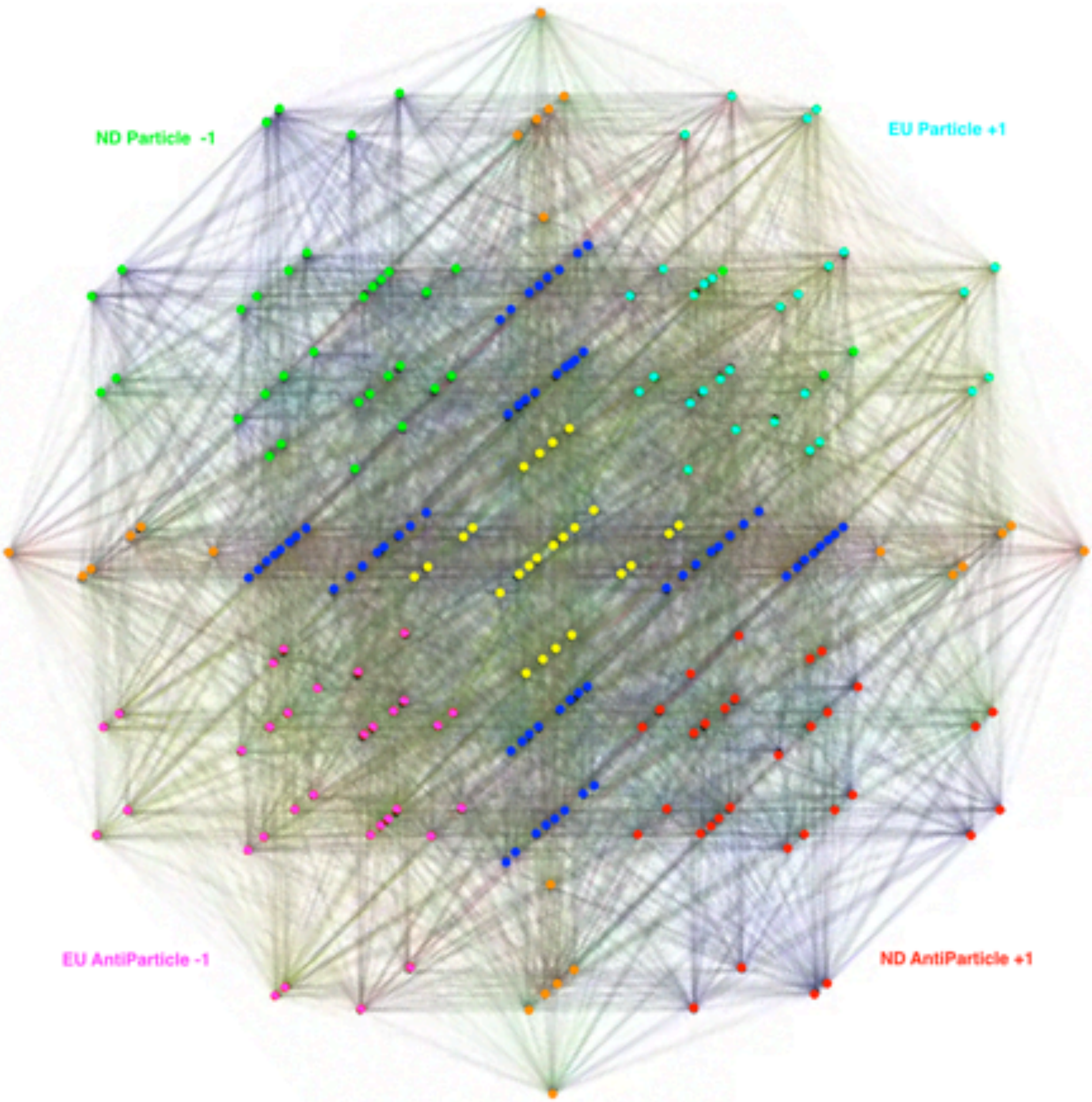
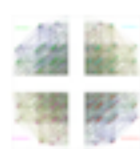
Cl(16) - E8 Lagrangian - AQFT



240

112



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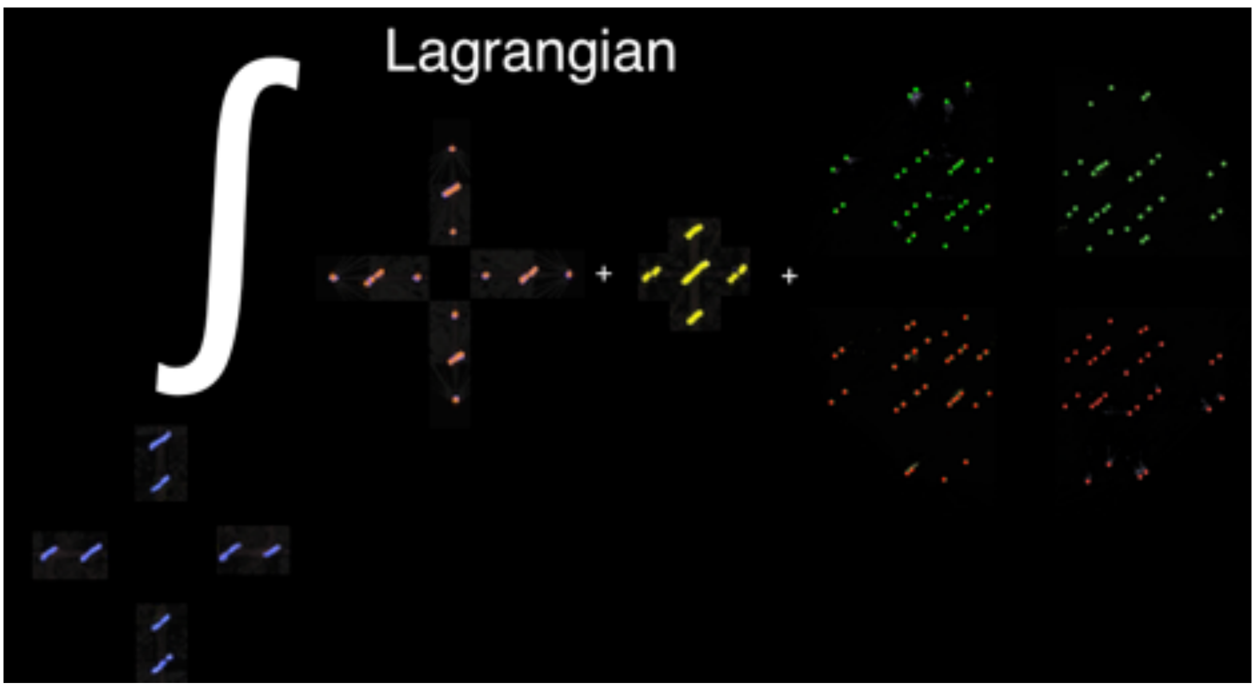
Frank Dodd (Tony) Smith, Jr. - 2014

viXra 1405.0030

Abstract:

Over the past 30 years or so I have been constructing Physics Models and writing about them as can be seen on my web sites at www.valdostamuseum.com/hamsmith/ www.tony5m17h.net/ and on viXra - list at vixra.org/author/frank_dodd_tony_smith_jr Due to experimental observations and my learning new techniques over those 30 years my Physics Models have been in a state of evolving flux - for example, 30 years ago their basis was the Lie Algebra Spin(8), then to contain vectors and spinors it was F4, then to contain the geometry of bounded complex domains it was E6, then Real Clifford Algebras were used to describe evolution from a Void Empty Set ø, then Periodicity showed the importance of Cl(8) and tensor product Cl(8)xCl(8) = Cl(16), then E8 emerged from Cl(16) to give the structure of a realistic local E8 Lagrangian, then completion of the union of all tensor products of Cl(16) local structures produced a realistic Algebraic Quantum Field Theory (AQFT). Since my works over those 30 years have been written from various points of view it is not easy to navigate among them. This paper is being written from a single point of view (that of May 2014) in the hope that it might be easier for readers to navigate. Although the nice math of my Cl(16)-E8 model is necessary, it is not sufficient. The Cl(16)-E8 model must be consistent with experimental observations. As of now, given that most calculations are tree-level, the model is substantially so consistent. An interesting test over the 2015-2016 time frame will be whether or not the LHC sees two additional Higgs mass states with cross section about 20% of that of a full Standard Model Higgs.



Preface

Over the past 30 years or so I have been constructing Physics Models and writing about them as can be seen on my web sites at http://www.valdostamuseum.com/hamsmith/ http://www.tony5m17h.net/ and on viXra - list at http://vixra.org/author/frank_dodd_tony_smith_jr
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Since my works over those 30 years have been written from various points of view it is not easy to navigate among them. This paper is being written from a single point of view (that of May 2014) in the hope that it might be easier for readers to navigate.

A lot of math is used in my Cl(16)-E8 model, some of which may be unfamiliar to many.
My efforts to find a single volume for the math of Cl(16) - E8 Lagrangian - AQFT led me to my Princeton University Advanced Calculus text by H. K. Nickerson, D. C. Spencer, and N. E. Steenrod. However, it is over 50 years old, so I have added some Supplementary Material to produce a 21 MB pdf file on the web at

http://www.valdostamuseum.com/hamsmith/NSS6313.pdf

TABLE OF CONTENTS OF THE SUPPLEMENTED TEXT:

Supplementary Material in Red

Although the nice math of my Cl(16)-E8 model is necessary, it is not sufficient.
My Cl(16)-E8 model must be, and is, consistent with experimental observations .

Here is a summary of E8 Physics model calculation results. Since ratios are calculated, values for one particle mass and one force strength are assumed. Quark masses are constituent masses. Most of the calculations are tree-level, so more detailed calculations might be even closer to observations.

Dark Energy : Dark Matter : Ordinary Matter = 0.75 : 0.21 : 0.04

Inflationary Gravitational Wave (IGW) tensor-to-scalar ratio r = 7/28 = 0.25

Fermions as Schwinger Sources have geometry of Complex Bounded Domains with Kerr-Newman Black Hole structure size about 10^(-24) cm.

Particle/Force	Tree-Level	Higher-Order
e-neutrino	0	0 for nu_1
mu-neutrino	0	9 x 10^(-3) eV for nu_2
tau-neutrino	0	5.4 x 10^(-2) eV for nu_3
electron	0.5110 MeV	
down quark	312.8 MeV	charged pion = 139 MeV
up quark	312.8 MeV	proton = 938.25 MeV
		neutron - proton = 1.1 MeV
muon	104.8 MeV	106.2 MeV
strange quark	625 MeV	
charm quark	2090 MeV	
tauon	1.88 GeV	
beauty quark	5.63 GeV	
truth quark (low state)	130 GeV	(middle state) 174 GeV
		(high state) 218 GeV
W+	80.326 GeV	
W-	80.326 GeV	
W0	98.379 GeV	Z0 = 91.862 GeV
Mplanck	1.217x10^19 GeV	
Higgs VEV (assumed)	252.5 GeV	
Higgs (low state)	126 GeV	(middle state) 182 GeV
		(high state) 239 GeV
Gravity Gg (assumed)	1	
(Gg)(Mproton^2 / Mplanck^2)		5 x 10^(-39)
EM fine structure	1/137.03608	
Weak Gw	0.2535	
Gw(Mproton^2 / (Mw+^2 + Mw-^2 + Mz0^2))		1.05 x 10^(-5)
Color Force at 0.245 GeV	0.6286	0.106 at 91 GeV

Kobayashi-Maskawa parameters for W+ and W- processes are:

	d	s	b
u	0.975	0.222	0.00249 -0.00388i
c	-0.222 -0.000161i	0.974 -0.0000365i	0.0423
t	0.00698 -0.00378i	-0.0418 -0.00086i	0.999

The phase angle d13 is taken to be 1 radian.

The 3-state system of Higgs and Tquark masses is a property of the Cl(16)-E8 model that can be tested at the **LHC 2015-2016 run by searching for Higgs middle and high mass states with cross section about 20% of that of a full SM Higgs**.

TABLE OF CONTENTS OF THIS PAPER:

1. The First Grothendieck Universe is the Empty Set ø which grows by Clifford Iteration to Cl(16) which contains E8

1					= Cl(0) = 1
ø					

1	1				= Cl(1) = 2
ø	(ø)				

1	2	1			= Cl(2) = 4
ø	(ø)	(ø (ø))			
	((ø))				

1	4	6	4	1	= Cl(4) = 16
ø	(ø)	(ø (ø))	((ø) ((ø)) (ø (ø)))	(ø (ø) ((ø)) (ø (ø)))	
	((ø))	(ø ((ø)))	(ø ((ø)) (ø (ø)))		
	(((ø)))	(ø (ø (ø)))	(ø (ø) (ø (ø)))		
	((ø (ø)))	((ø) ((ø)))	(ø (ø) ((ø)))		
		((ø) (ø (ø)))			
		(((ø)) (ø (ø)))			

1 16 120 560 1820 4368 8008 11440 12870 11440 8008 4368 1820 560 120 16 1

= Cl(16) = 2^16 = 65,536 =

= ((64+64) + (64+64)) x ((64+64) + (64+64))

Cl(16) BiVectors = D8 = 120 = 28 + 28 + 64

Cl(16) Spinors = (64+64) + (64+64)

28 + 28 + 64 + 64 + 64 = E8

From Cl(1,3)) = 16 to Cl(Cl(1,3)) = 65,536 with 16 $\wedge$ 16 = 120

(**Color Scheme on this page for Cl(1,3) is not the same used for Cl(16) and E8**)

1 4 6 4 1 $\wedge$ 1 4 6 4 1

1 $\wedge$ 4 = 4
4 $\wedge$ 6 = 24

1 $\wedge$ 4 = 4
6 $\wedge$ 4 = 24

1 $\wedge$ 6 = 6
1 $\wedge$ 1 = 1
6 $\wedge$ 6 = 15
6 $\wedge$ 1 = 6

4 $\wedge$ 4 = 6
4 $\wedge$ 4 = 16
4 $\wedge$ 4 = 6

4 $\wedge$ 1 = 4
4 $\wedge$ 1 = 4

28 D4 for Gravity +
28 D4 for Standard Model +
28 AntiSymmetric D4 rotations in 8-dim SpaceTime +
28 8x8 Symmetric Off-Diagonal +
8 8x8 Symmetric Diagonal for 4 + 4 Klauza-Klein M4 x CP2 = 120

E8 structure gives a Fundamental Local Lagrangian

E8 Root Vectors = 112 + 64 + 64 = 24 + 24 + 64 + 64 + 64

Fundamental Local Lagrangian =

$$= \int_{\text{8-dim SpaceTime}} \text{Standard Model Gauge Gravity} + \text{Fermion Particle-AntiParticle}$$

where E8 structure of the Lagrangian Terms is given by:

E8 / D8 = 64 + 64

64 = 8 Components of 8 Fermion Particles (first generation)

64 = 8 Components of 8 Fermion AntiParticles (first generation)

D8 / D4xD4 = 64

64 = 8-dim SpaceTime Position and Momentum

(Triality Automorphisms: 64 = 64 = 64)

D4xD4 = 24 + 4 + 24 + 4

24 + 4 = 28 = D4 for Gravity Gauge Bosons

24 + 4 = 28 = D4 for Standard Model Gauge Bosons

Standard Model Gauge Gravity term has total weight 28 x 1 = 28

12 generators for SU(3) and U(2) Standard Model

+

16 generators for U(2,2) of Conformal Gravity

=

28 D4 Gauge Bosons

each with 8-dim Lagrangian weight = 1

Fermion Particle-AntiParticle term also has total weight 8 x (7/2) = 28

8 Fermion Particle/Antiparticle types

each with 8-dim Lagrangian weight = 7/2

Since Boson Weight 28 = Fermion Weight 28

the Cl(16)-E8 model has a Subtle SuperSymmetry and is UltraViolet Finite.

The Cl(16)-E8 model has 8-dim Lorentz structure satisfying Coleman-Mandula because its fermionic fundamental spinor representations are built with respect to spinor representations for 8-dim Spin(1,7) spacetime.

(See pages 382-384 of Steven Weinberg's book "The Quantum Theory of Fields" Vol. III)

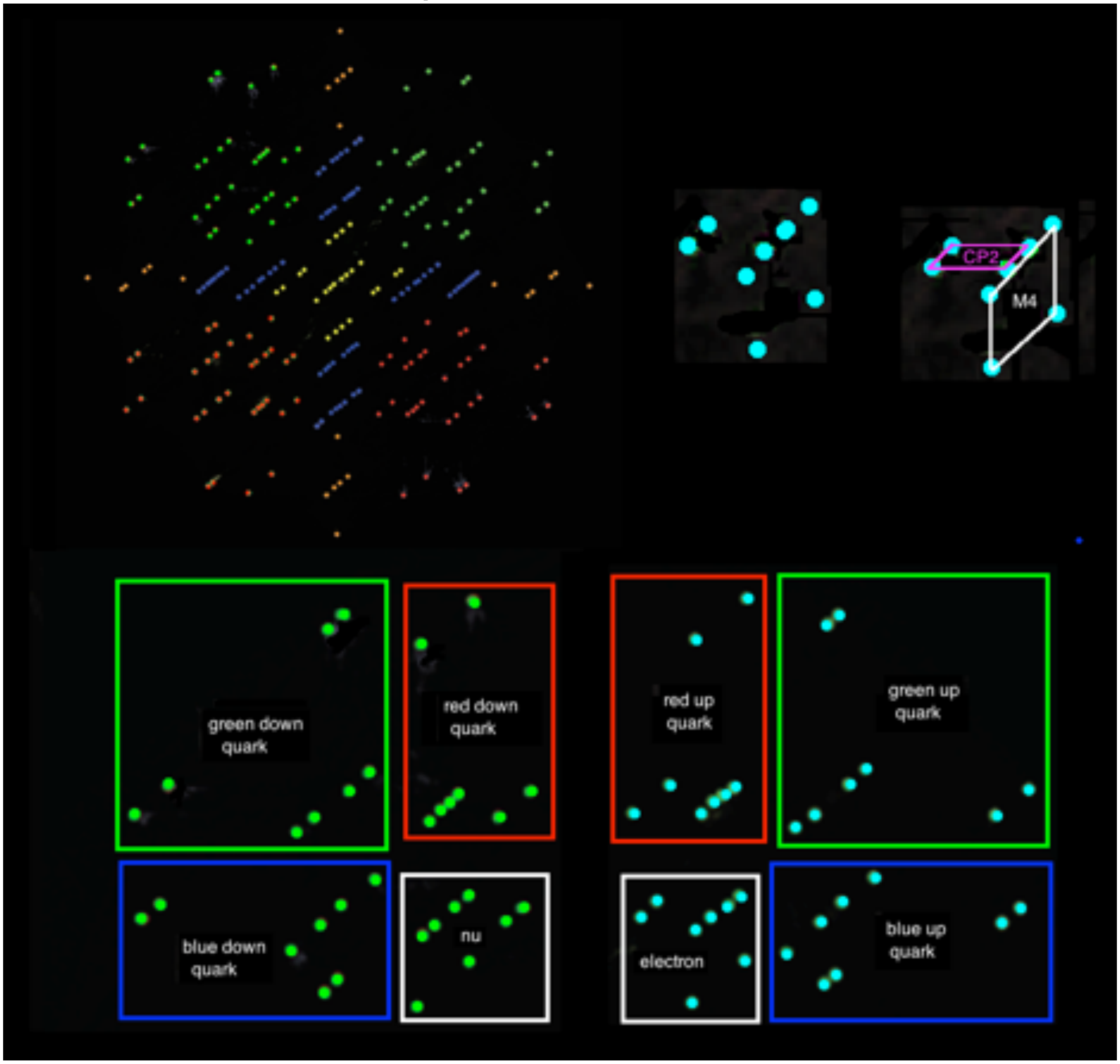
The Cl(16)-E8 model is Chiral because
E8 contains Cl(16) half-spinors (64+64) for a Fermion Generation
but does not contain Cl(16) Fermion AntiGeneration half-spinors (64+64).
Fermion +half-spinor Particles with high enough velocity are seen as left-handed.
Fermion -half-spinor AntiParticles with high enough velocity are seen as right-handed.

The Cl(16)-E8 model obeys Spin-Statistics because
the CP2 part of M4xCP2 Kaluza-Klein has index structure Euler number 2+1 = 3 and
Atiyah-Singer index -1/8 which is not the net number of generations because
CP2 has no spin structure but you can use a generalized spin structure
(Hawking and Pope (Phys. Lett. 73B (1978) 42-44))
to get (for integral m) the generalized CP2 index n_R - n_L = (1/2) m (m+1)
Prior to Dimensional Reduction: m = 1, n_R - n_L = (1/2)x1x2 = 1 for 1 generation
After Reduction to 4+4 Kaluza-Klein: m = 2, n_R - n_L = (1/2)x2x3 = 1 for 3 generations
(second and third generations emerge as effective composites of the first)

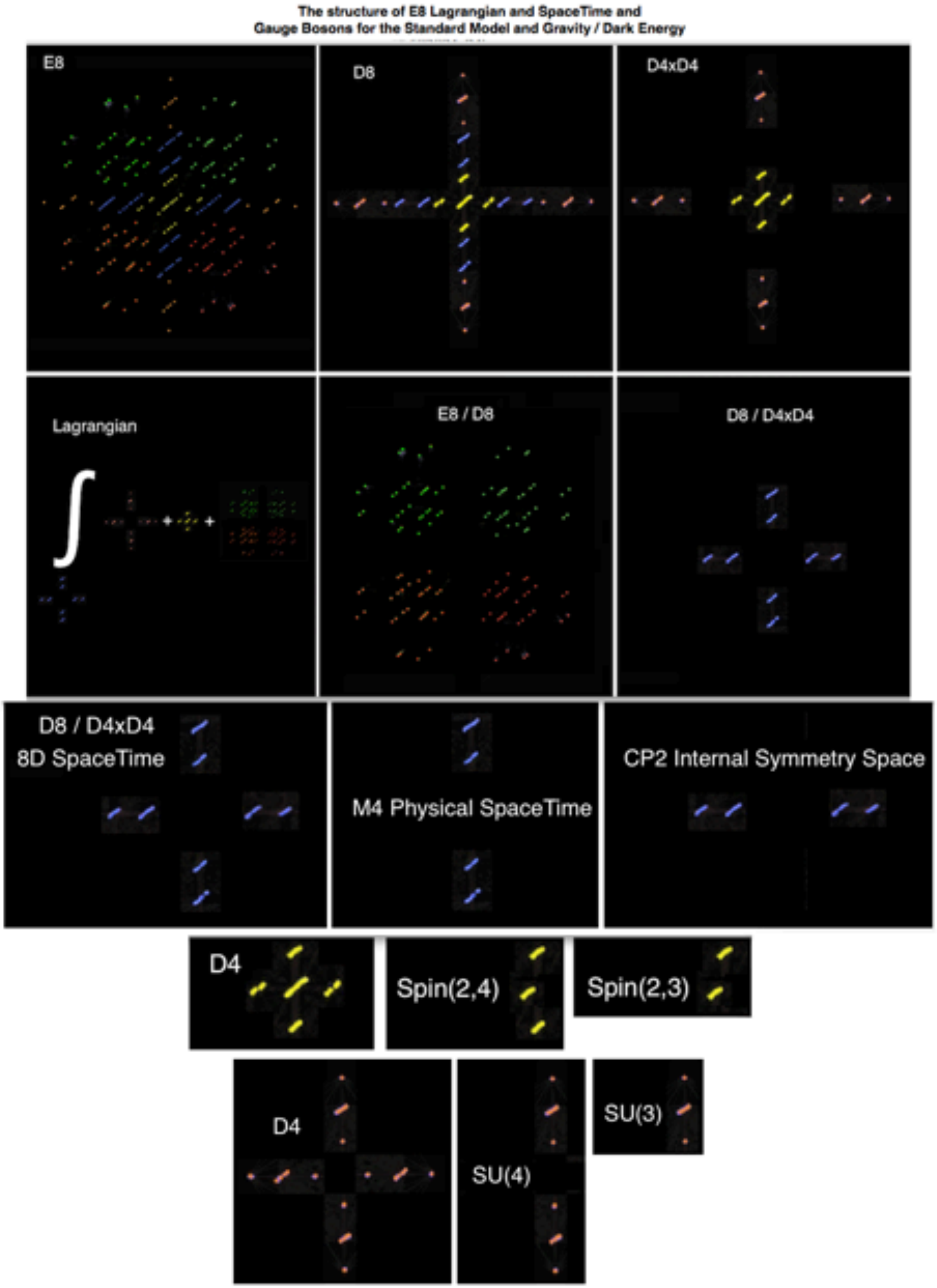
Hawking and Pope say: "Generalized Spin Structures in Quantum Gravity ...
what happens in CP2 … is a two-surface K which cannot be shrunk to zero. ...
However, one could replace the electromagnetic field by
a Yang-Mills field whose group G had a double covering G~.
The fermion field would have to occur in representations which changed sign
under the non-trivial element of the kernel of the projection … G~ -> G
while the bosons would have to occur in representations which did not change sign …".
For Cl(16)-E8 model gauge bosons are in the 28+28=56-dim D4 + D4 subalgebra of E8.
D4 = SO(8) is the Hawking-Pope G which has double covering G~ = Spin(8).

The 8 fermion particles / antiparticles are D4 half-spinors represented within E8
by anti-commutators and so do change sign while
the 28 gauge bosons are D4 adjoint represented within E8 by commutators
and so do not change sign. Further,
E8 inherits from F4 the property whereby
its Spinor Part need not be written as Commutators
but can also be written in terms of Fermionic AntiCommutators. (vixra 1208.0145)

The structure of E8 Spinor Fermions of the First Generation is:



Spinor 128 of 240 E8 Root Vectors are 64 red/magenta and 64 green/cyan dots.
64 Green Dots represent Fermion Particles. 64 Red Dots represent AntiParticles.

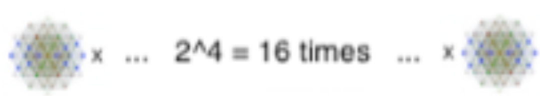


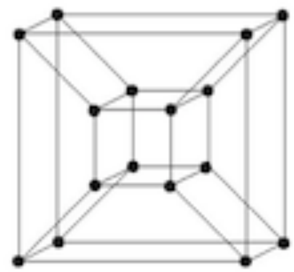
The D8 112 of the 240 E8 Root Vectors are 24 orange and 24 yellow and 64 blue dots.

2. The Second Grothendieck Universe is Hereditarily Finite Sets such as Discrete Clifford Algebras and Discrete Lattices.

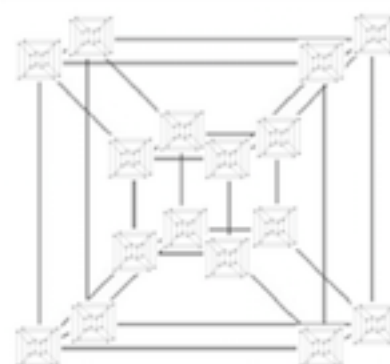
Each Local Lagrangian with Creation / Annihilation density terms
lives in an E8 which in turn lives in a Cl(16) Real Clifford Algebra.

