is said to be θ-open[θ-semi-open]. $Cl_\theta(A)[sCl_\theta(A)]$ is θ-closure [θ-semi-closure] of A.

(iv) v-dense in X if $vcl(A) = X$.

(v) The v-frontier of $A\subset X$; is defined by $v\,Fr(A) = vcl(A)-vcl(X-A) = vcl(A)-vint(A)$.

It is shown that $Cl_\theta(V) = cl(V)$ for every $V\in\tau$ and $Cl_\theta(S)$ is closed in X for every $S\subset X$.

Definition 2.2: A cover $\Sigma = \{U_\alpha: \alpha \in I\}$ of subsets of X is called a v-cover if U_α is v-open for each $\alpha\in I$.

Definition 2.3: A filter base Λ is said to be v-convergent (resp. rc-convergent) to a point x in X if for any $U\in vO(X, x)$(resp. $U\in RC(X, x)$), there exists a $B\in\Lambda$ such that $B\subset U$.

Definition 2.4: A function $f: X \to Y$ is called

(i) almost-[resp: almost-semi-; almost-pre-;almost-rα-; almost-α-; almost-β-; almost-ω-; almost-pre-semi-; almost-λ-]continuuos if $f^{-1}(V)$ is open[resp: semi-open; pre-open; rα-open; α-open; β-open; ω-open; pre-semi-open; λ-open] in X for every $V\in RO(Y)$.

(ii) regular set-connected if inverse image of every regular open set V in Y is clopen in X.

(iii) perfectly continuous inverse image of every open set V in Y is clopen in X.

(iv)almost s-continuous if for each $x\in X$ and each $V\in SO(Y)$ with $f(x) \in V$, there exists an open set U in X containing x such that $f(U) \subset scl(V)$.

(v) (p, s)-continuous(resp. (θ, s)-continuous) if for each $x \in X$ and each $V \in SO(Y, f(x))$, there exists $U \in PO(X, x)$ (resp. $U \in \tau$ containing x) such that $f(U) \subset Cl(V)$.

(vi) weakly continuous if for each $x \in X$ and each open set $V \in \sigma(Y), f(x))$, there exists an open set U of X containing x such that $f(U) \subset cl(V)$.

(vii) (θ, s)-continuous iff for each θ-semi-open set V of Y, $f^{-1}(V)$ is open in X.

3. Almost *v*-Continuous Functions:

Definition 3.1: A function f: X→ Y is said to be Almost *v*-continuous if the inverse image of every regular open set is *v*-open.

Note 1: Here onwards we call almost *v*-continuous as al.*v*.c., briefly.

Theorem 3.1: (i) f is al.*v*.c. iff f is al.*v*.c. at each $x \in X$.

(ii) If f is *v*.c., then f is al.*v*.c. Converse is true if X is discrete space.

(iii) If f is *v*-open and al.*v*.c. mapping, then $f^{-1}(A) \in vO(X)$ for each $A \in vO(Y)$

(iv) If f is al.*v*.c. and $A \in RO(X)$, then $f_{/A}$ is al.*v*.c.

Theorem 3.2: f is al.*v*.c. iff for every $x \in X$ and $U_Y \in vO(Y, f(x))$ $[U \in RO(Y, f(x))]$, $\exists$ $A \in vO(X, x)$ such that $f(A) \subset U$ $[f(A) \subset U_Y]$.

Proof: Let $U_Y \in RO(Y)$ and let $x \in f^{-1}(U_Y)$. Then $f(x) \in U_Y$ and thus there exists $A_x \in vO(X, x)$ and $f(A_x) \subset U_Y$. Then $x \in A_x \subset f^{-1}(U_Y)$ and $f^{-1}(U_Y) = \cup A_x$. Hence $f^{-1}(U_Y) \in vO(X)$.

Theorem 3.3: Let f_i: $X_i \to Y_i$ be al.*v*.c. for i = 1, 2. Let f: $X_1 \times X_2 \to Y_1 \times Y_2$ be defined as follows: $f(x_1, x_2) = (f_1(x_1), f_2(x_2))$. Then f: $X_1 \times X_2 \to Y_1 \times Y_2$ is al.*v*.c.

