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Inverse Heat Conduction technique in estimation of Heat Transfer through Rocket Nozzle throat (Cylindrical form)

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Abstract. Here we consider one dimensional heat conduction with thermo physical properties K, ROW, Cp of a material varying with temperature. The physical problem is that of a Rocket Nozzle Throat of finite thickness L, whose outer surface is insulated with a net heat flux at inner radius. The problem is solved by using Inverse Heat Conduction Technique (IHCP).

Keywords: Inverse technique, Direct method, Tridiogonal matrix algorithm , Rocket nozzle throat, Heat transfer characteristics.

1 Introduction

If the heat flux or Temperature histories at the surface of the solid are known as functions of time, then internal temperature distribution can be found. This is termed as direct problem. In many dynamic heat transfer problems the surface heat flux and temperature histories of a solid must be determined from transient temperature measurements at one or more locations. This is known as inverse problem. The performance of a rocket depends on the throat section of the system. In a rocket missile system, the missile is moving at high speeds due to high thrust developed by the exhaust gases leaving the nozzle. A insulator insert is used at the throat section to improve its performance.

An attempt is being made to measure the temperature distribution and heat transfer in rocket nozzle throat by direct and inverse methods.

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2 Theoretical Concepts for the solution of one Dimensional Inverse heat Conduction Problem

2.1 Function Estimation verses Parameter estimation

The word function estimation is used in connection with heat flux. In it heat flux is found to be an arbitrary, single valued function of time. The heat flux can be positive, negative, constant or abruptly changing periodic or not and so on. It may be influenced by human decisions. For example a pilot of a shuttle can change the reentry trajectory. In IHCP problem the surface heat flux is a function of time and may require hundreds of individually estimated heat flux components, q, to define it adequately. Related estimation problems are called parameter estimation problems which are inverse problems but with emphasis on the estimation of certain parameters or constants or physical properties. In the context of heat conduction one might be interested in determining the thermal conductivity of a body given some internal temperature histories and the surface heat flux and other boundary conditions. The thermal conductivity of iron near room temperature for example could be a parameter. It is not a function and does not require hundreds of values of K to describe it. The parameter estimation and function estimation problems start to merge if estimates are made of the thermal conductivity, K as a function of temperature. However, K(T) function is not arbitrary and is not adjustable by humans.

2.2 Measurements

In the IHCP problem there are a number of measured quantities in addition to temperature such as time, sensor location and specimen thickness. Each is assumed to be accurately known except the temperature. If this is not true, then it may be necessary, for example to simultaneously estimate sensor location and the surface heat flux. The latter problem would involve both inverse heat conduction and parameter estimation problem. If the thermal properties are not accurately known, they should be determined as accurately as possible using parameter estimation technique. The temperature measurements are assumed to contain the major sources of error or uncertainty. Any known systematic effective due to calibration errors presence of sensors, conduction or convection losses or whatever is assumed to be removed to the extent that the remaining errors may be considered at random. These random errors can statically describe.

2.3 Why is IHCP Difficult?

The IHCP is difficult because it is extremely sensitive to measurement errors. The difficulties are pronounced as one tries to obtain maximum amount of information from the data. For the one dimensional IHCP when discrete values of q curve are estimated maximizing the amount of information implies small time steps between q values.

However the uses of small time steps frequently introduces instability in the e solution of the IHCP unless restrictions are employed that will be discussed. Notice the conditions of small time steps have the opposite effects in the IHCP compared to the numerical solution of the heat conduction equation. In the latter the stability problems often can be corrected by reducing the size of time steps.

2.4 Damping & Lagging Effects

The transient temperature response of an internal point in an opaque heat conducting body is quite different from that of a point at the surface. The internal temperature excursions are much diminished internally compared to the surface changes. This is a damping effect. A large time delay or lag in the internal response can also be noted. These lagging and damping effects for the direct problem are important because they provide engineering insight into the difficulties encountered in the inverse problem.

2.5 Classification of methods

The methods for solving the inverse conduction problem can be classified in several ways, some of which are discussed in this section. One classification relates to the ability of a method to treat linear as well as non linear IHCP's. The two basic procedures given herein are the function specification and regularization methods. If the heat flux is varying with time, the method of solving IHCP is by function specification. The regularization method is a procedure which modifies the least square approach by adding factors that are intended to reduce excursions in the unknown function, such as the surface heat flux.

The method of solution of the heat conduction equation is another way to classify the IHCP. Methods of solution include the use of Duhumel's theorem, Finite elements and Finite control volumes. The use of Dhumel's theorem restricts the IHCP algorithm to a linear case, where as other two procedures can treat the non linear case also.