By 8-Periodicity of Real Clifford Algebras tensor products of N copies of Cl(16) form a Clifford Algebra **Cl(16N) = Cl(16) x ...(N times tensor product)... x Cl(16)** .
For N = 2^4 = 16 the 16 copies of Cl(16) form E8 Physics of a 4-dim HyperCube





corresponding to 4-dim M4 Physical SpaceTime.
For N = 2^8 = 256 the 256 copies of Cl(16) form E8 Physics of an 8-dim HyperCube

corresponding to 8-dim E8 SpaceTime (image by Conrad Schneiker in 1987 paper by Hameroff) and to M4 x CP2 Kaluza-Klein SpaceTime with each vertex of the 4-dim M4 HyperCube having a 4-dim CP2 Internal Symmetry Space.
For N = 2^16 = 65,536 = 4^8 the copies of Cl(16) fill in the 8-dim HyperCube as described by William Gilbert's web page: "... **The n-bit reflected binary Gray code will describe a path on the edges of an n-dimensional cube that can be used as the initial stage of a Hilbert curve that will fill an n-dimensional cube**. ...".
As N grows, the copies of Cl(16) continuue to fill the 8-dim HyperCube of E8 SpaceTime using higher Hilbert curve stages from the 8-bit reflected binary Gray code subdividing the initial 8-dim HyperCube into more and more sub-HyperCubes. If the edges of the sub-HyperCubes, equal to the distance between adjacent copies of Cl(16), remain constantly at the Planck Length, then the
full 8-dim HyperCube of our Universe expands as N grows to 2^16 and beyond.

The Union of all Cl(16) tensor products is the Union of all subdivided 8-HyperCubes and their Completion is a huge superposition of 8-HyperCube Continuous Volumes
which Completion belongs to the Third Grothendieck Universe.

H. S. M. Coxeter in his paper Regular and Semi-Regular Polyotpes III (Math. Z. 200, 3-45, 1988)

about the 240 units of an E8 Integral Domain said: "... "... the 16 + 16 + 16 octaves
±1, ±i, ±j, ±k, ±E, ±I, ±J, ±K,   (±1 ±I ±J ±K)/2,   (±E ±i ±j ±k)/2, and the 192 others derived from the last two expressions by ... the cyclic permutation (E, i, j, I, K, k, J), which preserves the integral domain ... the permutation (e I J i k K j), which is an automorphism of the whole ring of octaves (and of the finite [Fano] plane ...) transforms this particular integral domain into another one of R. H. Bruck's cyclic of seven such domains. ...". An 8th E8 Lattice (not a closed Integral Domain, Kirmse's mistake) can be taken to correspond the the 1 Real Element of the Octonion Basis { 1, i, j, k, E, I, J, K}.

```
There are 7 independent E8 Integral Domain Lattices
corresponding to the 7 Octonion Imaginary Basis Elements {i,j,k,E,I,J,K}

     Associative   Coassociative  Heptaverton
      Triangle         Square
                                      k J
        I              J---j          |/
i  -->  / \  ---------> |   | -->   I--i--E
      E---i            K---k         /|
                                    K j

                                      k I
        J              K---k          |/
j  -->  / \  ---------> |   | -->   J--j--E
      E---j            I---i         /|
                                    K i

                                      i J
        K              I---i          |/
k  -->  / \  ---------> |   | -->   K--k--E
      E---k            J---j         /|
                                    I j

                                      I k
        j              I---J          |/
E  -->  / \  ---------> |   | -->   J--E--j
      i---k            K---E         /|
                                    K i

                                      E j
        J              I---j          |/
I  -->  / \  ---------> |   | -->   J--I--k
      i---K            k---E         /|
                                    K i

                                      E k
        j              J---i          |/
J  -->  / \  ---------> |   | -->   K--J--i
      I---K            k---E         /|
                                    I i

                                      E i
        J              K---i          |/
K  -->  / \  ---------> |   | -->   I--K--j
      I---k            j---E         /|
                                    J k
```

E8 Lattices

E8 Lattices are based on Octonions, which have 480 different multiplication products. E8 Lattices can be combined to form 24-dimensional Leech Lattices and 26-dimensional Bosonic String Theory, which describes E8 Physics when the strings are physically interpreted as World-Lines. A basic String Theory Cell has as its automorphism group the Monster Group whose order is
2^46 .3^20 .5^9 .7^6 .11^2 .13^3 .17.19.23.29.31.41.47.59.71 = about 8 x 10^53.

For more about the Leech Lattice and the Monster and E8 Physics,
see viXra 1210.0072 and 1108.0027 .

E8 Root systems and lattices are discussed by Robert A. Wilson in his 2009 paper "Octonions and the Leech lattice":
"... The (real) octonion algebra is an 8-dimensional (non-division) algebra with an orthonomal basis { 1=ioo , i0 , i1 , i2 , i3 , i4 , i5 , i6 } labeled
by the projective line PL(7) = { oo } u F7
...
The E8 root system embeds in this algebra ... take the 240 roots to be ...
112 octonions ... +/- it +/- iu for any distinct t,u
... and ...
128 octonions (1/2)(+/- 1 +/- i0 +/- ... +/- i6) ...[with]... an odd number of minus signs.
Denote by L the lattice spanned by these 240 octonions
...
Let s = (1/2)(- 1 + i0 + ... + i6) so s is in L ... write R for Lbar ...
...
(1/2) (1 + i0) L = (1/2) R (1 + i0) is closed under multiplication ... Denote this ...by A
... Writing **B = (1/2) (1 + i0) A (1 + i0)** ...from ... Moufang laws ... we have
L R = 2 B , and ... B L = L and R B = R ...[also]... 2 B = L sbar
...
the roots of B are
[**16 octonions**]... +/- it for t in PL(7)
... together with
[**112 octonions**]... (1/2) (+/- 1 +/- it +/- i(t+1) +/- i(t+3)) ...for t in F7
... and ...
[**112 octonions**]... (1/2) (+/- i(t+2) +/- i(t+4) +/- i(t+5) +/- i(t+6)) ...for t in F7
...
B is not closed under multiplication ... Kirmse's mistake
...[but]... as Coxeter ... pointed out ...
... **there are seven non-associative rings At** = (1/2) (1 + it) B (1 + it) ,
obtained from B by swapping 1 with it ... for t in F7
...
LR = 2B and BL = L ...[which]... appear[s] not to have been noticed before ... some work ... by Geoffrey Dixon ...".

Geoffrey Dixon says in his book "Division Algebras, Lattices, Physics, Windmill Tilting"
using notation {e0,e1,e2,e3,e4,e5,e6,e7} for the Octonion basis elements
that Robert A. Wilson denotes by {1=ioo,i0,i1,i2,i3,i4,i5 ,i6}
and I sometimes denote by {1,i,j,k,E,I,J,K}: "...

$$
\begin{aligned}
\Xi_0 = &\quad \{\pm e_a\}, \\
\Xi_2 = &\quad \{(\pm e_a \pm e_b \pm e_c \pm e_d)/2 : a, b, c, d \text{ distinct}, \\
&\quad e_a(e_b(e_c e_d)) = \pm 1\},
\end{aligned}
$$

$$
\begin{aligned}
\Xi^{even} = &\quad \Xi_0 \cup \Xi_2, \\
\mathcal{E}_8^{even} = &\quad \text{span}\{\Xi^{even}\},
\end{aligned}
$$

$$
\begin{aligned}
\Xi_1 = &\quad \{(\pm e_a \pm e_b)/\sqrt{2} : a, b \text{ distinct}\}, \\
\Xi_3 = &\quad \{(\textstyle\sum_{a=0}^{7} \pm e_a)/\sqrt{8} : \text{ even number of +'s}\},
\end{aligned}
$$

$$
\begin{aligned}
\Xi^{odd} = &\quad \Xi_1 \cup \Xi_3, \\
\mathcal{E}_8^{odd} = &\quad \text{span}\{\Xi^{odd}\}
\end{aligned}
$$

(spans over integers)

Ξeven has 16+224 = 240 elements ... Ξodd has 112+128 = 240 elements ...

E8even does not close with respect to our given octonion multiplication
...[but]...
the set Ξeven[0-a], derived from Ξeven by replacing each occurrence of e0 ... with ea,
and vice versa, is multiplicatively closed. ...".

Geoffrey Dixon's Ξeven corresponds to Wilson's B which I denote as 1E8.

Geoffrey Dixon's Ξeven[0-a] correspond to Wilson's seven At
which I denote as iE8, jE8, kE8, EE8, IE8, JE8, KE8.

Geoffrey Dixon's Ξodd corresponds to Wilson's L.

My view is that **the E8 domains 1E8 = Ξeven = B is fundamental**
because
E8 domains iE8, jE8, kE8, EE8, IE8, JE8, KE = Ξeven[0-a] are derived from 1E8
and L and L s are also derived from 1E8 = Ξeven = B.

Using the notation {1,i,j,k,E,I,J,K} for Octonion basis
notice that in the Cl(16)-E8 model introduction of Quaternionic substructure
to produce (4+4)-dim M4 x CP2 Kaluza-Klein SpaceTime
requires breaking Octonionic light-cone elements
(+/- 1 +/- i +/- j +/- k +/- E +/- I +/- J +/- K) / 2
into Quaternionic 4-term forms like (+/- A +/- B +/- C +/- D) / 2.

To do that, consider that there are (8l4) = 70 ways to choose 4-term subsets
of the 8 Octonionic basis element terms. Using all of them produces
224 4-term subsets in each of the 7 Octonion Imaginary E8 lattices
iE8,jE8,kE8,EE8,IE8,JE8,KE8 each of which also has 16 1-term first-shell vertices.

56 of the 70 4-term subsets appear as 8 in each of the 7 Octonion Imaginary E8 lattices.

The other 70-56 = 14 4-term subsets occur in sets of 3 among 7x6 = 42 4-term subsets
as indicated in the following detailed list of the 7 Octonion Imaginary E8 lattices:

```
EE8:
112 of D8 Root Vectors
16 appear in all 7 of iE8,jE8,kE8,EE8,IE8,JE8,KE8
±1, ±i, ±j, ±k, ±E, ±I, ±J, ±K
96 appear in 3 of iE8,jE8,kE8,EE8,IE8,JE8,KE8
(±1 ±K ±E ±k)/2 (±i ±j ±I ±J)/2 kE8 , EE8 , KE8
(±1 ±J ±j ±E)/2 (±I ±K ±k ±i)/2 jE8 , EE8 , JE8
(±1 ±E ±I ±i)/2 (±K ±k ±J ±j)/2 iE8 , EE8 , IE8

128 of D8 half-spinors appear only in EE8
(±1 ±I ±J ±K)/2 (±E ±i ±j ±k)/2
(±1 ±k ±i ±J)/2 (±j ±I ±K ±E)/2
(±1 ±i ±K ±j)/2 (±k ±J ±E ±I)/2
(±1 ±j ±k ±I)/2 (±J ±E ±i ±K)/2
```

iE8:
112 of D8 Root Vectors
16 appear in all 7 of iE8,jE8,kE8,EE8,IE8,JE8,KE8
±1, ±i, ±j, ±k, ±E, ±I, ±J, ±K
96 appear in 3 of iE8,jE8,kE8,EE8,IE8,JE8,KE8
(±1 ±I ±i ±E)/2 (±j ±k ±J ±K)/2 iE8 , EE8 , IE8
(±1 ±K ±J ±i)/2 (±j ±k ±E ±I)/2 iE8 , JE8 , KE8
(±1 ±i ±k ±j)/2 (±E ±I ±J ±K)/2 iE8 , jE8 , kE8

128 of D8 half-spinors appear only in iE8
(±1 ±k ±K ±I)/2 (±i ±j ±E ±J)/2
(±1 ±E ±j ±K)/2 (±i ±k ±I ±J)/2
(±1 ±j ±I ±J)/2 (±i ±k ±E ±K)/2
(±1 ±J ±E ±k)/2 (±i ±j ±I ±K)/2

jE8:
112 of D8 Root Vectors
16 appear in all 7 of iE8,jE8,kE8,EE8,IE8,JE8,KE8
±1, ±i, ±j, ±k, ±E, ±I, ±J, ±K
96 appear in 3 of iE8,jE8,kE8,EE8,IE8,JE8,KE8
(±1 ±k ±j ±i)/2 (±E ±I ±J ±K)/2 iE8 , jE8 , kE8
(±1 ±I ±K ±j)/2 (±i ±k ±E ±J)/2 jE8 , IE8 , KE8
(±1 ±j ±E ±J)/2 (±i ±k ±I ±K)/2 jE8 , EE8 , JE8

128 of D8 half-spinors appear only in jE8
(±1 ±E ±I ±k)/2 (±i ±j ±J ±K)/2
(±1 ±i ±J ±I)/2 (±j ±k ±E ±K)/2
(±1 ±J ±k ±K)/2 (±i ±j ±E ±I)/2
(±1 ±K ±i ±E)/2 (±j ±k ±I ±J)/2

kE8:
112 of D8 Root Vectors
16 appear in all 7 of iE8,jE8,kE8,EE8,IE8,JE8,KE8
±1, ±i, ±j, ±k, ±E, ±I, ±J, ±K
96 appear in 3 of iE8,jE8,kE8,EE8,IE8,JE8,KE8
(±1 ±J ±k ±I)/2 (±i ±j ±E ±K)/2 kE8 , IE8 , JE8
(±1 ±j ±i ±k)/2 (±E ±I ±J ±K)/2 iE8 , jE8 , kE8
(±1 ±k ±K ±E)/2 (±i ±j ±I ±J)/2 kE8 , EE8 , KE8

128 of D8 half-spinors appear only in kE8
(±1 ±K ±j ±J)/2 (±i ±k ±E ±I)/2
(±1 ±I ±E ±j)/2 (±i ±k ±J ±K)/2
(±1 ±E ±J ±i)/2 (±j ±k ±I ±K)/2
(±1 ±i ±I ±K)/2 (±j ±k ±E ±J)/2

IE8:
112 of D8 Root Vectors
16 appear in all 7 of iE8,jE8,kE8,EE8,IE8,JE8,KE8
±1, ±i, ±j, ±k, ±E, ±I, ±J, ±K
96 appear in 3 of iE8,jE8,kE8,EE8,IE8,JE8,KE8
(±1 ±j ±I ±K)/2 (±i ±k ±E ±J)/2 jE8 , IE8 , KE8
(±1 ±i ±E ±I)/2 (±j ±k ±J ±K)/2 iE8 , EE8 , IE8
(±1 ±I ±J ±k)/2 (±i ±j ±E ±K)/2 kE8 , IE8 , JE8

128 of D8 half-spinors appear only in IE8
(±1 ±J ±i ±j)/2 (±k ±E ±I ±K)/2
(±1 ±K ±k ±i)/2 (±j ±E ±I ±J)/2
(±1 ±k ±j ±E)/2 (±i ±I ±J ±K)/2
(±1 ±E ±K ±J)/2 (±i ±j ±k ±I)/2

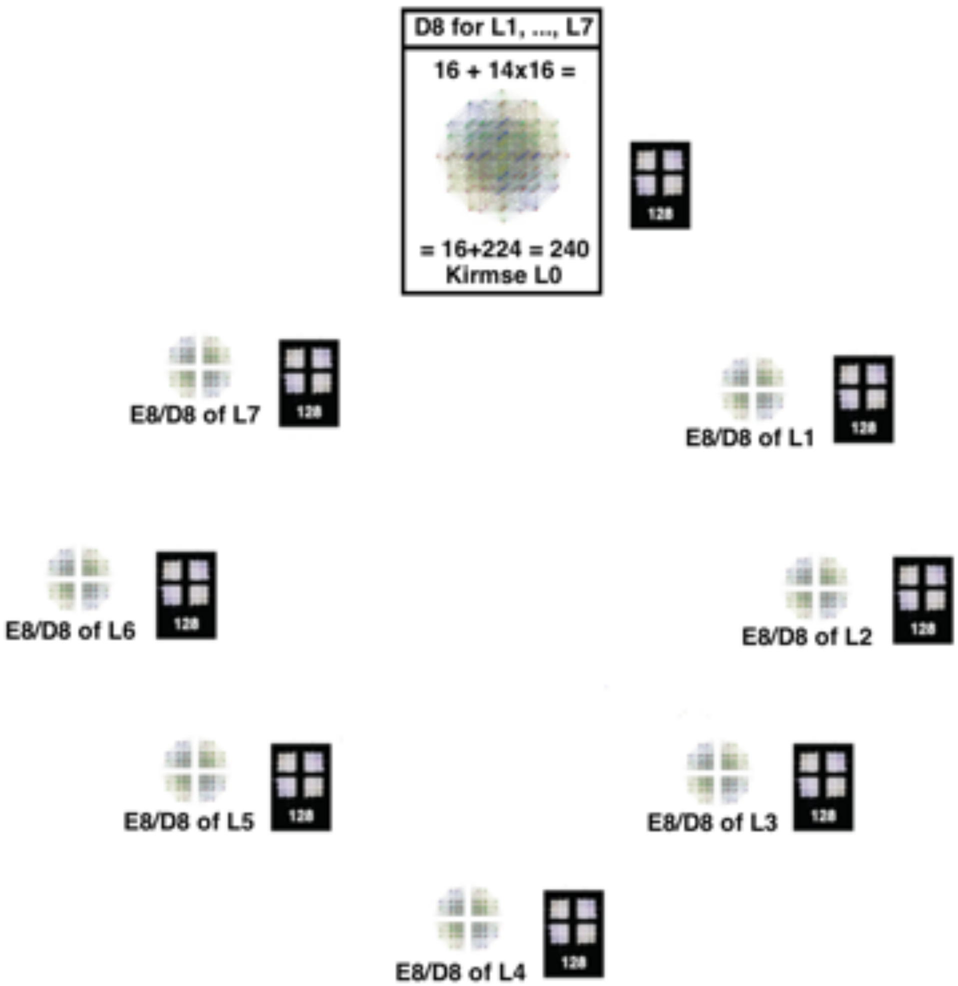
JE8:
112 of D8 Root Vectors
16 appear in all 7 of iE8,jE8,kE8,EE8,IE8,JE8,KE8
±1, ±i, ±j, ±k, ±E, ±I, ±J, ±K
96 appear in 3 of iE8,jE8,kE8,EE8,IE8,JE8,KE8
(±1 ±E ±J ±j)/2 (±i ±k ±I ±K)/2 jE8 , EE8 , JE8
(±1 ±k ±I ±J)/2 (±i ±j ±E ±I)/2 kE8 , IE8 , JE8
(±1 ±J ±i ±K)/2 (±j ±k ±E ±I)/2 iE8 , JE8 , KE8

128 of D8 half-spinors appear only in JE8
(±1 ±i ±k ±E)/2 (±j ±I ±J ±K)/2
(±1 ±j ±K ±k)/2 (±i ±E ±I ±J)/2
(±1 ±K ±E ±I)/2 (±i ±j ±k ±J)/2
(±1 ±I ±j ±i)/2 (±k ±E ±J ±K)/2

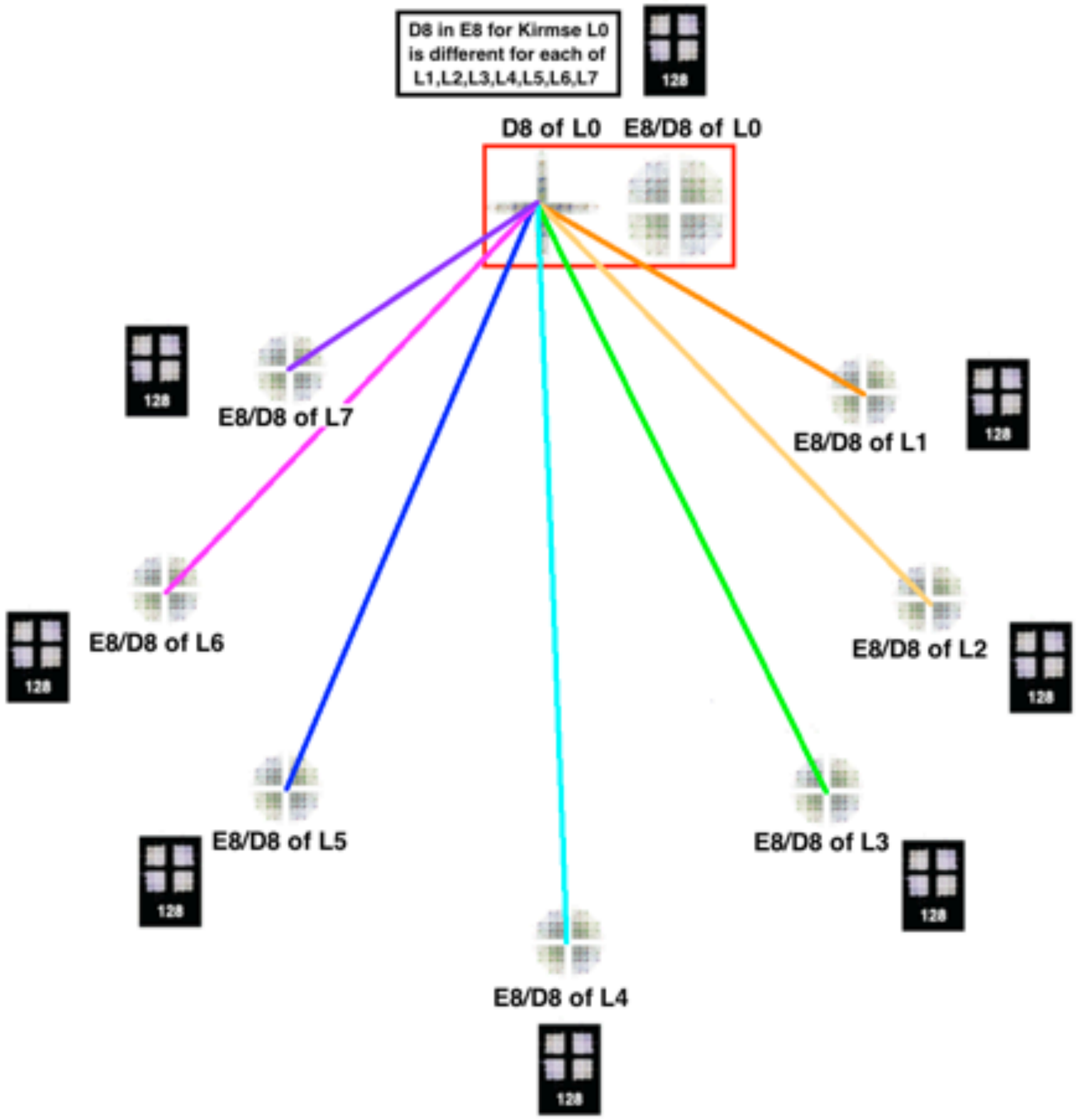
KE8:
112 of D8 Root Vectors
16 appear in all 7 of iE8,jE8,kE8,EE8,IE8,JE8,KE8
±1, ±i, ±j, ±k, ±E, ±I, ±J, ±K
96 appear in 3 of iE8,jE8,kE8,EE8,IE8,JE8,KE8
(±1 ±i ±K ±J)/2 (±j ±k ±E ±I)/2 iE8 , JE8 , KE8
(±1 ±E ±k ±K)/2 (±i ±j ±I ±J)/2 kE8 , EE8 , KE8
(±1 ±K ±j ±I)/2 (±i ±k ±E ±J)/2 jE8 , IE8 , KE8

128 of D8 half-spinors appear only in KE8
(±1 ±j ±E ±i)/2 (±k ±I ±J ±K)/2
(±1 ±J ±I ±E)/2 (±i ±j ±k ±K)/2
(±1 ±I ±i ±k)/2 (±j ±E ±J ±K)/2
(±1 ±k ±J ±j)/2 (±i ±E ±I ±K)/2

The vertices that appear in more than one lattice are:
16:
±1, ±i, ±j, ±k, ±e, ±ie, ±je, ±ke   in all of them;
6x16 = 96 (1 and e or neither 1 nor e):
(±1±i±e±ie)/2 and (±j±k±je±ke)/2   in 7E8, 1E8, and 3E8 ;
(±1±j±e±je)/2 and (±i±k±ie±ke)/2   in 7E8, 2E8, and 6E8 ;
(±1±k±e±ke)/2 and (±i±j±ie±je)/2   in 7E8, 4E8, and 5E8 ;
8x16 = 128 (either 1 or e singly):
(±1±i±j±k)/2 and (±e±ie±je±ke)/2   in 3E8, 4E8, and 6E8 ;
(±1±i±je±ke)/2 and (±j±k±e±ie)/2   in 2E8, 3E8, and 5E8 ;
(±1±j±ie±ke)/2 and (±i±k±e±je)/2   in 1E8, 5E8, and 6E8 ;
(±1±k±ie±je)/2 and (±i±j±e±ke)/2   in 1E8, 2E8, and 4E8 .
These 16 + 14x16 = 16 + 224 = 240 vertices
form the Kirmse E8 lattice L0 which is not an Integral Domain
because it is not closed under the chosen Octonion multiplication
but is the part of the 2160 vertices of the 2-shell of all E8 lattices
that provides the D8 part of the 7 distinct Integral Domains L1, ... , L7
whose E8/D8 half-spinor parts are in the 2-shell.



The identification of 1-shell E8 Lattices with their appearance in L0 2-shell
is up to a scale factor. The 2160-vertex 2-shell is effectively a
superposition of the 7 Integral Domains L1, ... , L7 plus the Kirmse L0.

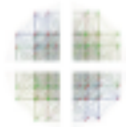


An E8 lattice 2-shell has 2160 vertices,

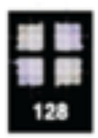
of which 8 x 

= 8x128 = 1024 are mirror D8 half-spinors that
are not in the E8 Lie algebra as they are not in any E8 1-shell E8 Root Vectors
and

7 x  = 7x128 = 896 are E8/D8 half-spinors that are in the E8 Lie Algebras of the
E8 1-shell Root Vectors of one of the 7 Integral Domain E8 Lattices L1, ... , L7
and

112 + 128   = 240 correspond to the D8 and E8/D8 half-spinors of the E8 1-shell Root Vectors of the Kirmse E8 Lattice L0.

For each of the 7 Integral Domain E8 Lattices L1, ... , L7 there is a different way to select 112  of the 240 of Kirmse L0 to add to the 128  of that Li and produce the 112 + 128 = 240 Root Vectors (up to scaling) of that Li E8 Lattice.

The 8 pairs  + 

of 128-element D8 half-spinors + mirror half-spinors
would represent the 128 + 128 = 256-dim full spinors of the Cl(16) Clifford Algebra
if the 128-dim E8/D8 half-spinors of the 8th pair, of Kirmse L0, were well-defined
but
since there are 7 different D8 sets of 112 and so 7 different E8/D8 sets of 128 for L0
depending on which of L1, ... , L7 E8/D8 is completed by the 112 D8 of Kirmse L0
the 128 E8/D8 of Kirmse L0 is not well-defined in the conventional 2160 E8 2-shell.
Therefore
the 2160 E8 2-shell should be considered to be a superposition of 7 shells
one for each of the 7 E8/D8 L1, ... , L7 that are completed to E8 by action of D8 of L0.
The superposition is expandable to an 8-shell superposition
by adding an 8th shell corresponding to Kirmse L0.

This is consistent with and justifies the superposition structures used in viXra 1210.0072 v4 and in viXra 1301.0150 v4 and in viXra 1405.0030 vF

Since the 1024 mirror D8 half-spinors are not in any E8,
the only part of the 2160 E8 2-shell that is directly relevant to E8 Physics
is the superposition structure
of its 2160 - 1024 = 1136 = 112 + 1024 = 112 + 128 + 896 vertex part.

If you look at the 2160 vertices of the second shell as expanded from the first shell,
then you see that the 2160 is made up of

112 that you might regard as a basic set of D8 root vectors plus

8 x 128 = 1024 as 8 sets of 128, each being a semi-8-hypercube that can be combined with the 112 to make the 112+128 = 240 of an E8 such that the 8 E8s that can be so formed correspond to the real 1 octonion (Kirmse) + 7 imaginary octonions (integral domains) plus

the remaining 2160 - 112 - 1024 = 1024 = 8 x 128 which are the other halves of the 8-hypercubes and so correspond to the mirror half-spinors that are NOT used in the E8s already constructed. They might in some sense be seen as the half-spinor parts of a mirror set of 8 E8s.