Theorem 3.4: Let $h: X \to X_1 \times X_2$ be al.*v*.c., where $h(x) = (h_1(x), h_2(x))$. Then $h_i: X \to X_i$ is al.*v*.c. for i = 1, 2.

In general we have the following extension of theorems 3.3 and 3.4:

Theorem 3.5: (i) f:$\Pi X_\lambda \to \Pi Y_\lambda$ is al.v.c, iff f_λ: $X_\lambda \to Y_\lambda$ is al.v.c for each $\lambda \in \Lambda$.
(ii)If f:$X \to \Pi Y_\lambda$ is al.v.c, then $P_\lambda \bullet f$: $X \to Y_\lambda$ is al.v.c for every $\lambda \in \Lambda$; P_λ:ΠY_λ onto Y_λ.

Note 2: With respect to usual topology on $\mathfrak{R}$, open sets and regular open sets are one and the same. So converse of theorem 3.5 is not true in general, as shown by.

Example 1: Let X = X_1 = X_2 = [0, 1]. Let f_1:$X \to X_1$ and f_2:$X \to X_2$ are defined as follows: f_1(x) = 1 if $0 \leq x \leq 1/2$ and f_1(x) = 0 if $1/2 < x \leq 1$. f_2(x) = 1 if $0 \leq x < 1/2$ and f_2(x) = 0 if 1/2 < x < 1. Then f_i: $X \to X_i$ is clearly al.v.c. for i = 1, 2., but h(x) = ($f_1(x_1)$, $f_2(x_2)$):$X \to X_1 \times X_2$ is not al.v.c., for $S_{1/2}(1, 0) \in RO(X_1 \times X_2)$, but $h^{-1}(S_{1/2}(1, 0)) = \{1/2\} \notin vO(X)$.

Remark 1: In general, (i) The algebraic sum; product and composition of two al.v.c. functions is not al.v.c. However the scalar multiple of al.v.c. function is al.v.c.
(ii)The pointwise limit of a sequence of al.v.c. functions is not al.v.c.
(iii) al.v.c. function of al.v.c. function is not al.v.c. as shown by the following examples.

Example 2: Let X = X_1 = X_2 = [0, 1]. Let f_1:$X \to X_1$ and f_2:$X \to X_2$ are defined as follows: f_1(x) = x if 0 < x < 1/2 and f_1(x) = 0 if 1/2 < x < 1; f_2(x) = 0 if 0 < x < 1/2 and f_2(x) = 1 if 1/2 < x < 1. Then their product is not al.v.c.

Example 3: Let X = Y = [0, 1]. Let f_n defined on X as follows: $f_n(x) = x_n$ for $n \geq 1$ then f is the limit of the sequence where f(x) = 0 if $0 \leq x < 1$ and f(x) = 1 if x = 1. Therefore f is not al.v.c. For (1/2, 1] is v-open in Y, f^{-1}((1/2, 1]) = (1) is not v-open in X.

However we can prove the following theorem.
Theorem 3.6: Uniform Limit of a sequence of al.v.c. functions is al.v.c.

Problem: (i) Are sup{f, g} and inf{f, g} are al.v.c if f, g are al.v.c
(ii) Is $C_{al.v.c}$(X, R), the set of all al.v.c functions,

(1) a Group. (2) a Ring. (3) a Vector space. (4) a Lattice.

Solution: No.

Example 4: Let X = Y = [0, 1]. Let *f*: X→ Y be defined as follows: *f*(x) = 1 if $0 \le x < 1/2$ and *f*(x) = 0 if $1/2 < x \le 1$. Then obviously *f* is al.*v*.c. but not r-continuous.

Example 5: Let X = Y = {a, b, c}; τ = {ϕ, {a}, {b}, {a, b}, X} and σ = {ϕ, {a}, {b}, {a, b}, {a, c}, X}. Then the identity map *f*:X→Y is al.s.c., and al.*v*.c. but not al.c., and r-irresolute.