The domain used n IHCP can also be used to classify the method of solution. Three time domains have been proposed (1) only the present time (2) to the present time plus few time steps and (#) the complete time domain. The last classification to be mentioned is related to the dimensionality of IHCP. If a single heat flux history is to be determined, the IHCP is considered as one dimensional. In the use of Dhumel's theorem, the physical dimensions are not of concern that is the same procedure is used for physically one, two or three dimension bodies provided a single heat flux history is to be estimated. If two or more heat flux histories are estimated and Dhumel's theorem is used, the problem is multidimensional. When the Finite difference or other methods are used for non linear, the dimensionality of the problem depends on the number of space coordinates needed to describe a heat conducting body, one coordinate would give one dimension problem and so on.

2.6 Sensitivity Coefficients

In function estimation as in parameter estimation a detailed examination of sensitivity coefficients can provide considerable insight into the estimation problem. These coefficients can show the possible areas of difficulty also lead to experimental design. The sensitivity coefficients are defined as first derivative of the dependent variable such as a heat flux component. If the sensitivity coefficients are either small or correlated to one another, the estimation problem is difficult and very sensitive to estimation errors.

For the IHCP problem, the sensitivity coefficients of interest are those of the first derivatives of temperature T, at location x and time t, with respect to a heat flux component q, and are defined by

$$X_{jm}(x_j, t_i) = \partial(x_j, t_j)/ \partial q_m$$

For j = 1,2 3 …………………n, and m = 1,2 3, ………………………n

Note that the number of times t, equals the number of heat flux components. If there is only one interior location, that is, j=1, the sensitivity coefficient is simply given by

$$X_m(t_i) = \partial(T_I)/ \partial q_m$$

For the transient problems considered in the IHCP, the sensitivity coefficients are zero for m>i. In other words the temperature at t is independent of a yet to occur future heat flux component of q_m, m>i. On e way to determine the linearity of an estimation problem is to inspect the sensitivity coefficients. If the sensitivity coefficients are not functions of the parameters, then the estimation problem is linear. If they are then the problem is non linear.. For example, consider the equation

$\partial T(\theta, t)/ \partial q_c = (L/K).2.(\alpha T/ \Pi L^2)^{½}$, T<+0.3 which is independent of qc. Thus it can be considered to be linear.

Sensitivity Coefficient approach for exactly matching data from a single sensor:

A single temperature sensor is considered to be located at a depth x, below the active surface. If the heat flux qm is constant over the time interval t m-1<t<tm, the value of qm that forces a matching of computed temperature at x with the measured temperature can be calculated.

The temperature field T(x,t) depends in a continues manner on the unknown heat flux qm. This dependence is written as T(x,t,t m-1,q m-1,qm) where qm-1 is the vector of all prevous heat values and t indicates the time that the heat flux step begins. Because the temperature field is continues in qm, it can be extended in Taylor series about an arbitrary but known value of q*.

For linear problems, only the first derivative is non zero, thus the following is an exact result for location x at time t

$T(x,t_{m-1},q_{m-1}, q_m) = T(x,t_m,t_{m-1},q_{m-1},q^*)+ (q_m-q^*)X(x,t_m,t_{m-1},q_{m-1})$ where the sensitivity Coefficient is defined by

$X(x,t_m,t_{m-1},q_{m-1}) = \partial T(x,t_m,t_{m-1},q_{m-1},q_m)/ \partial q_m$.

For the IHCP in which sensors are matched exactly, the left hand side of the equation is replaced by the experimental temperature.

Hence $Y_m = T_k + (q_m-q^*)X_{k,1}$

On solving $q_m= q^* +(y_m-T_k)/X_{k,1}$

Where Tk is the temperature at time t, for the sensor node k with q =q* over tm-1<t<tm. The calculation procedure is to assume an arbitrary value of q, calculate the temperature field T and knowing the sensitivity coefficient X, calculate the heat flux that exactly matches the temperature data Ym. Once qm is known, the complete field can be calculated.

The algorithm for exactly matching the temperature data from the single sensor can be summarized as follows.

1) Assume an arbitrary value of q* and calculate the entire temperature field.
2) Using the same matrix coefficients an, bn,cn,dn as were used for T calculation of step 1, calculate sensitivity coefficients.
3) Calculate the heat flux that exactly matches the experimental temperature data by using Taylor series expansion
   $Q_m = q^* + (Y_m - T_k) / X_{k,1}$
4) Calculate the temperature field from Taylor series expansion
   $T_j = T_j + (q_m - q^*) X_{j,1}$.