Given the framework of the basic D8 112 root vectors, there are 8 ways that you can fit a set of 128 into them so as to form 1 (Kirmse) + 7 (integral domain) E8 lattices.

7 E8 lattice integral domains E8i E8j E8k E8e E8ie E8je E8ke
correspond to the 7 imaginary octonions i j k e ie je ke

Scale them so that their Inner Shells have Unit Radius because as Geoffrey Dixon said "... the inner shell should ... consist of unit elements ...[since]... O multiplication of ... unit elements is closed ...".

Consider the 240-vertex Unit Radius Inner Shells of E8 Lattice Integral Domains corresponding to the algebraic generators i j e of the imaginary octonions with coordinates of the form (E8i , E8j , E8e)
(there are 3 E8s representing **8** in the 24-dim Leech Lattice)

E8i itself has 240 vertices (x , 0 , 0)
E8j itself has 240 vertices (0 , x , 0)
E8e itself has 240 vertices (0 , 0 , x)

Then, consider the 2x240 + 3840 = 4320 vertices of Unit Radius Inner Shells of Λ16 Barnes-Wall Lattices constructed from pairs of E8 Lattices using Dixon's XY-product
(there are 3 ways to choose a Barnes-Wall Λ16 representing **8**+**8** in Leech)

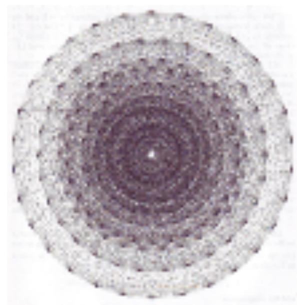
E8i x E8j = E8k with 16x240 = 3840 new vertices (x , y , 0)
E8i x E8e = E8ie with 16x240 = 3840 new vertices (x , 0 , y)
E8j x E8e = E8je with 16x240 = 3840 new vertices (0 , x , y)

Then, consider Unions of Unit Radius E8 Inner Shell with Unit Radius Λ16 Inner Shells,
rescaled by 1 / sqrt(2) so that the Unions have Unit Radius
(there are 3 ways to choose an E8 for (E8 + Barnes-Wall Λ16) to form Leech)

E8i x E8j x E8e x E8ie x E8je x E8k = E8ije x E8(-ijeek) = E8ke x E8ijk = E8ke x E8(-1) = E8(-ke) = 3x16x3840 = 3 x 61,440 = 184,320 vertices (x , y , z)
where 61,440 = vertices are in second shell of Barnes-Wall Λ16

The total number of inner vertices = 3 x (240 + 3840 + 61,440) = 196,560 which is the number of inner-shell vertices of the 24-dim Leech Lattice

Coxeter said in "Integral Cayley Numbers" (Duke Math. J. 13 (1946) 561-578 and
in "Regular and Semi-Regular Polytopes III" (Math. Z. 200 (1988) 3-45):
"... the 240 integral Cayley numbers of norm1 ... are the vertices of 4_21

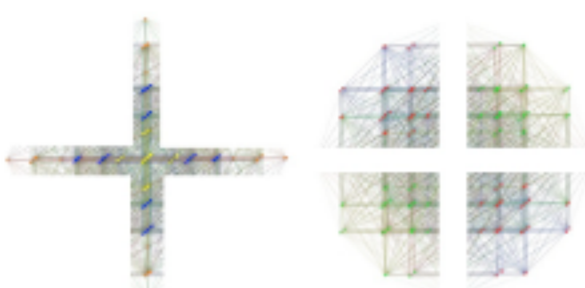


The polytope 4_21 ... has cells of two kinds ...
a seven-dimensional "cross polytope" (or octahedron-analogue) B_7
... there are ... 2160 B_7's ...
and ...
a seven-dimensional regular simplex A_7
... there are 17280 A_7's
...
the 2160 integral Cayley numbers of norm 2 are
the centers of the 2160 B_7's of a 4_21 of edge 2
...
the 17280 integral Cayley numbers of norm 4 (other than the doubles
of those of norm 1) are the centers of the 17280 A_7's of a 4_21 of edge 8/3 ...
[Using notation of {a1,a2,a3,a4,a5,a6,a7,a8} for Octonion basis elements we have]

norm 1

112 like (+/- a1 +/- a2)
[which correspond to 112 = 16 + 96 = 16 + 6x16 in each of the 7 E8 lattices]

128 like (1/2) (- a1 + a2 + a3 + ... + a8) with an odd number of minus signs
[which correspond to 128 = 8x16 in each of the 7 E8 lattices]



112 128

norm 2

16 like +/- 2 a1
[which correspond to 16 fo the 112 in each of the 7 E8 lattices]

1120 like +/- a1 +/- a2 +/- a3 +/- a4
[which correspond to 70x16 = (56+14)x16 that appear in the 7 E8 lattices

with each of the 14 appearing in three of the 7 E8 lattices so that
the 14 account for (14/7)x3x16 = 6x16 = 96 in each of the 7 E8 lattices
and for 14x16 = **224 of the 1120**
and
with each of the 56 appearing in only one of the 7 E8 lattices so that
the 56 account for (56/7)x16 = 128 in each of the 7 E8 lattices
and for 56x16 = **896 = 7x128 of the 1120**]

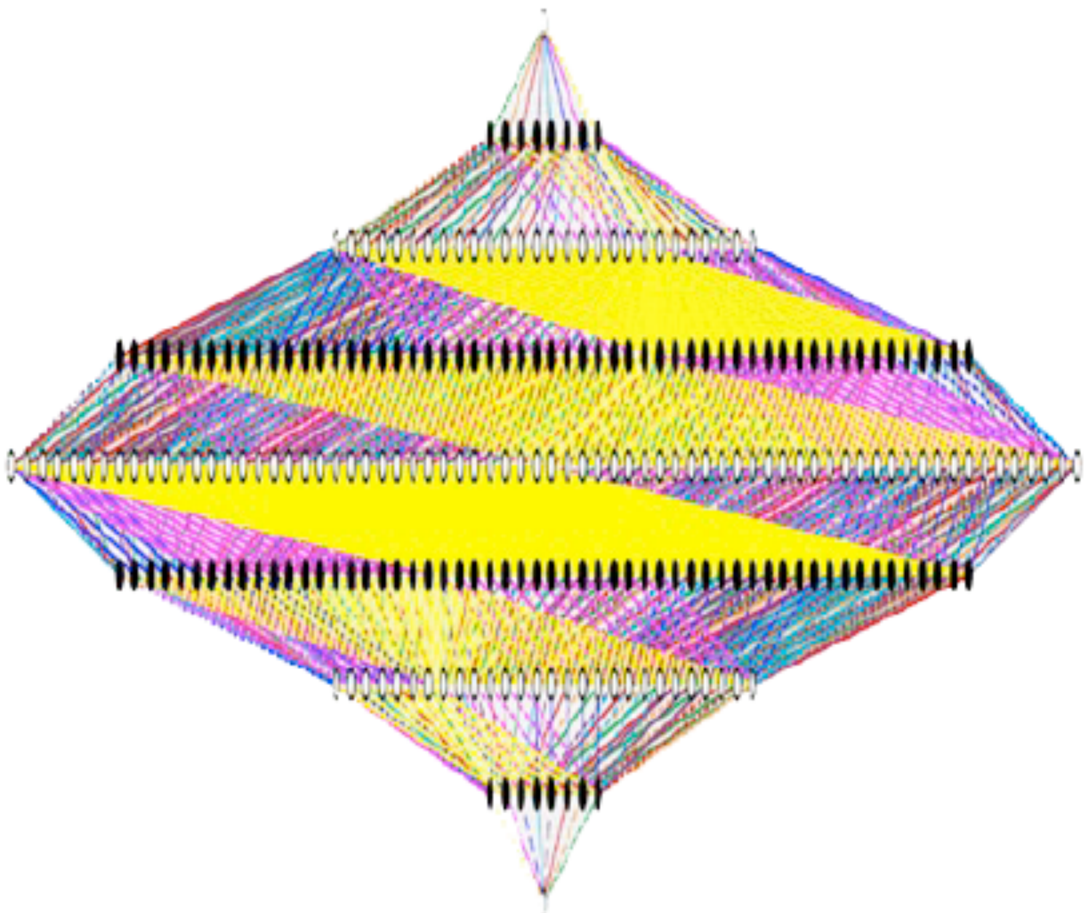
1024 like (1/2)(3a1 + 3a2 + a3 + a4 + ... + a8) with an even number of minus signs
[which correspond to **8x128** = 8 copies of the 128-dim Mirror D8 half-spinors that
are not used in the 7 E8 lattices. ...] ...".

One of the 128-dimensional Mirror D8 half-spinors from the 1024
combines with
the 128 from the 1120 corresponding to the one of the 7 E8 lattices that corresponds
to the central norm 1 240 = 112+128
and
the result is formation of a 128+128 = 256 corresponding to the Clifford Algebra Cl(8)
so that
the norm 2 second layer contains 7 copies of 256-dimensional Cl(8)

so the 2160 norm 2 vertices can be seen as

7(128+128) + 128 + 16 + 224 = 2160 vertices.

The 256 vertices of each pair 128+128 form an 8-cube with 1024 edges, 1792 square faces, 1792 cubic cells, 1120 tesseract 4-faces, 448 5-cube 5-faces, 112 6-cube 6-faces, and 16 7-cube 7-faces. The image format of African Adinkra for 256 Odu of IFA



shows Cl(8) graded structure 1 + 8 + 28 + 56 + 70 + 56 + 28 + 8 + 1 of 8-cube vertices. Physically they represent **Operators in H92 x SL(8)** Generalized Heisenberg Algebra that is the Maximal Contraction of E8:

Odd-Grade Parts of Cl(8) =
= **128 D8 half-spinors** of one of iE8, jE8, kE8, EE8, IE8, JE8, KE8
8+56 grades-1,3 = Fermion Particle 8-Component Creation (AntiParticle Annihilation)
56+8 grades-5,7 = Fermion AntiParticle 8-Component Creation (Particle Annihilation)

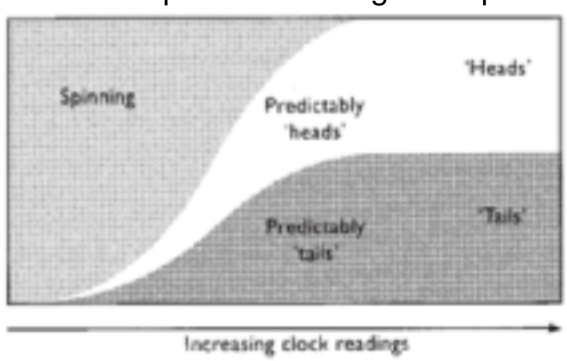
Even-Grade Subalgebra of Cl(8) = **128 Mirror D8 half-spinors** =
28 grade-2 = Gauge Boson Creation (16 for Gravity, 12 for Standard Model)
28 grade-6 = Gauge Boson Annihilation (16 for Gravity , 12 for Standard Model)
(each 28 = 24 Root Vectors + 4 of Cartan Subalgebra)
64 of grade-4 = 8-dim Position x Momentum
1+(3+3)+1 grades-0,4,8 = Primitive Idempotent:
(1+3) = Higgs Creation; (3+1) = Higgs Annihilation
= **112 D8 Root Vectors + 8 of E8 Cartan Subalgebra + 8 Higgs Operators**

3. The Third Grothendieck Universe is the Completion of Union of all tensor products of Cl(16) Real Clifford algebra

Since the Cl(16)-E8 Lagrangian is Local and Classical,
it is necessary to patch together Local Lagrangian Regions to form a Global Structure describing a Global Cl(16)-E8 Algebraic Quantum Field Theory (AQFT).

The usual Hyperfinite II1 von Neumann factor for creation and annihilation operators on Fermionic Fock Space over C^(2n) is constructed by completion of the union of all tensor products of 2x2 Complex Clifford algebra matrices, which have Periodicity 2,
so
for the Cl16)-E8 model based on Real Clifford Algebras with Periodicity 8,
whereby any Real Clifford Algebra, no matter how large,
can be embedded in a tensor product of factors of Cl(8) and of Cl(8)xCl(8) = Cl(16),
the completion of the union of all tensor products of Cl(16) = Cl(8)xCl(8)
produces a generalized Hyperfinite II1 von Neumann factor
that gives the Cl(16)-E8 model a natural Algebraic Quantum Field Theory.

The overall structure of Cl(160-E8 AQFT is similar to the Many-Worlds picture described by David Deutsch in his 1997 book "The Fabric of Reality" said (pages 276-283):
"… there is no fundamental demarcation between snapshots of other times and snapshots of other universes ... Other times are just special cases of other universes ... Suppose ... we toss a coin ... Each point in the diagram represents one snapshot



... in the multiverse there are far too many snapshots for clock readings alone to locate a snapshot relative to the others. To do that, we need to consider the intricate detail of which snapshots determine which others. …
in some regions of the multiverse, and in some places in space,
the snapshots of some physical objects do fall, for a period, into chains,
each of whose members determines all the others to a good approximation …".

The Real Clifford Algebra Cl(16) containing E8 for the Local Lagrangian of a Region is equivalent to a " snapshot" of the Deutsch "multiverse".
The completion of the union of all tensor products of all Cl(16)-E8 Local Lagrangian Regions forms a generalized hyperfinite II1 von Neumann factor AQFT
and emergently self-assembles into a structure = Deutsch multiverse.

For the Cl(16)-E8 model AQFT to be realistic, it must be consistent with **EPR entanglement relations**. Joy Christian in arXiv 0904.4259 said: "... a [geometrically] correct local-realistic framework ... provides exact, deterministic, and local underpinnings ... The alleged non-localities ... result from misidentified [geometries] of the EPR elements of reality. ... The correlations are ... the classical correlations [such as those] among the points of a 3 or 7-sphere ... S3 and S7 ... are ... parallelizable ... The correlations ... can be seen most transparently in the elegant language of Clifford algebra ...".
Since E8 is a Lie Group and therefore parallelizable and lives in Clifford Algebra Cl(16), **the Cl(16)-E8 model is consistent with EPR**.

The Creation-Annihilation Operator structure of Cl(16)-E8 AQFT is given by the

Maximal Contraction of E8 = semidirect product A7 x h92

where h92 = 92+1+92 = 185-dim Heisenberg algebra and A7 = 63-dim SL(8)
The Maximal E8 Contraction A7 x h92 can be written as a 5-Graded Lie Algebra

28 + 64 + (SL(8,R) + 1) + 64 + 28

Central Even Grade 0 = SL(8,R) + 1

The 1 is a scalar and SL(8,R) = Spin(8) + Traceless Symmetric 8x8 Matrices, so SL(8,R) represents a local 8-dim SpaceTime in Polar Coordinates.

Odd Grades -1 and +1 = 64 + 64

Each = 64 = 8x8 = Creation/Annihilation Operators for 8 components of 8 Fundamental Fermions.

Even Grades -2 and +2 = 28 + 28

Each = Creation/Annihilation Operators for 28 Gauge Bosons of Gravity + Standard Model.

The Cl(16)-E8 AQFT inherits structure from the Cl(16)-E8 Local Lagrangian

$\int$ Standard Model Gauge Gravity + Fermion Particle-AntiParticle
8-dim SpaceTime

The Cl(16)-E8 generalized Hyperfinite II1 von Neumann factor Algebraic Quantum Field Theory is based on the Completion of the Union of all Tensor Products of the form

Cl(16) x ...(N times tensor product)... x Cl(16) = Cl(16N)

For N = 2^8 = 256 the copies of Cl(16) are on the 256 vertices of the 8-dim HyperCube

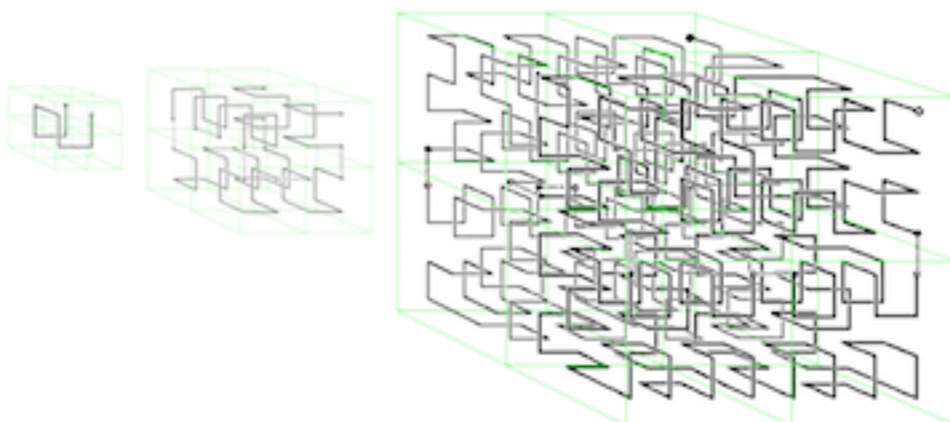
For N = 2^16 = 65,536 = 4^8 the copies of Cl(16) fill in the 8-dim HyperCube as described by William Gilbert's web page: "... **The n-bit reflected binary Gray code will describe a path on the edges of an n-dimensional cube that can be used as the initial stage of a Hilbert curve that will fill an n-dimensional cube**. ...".

The vertices of the Hilbert curve are at the centers of the 2^8 sub-8-HyperCubes whose edge lengths are 1/2 of the edge lengths of the original 8-dim HyperCube

As N grows, the copies of Cl(16) continue to fill the 8-dim HyperCube of E8 SpaceTime using higher Hilbert curve stages from the 8-bit reflected binary Gray code subdividing the initial 8-dim HyperCube into more and more sub-HyperCubes.

If edges of sub-HyperCubes, equal to the distance between adjacent copies of Cl(16), remain constantly at the Planck Length, then the

full 8-dim HyperCube of our Universe expands as N grows to 2^16 and beyond
similarly to the way shown by this 3-HyperCube example for N = 2^3, 4^3, 8^3
from Wiliam Gilbert's web page:



The Union of all Cl(16) tensor products is
the Union of all subdivided 8-HyperCubes
and
their Completion is a huge superposition of 8-HyperCube Continuous Volumes which Completion belongs to the Third Grothendieck Universe.

4. World-Line String Bohm Quantum Potential and Quantum Consciousness

Leech Lattice and E8 Bosonic String Theory

A Single Cell of E8 26-dimensional Bosonic String Theory,
in which Strings are physically interpreted as World-Lines,
can be described by taking the quotient of
its 24-dimensional O+, O-, Ov subspace modulo the 24-dimensional Leech lattice.
Its automorphism group is the largest finite sporadic group,
the Monster Group, whose order is
8080, 17424, 79451, 28758, 86459, 90496, 17107, 57005, 75436, 80000, 00000 =
= 2^46 .3^20 .5^9 .7^6 .11^2 .13^3 .17.19.23.29.31.41.47.59.71
or about 8 x 10^53.
A Leech lattice construction is described by Robert A. Wilson in his 2009 paper
"Octonions and the Leech lattice": "... The (real) octonion algebra is
an 8-dimensional ... algebra with an orthonomal basis { 1=ioo , i0 , i1 , i2 , i3 , i4 , i5 , i6 }
labeled by the projective line PL(7) = { oo } u F7
... **The E8 root system embeds in this algebra ... take the 240 roots to be ...**
112 octonions ... +/- it +/- iu for any distinct t,u ... and ...
128 octonions (1/2)(+/- 1 +/- i0 +/- ... +/- i6) which have an odd number of minus signs.nDenote by L the lattice spanned by these 240 octonions ...
Let s = (1/2)(- 1 + i0 + ... + i6) so s is in L ... write R for Lbar ...
(1/2) (1 + i0) L = (1/2) R (1 + i0) is closed under multiplication ... Denote this ...by A ...
Writing B = (1/2) (1 + i0) A (1 + i0) ...from ... Moufang laws ... we have
L R = 2 B , and ... B L = L and R B = R ...[also]... 2 B = L sbar
... **the roots of B are**
[16 octonions]... +/- it for t in PL(7) ... together with
[112 octonions]... (1/2) (+/- 1 +/- it +/- i(t+1) +/- i(t+3)) ...for t in F7 ... and ...
[112 octonions]... (1/2) (+/- i(t+2) +/- i(t+4) +/- i(t+5) +/- i(t+6)) ...for t in F7 ...

the octonionic Leech lattice ... contains the following 196560 vectors of norm 4 ,
where M is a root of L and j,k are in J = { +/- it l t in PL(7) },
and all permutations of the three coordinates are allowed:

(2 M, 0 , 0)
(M sbar, +/- (M sbar) j , 0)
((M s) j , +/- M k , +/- (M j) k) ...

Number: 3x240 = 720 Number: 3x240 x 16 = 11520: Number: 3x240 x 16 x 16 = 184320

The key to the simple proofs above is the observation that LR = 2B and BL = L:
these remarkable facts appear not to have been noticed before ... some work ...
by Geoffrey Dixon ...".

Geoffrey Dixon says in his book "Division Algebras, Lattices, Physics, Windmill Tilting" using notation {e0,e1,e2,e3,e4,e5,e6,e7} for the Octonion basis elements that Robert A. Wilson denotes by {1=ioo,i0,i1,i2,i3,i4,i5 ,i6} and I often denote by {1,i,j,k,E,I,J,K}:
"... (spans over integers) ...

Ξeven has 16+224 = 240 elements ...

Ξodd has 112+128 = 240 elements ...

E8even does not close with respect to our given octonion multiplication ...[but]...
the set Ξeven[0-a], derived from Ξeven by replacing each occurrence of e0 ... with ea, and vice versa, is multiplicatively closed. ...".

Geoffrey Dixon's Ξeven corresponds to B
Geoffrey Dixon's **Ξeven[0-a] corresponds to the seven At**
Geoffrey Dixon's Ξodd corresponds to L

Ignoring factors like 2 , j , k , and +/-1 the Leech lattice structure is:

(L , 0 , 0)
(B , B , 0)
(L s , L , L)

(Ξodd , 0 , 0)
(Ξeven , Ξeven , 0)
(Ξodd s , Ξodd , Ξodd)

Number: 3x240 = 720 Number: 3x240 x 16 = 11520: Number: 3x240 x 16 x 16 = 184320

Dixon Octonion XY, Fibrations, and 24-dim Leech Lattice

Frank Dodd (Tony) Smith, Jr. - 2014

Geoffrey Dixon, in his book “Division Algebras, Lattices, Physics, Windmill Tilting”,
defines an Octonionic XY-product and uses it to construct the $\Lambda 16$ Barnes-Wall Lattice
whose S15 Unit Sphere Inner Shell contains 2x240 +16x240 = 4320 vertices.
His construction makes use of the Last Hopf Fibration S7 -> S15 -> S8
and its lattice version E8 -> $\Lambda 16$ -> Z9

Ian Porteous, in his book “Clifford Algebras and the Classical Groups”,
notes that Spin(9) is transitive on S15 but Spin(10) is not transitive on S31
and constructs the non-Hopf Fibration S15 -> B24 -> S9
where S15 = Spin(9) / Spin(7) and S9 = Spin(10) / Spin(9)
and B24 = Spin(10) / Spin(7) = 24-dim orbit of 1
in the S31 Unit Sphere of the 32-dim Spinor Space of Spin(10)

Mikhail Postnikov, in “Lectures in Geometry Semester V Lie Groups and Lie Algebras”,
says Lie Algebra f4 = spin(8) + Mo24 where Tangent Space of B24 = Mo24 with basis

$$Y_1(\eta) = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 8 \\ 0 & -\bar{8} & 0 \end{pmatrix}, \quad Y_2(\eta) = \begin{pmatrix} 0 & 0 & -\bar{8} \\ 0 & 0 & 0 \\ 8 & 0 & 0 \end{pmatrix}, \quad Y_3(\eta) = \begin{pmatrix} 0 & 8 & 0 \\ -\bar{8} & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}.$$

where f4 inherits structure from the Real Clifford Algebra Cl(8) with grading
1 + 8 + **28** + 56 + 70 + 56 +28 + 8 + 1 = 256 = (8+8) x (8+8)

The resulting structure (here B24 = Mo24 means Mo24 is Tangent Space of B24):

$$\begin{array}{ccccc} \textbf{Spin(7)} & \rightarrow & \textbf{Spin(8)} & \rightarrow & \textbf{S7} \\ \downarrow & & \downarrow & & \\ \textbf{Spin(10)} & & \textbf{F4} & & \\ \downarrow & & \downarrow & & \\ \textbf{B24} & = & \begin{matrix} 0 & 8 & -\bar{8} \\ -\bar{8} & 0 & 8 \\ 8 & -\bar{8} & 0 \end{matrix} & & \end{array}$$

shows that B24 and Mo24, through F4 and its relationship to E8,
correspond to Cl(16)-E8 Physics. (see viXra 1405.0030)

So:

f4 = spin(8) + Mo24 = **28** + (**8** + (**8**+**8**))

where Spin(9) / Spin(8) = Spin(9) / **28** = OP1 = **8** with 8-dim E8 Lattice (**8** of B24)
and F4 / Spin(9) = OP2 = (**8**+**8**) with 16-dim Barnes-Wall Lattice Λ16 (Y and Z of B24)

All this is related to E8 Physics because the Leech Lattice B24 lives in F4
and F4 lives in Cl(8) and (by Real Clifford 8-Periodicity) Cl(16) = Cl(8) x Cl(8)
and **248-dim E8 lives in Cl(16) as 120-dim bivector D8 + 128-dim D8 half-spinor**
with F4 relationship coming from:

120 = **28**x1 + 1x**28** + **8**x**8**
128 = **8**x**8** + **8**x**8**

Note that the F4 **8**+**8** which represents Fermions can be written in terms of anti-commutators as well as commutators, so the same is true of E8 **8**x**8** + **8**x**8**
(see viXra 1208.0145)

Unit Sphere Inner Shell of B24 Λ24 Leech Lattice has 196,560 vertices
based on the fibration S15 -> B24 -> S9 where S9 is Unit Sphere in 10-dim.
Conway and Sloan in “Sphere Packings, Lattices, and Groups” 3rd ed),
Table 6.4, page 180, list a series of lattices:

8-dim E8 and 9-dim E8 A1 and 10-dim E8 A2

which series terminates with E8 A2.
(analogous to Minkowski Spin(1,3) and Spin(2,3) and Conformal Spin(2,4))

Leech Inner Shell Lattice Fibration Λ16 -> Postnikov Mo24 -> E8 A2
(Λ16 contains S15 and E8 A2 contains Unit Sphere of E8 A1 and
Tangent B24 = Mo24 contains S23 Leech Lattice Unit Sphere Inner Shell)

240 from the **8** E8 Inner Shell Unit Sphere of 9-dim E8 A1 of 10-dim E8 A2 lattice

2 x 240 = 720 from the Barnes-Wall E8 Inner Shell of **8**+**8** in Λ16

16 x 240 = 3840 from the XY-product, X and Y in **8** of E8 A2

16 x 240 = 3840 from the XY-product, X and Y in **8** of Λ16
16 x 240 = 3840 from the XY-product, X and Y in **8** of Λ16

16 x (3 x 16 x 240) = 184,320 from XY-product
of things outside **8** and **8**+**8**.

now all the Leech Lattice Unit Sphere Inner Shell points are accounted for:
3x240 + 3x16x240 + 16x3x16x240 = 196560

If you look at the 2160 vertices of the second shell as expanded from the first shell,
then you see that the 2160 is made up of

112 that you might regard as a basic set of D8 root vectors plus

8 x 128 = 1024 as 8 sets of 128, each being a semi-8-hypercube that can be combined with the 112 to make the 112+128 = 240 of an E8 such that the 8 E8s that can be so formed correspond to the real 1 octonion (Kirmse) + 7 imaginary octonions (integral domains) plus

the remaining 2160 - 112 - 1024 = 1024 = 8 x 128 which are the other halves of the 8-hypercubes and so correspond to the mirror half-spinors that are NOT used in the E8s already constructed. They might in some sense be seen as the half-spinor parts of a mirror set of 8 E8s.