Example 6: Let X = Y = {a, b, c}; τ = {ϕ, {b}, {a, b}, {b, c}, X} and σ = {ϕ, {a}, {b}, {a, b}, {a, c}, X} .Let *f*: X→Y be defined as *f*(a) = b; *f*(b) = a; *f*(c) = c is not al.s.c., al.c., al.*v*.c., and r-irresolute.

under usual topology on $\Re$ both continuous and nearly-continuous are same as well both al.s.c. and al.*v*.c. are same. In general, al.c and al.*v*.c. maps are independent to each other.

Theorem 3.7: (i) If RαO(X) = *v*O(X) then *f* is al.rα.c. iff *f* is al.*v*.c.

(ii) If *v*O(X) = RO(X) then *f* is al.*v*.c. iff *f* is r-irresolute.

(iii)If *v*O(X) = αO(X) then *f* is al. α.c. iff *f* is al.*v*.c.

(iv)If *v*O(X) = SO(X) then *f* is al.s.c. iff *f* is al.*v*.c.

(v) If *v*O(X) = βO(X) then *f* is al. β.c. iff *f* is al.*v*.c.

Theorem 3.8: (i) If *f* is al.*v*.c. and *g* is r-irresolute then *g*•*f* is al.*v*.c.

(ii) If *f* and *g* are r-irresolute then *g*•*f* is al.*v*.c.

(iii)If *f* is *v*.c.[al.*v*.c.]; *g* is al.g.c.[al.rg.c.] and Y is $T_{1/2}$[$rT_{1/2}$], then *g*•*f* is al.*v*.c.

(iv) If *f* is al.*v*.c.;[resp: *v*.c.;] *g* is g.c.[rg.c.] and every g-open set[rg-open] in Y is r-open, then *g*•*f* is al.*v*.c.

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Theorem 3.9: If f is v-irresolute, v-open and vO(X) = τ and g be a function, then $g\bullet f$ is al.v.c iff g is al.v.c.

Definition 3.2: f is said to be M-v-open if f(V) is v-open in Y whenever V is v-open in X.

Example 7: Let X = Y = $\Re$ with usual topology and f be defined by f(x) = 1 for all x$\in$ X then X is v-open in X but f(X) is not v-open in Y.

Theorem 3.10: Let X, Y, Z be spaces and every v-open set is r-open in Y, then the composition of two al.v.c.[resp:v-continuous] maps is al.v.c.

Corollary 3.1: (i)If f be r-open, al.v.c. and g be al.v.c., then $g\bullet f$ is al.v.c.

(ii) If f is v-irresolute, M-v-open and bijective, g is a function. Then g is al.v.c. iff $g\bullet f$ is al.v.c.

(iii) If f is al.v.c. and g is r-irresolute then $g\bullet f$ is al.s.c. and al.β.c.

(iv) If f is v.c.,[r.c.,];g is al.g.c.,[al.rg.c.,] and Y is $T_{1/2}$[r$T_{1/2}$], then $g\bullet f$ is al.s.c. and al.β.c.

Note 3: Pasting Lemma is not true with respect to al.v.c. functions. However we have the following weaker versions.

Theorem 3.11: Pasting Lemma: Let X; Y be such that X = A$\cup$B. Let $f_{/A}$ and $g_{/B}$ are al.v.c.[resp: r-irresolute] such that f(x) = g(x) for every x$\in$A$\cap$B. If A, B$\in$RO(X) and vO(X)[resp: RO(X)] is closed under finite unions, then the combination α:X$\to$Y is al.v.c.

4. Further Results on almost-v-continuous functions:

Theorem 4.1: The following statements are equivalent for a function f:

(1) f is al.v.c.;

(2) f^{-1}(F) $\in v$C(X) for every F$\in$ RC(Y);

(3) for each x$\in$X and each F$\in$ RC(Y,f(x)), there exists U$\in$ v C(X,x) such that f(U) $\subset$ F;

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(4) for each $x \in X$ and each $F \in RO(Y)$ non-containing $f(x)$, there exists $K \in vO(X)$ non-containing x such that $f^{-1}(V) \subset K$;

(5) $f^{-1}(\text{int}(cl(G))) \subset vO(X)$ for every r-open subset G of Y;

(6) $f^{-1}(cl(\text{int}(F))) \subset vC(X)$ for every r-closed subset F of Y.