2.7 Relevant work

Inverse Heat Conduction Problems (IHCPs) have been extensively studied over the last 50 years. They have numerous applications in many branches of science and technology.[14] The paper consists in determining the temperature and heat flux at inaccessible parts of the boundary of a 2- or 3-dimensional body from corresponding data The work explains ill posed problems with IHCP, which means that small perturbations in the data may cause extremely large errors in the solution. In their contribution they presented the problem and shown examples of calculations for 2-dimensional IHCP's where the direct problems are solved with the Finite Element method. As solution procedure they used Tikhonov's regularization in combination with the conjugate gradient method. [6] is the only commercially published work to deal with the engineering problem of determining surface heat flux and temperature history based on interior temperature measurements. The work provides the analytical techniques needed to arrive at otherwise difficult solutions. [13] In this paper, two matrix algebraic tools are discussed for studying the solution-stabilities of inverse heat conduction problems. The propagations of the computed temperature errors, as caused by a noise in temperature measurement, are presented. The spectral norm analysis reflects the effect of the computational time steps, the sensor locations and the number of future temperatures on the computed error levels. The Frobenius norm analysis manifests the dynamic propagations of the computed errors. [7] The radial and circumferential transient dependence of the strength of a volumetric heat source in a cylindrical rod is estimated with Alifanov's iterative regularization method. This inverse problem is solved as an optimization problem in which a squared residue functional is minimized with the conjugate gradient method. A sensitivity problem is used in the determination of the step size in the direction of descent, while an adjoint problem is solved to determine the gradient. [8] To determine unknowns such as thermal conditions, unknown geometries or thermo-physical properties from the temperature history and distribution of the sample.[10] A finite difference solution for the transient nonlinear heat conduction equation in a finite slab with a radiation boundary condition is proposed in this paper. An implicit finite difference approximation is used. A two-time level implicit method is used, while Taylor's forward projection method is employed for taking account of the nonlinearities. An example is illustrated which is typical of those that arise in practical applications. The results demonstrate that the method is remarkable in its stability to predict surface conditions without debilitation. [11] A technique for determining the heat transfer on the far surface of a wall based on measuring the heat flux and temperature on the near wall is presented. Although heat transfer measurements have previously been used to augment temperature measurements in inverse heat conduction methods, the sensors used alter the heat flow through the surface, disturbing the very quantity that is desired to be measured. The ideal sensor would not alter the boundary condition that would exist were the sensor not present. The innovation of this technique in that it has minimal impact on the wall boundary condition. Since the sensor is placed on the surface of the wall, no alteration of the wall is needed. The theoretical basis for the experimental technique as well as experimental results showing the heat flux sensor performance is presented in this work.

3 Methodology of Work

Here we consider one dimensional heat conduction with thermo physical properties K, ROW, Cp of a material varying with temperature. The physical problem is that of a rocket nozzle throat of finite thickness, whose outer surface is insulated, with a net heat flux at inner radius.

For every control volume, energy gaining per unit time =energy leaving the control volume – energy entering the control volume.

For the first grid point

$K/\delta r(T_2^{n+1} - T_1^{n+1}) +Q = \rho Cp\ \delta r/2*\delta t(T_1^{n+1} - T_1^{n})$

$T_1^{n+1} (\rho Cp\ \delta r/2*\delta t) + T_2^{n+1} (-K/ \delta r) = T_1^{n} (\rho Cp\ \delta r/2*\delta t)+Q$

$A(1) = \rho Cp\ \delta r/2*\delta t$

$B(1)= -K/ \delta r$

$D(1)= T_1^{n} (\rho Cp\ \delta r/2*\delta t)+Q$

For the second to n-1 grid points

$T_i^{n+1} (\rho Cp\ \delta r/\delta t + K/ \delta r) + T_{i+1}^{n+1} (-K/ \delta r((ri+ \delta r/2)/ri)) + T_{i-1}^{n+1} (-K/ \delta r((ri- \delta r/2)/ri)) = T_i^{n} (\rho Cp\ \delta r/\delta t)$

$A(i)= \rho Cp\ \delta r/\delta t + K/ \delta r$

$B(i)= -K/ \delta r((ri+ \delta r/2)/ri)$

$C(i)= -K/ \delta r((ri- \delta r/2)/ri)$

$D(i) = T_i^{n}(\rho Cp\ \delta r/\delta t)$

For the nth point

$-K/ \delta r(T_n^{n+1} – T_{n-1}^{n+1}) = \rho Cp\ \delta r/2*\delta t(T_n^{n+1} –T_n^{n})$