Given the framework of the basic D8 112 root vectors, there are 8 ways that you can fit a set of 128 into them so as to form 1 (Kirmse) + 7 (integral domain) E8 lattices.

7 E8 lattice integral domains E8i E8j E8k E8e E8ie E8je E8ke
correspond to the 7 imaginary octonions i j k e ie je ke

Scale them so that their Inner Shells have Unit Radius because as Geoffrey Dixon said "... the inner shell should ... consist of unit elements ...[since]... O multiplication of ... unit elements is closed ...".

Consider the 240-vertex Unit Radius Inner Shells of E8 Lattice Integral Domains corresponding to the algebraic generators i j e of the imaginary octonions with coordinates of the form (E8i , E8j , E8e)
(there are 3 E8s representing **8** in the 24-dim Leech Lattice)

E8i itself has 240 vertices (x , 0 , 0)
E8j itself has 240 vertices (0 , x , 0)
E8e itself has 240 vertices (0 , 0 , x)

Then, consider the 2x240 + 3840 = 4320 vertices of Unit Radius Inner Shells of $\Lambda 16$ Barnes-Wall Lattices constructed from pairs of E8 Lattices using Dixon's XY-product
(there are 3 ways to choose a Barnes-Wall $\Lambda 16$ representing **8**+**8** in Leech)

E8i x E8j = E8k with 16x240 = 3840 new vertices (x , y , 0)
E8i x E8e = E8ie with 16x240 = 3840 new vertices (x , 0 , y)
E8j x E8e = E8je with 16x240 = 3840 new vertices (0 , x , y)

Then, consider Unions of Unit Radius E8 Inner Shell with Unit Radius $\Lambda 16$ Inner Shells, rescaled by 1 / sqrt(2) so that the Unions have Unit Radius
(there are 3 ways to choose an E8 for (E8 + Barnes-Wall $\Lambda 16$) to form Leech)

E8i x E8j x E8e x E8ie x E8je x E8k = E8ije x E8(-ijeek) = E8ke x E8ijk = E8ke x E8(-1) = E8(-ke) = 3x16x3840 = 3 x 61,440 = 184,320 vertices (x , y , z)
where 61,440 = vertices are in second shell of Barnes-Wall $\Lambda 16$

The total number of inner vertices = 3 x (240 + 3840 + 61,440) = 196,560
which is the number of inner-shell vertices of the 24-dim Leech Lattice

Ian Porteous, in his book "Clifford Algebras and the Classical Groups" Chapter 24 "Triality", says (quoted/paraphrased):

"... The induced Clifford or spinor actions of

Spin(1)	on S0
Spin(2)	on S1
Spin(3) and Spin(4)	on S3
Spin(5), Spin(6), Spin(7) and Spin(8)	on S7
Spin(9)	on S15

are … all transitive although the Clifford action of Spin(10) on S31 is not ...

The action of $Spin(10)$ *on* S^{31}

All the Clifford actions on spheres discussed up until now have been transitive. By contrast, the Clifford action of $Spin(10)$ on S^{31} is not, for the isotropy subgroup at 1 at least contains a Clifford copy of $Spin(7)$ as a subgroup, from which it follows that the dimension of the orbit of 1 is at most equal to

$$\dim Spin(10) - \dim Spin(7) = 45 - 21 = 24.$$

In fact the space of orbits, assigned the quotient topology, can be shown to be homeomorphic to a closed interval of the real line, one end-point of which represents an orbit A_{21} of dimension 21, homeomorphic both to $Spin(9)/Spin(6)$ and to $Spin(10)/SU(5)$, the embedding of $Spin(6)$ in $Spin(9)$ in the former case being a Clifford one, while the other end-point represents an orbit B_{24} of dimension 24, homeomorphic to $Spin(10)/Spin(7)$, the embedding of $Spin(7)$ in $Spin(10)$ being Clifford, $Spin(7) = H_0$ being indeed the isotropy subgroup at 1. Each of the interior points of the interval represents an orbit of dimension 30, homeomorphic to

$$C_{30} = Spin(10)/Spin(6) \cong A_{21} \times S^9$$

(the embedding of $Spin(6)$ in $Spin(10)$ being Clifford).

$$\begin{tikzcd}
Spin(6) \cong SU(4) \arrow[r, equal] \arrow[d] & Spin(6) \arrow[d] & \\
SU(5) \arrow[r] \arrow[d] & Spin(10) \arrow[r] \arrow[d] & A_{21} \arrow[d, equal] \\
S^9 \arrow[r] & C_{30} \arrow[r] & A_{21}
\end{tikzcd}$$

$$\begin{tikzcd}
Spin(6) \arrow[r, equal] \arrow[d] & Spin(6) \arrow[d] & \\
Spin(7) \arrow[r] \arrow[d] & Spin(10) \arrow[r] \arrow[d] & B_{24} \arrow[d, equal] \\
S^6 \arrow[r] & C_{30} \arrow[r] & B_{24}
\end{tikzcd}
\qquad
\begin{tikzcd}
Spin(7) \arrow[r, equal] \arrow[d] & Spin(7) \arrow[d] & \\
Spin(9) \arrow[r] \arrow[d] & Spin(10) \arrow[r] \arrow[d] & S^9 \arrow[d, equal] \\
S^{15} \arrow[r] & B_{24} \arrow[r] & S^9
\end{tikzcd}$$

...”.

J. F. Adams, in “Lectures on Exceptional Lie Groups”, said: “...

F_4 contains $Spin(9)$ as a subgroup of maximal rank:

$$F_4 \supset Spin(9) \supset Spin(8) \supset T.$$

the roots are

$\pm x_i \pm x_j$, $1 \leq i < j \leq 4$: there are 24 of these, all long;

$\pm x_i$, $1 \leq i \leq 4$: there are 8 of these short roots;

(These 32 roots come from $Spin(9)$.)

$\frac{1}{2}(\pm x_1 \pm x_2 \pm x_3 \pm x_4)$: there are 16 of these short roots, all from Δ.

Theorem 16.7. $F_4 \cong Aut(J)$

The subgroup of $Aut(J)$ *fixing* $e_1 = diag(1,0,0)$ *is* $Spin(9)$.

$$\theta(e_2 - e_3) = \begin{pmatrix} 0 & 0 & 0 \\ 0 & \lambda & x_1 \\ 0 & \bar{x}_1 & -\lambda \end{pmatrix} \quad \text{where } \lambda^2 + \bar{x}_1 x_1 = 2.$$

The subgroup of $Aut(J)$ *fixing* e_1, $e_2 = diag(0,1,0)$ *and* $e_3 = diag(0,0,1)$ *is* $Spin(8)$.

Its $\frac{1}{2}$-eigenspace, of dimension 16, is $V_2 + V_3$,

the space of matrices of the form $\begin{pmatrix} 0 & x_3 & \bar{x}_2 \\ \bar{x}_3 & 0 & 0 \\ x_2 & 0 & 0 \end{pmatrix}$

...".

Here are some details about Dixon's XY-product:

Geoffrey Dixon, in his book "Division Algebras, Lattices, Physics, Windmill Tilting", says:
"... use the product ...

$\mathbf{O}^{+3}$: $e_a e_{a+1} = e_{a+3}$;
$(124), (235), (346), (457), (561), (672), (713),$

the X-product could be used to generate all the 480 renumberings of the e_a, $a = 1, ..., 7$, which leave $e_0 = 1$ fixed as the identity (starting from either $\mathbf{O}^{\pm 3}$). There are 7680 renumberings of the entire collection, e_a, $a = 0, ..., 7$, and the XY-product plays exactly the same role in this context. In addition, in the X-product case the 480 renumberings arose from a pair of octonion Λ_8 lattices. The XY-product renumberings are related in a similar fashion to the pair of octonion Λ_{16} lattices.

there are 16 possible values for $\pm e_o$
$16 \times 240 = 3840$ form part of the inner shell of a Λ_{16} lattice (of radius $\sqrt{2}$).
an additional $2 \times 240 = 480$ elements for inner shell
bring the total to $3840 + 480 = 4320$ which is the kissing number of Λ_{16}.

Let $A, B, X \in \mathbf{O}$, with X a unit octonion:

$$XX^\dagger = 1 \Longrightarrow X \in S^7,$$

where S^7 is the 7-sphere in the 8-dimensional space of $\mathbf{O}$ Define

$$\boxed{A \circ_X B = (AX)(X^\dagger B) = (A(BX))X^\dagger = X((X^\dagger A)B),}$$

the X-product of A and B.

Because of the nonassociativity of $\mathbf{O}$, $A \circ_X B \neq AB$ in general.

The X-product changes $\mathbf{O}$ to $\mathbf{O}_X$, which is isomorphic to $\mathbf{O}$.
the identities of $\mathbf{O}$ and $\mathbf{O}_X$ are both $e_0 = 1$.

However, it is possible to modify the octonion product in such a way that e_0 is not the identity of the result.

In particular, define

$$\boxed{\begin{aligned} A \circ_{X,Y} B &= (AX)(Y^\dagger B) \\ &= A \circ_X ((XY^\dagger) \circ_X B) \\ &= (A \circ_Y (XY^\dagger)) \circ_Y B, \end{aligned}}$$

where as usual we assume that both $X, Y \in S^7$ (the X-product is obtained by setting $X = Y$). Let $\mathbf{O}_{X,Y}$ be $\mathbf{O}$ with this modified product.

let

$$E_a, \ a = 0, ..., 7,$$

be a basis for $\mathbf{O}_{X,Y}$, for arbitrarily chosen $X, Y \in S^7$.

$YX^\dagger = Z$ is the identity of $\mathbf{O}_{X,Y}$.

We will establish $e_a \longrightarrow E_a$, $a = 0, ..., 7$, as an isomorphism from $\mathbf{O}$ to $\mathbf{O}_{X,Y}$.

...". Lattice Fibrations he uses include

For E8:
inner shell lattice fibration D4 -> E8 -> Z5 (D4 contains S3 and Z5 contains S4 and E8 contains S7 inner shell)

For Barnes-Wall:
inner shell lattice fibration E8 -> /\16 -> Z9 (E8 contains S7 and Z9 contains S8 and /\16 contains S15 inner shell)

Spheres and Physics

Frank Dodd (Tony) Smith, Jr. - 2014

Parallelizable Spheres:

S0 = equator of S1 = RP1 (… infinite sequence of RPn … = useless Category stuff)

S1 = equator of S2 = CP1 (… infinite sequence of CPn … = useless Category stuff)

S3 = equator of S4 = HP1 (… infinite sequence of HPn … = useless Category stuff)

S7 = equator of S8 = OP1 G2 / SU(3) = S6 = equator of S7

OP2 = F4 / Spin(9)

(CxO)P2 = E6 / Spin(10) x U(1)

(HxO)P2 = E7 / Spin(12) x SU(2)

(OxO)P2 = E8 / Spin(16) (finite exceptional stuff stops here = God says is important)

Spin(16) = bivector of real Clifford Algebra Cl(16) = Cl(8) x Cl(8)

F4 = 8 + 28 + (8+8) of Cl(8) = 1+8+28+56+70+56+28+8+1 = 256 = (8+8) x (8+8)

F4 = (28 = Spin(8)) + (8 + (8+8) = E8 Lattice + Λ16 Lattice = Λ24 Leech)

also

F4 acts on 26-dim rep = J3(O)o = traceless part of Jordan J3(O) = Λ25,1 Lorentz Leech

F4 + J3(O)o = E6

from string theory (with strings physically interpreted as World-Lines) point of view
26-dim J3(O)o = 16-dim orbifold fermions + 10-dim spacetime

10-dim spacetime = 4+6
where 4 = CP2 = SU(3) / SU(2) x U(1) Internal Symmetry Space of Kaluza-Klein
and
6 = Conformal (2,4) space over Minkowski (1,3) physical spacetime of Kaluza-Klein

Exceptional Groups and Physics

hep-th/0301050 **Pierre Ramond**

Introduce the sixteen (256×256) Dirac matrices

$$\{\Gamma^a, \Gamma^b\} = 2\delta^{ab}, \qquad a, b = 1, 2 \ldots 16,$$

with vector indices transforming as the $SO(9)$ spinor(recall the anomalous Dynkin embedding). These are not to be confused with the (16×16) nine Dirac matrices which transform as $SO(9)$ vectors

$$\{\gamma^i, \gamma^j\} = 2\delta^{ij}, \qquad i, j = 1, 2 \ldots, 9.$$

Together they allow for a neat way of writing the $SO(9)$ generators

$$S^{ij} = -\frac{i}{4}\left(\gamma^{ij}\right)_{ab}\Gamma^{ab},$$

where in the usual notation $\gamma^{ij} = \gamma^i\gamma^j$, $i \neq j$, $\Gamma^{ab} = \Gamma^a\Gamma^b$, $a \neq b$. The 52 F_4 parameters split into the 36 S^{ij} which generate $SO(9)$, and sixteen $SO(9)$ spinors, T^a. Algebraic closure is given by

$$[T^a, T^b] = \frac{i}{2}\left(\gamma^{ij}\right)^a S^{ij},$$

there are three equivalent ways to embed $SO(9)$ inside F_4

This is the octonionic equivalent I-spin, U-spin and V-spin which label three equivalent ways to embed $SU(2)$ inside $SU(3)$. The F_4 Weyl chamber is 1/3 that of $SO(9)$. Take a highest weight in the F_4 Weyl chamber, λ. Let ρ be the sum of the fundamental weights. There exist two Weyl reflections C, which map λ outside the F_4 Weyl chamber, but stay inside that of $SO(9)$. Hence there is a unique way to associate one F_4 representation to three $SO(9)$ irreps. The mapping is

$$C \bullet \lambda = C\left(\lambda + \rho_{F_4}\right) - \rho_{SO(9)}.$$

This mapping associates with each F_4 irrep, a set of three $SO(9)$ representations called Euler triplets. Equality betwen its Dynkin indices is guaranteed by the character formula

$$V_\lambda \otimes S^+ - V_\lambda \otimes S^- = \sum_C sgn(C)\,\mathcal{U}_{C\bullet\lambda},$$

where V_λ is any F_4 representation written in terms of its $SO(9)$ content, $S^\pm$ are the two spinor irreps of $SO(16)$

S^+ and S^- have different Pfaffian invariants

One recognizes the "trivial" Euler triplet as

akin to an index formula for Kostant's operator associated with the coset $F_4/SO(9)$, the sixteen-dimensional projective Cayley-Moufang plane. Euler triplets are solutions of Kostant's equation

$$\not{K}\,\Psi \;\equiv\; \Gamma^a\,T^a\,\Psi \;=\; 0\;,$$

where the T^a generate the $F_4/SO(9)$ tranformations.

$$[\,T^a\,,\,T^b\,] \;=\; i\,f^{[ij]\,ab}\,T^{ij}\;.$$

Kostant's operator commutes with the generalized $SO(9)$ generator made up of an "orbital" and the previously defined "spin" part

$$L^{ij} \;\equiv\; T^{ij} + S^{ij}\;.$$

The solutions to Kostant's equation are the Euler triplets

The number of representations in each Euler set is the ratio of the order of the F_4 and $SO(9)$ Weyl groups. It is also the Euler number of the coset manifold, hence the name.

It is convenient to express the F_4 in terms of three sets of 26 real coordinates: u_i which transform as transverse space vectors, u_0 as scalars, and ζ_a as space spinors. This enables us to write the Euler triplets as chiral superfields of the form

$$\Phi(y^-,\vec{x}\,,\theta^\alpha) \;=\; \theta^1\theta^8\Big(h(y^-,\vec{x}\,,u_i\,,\zeta_a) + \theta^4\,\psi(y^-,\vec{x}) + \theta^4\theta^5\,A(y^-,\vec{x})\Big)\;,$$

where now the components h, ψ and A are the highest weight components of the three irreps with definite polynomial dependence on the new coordinates. For the proper spin-statistics interpretation, the twistor-like variables ζ_a must appear quadratically. It turns out that the ζ's appear in even powers only for those Euler triplets that have the same number of bosons and fermions!

ALGEBRAIC DREAMS

hep-th/0112261 **Pierre Ramond**

sixteen (256×256) matrices, Γ^a satisfy the Dirac algebra

$$\{\Gamma^a, \Gamma^b\} = 2\delta^{ab} .$$

This leads to an elegant representation of the $SO(9)$ generators

$$S^{ij} = -\frac{i}{4}(\gamma^{ij})^{ab}\Gamma^a\Gamma^b \equiv -\frac{i}{2}f^{ij\,ab}\Gamma^a\Gamma^b .$$

The coefficients

$$f^{ij\,ab} \equiv \frac{1}{2}(\gamma^{ij})^{ab} ,$$

naturally appear in the commutator between the generators of $SO(9)$ and any spinor operator T^a, as

$$[T^{ij}, T^a] = \frac{i}{2}\left(\gamma^{ij}T\right)^a = if^{ij\,ab}T^b .$$

the $(\gamma^{ij})^{ab}$ can also be viewed as structure constants of a Lie algebra. Manifestly antisymmetric under $a \leftrightarrow b$, they can appear in the commutator of two spinors into the $SO(9)$ generators

$$[T^a, T^b] = \frac{i}{2}(\gamma^{ij})^{ab}T^{ij} = f^{a\,b\,ij}T^{ij} ,$$

and one easily checks that they satisfy the Jacobi identities.

the 52 operators T^{ij} and T^a generate the exceptional Lie algebra F_4

3.3 The Kostant Operator

This character formula can be viewed as the index formula of a Dirac-like operator formed over the coset $F_4/SO(9)$. This coset is the sixteen-dimensional Cayley projective plane, over which we introduce the previously considered Clifford algebra

$$\{\,\Gamma^a\,,\Gamma^b\,\}\;=\;2\,\delta^{ab}\;,\quad a,b=1,2,\ldots,16\;,$$

generated by (256×256) matrices. The Kostant equation is defined as

$$\not{\mathcal{K}}\,\Psi\;=\;\sum_{a=1}^{16}\Gamma^a\,T^a\,\Psi\;=\;0\;,$$

where T_a are F_4 generators not in $SO(9)$, with commutation relations

$$[\,T^a\,,T^b\,]\;=\;i\,f^{\,ab\,ij}\,T^{ij}\;.$$

Although it is taken over a compact manifold, it has non-trivial solutions. To see this, we rewrite its square as the difference of positive definite quantities,

$$\not{\mathcal{K}}\not{\mathcal{K}}\;=\;C^2_{F_4}-C^2_{SO(9)}+72\;,$$

where

$$C^2_{F_4}\;=\;\frac{1}{2}T^{ij}\,T^{ij}+T^a\,T^a\;,$$

is the F_4 quadratic Casimir operator, and

$$C^2_{SO(9)}\;=\;\frac{1}{2}\left(T^{ij}-i\,f^{ab\,ij}\,\widetilde{\Gamma}^{ab}\right)^2\;,$$

is the quadratic Casimir for the sum

$$L^{ij}\;\equiv\;T^{ij}+S^{ij}\;,$$

where S^{ij} is the previously defined $SO(9)$ generator

We have also used the quadratic Casimir on the spinor representation

$$\frac{1}{2} S^{ij} S^{ij} \;=\; 72\;.$$

Kostant's operator commutes with the sum of the generators,

$$[\not{\mathcal{K}}, L^{ij}] \;=\; 0\;,$$

allowing its solutions to be labelled by $SO(9)$ quantum numbers.

The same construction of Kostant's operator applies to all equal rank embeddings

In particular we note the cases $E_6/SO(10) \times SO(2)$, with Euler number 27, $E_7/SO(12) \times SO(3)$ with Euler number 63, and $E_8/SO(16)$, where the Euler triplets contain 135 representations These cosets with dimensions 32 ,64, and 128 could be viewed as complex, quaternionic and octonionic Cayley plane

3.4 Oscillator Representation of F_4

Schwinger's celebrated representation of $SU(2)$ generators of in terms of one doublet of harmonic oscillators can be extended to other Lie algebras

The generalization involves several sets of harmonic oscillators, each spanning the fundamental representation. For example, $SU(3)$ is generated by two sets of triplet harmonic oscillators, $SU(4)$ by two quartets. In the same way, all representations of the exceptional group F_4 are generated by three sets of oscillators transforming as **26**. We label each copy of 26 oscillators as $A_0^{[\kappa]}$, $A_i^{[\kappa]}$, $i = 1, \cdots, 9$, $B_a^{[\kappa]}$, $a = 1, \cdots, 16$, and their hermitian conjugates, and where $\kappa = 1, 2, 3$. Under $SO(9)$, the $A_i^{[\kappa]}$ transform as **9**, $B_a^{[\kappa]}$ transform as **16**, and $A_0^{[\kappa]}$ is a scalar. They satisfy the commutation relations of ordinary harmonic oscillators

$$[\,A_i^{[\kappa]}\,,\, A_j^{[\kappa']\,\dagger}\,] \;=\; \delta_{ij}\,\delta^{[\kappa]\,[\kappa']}\;, \qquad [\,A_0^{[\kappa]}\,,\, A_0^{[\kappa']\,\dagger}\,] \;=\; \delta^{[\kappa\,\kappa']}\;.$$

Note that the $SO(9)$ spinor operators satisfy Bose-like commutation relations

$$[\,B_a^{[\kappa]}\,,\,B_b^{[\kappa']\,\dagger}\,]\;=\;\delta_{ab}\,\delta^{[\kappa]\,[\kappa']}\;.$$

The generators T_{ij} and T_a

$$T_{ij}\;=\;-i\sum_{\kappa=1}^{4}\left\{\left(A_i^{[\kappa]\dagger}A_j^{[\kappa]}-A_j^{[\kappa]\dagger}A_i^{[\kappa]}\right)+\frac{1}{2}\,B^{[\kappa]\dagger}\,\gamma_{ij}B^{[\kappa]}\right\}\;,$$

$$T_a\;=\;-\frac{i}{2}\sum_{\kappa=1}^{4}\left\{(\gamma_i)^{ab}\left(A_i^{[\kappa]\dagger}B_b^{[\kappa]}-B_b^{[\kappa]\dagger}A_i^{[\kappa]}\right)-\sqrt{3}\left(B_a^{[\kappa]\dagger}A_0^{[\kappa]}-A_0^{[\kappa]\dagger}B_a^{[\kappa]}\right)\right\}\;,$$

satisfy the F_4 algebra,

$$[\,T_{ij}\,,T_{kl}\,]\;=\;-i\,(\delta_{jk}\,T_{il}+\delta_{il}\,T_{jk}-\delta_{ik}\,T_{jl}-\delta_{jl}\,T_{ik})\;,$$
$$[\,T_{ij}\,,T_{a}\,]\;=\;\frac{i}{2}\,(\gamma_{ij})_{ab}\,T_b\;,$$
$$[\,T_{a}\,,T_{b}\,]\;=\;\frac{i}{2}\,(\gamma_{ij})_{ab}\,T_{ij}\;,$$

so that the structure constants are given by

$$f_{ij\,ab}\;=\;f_{ab\,ij}\;=\;\frac{1}{2}\,(\gamma_{ij})_{ab}\;.$$

The last commutator requires the Fierz-derived identity

$$\frac{1}{4}\,\theta\,\gamma^{ij}\,\theta\,\chi\,\gamma^{ij}\,\chi\;=\;3\,\theta\,\chi\,\chi\,\theta+\theta\,\gamma^{i}\,\chi\,\chi\,\gamma^{i}\theta\;,$$

from which we deduce

$$3\,\delta^{ac}\delta^{db}+(\gamma^{i})^{ac}\,(\gamma^{i})^{db}-(a\leftrightarrow b)\;=\;\frac{1}{4}\,(\gamma^{ij})^{ab}\,(\gamma^{ij})^{cd}\;.$$

To satisfy these commutation relations, we have required both A_0 and B_a to obey Bose commutation relations

(Curiously, if both are anticommuting, the F_4 algebra is still satisfied).

The traceless Jordan matrices span the **26** representations of F_4. One can supplement the F_4 transformation by an additional 26 parameters, and define

$$\mathcal{D}_X\,J\;\equiv\;X\circ J\;,$$

leading to a group with 78 parameters. These extra transformations are non-compact, and close on the F_4 transformations, leading to the exceptional group $E_{6(-26)}$. The subscript in parenthesis denotes the number of non-compact minus the number of compact generators.

The Cl(16)-E8 AQFT inherits structure from the Cl(16)-E8 Local Lagrangian

$\int$ Standard Model Gauge Gravity + Fermion Particle-AntiParticle
8-dim SpaceTime

whereby World-Lines of Particles are represented by Strings moving in a space whose dimensionality includes 8v = 8-dim SpaceTime Dimensions + + 8s+ = 8 Fermion Particle Types + 8s- = 8 Fermion AntiParticle Types combined in the traceless part J(3,O)o of the 3x3 Octonion Hermitian Jordan Algebra

a	8s+	8v
8s+*	b	8s-
8v*	8s-*	-a-b

which has total dimension 8v + 8s+ + 8s- + 2 = 26 and is the space of a
26D String Theory with Strings seen as World-Lines.

Slices of 8v SpaceTime are represented as D8 branes. Each D8 brane has Planck-Scale Lattice Structure superpositions of 8 types of E8 Lattice
denoted by 1E8, iE8, jE8, kE8, EE8, IE8, JE8, KE8

Stack D8 branes to get SpaceTime with Strings = World-Lines
with
a and b representing
ordering of D8 brane stacks and Bohm-type Quantum Potential

Let Oct16 = discrete mutiplicative group { +/-1, +/-i, +/-j, +/-k, +/-E, +/-I, +/-J, +/-K}.
Orbifold by Oct16 the 8s+ to get 8 Fermion Particle Types
Orbifold by Oct16 the 8s- to get 8 Fermion AntiParticle Types

Gauge Bosons from 1E8 and EE8 parts of a D8 give U(2) Electroweak Force
Gauge Bosons from IE8, JE8, and KE8 parts of a D8 give SU(3) Color Force
Gauge Bosons from 1E8, iE8, jE8, and kE8 parts of a D8 give U(2,2) Conformal Gravity

The 8x8 matrices for collective coordinates linking one D8 to the next D8
give Position x Momentum

Green, Schwartz, and Witten say in their book "Superstring Theory" vol. 1 (Cambridge 1986)
"... For the ... closed ... bosonic string .... The first excited level ... consists of ...
the ground state ... tachyon ... and ... a scalar ... 'dilaton' ... and ...
SO(24) ... little group of a ...[26-dim]... massless particle ... and ...
a ... massless ... spin two state ...".
Closed string tachyons localized at orbifolds of fermions produce virtual clouds of particles / antiparticles that dress fermions.

Dilatons are Goldstone bosons of spontaneously broken scale invariance that (analagous to Higgs) go from mediating a long-range scalar gravity-type force to the nonlocality of the Bohm-Sarfatti Quantum Potential.

The SO(24) little group is related to the Monster automorphism group that is the symmetry of each cell of Planck-scale local lattice structure.

The massless spin two state is what I call the Bohmion:
the carrier of the Bohm Force of the Bohm-Sarfatti Quantum Potential.
Peter R. Holland says in his book "The Quantum Theory of Motion" (Cambridge 1993)
"... the total force ... from the quantum potential ... does not ... fall off with distance ... because ... the quantum potential ... depends on the form of ...[the quantum state]... rather than ... its ... magnitude ...".