Example 8: Let $X = \{a, b, c\}$, $\tau = \sigma = \{\phi, \{b\}, \{c\}, \{b, c\}, X\}$. Then the identity function f on X is al.v.c. But it is not regular set-connected.

Example 9: Let $X = \{a, b, c\}$, $\tau = \{\phi, X\}$ and $\sigma = \{\phi, \{a\}, X\}$. The identity function f on X and f defined as $f(a) = b; f(b) = c; f(c) = a$ are al.v.c. function which is not c.v.c., and v.c.

Remark 2: Every restriction of an al.v.c. function is not necessarily al.v.c.

Theorem 4.2: Let f be a function and $\Sigma = \{U_\alpha : \alpha \in I\}$ be a v-cover of X. If for each $\alpha \in I$, $f_{|U\alpha}$ is al.v.c., then f is an al.v.c.

Proof: Let $F \in RO(Y)$. $f_{|U\alpha}$ is al.v.c. for each $\alpha \in I$, $f_{|U\alpha}^{-1}(F) \in vO_{|U\alpha}$. Since $U_\alpha \in vO(X)$, $f_{|U\alpha}^{-1}(F) \in vO(X)$ for each $\alpha \in I$. Then $f^{-1}(F) = \cup_{\alpha \in I} f_{|U\alpha}^{-1}(F) \in vO(X)$. Thus f is al.v.c.

Theorem 4.3: Let f be a function and $x \in X$. If $\exists\ U \in RO(X, x)$ and $f_{|U}$ is al.v.c. at x, then f is al.v.c. at x.

Proof: Let $F \in RO(Y, f(x))$. Since $f_{|U}$ is al.v.c. at x, there exists $V \in vO(U, x)$ such that $f(V) = (f_{|U})(V) \subset F$. Since $U \in RO(X, x)$, it follows that $V \in vO(X, x)$. Therefore f is al.v.c. at x.

Theorem 4.4: Let f be a function and let $g: X \to X \times Y$ be the graph function of f, defined by $g(x) = (x, f(x))$ for every $x \in X$. If g is al.v.c., then f is al.v.c.

Proof: Let $V \in RC(Y)$, then $X \times V = X \times cl(\text{int}(V)) = cl(\text{int}(X)) \times cl(\text{int}(V)) = cl(\text{int}(X \times V)) \in RC(X \times Y)$. Since g is al.v.c., then $f^{-1}(V) = g^{-1}(X \times V) \in vC(X)$. Thus, f is al.v.c.

Theorem 4.5: For f and g. The following properties hold:

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(1) If f is al.v.c.[c.v.c.] and g is regular set-connected, then $g\bullet f$ is al.v.c.

(2) If f is al.v.c. and g is perfectly continuous, then $g\bullet f$ is v.c. and c.v.c.

Theorem 4.6: If f is a surjective M-v-open[resp:M-v-closed] and g is a function such that $g\bullet f$ is al.v.c., then g is al.v.c.

Theorem 4.7: If f is al.v.c., then for each point $x\in X$ and each filter base Λ in X v-converging to x, the filter base $f(\Lambda)$ is rc-convergent to $f(x)$.

Definition 4.2: A function f is called (v, s)-continuous if for each $x\in X$ and each $V\in SO(Y, f(x))$, there exists $U\in v\,O(X, x)$ such that $f(U) \subset cl\{V\}$.

Theorem 4.8: For f, the following properties are equivalent:

(1) f is (v, s)-continuous;

(2) f is al.v.c.;

(3) $f^{\,1}(V)$ is v-open in X for each θ-semi-open set V of Y;

(4) $f^{-1}(F)$ is v-closed in X for each θ-semi-closed set F of Y.

Theorem 4.9: For f, the following properties are equivalent:

(1) f is al.v.c.;

(2) $f(v(cl\,A)) \subset sCl_\theta(f(A))$ for every subset A of X;

(3) $vcl\{(f^{\,1}(B))\} \subset f^{-1}(sCl_\theta(B))$ for every subset B of Y.

5. The preservation theorems:

Theorem 5.1: If f is al.v.c.[r-irresolute] surjection and X is v-compact, then Y is compact[resp: nearly compact].