$T_n^{n+1}(\rho Cp\ \delta r/2*\delta t + K/ \delta r) + T_{n-1}^{n+1}(-K/ \delta r) = \rho Cp\ \delta r/2*\delta t* T_n^{n}$

A(n)= ρCp δr/2*δt + K/ δr
C(n)= - K/ δr
D(n)= ρCp δr/2*δt* $T_n^{\ n}$

Iterative schemes:

The tridiogonal system for equations above is solved using algorithm. But in the above equations**,** Q is an unknown parameter, thus the solution of complete problem from x=0 to x=L, cannot be obtained readily because the boundary condition is not known at x=0, but rather an interior temperature history is given. In estimating one minimizes

F(Q) = (T_c(x,t)-T_m(x,t))

Where T_c & T_m are respectively, calculated and measured thermocouple temperatures at (x,t)

The calculated temperature is in general a non linear function of Q but it can be solved by using iteration with a linear approximation. Then for the iteration the taylor series approximation

$T^{n+1}(Q) = T^{n}(Q) + (Q^{n+1} - Q^{n})\,\partial T/\partial Q$ is used. The subscript is an index related to the number of iteration. The partial derivative in the above equation can be calculated using

$\partial T/\partial Q = T(Q(1+e)-T(Q))/e*Q$

Where e is made equal to 10 % of Q, ∂t/∂Q is approximated accurately. The temperature on the right hand side of the equation is calculated. The above tri diagonal equations twice with Q & Q(1+e). Using (∂F/∂Q) a correction in Q is given by

$Q = -F(Q)/\ \partial T/\partial Q.$

The iteration procedure begins with the estimated value of Q and continues until F is less than say 10 -3.

4 Effect of Measurement errors on the estimate

The measurements that one makes of temperature using thermocouples will be corrupted to some extent by errors due to various noise sources. These errors can affect the accuracy of IHCP estimates. The effect of these errors is checked in the present study by modeling the errors as a random number having a Gaussian Probability density function. These errors were added to the temperature variation calculated from the direct problem before using them in IHCP estimates. A typical considered has a standard deviation of 2.5 Celsius. The effect of which is shown in fig. 2.

5. Results

slab condition and material properties taken are

L=0.018
Cp=1750 W sec/Kg. K
Row= 1900 Kg/m, T=300.

Table 1: Heat transfer thru Rocket nozzle throat by Direct & Inverse methods

Time in seconds	Q w/m2 (Direct)	Q w/m2(Inverse)
0.2	8.52 E +007	8.52 E +007
0.4	5.3 E +007	5.35 E +007
0.6	4.37 E +007	4.28 E +007
0.8	3.82 E +007	3.91 E +007
1.0	3.45 E +007	3.45 E +007
1.2	3.16 E +007	3.13 E +007
1.4	2.94 E +007	2.94E +007
1.6	2.76E +007	2.76 E +007
1.8	2.6 E +007	2.6 E +007
2.0	2.46 E +007	2.5 E +007