Quantum Consciousness is due to Resonant Quantum Potential Connections among Quantum State Forms. The Quantum State Form of a Conscious Brain is determined by the configuration of a subset of its 10^{18} to 10^{19} Tubulin Dimers with math description in terms of a large Real Clifford Algebra.

First consider Superposition of States involving one tubulin
with one electron of mass m and two different position states separated by a .
The Superposition Separation Energy Difference is the gravitational energy

$$E_{electron} = G m^2 / a$$

For any single given tubulin a = 1 nanometer = 10^{-7} cm so that for a single Electron

$$T = h / E_{electron} = (\text{Compton} / \text{Schwarzschild}) (a / c) = 10^{26} \text{ sec} = 10^{19} \text{ years}$$

Now consider the case of N Tubulin Electrons in Coherent Superposition
Jack Sarfatti defines coherence length L by $L^3 = N a^3$ so that
the Superposition Energy E_N of N superposed Conformation Electrons is

$$E_N = G M^2 / L = N^{5/3} E_{electron}$$

The decoherence time for the system of N Tubulin Electrons is

$$T_N = h / E_N = h / N^{5/3} E_{electron} = N^{-5/3} 10^{26} \text{ sec}$$

So we have the following rough approximate Decoherence Times T_N

Time T_N	Number of Involved Tubulins
10^{-5} sec	10^{18}
25×10^{-3} sec (40 Hz)	10^{16}

Quantum Resonant States in Superposition

A Quantum Resonant Consciousness (QRC) Superposition State is
a Tubulin Configuration of up to 2^64 = 10^19 Tubulins (each Tubulin = 1 qubit)
with each QRC State in the Superposition being organized
with respect to the E8 inside Cl(16) Clifford Algebras.

Each QRC State, analagous to a Possible Conscious Thought,
is represented by a Chain of Local E8-Cl(16) Deutsch-type Multiverse Snapshots
in which each Link in the Chain is
a Central Local E8-Cl(16) Multiverse Shapshot
connected to
a Past Local E8-Cl(16) Multiverse Snapshot
and
a Future Local E8-Cl(16) Multiverse Snapshot.

Since Cl(16) is 2^16 = 65,536-dimensional each Link in the QRC State Chain requires
the information of 2^16 x 2^16 x 2^16 = 2^48 Tubulin qubits.

The remaining 2^(64-48) = 2^16 = 2^6 x 2^10 = 64 x 1024 Tubulin qubits represent:
64 Links in each Chain of a Possible Conscious Thought
and
1024 Possible Conscious Thoughts in the QRC Superposition.

After Decoherence of the QRC Superposition there emerges the One Actual Thought.

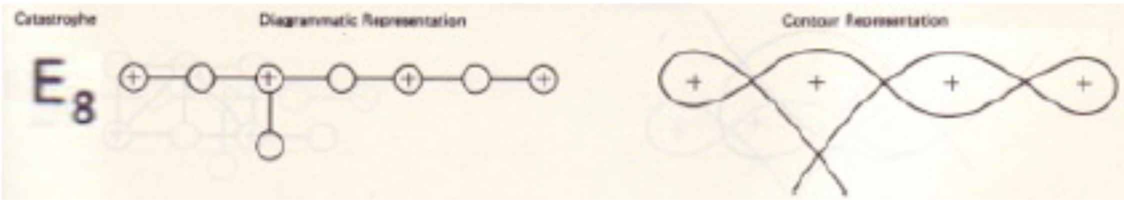
Each of the Local E8-Cl(16) Multiverse Snapshots is described by an E8 State.
Since E8 has 240 Root Vectors
and
the 240 Root Vectors correspond to the 240-Polytope (see “Geometric Frustration” by Sadoc and Mosseri (Cambridge 2006) where they say “The polytope 240 ...[is]... not a regular polytope ... but ... an ordered structure on a hypersphere ... S3 ... which is chiral ... generated by adding two replicas of the {3,3,5}, displaced along a screw axis of S3 ...”.)
each Local E8-Cl(16) Multiverse Snapshot is represented by a pair of {3,3,5} 600-cells.

Each of the 600-cells has 120 vertices corresponding to the 120-dimensional Icosahedral Double (ID) group which in turn corresponds to E8 (John McKay said on usenet sci.math in 1993:
"... For each finite subgroup of SU2, we get an affine Dynkin diagram ...

```
                     3
                     |
E[8]  1-2-3-4-5-6-4-2
```

... The [McKay] correspondence is ...
E[8] ...[corresponds to] 2.Alt[5] = SL(2,5) binary icosahedral [ID group] ...
There are [8+1 = 9 balance numbers for E8]...
The sum of the numbers [1+2+3+4+5+6+4+2+3 = 30 is] h = Coxeter number.
The sum of the squares is the order of ...[120-element ID for E8]...
They are the periods of products of pairs of Fischer involutions mod centre ... E[8] ...[for]... Monster
... for the E8 - icosahedral ... case, the singularity is x^2+y^3+z^5=0 ...”.

Robert Gilmore, in his book “Catastrophe Theory” (Dover 1981) said:
"...[The Icosahedral Double Group Catastrophe]... E8 ...[has]... Catastrophe Germ ... X^3 + Y^5
...[with]... Perturbation ... a1 Y + a2 Y^2 + a3 Y^3 + a4 X + a5 X Y + a6 X Y^2 + a7 X Y^3



...

The germs E_6, E_8 are

$$E_6: \quad f(x, y) = x^3 + y^4$$

$$E_8: \quad f(x, y) = x^3 + y^5$$

The rules for determinacy and unfolding are particularly easy to carry out for E_6 and E_8 because both $\partial f/\partial x$ and $\partial f/\partial y$ are monomials. These calculations are summarized diagrammatically in Fig. 23.4.

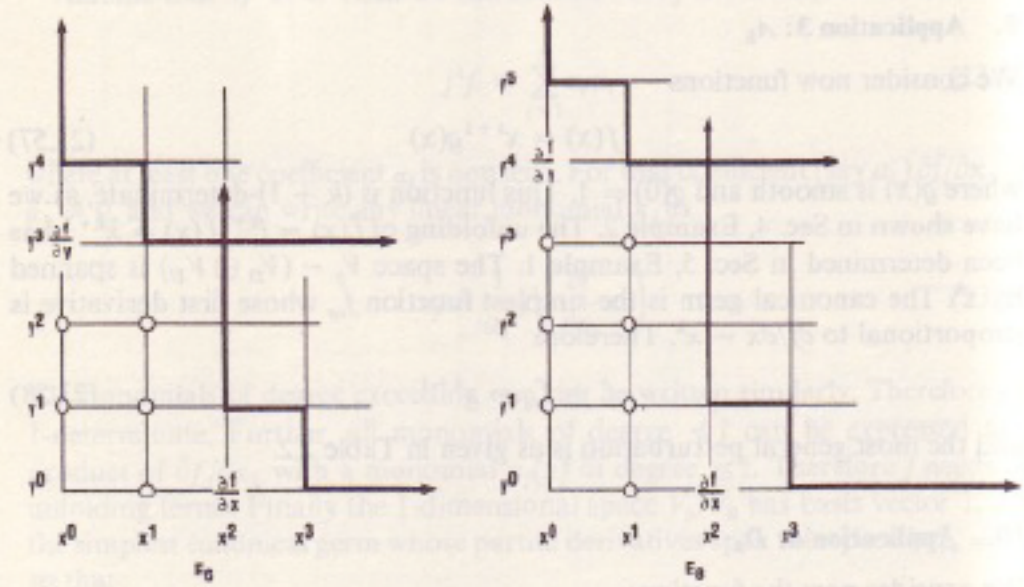


Figure 23.4 For E_6 and E_8 all monomials of degree 4 and 5 can be expressed in the form $(\partial f/\partial x_i)m_j$. The unfolding terms are represented by open circles. We exclude the constant term.

for E8 ...[with]... control parameter space R7 ...[basis {a1,a2,a3,a4,a5,a6,a7}]...
the maximum number ... of isolated critical points ..[is]... 8 ...”.

In his Appendix to Jeffrey Mishlove’s book “Roots of Consciousness”, Saul-Paul Sirag did not “exclude the constant term” as Robert Gilmore did, so, if we add a control parameter a0, we see that the ID E8 Catastrophe Control Parameter Space is R8 with basis {a0,a1,a2,a3,a4,a5,a6,a7}.
Adding two basis elements {X,Y} of ID Catastrophe Germ space whose polynomials are invariant under the Icosahedral Double Group ID results in 8+2 = 10 dimensions.

David Ford and John McKay wrote in the book "The Geometric Vein"
(Springer-Verlag 1981):
"... The columns of the character tables of ... the binary icosahedral group
...[ID Icosahedral Double Group]... of order 120
are the (suitably normalized) eigenvectors of the Cartan matrices of type ... E8 ...
[Let gr = (1/2) (- 1 - sqrt(5)) and GR = (1/2) (- 1 + sqrt(5)) and note that gr + GR = -1]
...

1	1	1	1	1	1	1	1	1
2	-2	0	-1	1	GR	gr	-gr	-GR
2	-2	0	-1	1	gr	GR	-GR	-gr
3	3	-1	0	0	-gr	-GR	-GR	-gr
3	3	-1	0	0	-GR	-gr	-gr	-GR
4	4	0	1	1	-1	-1	-1	-1
4	-4	0	1	-1	-1	-1	1	1
5	5	1	-1	-1	0	0	0	0
6	-6	0	0	0	1	1	-1	-1

...".

A Cartan matrix for E8 is

2	0	-1	0	0	0	0	0
0	2	0	-1	0	0	0	0
-1	0	2	-1	0	0	0	0
0	-1	-1	2	-1	0	0	0
0	0	0	-1	2	-1	0	0
0	0	0	0	-1	2	-1	0
0	0	0	0	0	-1	2	-1
0	0	0	0	0	0	-1	2

Note that E8 can be constructed from the representations of E6 and D8.
The grade-1 vector representation of D8 is 120-dimensional.
The half-spinor representation of D8 is 128-dimensional.
The adjoint representation of E8 is 120 + 128 = 248-dimensional.
E6 has 27-dimensional and 78-dimensional representations.

E8 Dynkin representations are:

```
                                                147,250
                                                  |
248 - 30,380 - 2,450,240 - 146,325,270 - 6,899,079,264 - 6,696,000 - 3,875
```

To construct them:
First, construct the exterior/wedge products of the E8 adjoint 248:
The grade-1 part has dimension 248.
The grade-2 part has dimension 248/\248 = 30,628.
The grade-3 part has dimension 248/\248/\248 = 2,511,496.
The grade-4 part has dimension 248/\248/\248/\248 = 153,829,130.
The grade-5 part has dimension 248/\248/\248/\248/\248 = 7,506,861,544.
Now:
Keep the grade-1 part of dimension 248.
Subtract off 248 from 248/\248 = 30,628 to get 30,380.
Subtract off 2 x 248/\248 = 2 x 30,628 from 248/\248/\248 = 2,511,496 to get 2,450,240.
Subtract off 2 x 2,511,496 and 2,450,240 and 30,628
from 248/\248/\248/\248 = 153,829,130 to get 146,325,270.
Subtract off 2 x 153,829,130 and 2 x 146,325,270 and 2 x 2,511,496 and 2,450,240 and 248
from 248/\248/\248/\248/\248 = 7,506,861,544 to get 6,899,079,264.
These are 5 of the 8 fundamental representations of E8.

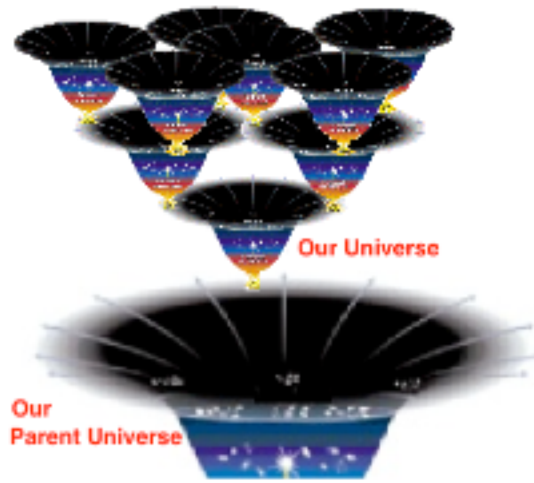
They, like the D(N) and A(N) series constructions,
are all in the same exterior algebra (of /\248),
and so can be represented as the vertices of a pentagon

What about the 6th and 7th fundamental representations of E8?
Consider the 27-dimensional E6 representation space. Add 32 copies of the 128-dimensional D8 half-spinor space, and subtract off one copy of the 248-dimensional E8 representation space
to get a 27 + 32 x 128 - 248 = 3,875-dimensional representation space.
Now, consider the antisymmetric exterior wedge algebra of that 3,875-dimensional space.
The grade-1 part has dimension 3,875. The grade-2 part has dimension 3,875/\3,875 = 7,505,875.
Now:
Keep the grade-1 part of dimension 3,875.
From the grade-2 part, subtract off 5 x 147,250 and 2 x 30,628 and 3 x 3,875 and 3 x 248
from 3,875/\3,875 = 7,505,875 to get 6,696,000. They are the 6th and 7th fundamental representations.
Since they are not in the same /\248 exterior algebra as the 5 pentagon-vertex fundamental representations of E8, they should not be vertices in the same plane as the pentagon.
However, since they are in the same /\3,875 exterior algebra, they should be collinear, one above and one below the pentagon, thus forming a pentagonal bipyramid.

What about the 8th fundamental representation of E8?
Consider 2 x 24x24 - 1 = 2 x 576 - 1 = 1,151 copies of the 128-dimensional D8 half-spinor space,
and subtract off one copy of the 78-dimensional E6 representation space
to get a representation space of dimension 1,151 x 128 - 78 = 147,328 - 78 = 147,250.
Now, consider the antisymmetric exterior wedge algebra of that 147,250-dimensional space.
The grade-1 part has dimension 147,250. It is the 8th fundamental representation of E8.
Since it is not in the same /\248 exterior algebra as the 5 pentagon-vertex fundamental representations of E8, it should not be a vertex in the same plane as the pentagon.
Also, since it is not in the same /\3,875 exterior algebra as the two bipyramid-peak-vertex fundamental representations of E8, it should not be a vertex on the same line as the pentagonal bipyramid axis.
It should represent a vertex creating a triangle whose base is one of the sides of the pentagon
and whose top is near one of the bipyramid-peak-vertices, to which it is connected by a line.
To produce a symmetric figure, the vertex must be reproduced in 5 copies,
one over each of the 5 sides of the pentagon.
Then, for the entire figure to be symmetric, it must form an icosahedron.
The binary icosahedral group {2,3,5} is of order 120.

Another way to look at it is:
The graded sequence 248 248/\248 248/\248/\248 248/\248/\248/\248 248/\248/\248/\248/\248
has symmetry Cy(5) of order 5 for cyclic permutations,but do not use Hodge duality
since 248/\248/\248/\248/\248 is fixed by its relation to 3,875/\3,875/\3,875.
The graded sequence 3,875 3,875/\3,87 3,875/\3,875/\3,875
has symmetry Cy(3) of order 3 for cyclic permutations, but do not use Hodge duality
since 3,875/\3,875/\3,875 is fixed by its relation to 248/\248/\248/\248/\248.
The graded sequence 147,250 147,250/\147,250
has symmetry Cy(2) of order 2 for cyclic permutations, but do not use Hodge duality
since 147,250/\147,250 is fixed by its relation to 248/\248/\248/\248/\248.
The +/- signs for the D5 half-spinors inherited from E6 through E7 have symmetry of order 2.
Since E8 is the sum of the 120-dimensional adjoint representation of D8
plus ONE of the 128-dimensional half-spinor representations of D8,
there is a choice to be made as to which of the two half-spinor representations of D8 are used.
As they are mirror images of each other, that choice has a symmetry of order 2.
Therefore:
the total symmetry group is of order 5x3x2x2x2 = 120,
the symmetry of the binary icosahedral group {2,3,5} corresponding by McKay to the E8 Lie Algebra.

5. Our Universe emerged from its parent in Octonionic Inflation



As Our Parent Universe expanded to a Cold Thin State Quantum Fluctuations occurred.
Most of them just appeared and disappeared as Virtual Fluctuations,
but at least one Quantum Fluctuation had enough energy to produce 64 Unfoldings
and reach Paola Zizzi's State of Decoherence
thus making it a Real Fluctuation that became Our Universe.

As Our Universe expands to a Cold Thin State, it will probably give birth
to Our Child, GrandChild, etc, Universes.

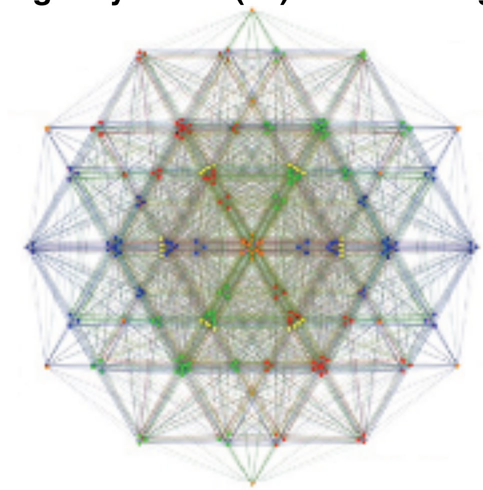
Unlike "the inflationary multiverse" decribed by Andrei Linde in arXiv 1402.0526 as
"a scientific justification of the anthropic principle",
in the Cl(16)-E8 model ALL Universes (Ours, Ancestors, Descendants)
have the SAME Physics Structure as E8 Physics (viXra 1312.0036 and 1310.0182)

In the Cl(16)-E8 model,
our SpaceTime remains Octonionic 8-dimensional throughout inflation.

Stephen L. Adler in his book Quaternionic Quantum Mechanics and Quantum Fields (1995) said at pages 50-52, 561: "... If the multiplication is associative, as in the complex and quaternionic cases, we can remove parentheses in ... Schroedinger equation dynamics ... to conclude that ... the inner product < f(t) | g(t) > ... is invariant ... this proof fails in the octonionic case, and hence one cannot follow the standard procedure to get a unitary dynamics. ...[so there is a]...
failure of unitarity in octonionic quantum mechanics ...".

The NonAssociativity and Non-Unitarity of Octonions
accounts for particle creation without the need for a conventional inflaton field.

Inflation begins in Octonionic Cl(16)-E8 Physics with a Quantum Fluctuation initially containing only one Cl(16) E8 Local Lagrangian Region



The Fermion Representation Space for a Cl(16) E8 Local Lagrangian Region
is E8 / D8 = the 64+64 = 128-dim +half-spinor space 64s++ + 64s+- of Cl(16)

64s++ = 8 components of 8 Fermion Particles

64s+- = 8 components of 8 Fermion Antiparticles

By 8-Periodicity of Real Clifford Algebras Cl(16) = tensor product Cl(8) x Cl(8)
where the two copies of Cl(8) can be denoted by Cl(8)G and Cl(8)SM

(in E8 Physics Cl(8)G gives Gravity with Dark Energy and Cl(8)SM gives the Standard Model)

Cl(8)G and Cl(8)SM each have 8-dim half-spinor spaces 8Gs+ 8Gs- and 8SMs+ 8SMs-

8Gs+ and 8SMs+ representing 8 Fermion Particles

8Gs- and 8SMs- representing 8 Fermion Antiparticles

so that

64s++ = 8Gs+ x 8SMs+ for First Generation Particles of E8 Physics

64s+- = 8Gs+ x 8SMs- for First Generation AntiParticles of E8 Physics

64s-+ = 8Gs- x 8SMs+ for AntiGeneration Particles (NOT in E8 Physics)

64s-- = 8Gs- x 8SMs- for AntiGeneration AntiParticles (NOT in E8 Physics)

where

+/- half-spinor of Cl(8)G determines +/- half-spinor of Cl(16)
and Generation or AntiGeneration (only +half-spinor Generation is in E8)

+/- half-spinor of Cl(8)SM determines Particle or AntiParticle

E8 Physics has Representation space for 8 Fermion Particles + 8 Fermion Antiparticles on the original Cl(16) E8 Local Lagrangian Region that is 64s++ + 8 of 64s+- =

_ _ _ _ _ _ _ _ + _ _ _ _ _ _ _ _

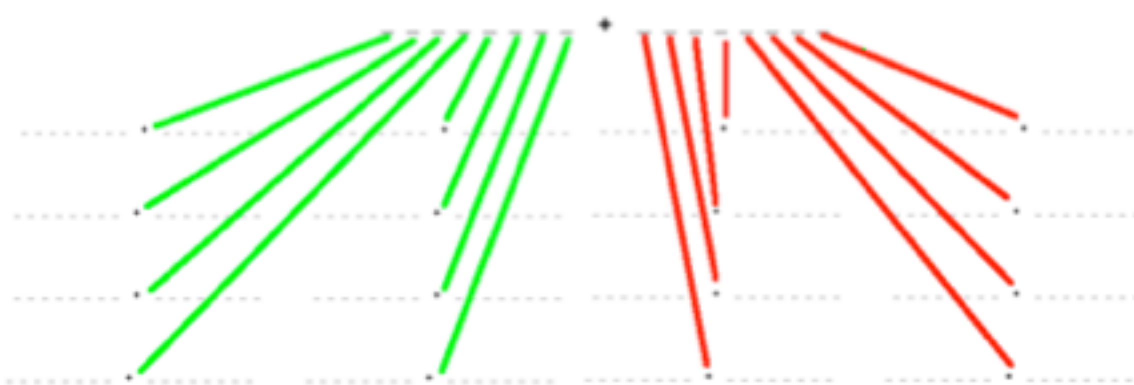
where a Fermion Representation slot _ of the 8+8 = 16 slots can be filled

by Real Fermion Particles or Real Fermion Antiparticles

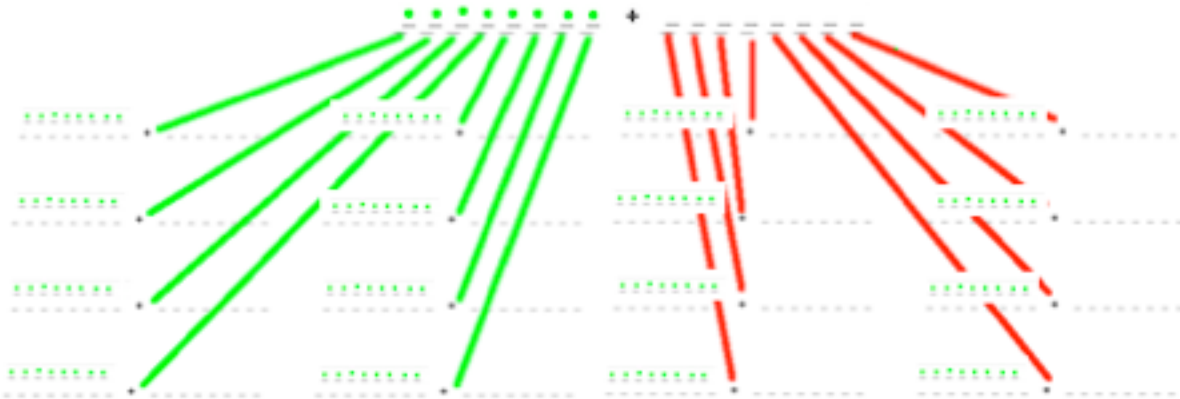
IF the Quantum Fluctuation(QF) has enough Energy to produce them as Real and IF the Cl(16) E8 Local Lagrangian Region has an Effective Path from its QF Energy to that Particular slot. (see Appendix III for Geoffrey Dixon's ideas and Effective Path of QF Energy)

Since E8 contains only the 128 +half-spinors and none of the 128 -half-spinors of Cl(16) the only Effective Path of QF Energy to E8 Fermion Representation slots goes to the only Fermion Particle slots that are also of type + that is, to the 8 Fermion Particle Representation slots

Next, consider **the first Unfolding step of Octonionic Inflation**.It is based on all 16 = 8 Fermion Particle slots + 8 Fermion Antiparticle Representation slots whether or not they have been filled by QF Energy.
7 of the 8 Fermion Particle slots correspond to the 7 Imaginary Octonions and therefore to the 7 Independent E8 Integral Domain Lattices and therefore to 7 New Cl(16) E8 Local Lagrangian Regions.
The 8th Fermion Particle slot corresponds to the 1 Real Octonion and therefore to the 8th E8 Integral Domain Lattice (not independent - see Kirmse's mistake) and therefore to the 8th New Cl(16) E8 Local Lagrangian Region.
Similarly, the 8 Fermion Antiparticle slots Unfold into 8 more New New Cl(16) E8 Local Lagrangian Regions, so that one Unfolding Step is a 16-fold multiplication of Cl(16) E8 Local Lagrangian Regions:



If the QF Energy is sufficient, the Fermion Particle content after the first Unfolding is



so it is clear that **the Octonionic Inflation Unfolding Process creates Fermion Particles with no Antiparticles, thus explaining the dominance of Matter over AntiMatter in Our Universe**.

Each Unfolding has duration of the Planck Time Tplanck
and none of the components of the Unfolding Process Components are simultaneous,
so that **the total duration of N Unfoldings is 2^N Tplanck**.

Paola Zizzi in gr-qc/0007006 said: "... **during inflation,**
the universe can be described as a superposed state of quantum ... [qubits].
the self-reduction of the superposed quantum state is ... reached at the end of inflation ...[at]... **the decoherence time ... [Tdecoh = 10^9 Tplanck = 10^(-34) sec]** ... and **corresponds to a superposed state of ... [10^19 = 2^64 qubits]**. ...".

Why decoherence at 64 Unfoldings = 2^64 qubits ?

2^64 qubits corresponds to the Clifford algebra Cl(64) = Cl(8x8).
By the periodicity-8 theorem of Real Clifford algebras, Cl(64) is the smallest Real Clifford algebra for which we can reflexively identify each component Cl(8) with a vector in the Cl(8) vector space. This reflexive identification/reduction causes our universe to decohere at N = 2^64 = 10^19
which is roughly the number of Quantum Consciousness Tubulins in the Human Brain.

The Real Clifford Algebra Cl(8) is the basic building block of Real Clifford Algebras
due to 8-Periodicity whereby Cl(8N) = Cl(8) x ...(N times tensor product)... x Cl(8)

An Octonionic basis for the Cl(8) 8-dim vector space is {1,i,j,k,E,I,J,K}

NonAssociativity, NonUnitarity, and Reflexivity of Octonions is exemplified by
the 1-1 correspondence between Octonion Basis Elements and E8 Integral Domains

1 <=> 0E8
i <=> 1E8
j <=> 2E8
k <=> 3E8
E <=> 4E8
I <=> 5E8
J <=> 6E8
K <=> 7E8

where 1E8,2E8,3E8,4E8,5E8,6E8,7E8 are 7 independent Integral Domain E8 Lattices
and 0E8 is an 8th E8 Lattice (Kirmse's mistake) not closed as an Integral Domain.
Using that correspondence expands the basis {1,i,j,k,E,I,J,K} to
{0E8,1E8,2E8,3E8,4E8,5E8,6E8,7E8}

Each of the E8 Lattices has 240 nearest neighbor vectors so the total dimension of the
Expanded Space is 240 x 240 x 240 x 240 x 240 x 240 x 240 x 240

Everything in the Expanded Space comes directly from the original Cl(8) 8-dim space
so all Quantum States in the Expanded Space can be held in Coherent Superposition.
However,
if further expansion is attempted, there is no direct connection to original Cl(8) space
and any Quantum Superposition undergoes Decoherence.