Theorem 5.2: If f is al.v.c.[r-irresolute], surjection. Then the following statements hold:

(i) If X is v-compact[v-lindeloff; s-closed] then Y is mildly compact[mildly lindeloff].

(ii) If X is locally *v*-compact, then Y is locally compact[locally nearly compact; locally mildly compact.]

(iii)If X is *v*-Lindeloff[locally *v*-lindeloff], then Y is Lindeloff[resp: locally Lindeloff; nearly Lindeloff; locally nearly Lindeloff; locally mildly lindeloff].

(v) If X is *v*-compact[resp: countably *v*-compact], then Y is S-closed[resp: countably S-closed].

(vi) If X is *v*-Lindelof, then Y is S-Lindelof[resp: nearly Lindeloff].

Theorem 5.3: If *f* is an r-irresolute and al.c. surjection and X is mildly compact (resp. mildly countably compact, mildly Lindelof), then Y is nearly compact (resp. nearly countably compact, nearly Lindelof) and S-closed (resp. countably S-closed, S-Lindelof).

Theorem 5.4: (i) If *f* is al.*v*.c.[contra *v*-irreolute] surjection and X is *v*-connected, then Y is connected[*v*-connected]

(ii) If X is *v*-ultra-connected and *f* is al.*v*.c. and surjective, then Y is hyperconnected.

(iii) The inverse image of a disconnected[*v*-disconnected] space under al.*v*.c.,[contra *v*-irreolute] surjection is *v*-disconnected.

Theorem 5.6: If *f* is al.*v*.c., injection and

(i) Y is UT_i[resp: UC_i; UD_i], then X is *v* T_i[resp:*v* C_i; *v*D_i] and hence semi T_i[resp: semi C_i; semi D_i] and βT_i[resp: βC_i; βD_i] i = 0,1,2.

(ii) Y is UR_i, then X is *v*-R_i[hence semi-R_i and βR_i] i = 0, 1.

(iii)If *f* is closed, Y is UT_i, then X is *v*-T_i[hence semi-T_i and βT_i] i = 3, 4.

Theorem 5.7: (i) If *f* is al.*v*.c.[resp: al.r.c] and Y is UT_2, then the graph G(*f*) of *f* is *v*-closed[resp:semi-closed; β-closed and semi-θ-closed] in X× Y.

(ii) If *f* is al.*v*.c.[al.r.c] and Y is UT_2, then A = $\{(x_1, x_2) | f(x_1) = f(x_2)\}$ is *v*-closed[and hence semi-closed and β-closed] in X× X.

(iii) If *f* is r-irresolute{al.c.}; *g* is c.*v*.c; and Y is UT_2, then E = $\{x \in X : f(x) = g(x)\}$ is *v*-closed[and hence semi-closed and β-closed] in X.

(iii) If f is al.v.c. injection and Y is rT_2, then X is v-T_i; i = 0,1,2.

6. Relations to weak forms of continuity:

Definition 6.1: A function f is said to be faintly v-continuous if for each $x \in X$ and each θ-open set V of Y containing $f(x)$, there exists $U \in vO(X, x)$ such that $f(U) \subset V$.

Example 10: Let X = {a, b, c}, τ = {ϕ, {a, b}, X} and σ = {ϕ, {a}, {b, c}, X}. Then, the identity function f is not al.v.c and is not weakly continuous.

Example 11: Let X = {a, b, c}, τ = {ϕ, {a}, {a, b}, {a, c}, X} and σ = {ϕ, {a}, {a, b}, X}. Then, the identity function f is (θ, s)-continuous and al.v.c.

Example 12: Let $\Re$ be the reals with the usual topology and f: $\Re \to \Re$ the identity function. Then f is continuous; weakly continuous; al.p.c., and al.v.c.

Example 13: Let X = {a, b, c}, τ = {ϕ, {a}, {b}, {a, b}, {a, c}, X} and σ = {ϕ, {b}, {c}, {b, c}, X}. Then the identity function f on X is c.c., c.s.c., but it is not al.v.c.

Corollary 6.1: If f is M-v-open and c.v.c., then f is al.v.c.

Lemma 6.1: For f, the following properties are equivalent:
(1) f is faintly-v-continuous;
(2) $f^{-1}(V) \in vO(X)$ for every θ-open set V of Y;
(3) $f^{-1}(K) \in vC(X)$ for every θ-closed set K of Y.