Fig. 1: Heat transfer thru Rocket nozzle throat by Direct & Inverse methods

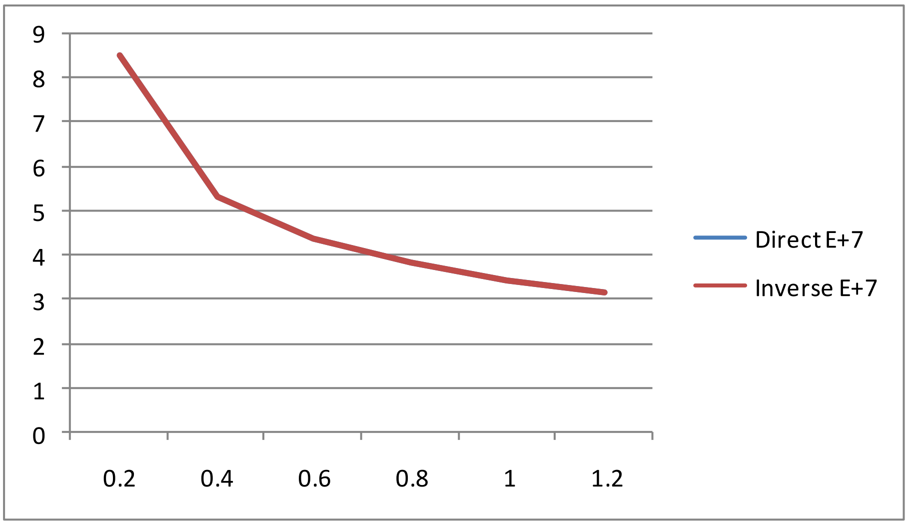
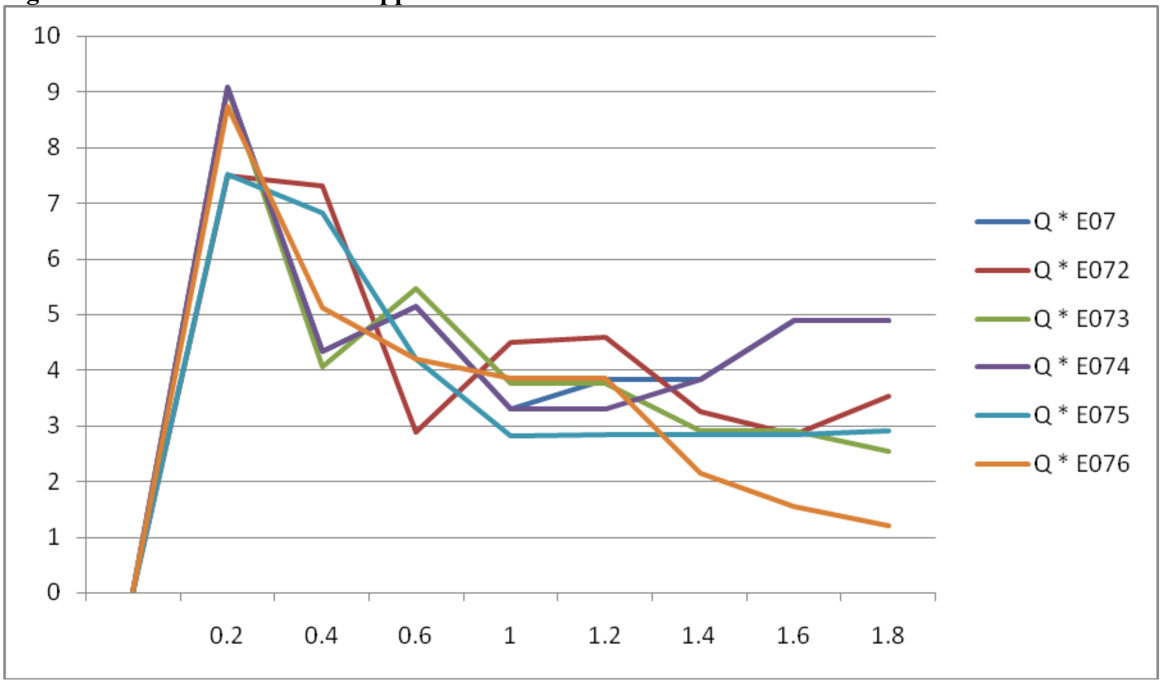


Table 2: Heat Transfer rates with supposed errors

Time Sec	Q * E07 ISEE D 70	Q * E07 ISEE D 80	Q * E07 ISEE D 90	Q * E07 ISEE D 100	Q * E07 ISEE D 110	Q * E07 ISEE D 120
0.2	9.09	7.49	9.05	9.09	7.52	8.73
0.4	4.33	7.3	4.06	4.33	6.83	5.11
0.6	5.15	2.87	5.45	5.15	4.2	4.19
1	3.29	4.5	3.76	3.29	2.82	3.84
1.2	3.83	4.59	3.76	3.29	2.83	3.84
1.4	3.83	3.25	2.9	3.83	2.83	2.14
1.6	4.88	2.83	2.9	4.88	2.84	1.54
1.8	4.88	3.53	2.53	4.88	2.91	1.2

Fig 2 Heat Transfer rates with supposed errors



6. Conclusion & Future work

A finite control volume method has been evolved to calculate temperature distribution & heat transfer in a slab of infinite thickness and finite length. An IHCP method was evolved to calculate heat transfer in the slab which is considered as part of rocket nozzle throat. Single temperature sensor, Sensitivity coefficient method has been used to estimate heat transfer in rocket nozzle throat. By assuming some surface heat transfer, the temperature at certain depth below the actual surface had been calculated. This temperature is compared with the measured temperature from a thermo couple, placed at certain depth, and the incremental heat transfer has been calculated. This incremental heat transfer was added to the surface heat transfer. The process was repeated until the measured temperature and calculated temperature at the same depth were equal. In estimating the IHCP problem, past and present time steps had been used.

Errors were also introduced in IHCP estimation problem. These errors were added to temperature variation calculated from the direct problem and their effect had been checked.

A single sensor has been used to calculate heat flux. But by using multiple sensors more accurate results can be obtained. In the present case only past & present times had been used. To get more accurate estimates of wall temperature and surface heat transfer future time steps may also be used.

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