If each 240 is embedded reflexively into the 256 elements of Cl(8) the total dimension is
256 x 256 x 256 x 256 x 256 x 256 x 256 x 256 = 256^8 = 2^(8x8) = 2^64 =
= Cl(8) x Cl(8) x Cl(8) x Cl(8) x Cl(8) x Cl(8) x Cl(8) x Cl(8) = Cl(8x8) = Cl(64)
so the largest Clifford Algebra that can maintain Coherent Superposition is Cl(64)
which is why Zizzi Quantum Inflation ends at the Cl(64) level.

At the end of 64 Unfoldings, Non-Unitary Octonionic Inflation ended having produced about (1/2) 16^64 = (1/2) (2^4)^64 = 2^255 = 6 x 10^76 Fermion Particles

The End of Inflation time was at about 10^(-34) sec = 2^64 Tplanck
and
the size of our Universe was then about 10^(-24) cm
which is **about the size of a Fermion Schwinger Source Kerr-Newman Cloud**.
(see viXra 1311.0088)

Octonion Inflation produces Gravitational Waves that can now be observed in Polarization Patterns of the Cosmic Microwave Background.

BICEP2 in arXiv 1403.3985 said:
"... Inflation predicts ... a primordial background of ... gravitational waves ...[that]... would have imprinted a unique signature upon the CMB. **Gravitational waves induce local quadrupole anisotropies** in the radiation field within the last-scattering surface, inducing polarization in the scattered light ... **This polarization pattern will include a “curl” or ... inflationary gravitational wave (IGW) B-mode ... component** at degree angular scales that cannot be generated primordially by density perturbations. The amplitude of this signal depends upon the **tensor-to-scalar ratio ... r = 0.20 +0.07 -0.05** ... which itself is a function of the energy scale of inflation. ...".

In the Cl(16)-E8 model,
Inflation is due to Non-Unitarity of Octonion Quantum Processes
that occur in 8-dim SpaceTime before freezing out of a preferred Quaternionic Frame ends Inflation and begins Ordinary Evolution in (4+4)-dim M4 x CP2 Kaluza-Klein.
The unit sphere in the Euclidean version of 8-dim SpaceTime (see viXra 1311.0088 for Schwinger's "unitary trick" to allow use of Euclidean SpaceTime) is the 7-sphere S7.

Curl-type B-modes (tensor) are Octonionic Quantum Processes on the surface of SpaceTime S7 which is a **7-dim NonAssociative Moufang Loop Malcev Algebra**.

(image below from Sky and Telescope)

B-modes look like  Spirals on the Surface of S7

Divergence-type E modes (scalar and tensor) are Octonionic Quantum Processes from SpaceTime S7
plus a spinor-type S7 representing Dirac Fermions living in SpaceTime
plus a 14-dim G2 Octonionic Derivation Algebra connecting the two S7 spheres
all of which is a **28-dim D4 Lie Algebra Spin(8)**.

(image below from Sky and Telescope)

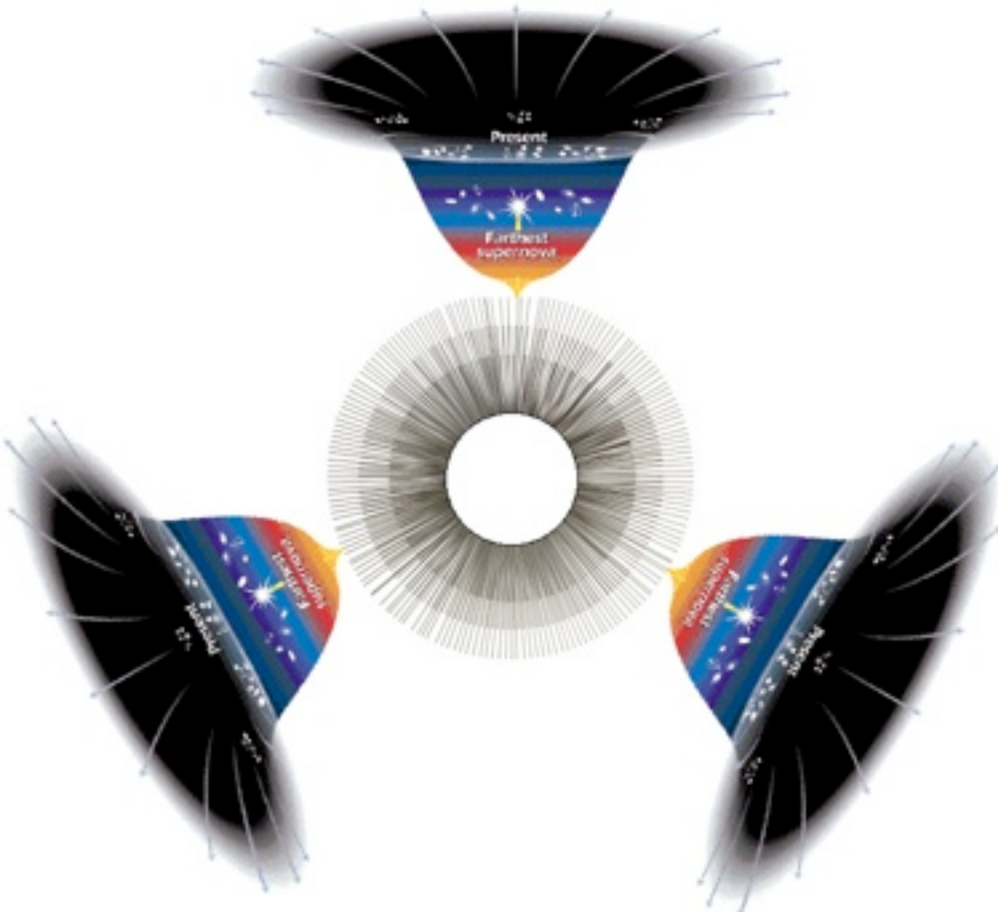
E-modes look like Fermion Pair Creation either

off (scalar)  or on (tensor)  the Surface of S7

Therefore: **for E8 Physics Octonionic Inflation the ratio r = 7 / 28 = 0.25**

End of Inflation and Low Initial Entropy in Our Universe:

Roger Penrose in his book The Emperor's New Mind (Oxford 1989, pages 316-317) said:
"... in our universe ... Entropy ... increases ... Something forced the entropy to be low in the past. ... **the low-entropy states in the past are a puzzle**. ...".
The key to solving Penrose's Puzzle is given by Paola Zizzi in gr-qc/0007006:
"... **The self-reduction of the superposed quantum state is ... reached at the end of inflation ...[at]... the decoherence time ... [Tdecoh = 10^9 Tplanck = 10(-34) sec] ... and corresponds to a superposed state of ... [10^19 = 2^64 qubits]**. ...
... **This is also the number of superposed tubulins-qubits in our brain ... leading to a conscious event**. ...".
The Zizzi Inflation phase of our universe ends with decoherence "collapse" of the 2^64 Superposition Inflated Universe into Many Worlds of Quantum Theory,



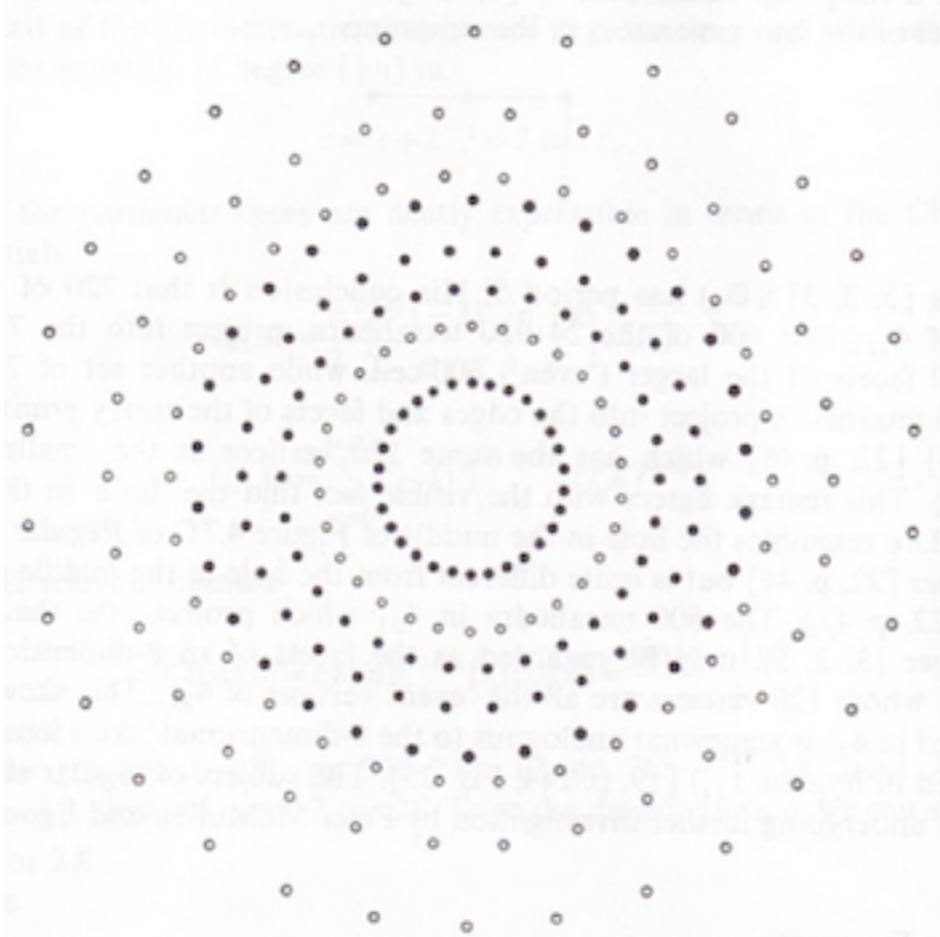
only one of which Worlds is our World. The central white circle is the Inflation Era in which everything is in Superposition; the boundary of the central circle marks the decoherence/collapse at the End of Inflation; and each line radiating from the central circle corrresponds to one decohered/collapsed Universe World (of course, there are many more lines than actually shown), only three of which are explicitly indicated in the image, and only one of which is Our Universe World.

Since our World is only a tiny fraction of all the Worlds, it carries only a tiny fraction of the entropy of the 2^64 Superposition Inflated Universe, thus solving Penrose's Puzzle.

6. Quaternionic M4xCP2 Kaluza-Klein SpaceTime

At the end of Non-Unitary Octonionic Inflation Our Universe
had about (1/2) 16^64 = (1/2) (2^4)^64 = 2^255 = 6 x 10^76 Fermion Particles
The End of Inflation time was at about 10^(-34) sec = 2^64 Tplanck
and
the size of our Universe was then about 10^(-24) cm
which is about the size of a Fermion Schwinger Source Kerr-Newman Cloud
and
the Real Clifford Algebra of 8-dim SpaceTime was Cl(1,7) = Cl(0,8) = M(16,R)

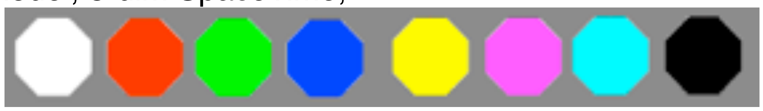
The Event that Ended Inflation was Decoherence of Zizzi Quantum Inflation that also produced decoherence of the D8 brane SpaceTime Planck-Scale Lattice superpositions of the 8 types of E8 Lattice 1E8, iE8, jE8, kE8, EE8, IE8, JE8, KE8 which resulted in a decoherence choice of a particular E8 Lattice. The 240 origin-nearest-neighbor Root Vectors of such a chosen E8 Lattice can be represented as 8 circles of 30 vertices each

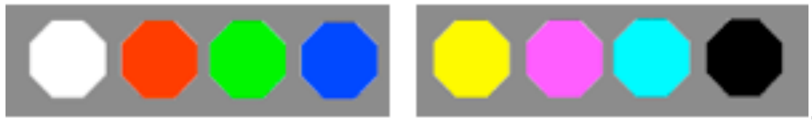


with 4x30 = 120 vertices (black dots) forming a 600-cell and
the other 4x30 = 120 vertices (white dots) forming another 600-cell at radii expanded from that of the black dots by a Golden Ratio factor. Since each 600-cell is 4-dim,
the Octonionic 8-dim E8 SpaceTime is decomposed into 2 Quaternionic 4-dim parts,

giving the Post-Inflation Cl(16)-E8 model a (4+4)-dim Kaluza-Klein SpaceTime
of the form M4 x CP2 where
M4 is 4-dim Physical Minkowski SpaceTime on which Gravity acts and
CP2 = SU(3) / U(2) is 4-dim Internal Symmetry Space for Standard Model Forces.

In the Cl(16)-E8 model, 8-dim SpaceTime,

both Octonionic 

and Quarternionic ,

is represented by the 64-dim Adjoint D8 / D4xD4 part of E8
which is the A7 x R grade-0 part of the Maximal Contraction A7 x h92 with 5-grading

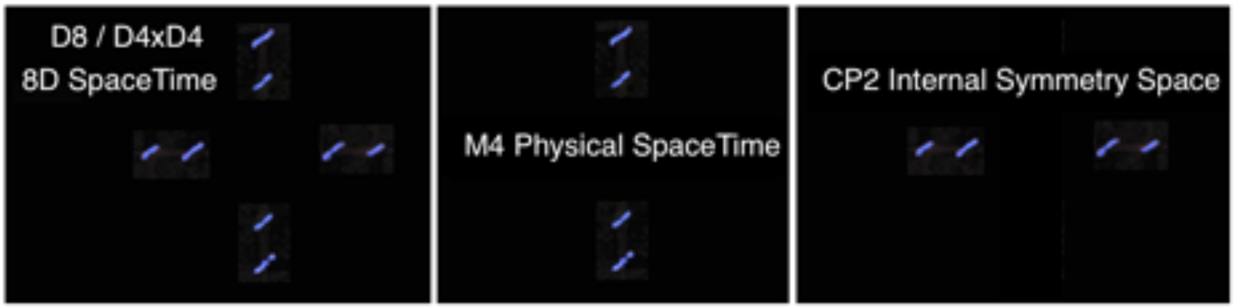
28 + 64 + (SL(8,R) + 1) + 64 + 28

In the Cl(16)-E8 model Gravity is most often written as in Chapter 18 of this paper
in terms of the MacDowell-Mansouri Conformal Group Spin(2,4) which is
the 15-dimensional Conformal BiVector Group of the 64-dim Cl(2,4) Clifford Algebra
but
it can also be written in terms of 64-dim grade-0 Maximal Contraction term SL(8,R) + 1
in which case it is known as Unimodular SL(8,R) Gravity which effectively describes
a generalized checkerboard of 8-dim SpaceTime HyperVolume Elements and,
with respect to Cl(16) = Cl(8)xCl(8), is the tensor product of the two 8v vector spaces
of the two Cl(8) factors of Cl(16). If those two Cl(8) factors are regarded
as Fourier Duals, then 8v x 8v describes Position x Momentum in 8-dim SpaceTime.

Conformal Spin(2,4) = SU(2,2) Gravity and Unimodular SL(4,R) = Spin(3,3) Gravity
seem to be effectively equivalent since, as Bradonjic and Stachel in arXiv 1110.2159
said: "... in ... Unimodular relativity ... the symmetry group of space-time is ...
the special linear group SL(4,R) ... the metric tensor ... break[s up] ... into
the conformal structure represented by a conformal metric ... with det = -1
and a four-volume element ... at each point of space-time ...[that]... may be
the remnant, in the ... continuum limit,
of a more fundamental discrete quantum structure of space-time itself ...".
Further,
Frampton, Ng, and Van Dam in J. Math. Phys. 33 (1992) 3881-3882 said:
"... Because of the existence of topologically nontrivial solutions, instantons, of the
classical field equations associated with quantum chromodynamics (QCD), the
quantized theory contains a dimensionless parameter ø ($0 < ø < 2\pi$) not explicit in the
classical lagrangian. Since ø multiplies an expression odd in CP, QCD predicts violation
of that symmetry unless the phase ø takes one of the special values ... 0 (mod π) ...
this fine tuning is the strong CP problem ... the quantum dynamics of ... unimodular
gravity ... may lead to the relaxation of ø to ø = 0 (mod π)
without the need ... for a new particle ... such as the axion ...".

End of Inflation and Quaternionic Structure

In Cl(16)-E8 Physics (vixra 1405.0030) Octonionic symmetry of 8-dim spacetime is broken at the End of Non-Unitary Octonionic Inflation to Quaternionic symmetry of (4+4)-dim Kaluza-Klein M4 x CP2 physical spacetime x internal symmetry space.



Here are some details about that transition:

The basic local entity of Cl(16)-E8 Physics is
Cl(0,16) = Cl(1,15) = Cl(4,12) = Cl(5,11) = Cl(8,8) = M(R,256) = 256x256 Real Matrices
which contains E8 with 8-dim Octonionic spacetime
and is the tensor product Cl(0,8) x Cl(0,8) = Cl(1,7) x Cl(1,7)
where Cl(0,8) = Cl(1,7) = M(R,16) is the Clifford Algebra of the 8-dim spacetime.

Non-Unitary Octonionic Inflation is based on Octonionic spacetime structure with superposition of E8 integral domain lattices. At the End of Inflation the superposition ends and Octonionic 8-dim structure is replaced by Quaternionic (4+4)-dim structure.

Since M(R,16) = M(Q,2) x M(Q,2) and M(Q,2) = Cl(1,3) = Cl(0,4)
Cl(0,8) = Cl(1,7) can be represented as Cl(1,3) x Cl(0,4)
where
Cl(1,3) is the Clifford Algebra for M4 physical spacetime
and
Cl(0,4) is the Clifford Algebra for CP2 = SU(3) / U(2) internal symmetry space
thus
making explicit the Quaternionic structure of (4+4)-dim M4 x CP2 Kaluza-Klein.

Cl(1,3) = Cl(0,4) = M(Q,2) has graded structure based on 1 2 1 grading of 2x2 matrices and 1 2 1 grading of the Quaternions, so that its total graded structure is

```
1  2  1
   2  4  2
      1  2  1
1  4  6  4  1
```

and its Spinor structure is 2x1 Quaternion matrices

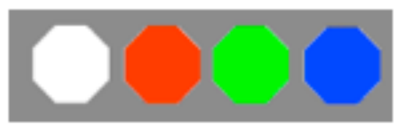
```
1 2 1
1 2 1
2 4 2   =  1 2 1  +  1 2 1
```

1 2 1 = 4-dim Shilov Boundary for Lie Sphere Spin(6) / Spin(4)xU(1) =
= half-spinors for First Generation Lepton + 3 Quarks

4s+ for Electron + 3 Up Quarks
and

4s- for Neutrino + 3 Down Quarks

One copy of Cl(1,3) only has room for Particles, no AntiParticles



Cl(1,3) vectors can represent M4 physical spacetime
but

the CP2 part of M4 x CP2 Kaluza-Klein is not directly represented by Cl(1,3).

Note that Cl(3,1) = Cl(2,2) = M(R,4) has the same Clifford Algebra dimension = 16
as does Cl(1,3) = Cl(0,4) = M(Q,2)
but
Cl(3,1) = Cl(2,2) = M(R,4) Spinors are 4x1 = 4-dimensional (Real Dirac Gammas)
so physicists had to Complexify them in order to get realistic results
while
Cl(1,3) = Cl(0,4) = M(Q,2) Spinors are 2x4 = 8-dimensional and directly give
the same realistic physical results of Complex Dirac Gammas.
Roughly,
Quaternification of the Clifford Algebra is like Complexification of Spinors.

Cl(0,8) = Cl(1,7) = M(R,16) = M(Q,2) x M(Q,2) has graded structure

1	4	6	4	1				
	4	16	24	16	4			
		6	24	36	24	6		
			4	16	24	16	4	
				1	4	6	4	1
1	8	28	56	70	56	28	8	1

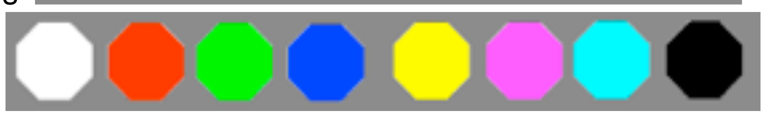
and its Spinor structure based on M(Q,2) = 16-dim is

Spinors = sqrt(16x16) = 16 = 8+8

Their Real / Octonionic M(R,16) structure is:

with 8-dim +half-spinors 

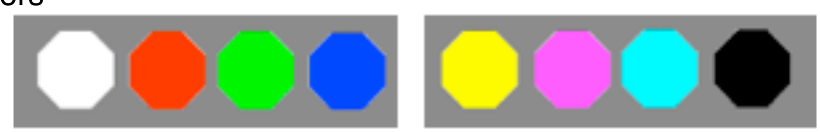
and 8-dim -half-spinors 

and 8-dim vectors 
related to each other by Triality

Their Quaternionic M(Q,2) x M(Q,2) structure is

with 2-Quaternionic +half-spinors 

and 2-Quaternionic -half-spinors 

and 2-Quaternionic vectors 
representing (4+4)-dim Kaluza-Klein M4 x CP2, related to half-spinors by Triality.

The 8-dim vectors of Cl(0,8) = Cl(1,7) correspond to B4 / D4 = OP1

Spinors = 8+8 = F4 / B4 = 52-36 = OP2

F4 = 8 + 28 + (8+8)

8 = Shilov Boundary for Lie Sphere Spin(10) / Spin(8)xU(1) =
= half-spinors for First Generation Fermion Particles / AntiParticles
8s+ for Particles and 8s- for AntiParticles
One copy of Cl(8) only has room for one Generation, no AntiGeneration
The AntiGeneration appears for Cl(16) = Cl(8) x Cl(8)
but is not in E8 which omits the AntiGeneration half-spinors of Cl(16)

Cl(0,16) = Cl(1,15) = M(R,256) = M(Q,2) x M(Q,2) x M(Q,2) x M(Q,2)
has graded structure

1	8	28	56	70	56	28	8	1
	8	64	...					
		28	...					
			...					...
1	16	120	...					

and its Spinor structure based on M(Q,2) = 16-dim is

Spinors = sqrt(16x16x16x16) = 16x16 = 256 = 128+128

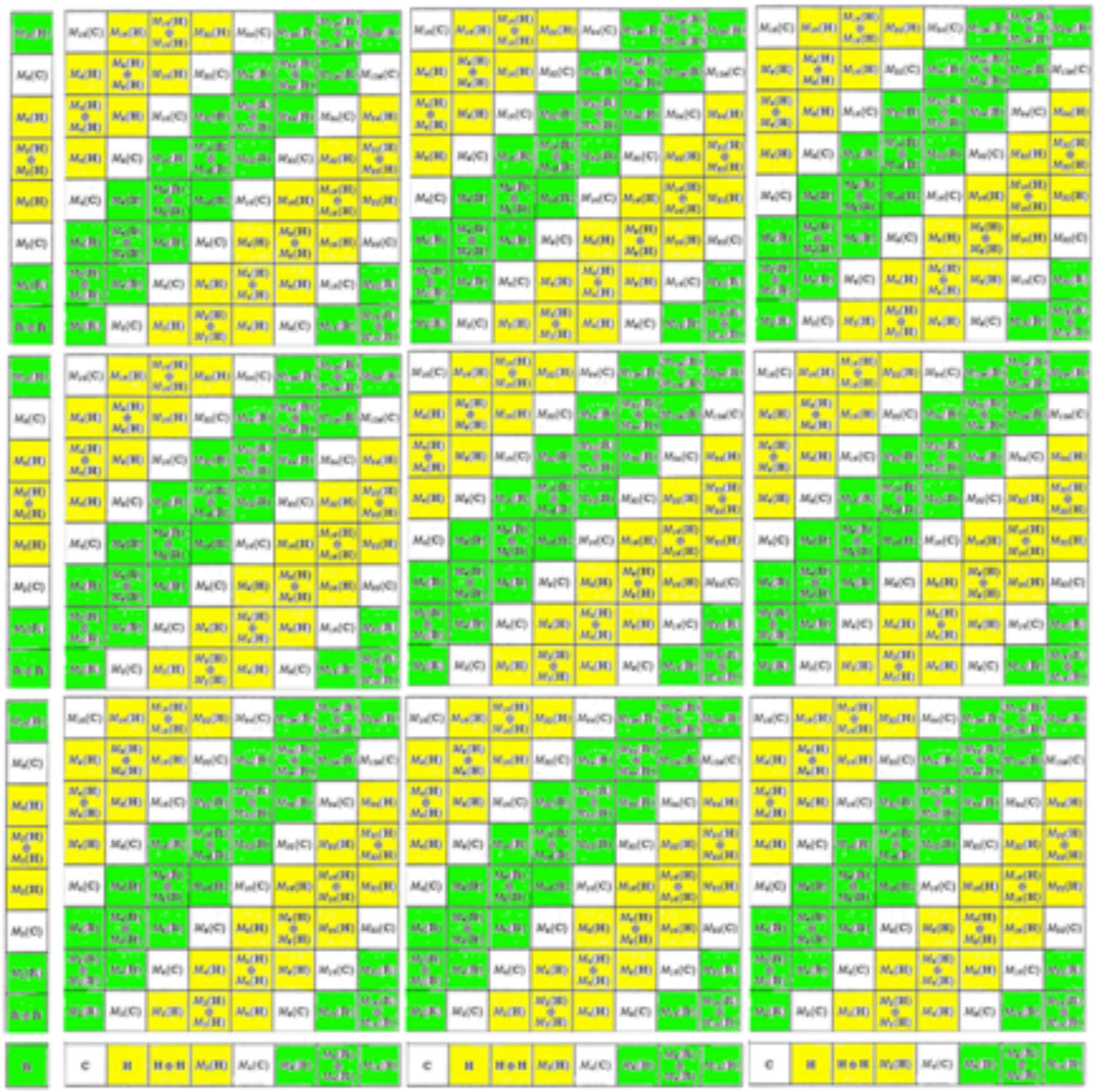
(equivalent to M(R,256) Spinors = 256x1 Real = 256 = 128 + 128)

Spinors = 128+128

E8 = 120 + 128

128 = Cl(16) half-spinors for One Generation Fermion Particles and AntiParticles

The other 128 is for One AntiGeneration that is not in E8



Quaternionic structure similar to that of Cl(1,3) = Cl(0,4) = M(Q,2) is seen in

Cl(2,4) = M(Q,4) = 4x4 Quaternion matrices with grading based on 4x4 = 1 4 6 4 1

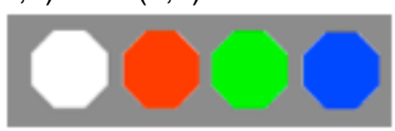
1	2	1				
	4	8	4			
		6	12	6		
			4	8	4	
				1	2	1
1	6	15	20	15	6	1

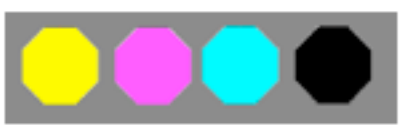
Conformal Gravity Spin(2,4) = SU(2,2) of Cl(2,4) = M(Q,4) 4x4 Quaternionic Matrices
have (4+4)x4 = 32-dim spinors
with

2-Quaternionic +half-spinors  
and

2-Quaternionic -half-spinors  

Cl(2,4) vectors are 6-dim but Spin(2,4) = SU(2,2) so the Twistor Correspondence

produces 1-Quaternionic Twistors 
that represent the M4 part of M4xCP2 Kaluza-Klein

with the CP2 part  not directly represented by Cl(2,4).