Theorem 6.1: If for each $x_1 \neq x_2 \in X$ there exists a function f of X into a Urysohn space Y such that $f(x_1) \neq f(x_2)$ and f is al.v.c., at x_1 and x_2, then X is v-T_2.

Proof: Let $x_1 \neq x_2$. By hypothesis, $\exists\ V_i \in (\sigma, f(x_i))$ s.t., $\cap cl(V_i) = \phi$ for i = 1,2. For f is al.v.c., at x_i, $\exists\ U_i \in vO(X, x_i)$ s.t., $f(U_i) \subset cl(V_i)$ for i = 1, 2., and $\cap U_i = \phi$. Hence X is vT_2.

Corollary 6.2: If f is al.v.c. injection and Y is Urysohn, then X is v T_2.

Theorem 6.2: $\{x\in X: f$ is not al.v.c.$\}$ is identical with the union of the v-frontier of the inverse images of regular closed sets of Y containing $f(x)$.

Proof: If f is not al.v.c. at $x\in X$. By Theorem 4.1,, $\exists$ $F\in RC(Y, f(x))$ s.t., $f(U)\cap(Y -F) \neq \phi$ for every $U\in vO(X, x)$. Then $x\in vcl(f^{-1}(Y -F)) = vcl(X - f^{-1}(F))$. On the other hand, we get $x\in f^{-1}(F) \subset vcl(f^{-1}(F))$ and hence $x\in v\,Fr(f^{-1}(F))$.

Conversely, If f is al.v.c. at x and $F\in RO(Y, f(x))$. By Thm. 4.1, $\exists$ $U\in vO(X, x)$ s.t $x\in U\subset f^{-1}(F)$. Hence $x\in vint(f^{-1}(F))$, which contradicts $x\in vFr(f^{-1}(F))$. Thus f is not al.v.c.

Definition 6.2: A function f is said to have a strongly contra-v-closed graph if for each $(x, y)\in(X\times Y) - g(f)$ there exists $U\in vO(X, x)$ and $V\in RC(Y,y)$ such that $(U\times V)\cap\{g(f)\} = \phi$.

Lemma 6.2: f has a strongly contra-v-closed graph iff for each $(x, y) \in (X\times Y) - g(f)$ $\exists$ $U\in vO(X, x)$ and $V\in RC(Y,y)$ such that $f(U)\cap V = \phi$.

Theorem 6.3: If f is al.v.c. and Y is Hausdorff, then $g(f)$ is strongly contra-v-closed.

Proof: If $(x, y) \in (X\times Y)-g(f)$, then $y \neq f(x)$. Since Y is T_2, $\exists$ $V\in(\sigma, y)$ and $W\in(\sigma, f(x))$, s.t., $V\cap W = \phi$; hence $cl(V)\cap int(cl(W)) = \phi$. Since f is al.v.c., by Lemma 6.3 $\exists$ $U\in vO(X, x)$ s.t., $f(U)\subset int(cl(W))$. Thus $f(U)\cap cl(V) = \phi$ and hence $g(f)$ is strongly contra$v$-closed.

Theorem 6.4: If f is injective al.v.c. with strongly contra-v-closed graph, then X is vT_2.

Proof: Let $x \neq y\in X$. Since f is injective, we have $f(x) \neq f(y)$ and $(x, f(y)) \in (X\times Y) - g(f)$. Since $g(f)$ is strongly contra-v-closed, by Lemma 6.2 there exists $U\in vO(X, x)$ and $V\in RC(Y, f(y))$ such that $f(U)\cap V = \phi$. Since f is al.v.c., by Theorem 4.1, there exists $G\in vO(X, y)$ such that $f(G)\subset V$. Therefore $f(U)\cap f(G) = \phi$; hence $U\cap G = \phi$. Thus X is v T_2.

Corollary 6.3: If f is al.v.c. and Y is Urysohn, then $g(f)$ is strongly contra-v-closed and contra-v-closed.

CONCLUSION: In this paper we defined almost v-continuous functions, studied its properties and their interrelations with other types of almost-continuous functions.

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