Spinors = 4x1 Quaternion 16 = 4 8 4 = 2 4 2 + 2 4 2

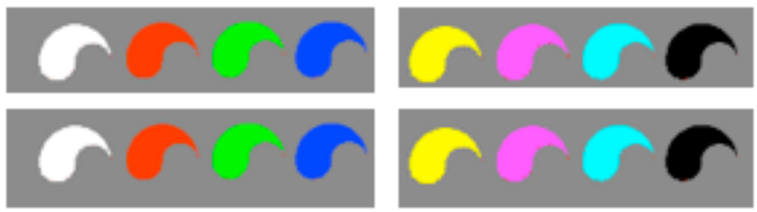
2 4 2 = 8 = Lie Sphere Spin(6) / Spin(4)xU(1) Complex Domain has Cl(1,3) half-spinor Shilov Boundary
Cl(2,4) is in some sense a (1,1) Complexification of Cl(1,3)

and in

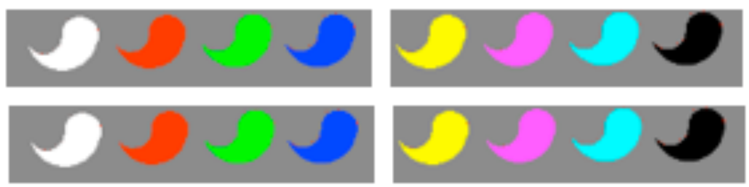
Cl(2,6) = Cl(3,5) = M(Q,8) = 8x8 Q-matrix grading based on 8x8 = 1 6 15 20 15 6 1

1	2	1						
	6	12	6					
		15	30	15				
			20	40	20			
				15	30	15		
					6	12	6	
						1	2	1
1	8	28	56	70	56	28	8	1

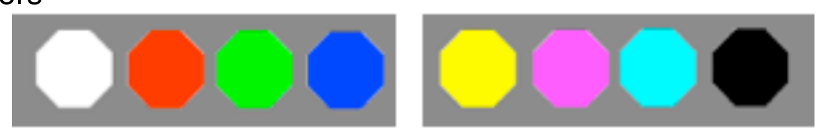
Quaternionic M(Q,8) 8x8 Quaternionic Matrices have (4+4)x4 = 32-dim spinors



with 4-Quaternionic +half-spinors



and 4-Quaternionic -half-spinors



and 2-Quaternionic vectors
that represent the two 4-dim spaces of Kaluza-Klein M4 x CP2
The 8-dim vectors do not correspond to 16-dim D5 / D4xU(1) = (CxO)P1
If you were to expand the vectors to 16-dim you would go to Cl(16) = Cl(8)xCl(8)

Spinors = 8x1 Quaternion = E6 / D5xU(1) = 78-45-1 = 32

E6 = 15 + 30 + 32

32 = 8 16 8 = 4 8 4 + 4 8 4

4 8 4 = Lie Sphere Spin(10) / Spin(8)xU(1)

The Quaternionic half-spinors in Cl(2,6) correspond to Lie Sphere Complex Domains whereas
the half-spinors in Cl(1,7) = Cl(0,8) correspond to Shilov Boundaries
and
the E6 of Cl(2,6) is in some aspects a Complexification of the F4 of Cl(1,7) = Cl(0,8).

7. Batakis Standard Model Gauge Groups and Mayer-Trautman Higgs

The Mayer-Trautman Mechanism reduces
the Lagrangian integral over the 8-dim SpaceTime whose 8-Position x 8-Momentum is represented by 64-dim D8 / D4xD4 where D8 is the Adjoint part of E8.

$\int_{\text{8-dim SpaceTime}}$ Standard Model Gauge Gravity + Fermion Particle-AntiParticle .

to

a Lagrangian integral over the 4-dim M4 Minkowski Physical SpaceTime part of Kaluza-Klein M4 x CP2

$\int_{\text{4-dim M4}}$ SM GG + Fermion Particle-AntiParticle + Higgs .

by integrating out the Lagrangian Density over the CP2 Internal Symmetry Space and so creating a new Higgs term in the Lagrangian Density integrated only over M4.

Since the D4 = U(2,2) of Gauge Gravity acts on the M4, there is no problem with it.

As to the D4 = U(4) of the Standard Model, U(4) contains as a subgroup color SU(3) which is also the global symmetry group of the CP2 = SU(3) / SU(2)xU(1) Internal Symmetry Space of M4 X CP2 Kaluza-Klein SpaceTime.

A. Batakis in Class. Quantum Grav. 3 (1986) L99-L105 said:
"... In a standard Kaluza-Klein framework, M4 x CP2 allows
the classical unified description of an SU(3) gauge field with gravity ...
[and]
the possibility of an additional SU(2) x U(1) gauge field structure is uncovered. ...".

Since the CP2 = SU(3) / U(2) has global SU(3) action,
the SU(3) can be considered as a local gauge group acting on the M4,
so there is no problem with it.

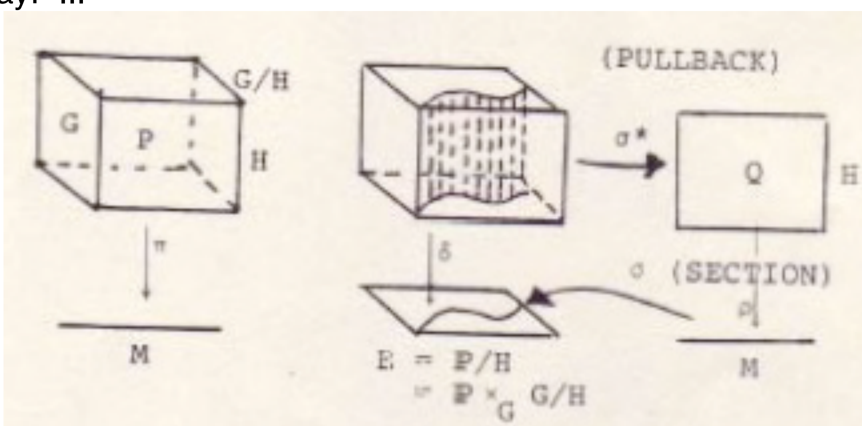
However, the U(2) acts on the CP2 = SU(3) / U(2) as little group, and so has local action on CP2 and then on M4, so **the local action of U(2) on CP2 must be integrated out to get the desired U(2) = SU(2)xU(1) local action directly on M4**.

Since the U(1) part of U(2) = U(1) x SU(2) is Abelian, its local action on CP2 and then M4 can be composed to produce a single U(1) local action on M4, so there is no problem with it.

That leaves non-Abelian SU(2) with local action on CP2 and then on M4, and the necessity to integrate out the local CP2 action to get something acting locally directly on M4.

This is done by a mechanism due to Meinhard Mayer and A. Trautman in
“A Brief Introduction to the Geometry of Gauge Fields” and
“The Geometry of Symmetry Breaking in Gauge Theories”,
Acta Physica Austriaca, Suppl. XXIII (1981)

where they say: "...



... We start out from ... four-dimensional M [M4] ...[and]... R ...[that is]... obtained from ... G/H [CP2 = SU(3) / U(2)] ... the physical surviving components of A and F, which we will denote by A and F, respectively, are a one-form and two form on M [M4] with values in H [SU(2)] ... the remaining components will be subjected to symmetry and gauge transformations, thus reducing the Yang-Mills action ...[on M4 x CP2]... to a Yang-Mills-Ginzburg-Landau action on M [M4] ... Consider the Yang-Mills action on R ...
S_YM = Integral Tr (F /\ *F)
... We can ... split the curvature F into components along M [M4] (spacetime) and those along directions tangent to G/H [CP2] .
We denote the former components by F_!! and the latter by F_?? , whereas the mixed components (one along M, the other along G/H) will be denoted by F_!? ...
Then the integrand ... becomes
Tr(F_!! F^!! + 2 F_!? F^!? + F_?? F^??)
...

The first term .. becomes the [SU(2)] Yang-Mills action for the reduced [SU(2)] Yang-Mills theory
...
the middle term .. becomes, symbolically,
Tr Sum D_! PHI(?) D^! PHI(?)
where PHI(?) is the Lie-algebra-valued 0-form corresponding to the invariance of A with respect to the vector field ? , in the G/H [CP2] direction
...
the third term ... involves the contraction F_?? of F with two vector fields lying along G/H [CP2] ... we make use of the equation [from Mayer-Trautman, Acta Physica Austriaca, Suppl. XXIII (1981) 433-476, equation 6.18]
2 F_?? = [PHI(?) , PHI(?)] - PHI([?,?])
... Thus,
the third term ... reduces to what is essentially a Ginzburg-Landau potential in the components of PHI:
Tr F_?? F^?? = (1/4) Tr ([PHI , PHI] - PHI)^2
...
special cases which were considered show that ...[the equation immediately above]... has indeed the properties required of a Ginzburg_Landau-Higgs potential, and moreover the relative signs of the quartic and quadratic terms are correct, and only one overall normalization constant ... is needed. ...".

See S. Kobayashi and K. Nomizu, Foundations of Differential Geometry, Volume I, Wiley (1963), especially section II.11: "...

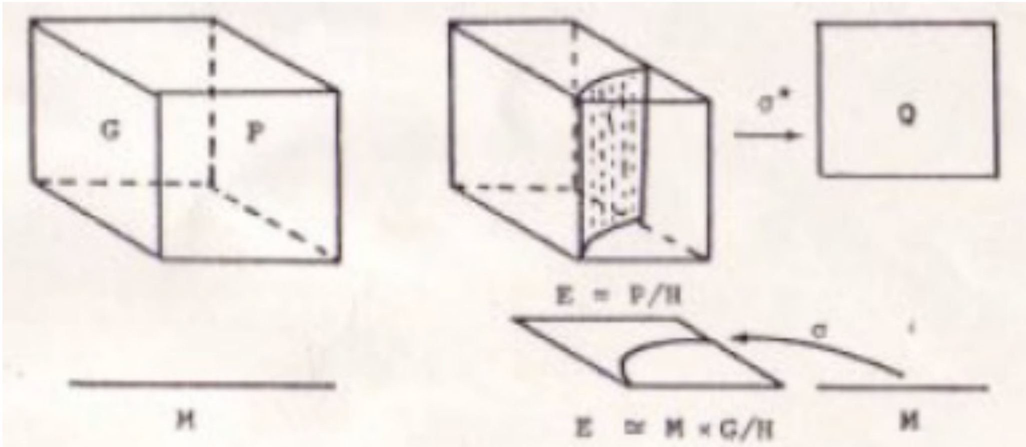
THEOREM 11.7. *Assume in Theorem* 11.5 *that* $\mathfrak{k}$ *admits a subspace* $\mathfrak{m}$ *such that* $\mathfrak{k} = \mathfrak{j} + \mathfrak{m}$ (*direct sum*) *and* $\mathrm{ad}\,(J)(\mathfrak{m}) = \mathfrak{m}$, *where* $\mathrm{ad}\,(J)$ *is the adjoint representation of* J *in* $\mathfrak{k}$. *Then* ...

The curvature form Ω *of the K-invariant connection defined by* $\Lambda_{\mathfrak{m}}$ *satisfies the following condition:*

$$2\Omega_{u_0}(\tilde{X}, \tilde{Y}) = [\Lambda_{\mathfrak{m}}(X), \Lambda_{\mathfrak{m}}(Y)] - \Lambda_{\mathfrak{m}}([X, Y]_{\mathfrak{m}}) - \lambda([X, Y]_{\mathfrak{j}}) \quad \text{for } X, Y \in \mathfrak{m},$$

...".

Along the same lines, Meinhard E. Mayer said (Hadronic Journal 4 (1981) 108-152): “...



... each point of ... the ... fibre bundle ... E consists of a four- dimensional spacetime point x [in M4] to which is attached the homogeneous space G / H [SU(3) / U(2) = CP2] ... the components of the curvature lying in the homogeneous space G / H [= SU(3) / U(2)] could be reinterpreted as Higgs scalars (with respect to spacetime [M4]) ...
the Yang-Mills action reduces to a Yang-Mills action for the h-components [U(2) components] of the curvature over M [M4] and a quartic functional for the “Higgs scalars”, which not only reproduces the Ginzburg-Landau potential, but also gives the correct relative sign of the constants, required for the BEHK ... Brout-Englert-Higgs-Kibble ... mechanism to work. ...”.

8. 2nd and 3rd Generation Fermions

The 8 First Generation Fermion Particles
can each be represented by the 8 basis elements {1,i,j,k,E,I,J,K} of the Octonions O

1 <=> e-neutrino
i <=> red down quark
j <=> green down quark
k <=> blue down quark

E <=> electron
I <=> red up quark
J <=> green up quark
K <=> blue up quark with AntiParticles being represented similarly.

The Second and Third Generations can be represented by
Pairs of Octonions OxO and Triples of Octonions OxOxO
respectively.

When the non-unitary Octonionic 8-dim spacetime is reduced to the
Kaluza-Klein M4 x CP2 at the End of Inflation,
there are 3 possibilities for a fermion propagator from point A to point B:

1 - A and B are both in M4,
so its path can be represented by the single O;

2 - Either A or B, but not both, is in CP2,
so its path must be augmented by one projection from CP2 to M4,
which projection can be represented by a second O,
giving a second generation OxO;

3 - Both A and B are in CP2,
so its path must be augmented by two projections from CP2 to M4,
which projections can be represented by a second O and a third O,
giving a third generation OxOxO.

Combinatorics contributes to Fermion mass ratios. For example:

Blue Down Quark is 1 out of 8 and Blue Up Quark is 1 out of 8
so the Down Quark : Up Quark mass ratio is 1 : 1

Blue Strange Quark is 3 out of 8x8 = 64 and Blue Charm Quark is 17 out of 8x8 = 64
so the Strange Quark : Charm Quark mass ratio is 3 : 17

Blue Beauty Quark is 7 out of 8x8x8 = 512 and Blue Truth Quark is 161 out of 8x8x8 = 512
so the Beauty Quark : Truth Quark mass ratio is 7 : 161

9. Schwinger Sources with inherited Monster Group Symmetry have Kerr-Newman Black Hole structure size about 10^(-24) cm and Geometry of Bounded Complex Domains and Shilov boundaries

The Cl(16)-E8 model Lagrangian over 4-dim Minkowski SpaceTime M4 is

$$\int_{\text{4-dim M4}} \text{SM GG} + \text{Fermion Particle-AntiParticle} + \text{Higgs}$$

Consider the Fermion Term.

In the conventional picture, the spinor fermion term is of the form m S S* where m is the fermion mass and S and S* represent the given fermion.
The Higgs coupling constants are, in the conventional picture, ad hoc parameters, so that effectively themass term is, in the conventional picture, an ad hoc inclusion.

The Cl(16)-E8 model does not put in the mass m in an ad hoc way,
but constructs the Lagrangian integral such that
the mass m emerges naturally from the geometry of the spinor fermions by setting the spinor fermion mass term as the volume of the Schwinger Source Fermions.

Effectively the integral over the Schwinger Source spacetime region
of its Kerr-Newman cloud of virtual particle/antiparticle pairs plus the valence fermion gives the volume of the Schwinger Source fermion and defines its mass,
which, since it is dressed with the particle/antiparticle pair cloud,
gives quark mass as constituent mass.

The Cl(16)-E8 model constructs the Lagrangian integral such that
the mass m emerges as the integral over the Schwinger Source spacetime region
of its Kerr-Newman cloud of virtual particle/antiparticle pairs plus the valence fermion so that the volume of the Schwinger Source fermion defines its mass, which, being dressed with the particle/antiparticle pair cloud, gives quark mass as constituent mass.

Fermion Schwinger Sources correspond to the Lie Sphere Symmetric space
Spin(10) / Spin(8)xU(1)
which has local symmetry of the Spin(8) gauge group
from which the first generation spinor fermions are formed as +half-spinor and -half-spinor spaces
and
Bounded Complex Domain D8 of type IV8 and Shilov Boundary Q8 = RP1 x S7

Consider the SM GG term from Gauge Gravity and Standard Model Gauge Bosons.
The process of breaking Octonionic 8-dim SpaceTime
down to Quaternionic (4+4)-dim M4 x CP2 Kaluza-Klein
creates differences in the way gauge bosons "see" 4-dim Physical SpaceTime

There 4 equivalence classes of 4-dimensional Riemannian Symmetric Spaces
with Quaternionic structure consistent with 4-dim Physical SpaceTime:

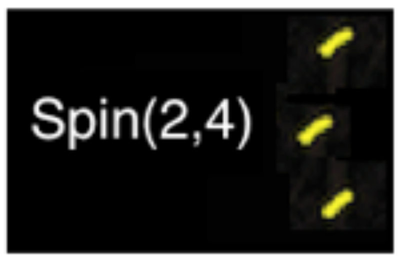
S4 = 4-sphere = Spin(5) / Spin(4) where Spin(5) = Schwinger-Euclidean version of the Anti-DeSitter subgroup of the Conformal Group that gives MacDowell-Mansouiri Gravity

CP2 = complex projective 2-space = SU(3) / U(2) with the SU(3) of the Color Force

S2 x S2 = SU(2)/U(1) x SU(2)/U(1) with two copies of the SU(2) of the Weak Force

S1 x S1 x S1 x S1 = U(1) x U(1) x U(1) x U(1) = 4 copies of the U(1) of the EM Photon
(1 copy for each of the 4 covariant components of the Photon)

The Gravity Gauge Bosons (Schwinger-Euclidean versions) live in
a Spin(5) subalgebra of the Spin(6) Conformal subalgebra of D4 = Spin(8).

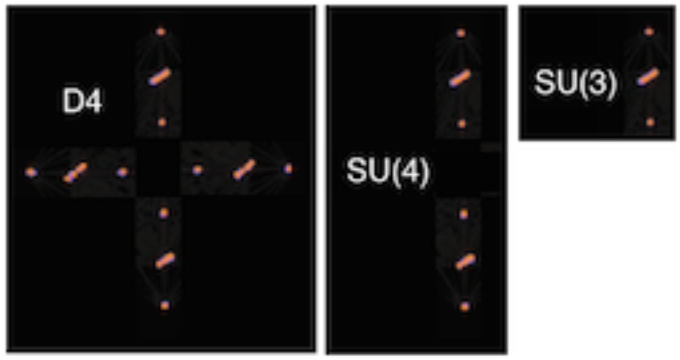


They "see" M4 Physical spacetime as the 4-sphere S4
so that their part of the Physical Lagrangian is

$$\int_{S4} \text{Gravity Gauge Boson Term} \quad .$$

an integral over SpaceTime S4.
The Schwinger Sources for GRb bosons are the Complex Bounded Domains and Shilov Boundaries for Spin(5) MacDowell-Mansouri Gravity bosons.
However, due to Stabilization of Condensate SpaceTime
by virtual Planck Mass Gravitational Black Holes,
for Gravity, the effective force strength that we see in our experiments
is not just composed of the S4 volume and the Spin(5) Schwinger Source volume,
but is suppressed by the square of the Planck Mass.
The unsuppressed Gravity force strength is the Geometric Part of the force strength.

The Standard Model SU(3) Color Force bosons live in
a SU(3) subalgebra of the SU(4) subalgebra of D4 = Spin(8).



They "see" M4 Physical spacetime as the complex projective plane CP2
so that their part of the Physical Lagrangian is

$$\int_{CP2} \text{SU(3) Color Force Gauge Boson Term} \quad .$$

an integral over SpaceTime CP2.
The Schwinger Sources for SU(3) bosons are the Complex Bounded Domains and Shilov Boundaries for SU(3) Color Force bosons.
The Color Force Strength is given by
the SpaceTime CP2 volume and the SU(3) Schwinger Source volume.
Note that since the Schwinger Source volume is dressed with the particle/antiparticle pair cloud, the calculated force strength is
for the characteristic energy level of the Color Force (about 245 MeV).

The Standard Model SU(2) Weak Force bosons live in
a SU(2) subalgebra of the U(2) local group of CP2 = SU(3) / U(2)
They "see" M4 Physical spacetime as two 2-spheres S2 x S2
so that their part of the Physical Lagrangian is

$$\int_{S2xS2} \text{SU(2) Weak Force Gauge Boson Term} \quad .$$

an integral over SpaceTime S2xS2.
The Schwinger Sources for SU(2) bosons are the Complex Bounded Domains and Shilov Boundaries for SU(2) Weak Force bosons.
However, due to the action of the Higgs mechanism,
for the Weak Force, the effective force strength that we see in our experiments
is not just composed of the S2xS2 volume and the SU(2) Schwinger Source volume,
but is suppressed by the square of the Weak Boson masses.
The unsuppressed Weak Force strength is the Geometric Part of the force strength.

The Standard Model U(1) Electromagnetic Force bosons (photons) live in
a U(1) subalgebra of the U(2) local group of CP2 = SU(3) / U(2)
They "see" M4 Physical spacetime as four 1-sphere circles S1xS1xS1xS1 = T4
(T4 = 4-torus) so that their part of the Physical Lagrangian is

$$\int_{T4} \text{(U(1) Electromagnetism Gauge Boson Term} \quad .$$

an integral over SpaceTime T4.
The Schwinger Sources for U(1) photons
are the Complex Bounded Domains and Shilov Boundaries for U(1) photons.
The Electromagnetic Force Strength is given by
the SpaceTime T4 volume and the U(1) Schwinger Source volume.

Schwinger Sources as described above are continuous manifold structures
of Bounded Complex Domains and their Shilov Boundaries
but
the Cl(16)-E8 model at the Planck Scale has spacetime condensing out of Clifford structures forming a Leech lattice underlying 26-dim String Theory of World-Lines
with 8 + 8 + 8 = 24-dim of fermion particles and antiparticles and of spacetime.

The automorphism group of a single 26-dim String Theory cell modulo the Leech lattice
is the Monster Group of order about 8 x 10^53.

When a fermion particle/antiparticle appears in E8 spacetime it does not remain a single
Planck-scale entity becauseTachyons create a cloud of particles/antiparticles.
The cloud is one Planck-scale Fundamental Fermion Valence Particle plus an effectively
neutral cloud of particle/antiparticle pairs forming a Kerr-Newman black hole.

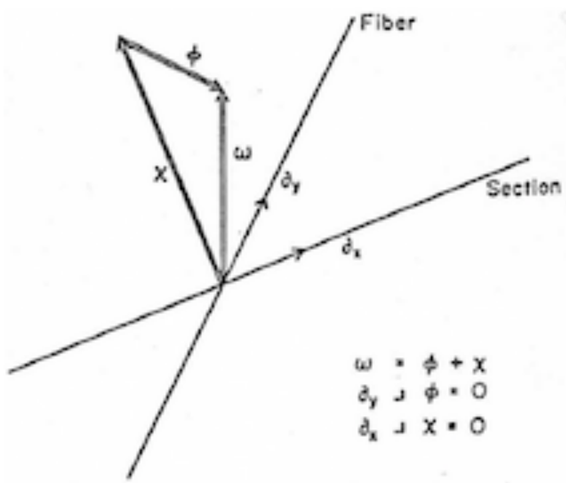
That cloud constitutes the Schwinger Source.
Its structure comes from the 24-dim Leech lattice part of the Monster Group which is
2^(1+24) times the double cover of Co1, for a total order of about 10^26.

(Since a Leech lattice is based on copies of an E8 lattice and since there are 7 distinct E8 integral domain
lattices there are 7 (or 8 if you include a non-integral domain E8 lattice)mdistinct Leech lattices.
The physical Leech lattice is a superposition of them, effectively adding a factor of 8 to the order.)

The volume of the Kerr-Newman Cloud is on the order of 10^27 x Planck scale,
so the Kerr-Newman Cloud should contain about 10^27 particle/antiparticle pairs
and its size should be about 10^(27/3) x 1.6 x 10^(-33) cm =
= roughly 10^(-24) cm.

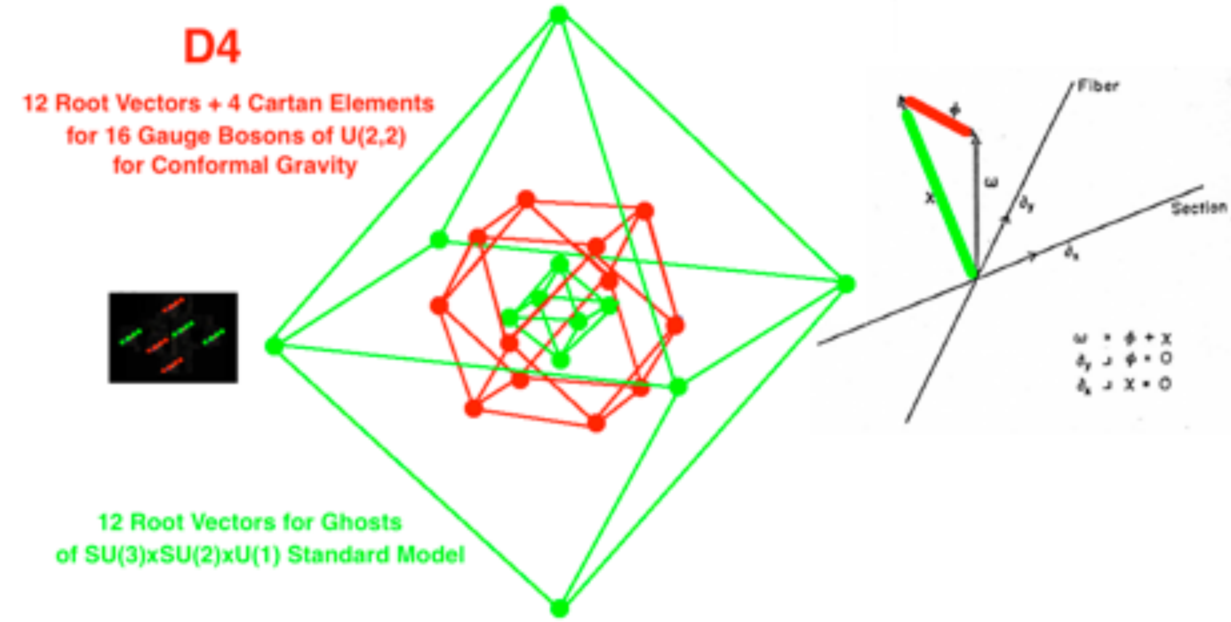
Ghosts

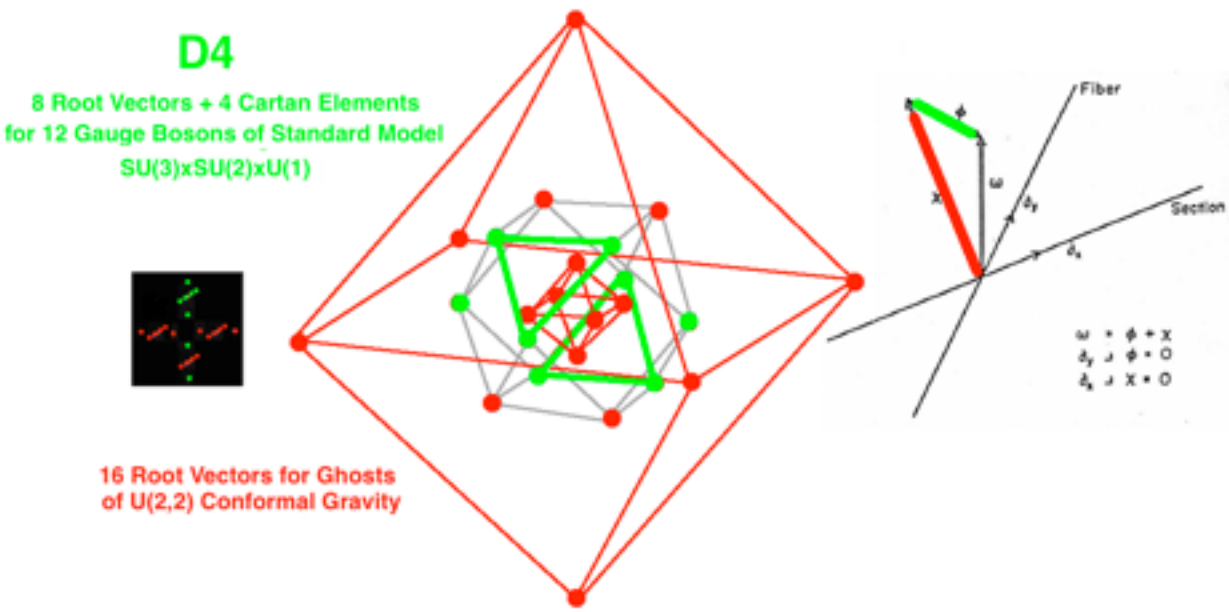
AQFT of Cl(16)-E8 Physics comes from the generalized von Neumann factor algebra constructed by completion of the union of all tensor products of Cl(16) Clifford Algebra where each Cl(16) contains E8 and a local Lagrangian constructed from E8.
The tensor product structure of Cl(16)-E8 AQFT is analogous to the sum-over-histories structure of Path Integral Quantization.
Jean Thierry-Mieg in J. Math. Phys. 21 (1980) 2834-2838 said: "... Because of gauge invariance, the classical Yang-Mills Lagrangian does not define a propagator for the gauge field. Using the path integral formulation of quantum field theory, Faddeev and Popov attributed this effect to the overcounting of gauge equivalent configurations. By fixing the gauge, Feynman diagrams are generated but unitarity is lost unless additional quantum fields are introduced: the ghost particles ...

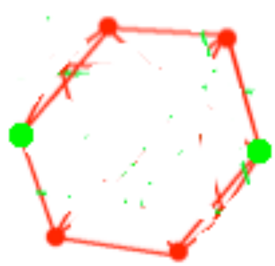


... FIG. 1. The ghost and the gauge field: The single lines represent a local coordinate system of a principal fiber bundle of base space-time. The double lines are 1 forms. The connection of the principle bundle w is assumed to be vertical. Its contravariant components PHI and X are recognized, respectively, as the Yang-Mills gauge field and the Faddeev-Popov ghost form ... By assumption, the ghost does not contribute to the description of motions tangent to the section. The exterior differential over ... the principal bundle ... of a function also splits, and its component normal to the section is recognized as the BRS operator ... the Cartan-Maurer structural theorem, which states the compatibility of the connection with the fibration, implies the BRS transformation rules of the gauge and ghost fields ... the ghost does not contribute to the curvature 2 form (field strength) and may thus be eliminated from the description of the classical theory. ... In ... the construction of the effective Lagrangian by using the generating functional ... No infinite constant has to be extracted, as the differential of the volume element of the group is actually lifted into the effective Lagrangian in the form of the ghost. The nongeometric transformation of the antighost, a Lagrange multiplier, is not recovered. However, the proof of renormalizability is not altered by the noninvariance of the effective Lagrangian, as one usually cancels the antighost variation via its equations of motion. On the contrary, the renormalized BRS operator is shown, as geometry suggests, not to act on the antighost ...".

There are two D4 in D8 in E8 in Cl(16): D4 Gravity and D4 Standard Model







SU(4) / SU(3) = 1 Cartan + CP3 = SU(2) + CP2

CP3 = Projective Twistors contains SU(2) and is Chiral **(Andrew Hodges "One to Nine")**

CP2 = SU(3) / SU(2)xU(1)

10. Fermion Mass Calculation

In the Cl(16)-E8 model, the first generation spinor fermions are
seen as +half-spinor and -half-spinor spaces of Cl(1,7) = Cl(8).
Due to Triality,
Spin(8) can act on those 8-dimensional half-spinor spaces
similarly to the way it acts on 8-dimensional vector spacetime.

Take the the spinor fermion volume to be the Shilov boundary corresponding
to the same symmetric space on which Spin(8) acts as a local gauge group
that is used to construct 8-dimensional vector spacetime:
the symmetric space Spin(10) / Spin(8)xU(1)
corresponding to a bounded domain of type IV8
whose Shilov boundary is RP^1 x S^7

Since all first generation fermions see the spacetime over which the integral is
taken in the same way (unlike what happens for the force strength calculation),
the only geometric volume factor relevant for calculating first generation fermion
mass ratios is in the spinor fermion volume term.
Cl(16)-E8 model fermions correspond to Schwinger Source Kerr-Newman Black Holes,
so the quark mass in the Cl(16)-E8 model is a constituent mass.

Fermion masses are calculated as a product of four factors:

V(Qfermion) x N(Graviton) x N(octonion) x Sym

V(Qfermion) is the volume of the part of the half-spinor fermion particle manifold
S^7 x RP^1 related to the fermion particle by photon, weak boson, or gluon interactions.

N(Graviton) is the number of types of Spin(0,5) graviton related to the fermion.
The 10 gravitons correspond to the 10 infinitesimal generators of Spin(0,5) = Sp(2).
2 of them are in the Cartan subalgebra.
6 of them carry color charge, and therefore correspond to quarks.
The remaining 2 carry no color charge, but may carry electric charge and so
may be considered as corresponding to electrons.
One graviton takes the electron into itself, and the other can only take the first-generation electron into the massless electron neutrino. Therefore only one graviton should correspond to the mass of the first-generation electron. The graviton number ratio of the down quark to the first-generation electron is therefore 6/1 = 6.

N(octonion) is an octonion number factor relating up-type quark masses to
down-type quark masses in each generation.

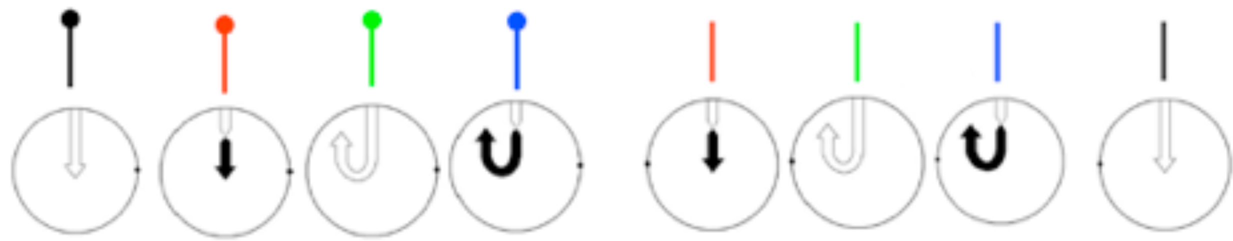
Sym is an internal symmetry factor, relating 2nd and 3rd generation massive
leptons to first generation fermions. It is not used in first-generation calculations.

3 Generation Fermion Combinatorics

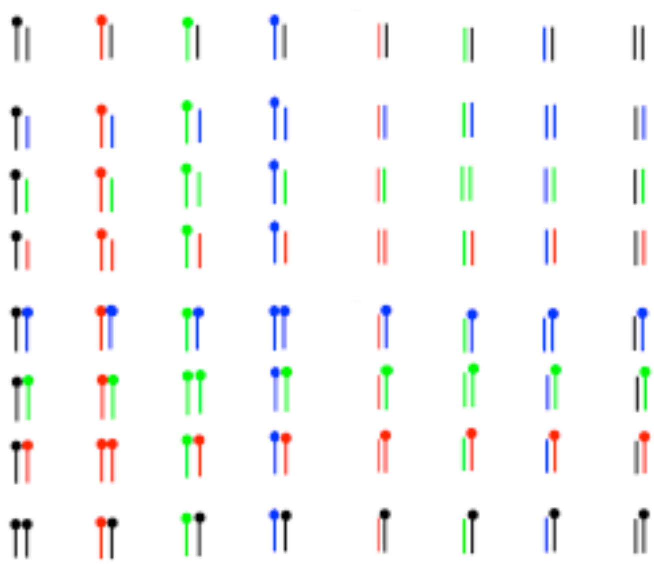
First Generation (8)

(geometric representation of Octonions is from arXiv 1010.2979)

electron	red up quark	green up quark	blue up quark	red down quark	green down quark	blue down quark	neutrino
E	I	J	K	i	j	k	1



Second Generation (64)

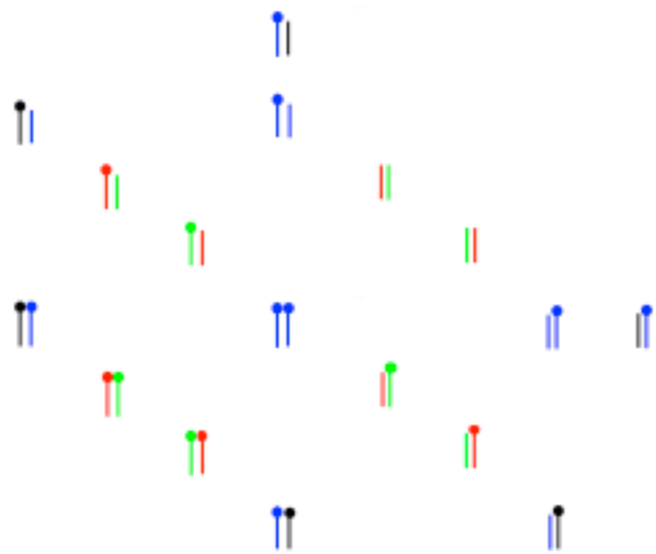


Mu Neutrino (1)
Rule: a Pair belongs to the Mu Neutrino if:
All elements are Colorless (black)
and all elements are Associative
(that is, is 1 which is the only Colorless Associative element) .

Muon (3)
Rule: a Pair belongs to the Muon if:
All elements are Colorless (black)
and at least one element is NonAssociative
(that is, is E which is the only Colorless NonAssociative element).

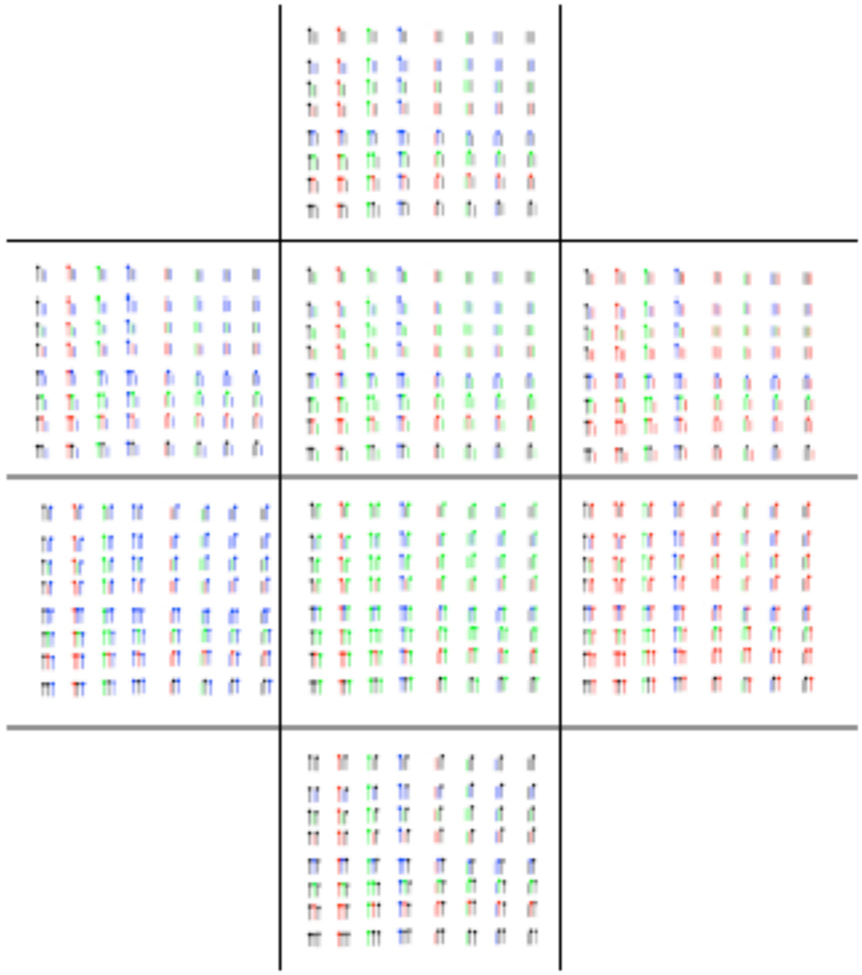
Blue Strange Quark **(3)**
Rule: a Pair belongs to the Blue Strange Quark if:
There is at least one Blue element and the other element is Blue or Colorless (black)
and all elements are Associative (that is, is either 1 or i or j or k).

Blue Charm Quark **(17)**
Rules: a Pair belongs to the Blue Charm Quark if:
1 - There is at least one Blue element and the other element is Blue or Colorless (black)
and at least one element is NonAssociative (that is, is either E or I or J or K)
2 - There is one Red element and one Green element (Red x Green = Blue).



(Red and Green Strange and Charm Quarks follow similar rules)

Third Generation (512)

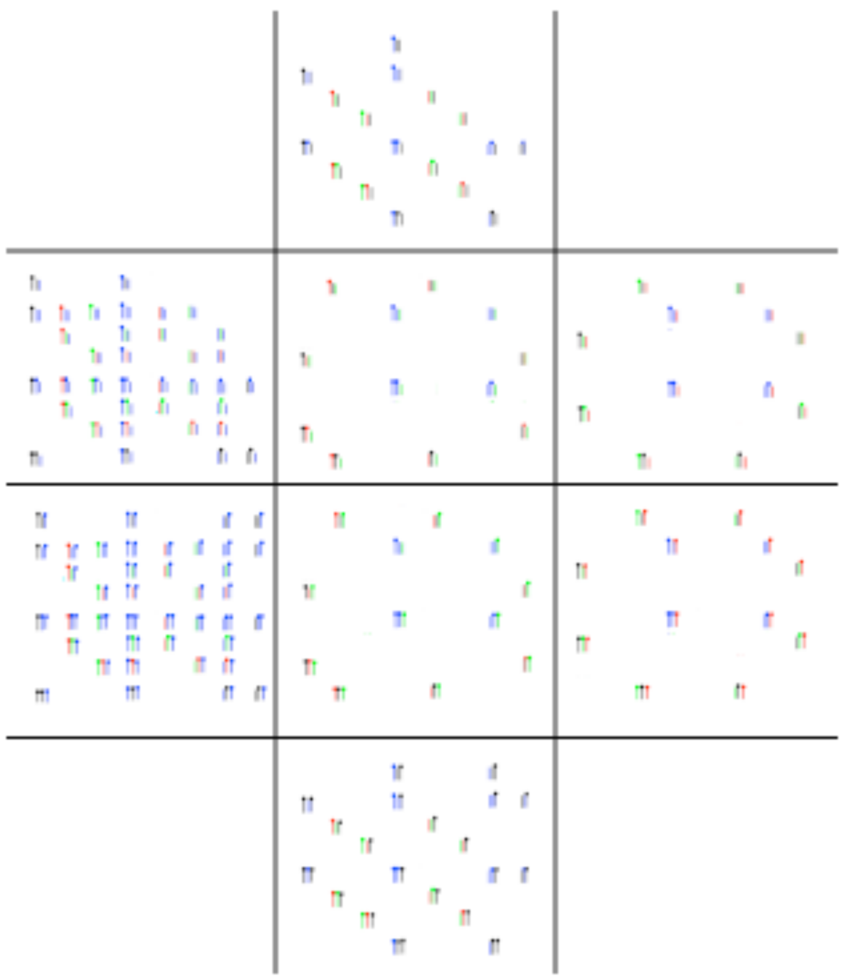


Tau Neutrino (1)
Rule: a Triple belongs to the Tau Neutrino if:
All elements are Colorless (black)
and all elements are Associative
(that is, is 1 which is the only Colorless Associative element)

Tauon (7)
Rule: a Triple belongs to the Tauon if:
All elements are Colorless (black)
and at least one element is NonAssociative (that is, is E which is the only Colorless NonAssociative element)

Blue Beauty Quark (7)
Rule: a Triple belongs to the Blue Beauty Quark if:
There is at least one Blue element and all other elements are Blue or Colorless (black)
and all elements are Associative (that is, is either 1 or i or j or k).

Blue Truth Quark (161)
Rules: a Triple belongs to the Blue Truth Quark if:
1 - There is at least one Blue element and all other elements are Blue or Colorless (black)
and at least one element is NonAssociative (that is, is either E or I or J or K)
2 - There is one Red element and one Green element and the other element is Colorless (Red x Green = Blue)
3 - The Triple has one element each that is Red, Green, or Blue,
in which case the color of the Third element (for Third Generation) is determinative
and must be Blue.



(Red and Green Beauty and Truth Quarks follow similar rules)

The first generation down quark constituent mass : electron mass ratio is:

The electron, E, can only be taken into the tree-level-massless neutrino, 1, by photon, weak boson, and gluon interactions.
The electron and neutrino, or their antiparticles, cannot be combined to produce any of the massive up or down quarks.
The neutrino, being massless at tree level, does not add anything to the mass formula for the electron.
Since the electron cannot be related to any other massive Dirac fermion,
its volume V(Qelectron) is taken to be 1.

Next consider a red down quark i.
By gluon interactions, i can be taken into j and k, the blue and green down quarks.
By also using weak boson interactions,
it can also be taken into I, J, and K, the red, blue, and green up quarks.
Given the up and down quarks, pions can be formed from quark-antiquark pairs,
and the pions can decay to produce electrons and neutrinos.
Therefore the red down quark (similarly, any down quark)
is related to all parts of S^7 x RP^1,
the compact manifold corresponding to { 1, i, j, k, E, I, J, K }
and therefore
a down quark should have
a spinor manifold volume factor V(Qdown quark) of the volume of S^7 x RP^1.

The ratio of the down quark spinor manifold volume factor
to the electron spinor manifold volume factor is

V(Qdown quark) / V(Qelectron) = V(S^7x RP^1)/1 = pi^5 / 3.

Since the first generation graviton factor is 6,

md / me = 6 V(S^7 x RP^1) = 2 pi^5 = 612.03937

As the up quarks correspond to I, J, and K, which are the octonion transforms under E of i, j, and k of the down quarks, the up quarks and down quarks have the same constituent mass

mu = md.

Antiparticles have the same mass as the corresponding particles.
Since the model only gives ratios of masses,
the mass scale is fixed so that the electron mass me = 0.5110 MeV.

Then, the constituent mass of the down quark is md = 312.75 MeV,
and the constituent mass for the up quark is mu = 312.75 MeV.

These results when added up give a total mass of first generation fermion particles:

Sigmaf1 = 1.877 GeV

As the proton mass is taken to be the sum of the constituent masses of its constituent quarks

mproton = mu + mu + md = 938.25 MeV

which is close to the experimental value of 938.27 MeV.

The third generation fermion particles correspond to triples of octonions. There are 8^3 = 512 such triples.

The triple { 1,1,1 } corresponds to the tau-neutrino.

The other 7 triples involving only 1 and E correspond to the tauon:

{ E, E, E }
{ E, E, 1 }
{ E, 1, E }
{ 1, E, E }
{ 1, 1, E }
{ 1, E, 1 }
{ E, 1, 1 }

The symmetry of the 7 tauon triples is the same
as the symmetry of the first generation tree-level-massive fermions,
3 down, quarks, the 3 up quarks, and the electron,
so by the Sym factor the tauon mass should be the same as
the sum of the masses of the first generation massive fermion particles.

Therefore the tauon mass is calculated at tree level as 1.877 GeV.

The calculated tauon mass of 1.88 GeV is a sum of first generation fermion masses, all of which are valid at the energy level of about 1 GeV.

However, as the tauon mass is about 2 GeV,
the effective tauon mass should be renormalized
from the energy level of 1 GeV at which the mass is 1.88 GeV
to the energy level of 2 GeV.
Such a renormalization should reduce the mass.

If the renormalization reduction were about 5 percent,
the effective tauon mass at 2 GeV would be about 1.78 GeV.
The 1996 Particle Data Group Review of Particle Physics gives a tauon mass of 1.777 GeV.

All triples corresponding to the tau and the tau-neutrino are colorless.

The beauty quark corresponds to 21 triples.
They are triples of the same form as the 7 tauon triples involving 1 and E,
but for 1 and I, 1 and J, and 1 and K,
which correspond to the red, green, and blue beauty quarks,
respectively.

The seven red beauty quark triples correspond to the seven tauon triples,
except that
the beauty quark interacts with 6 Spin(0,5) gravitons
while the tauon interacts with only two.

The red beauty quark constituent mass should be the tauon mass times
the third generation graviton factor 6/2 = 3,
so the red beauty quark mass is mb = 5.63111 GeV.

The blue and green beauty quarks are similarly determined to also be 5.63111 GeV.

The calculated beauty quark mass of 5.63 GeV is a consitituent mass,
that is, it corresponds to the conventional pole mass plus 312.8 MeV.
Therefore, the calculated beauty quark mass of 5.63 GeV
corresponds to a conventional pole mass of 5.32 GeV.

The 1996 Particle Data Group Review of Particle Physics gives
a lattice gauge theory beauty quark pole mass as 5.0 GeV.

The pole mass can be converted to an MSbar mass
if the color force strength constant alpha_s is known.
The conventional value of alpha_s at about 5 GeV is about 0.22.

Using alpha_s (5 GeV) = 0.22, a pole mass of 5.0 GeV
gives an MSbar 1-loop beauty quark mass of 4.6 GeV,
and
an MSbar 1,2-loop beauty quark mass of 4.3, evaluated at about 5 GeV.

If the MSbar mass is run from 5 GeV up to 90 GeV,
the MSbar mass decreases by about 1.3 GeV,
giving an expected MSbar mass of about 3.0 GeV at 90 GeV.

DELPHI at LEP has observed the Beauty Quark
and found a 90 GeV MSbar beauty quark mass of about 2.67 GeV,
with error bars +/- 0.25 (stat) +/- 0.34 (frag) +/- 0.27 (theo).

The theoretical model calculated Beauty Quark mass of 5.63 GeV
corresponds to a pole mass of 5.32 GeV,
which is somewhat higher than the conventional value of 5.0 GeV.

However, the theoretical model calculated value
of the color force strength constant alpha_s at about 5 GeV is about 0.166,
while the conventional value
of the color force strength constant alpha_s at about 5 GeV is about 0.216,
and
the theoretical model calculated value
of the color force strength constant alpha_s at about 90 GeV is about 0.106,
while the conventional value
of the color force strength constant alpha_s at about 90 GeV is about 0.118.

The theoretical model calculations gives a Beauty Quark pole mass (5.3 GeV) that
is about 6 percent higher than the conventional Beauty Quark pole mass (5.0 GeV),
and a color force strength alpha_s at 5 GeV (0.166)
such that 1 + alpha_s = 1.166 is about 4 percent lower
than the conventional value of 1 + alpha_s = 1.216 at 5 GeV.

Triples of the type { 1, I, J } , { I, J, K }, etc.,
do not correspond to the beauty quark, but to the truth quark.
The truth quark corresponds to those 512 - 1 - 7 - 21 = 483 triples,
so the constituent mass of the red truth quark
is 161 / 7 = 23 times the red beauty quark mass,
and the red T-quark mass is
mt = 129.5155 GeV

The blue and green truth quarks are similarly determined to also be 129.5155 GeV.

This is the value of the Low Mass State of the Truth calculated in the Cl(16)_E8 model.
The Middle Mass State of the Truth Quark has been observed by Fermilab since 1994.
The Low and High Mass States of the Truth Quark have, in my opinion, also been observed by Fermilab (see Chapter 17 of this paper) but the Fermilab and CERN establishments disagree.

All other masses than the electron mass
(which is the basis of the assumption of the value of the Higgs scalar field vacuum expectation value v = 252.514 GeV),
including the Higgs scalar mass and Truth quark mass,
are calculated (not assumed) masses in the Cl(16)-E8 model.
These results when added up give a total mass of third generation fermion particles:

Sigmaf3 = 1,629 GeV

The second generation fermion particles correspond to pairs of octonions.
There are 8^2 = 64 such pairs.

The pair { 1,1 } corresponds to the mu-neutrino.

The pairs { 1, E }, { E, 1 }, and { E, E } correspond to the muon.

For the Sym factor, compare the symmetries of the muon pairs
to the symmetries of the first generation fermion particles:
The pair { E, E } should correspond to the E electron.
The other two muon pairs have a symmetry group S2,
which is 1/3 the size of the color symmetry group S3
which gives the up and down quarks their mass of 312.75 MeV.

Therefore the mass of the muon should be the sum of
the { E, E } electron mass and
the { 1, E }, { E, 1 } symmetry mass, which is 1/3 of the up or down quark mass.
Therefore, mmu = 104.76 MeV .

According to the 1998 Review of Particle Physics of the Particle Data Group,
the experimental muon mass is about 105.66 MeV which may be consistent with
radiative corrections for the calculated tree-level mmu = 104.76 MeV as
Bailin and Love, in “Introduction to Gauge Field Theory”, IOP (rev ed 1993), say:
"... considering the order alpha radiative corrections to muon decay ... Numerical details are contained in Sirlin ... 1980 Phys. Rev. D 22 971 ... who concludes that the order alpha corrections have the effect of increasing the decay rate about 7% compared with the tree graph prediction ...". Since the decay rate is proportional to mmu^5 the corresponding effective increase in muon mass would be about 1.36%, which would bring 104.8 MeV up to about 106.2 MeV.

All pairs corresponding to the muon and the mu-neutrino are colorless.

The red, blue and green strange quark each corresponds
to the 3 pairs involving 1 and i, j, or k.

The red strange quark is defined as the three pairs { 1, i }, { i, 1 }, { i, i }
because i is the red down quark.
Its mass should be the sum of two parts:
the { i, i } red down quark mass, 312.75 MeV, and
the product of the symmetry part of the muon mass, 104.25 MeV,
times the graviton factor.

Unlike the first generation situation,
massive second and third generation leptons can be taken,
by both of the colorless gravitons that may carry electric charge,
into massive particles.

Therefore the graviton factor for the second and third generations is 6/2 = 3.

So the symmetry part of the muon mass times the graviton factor 3 is 312.75 MeV, and
the red strange quark constituent mass is ms = 312.75 MeV + 312.75 MeV = 625.5 MeV

The blue strange quarks correspond to the three pairs involving j,
the green strange quarks correspond to the three pairs involving k,
and their masses are similarly determined to also be 625.5 MeV.
The charm quark corresponds to the remaining 64 - 1 - 3 - 9 = 51 pairs.

Therefore, the mass of the red charm quark should be the sum of two parts:
the { i, i }, red up quark mass, 312.75 MeV;
and
the product of the symmetry part of the strange quark mass, 312.75 MeV,
and the charm to strange octonion number factor 51 / 9,
which product is 1,772.25 MeV.

Therefore the red charm quark constituent mass is
mc = 312.75 MeV + 1,772.25 MeV = 2.085 GeV

The blue and green charm quarks are similarly determined to also be 2.085 GeV.

The calculated Charm Quark mass of 2.09 GeV is a consitituent mass,
that is, it corresponds to the conventional pole mass plus 312.8 MeV.

Therefore, the calculated Charm Quark mass of 2.09 GeV corresponds to a
conventional pole mass of 1.78 GeV.

The 1996 Particle Data Group Review of Particle Physics gives a range for the
Charm Quark pole mass from 1.2 to 1.9 GeV.

The pole mass can be converted to an MSbar mass if the color force strength constant alpha_s is known.
The conventional value of alpha_s at about 2 GeV is about 0.39,
which is somewhat lower than the theoretical model value.
Using alpha_s (2 GeV) = 0.39, a pole mass of 1.9 GeV
gives an MSbar 1-loop mass of 1.6 GeV, evaluated at about 2 GeV.

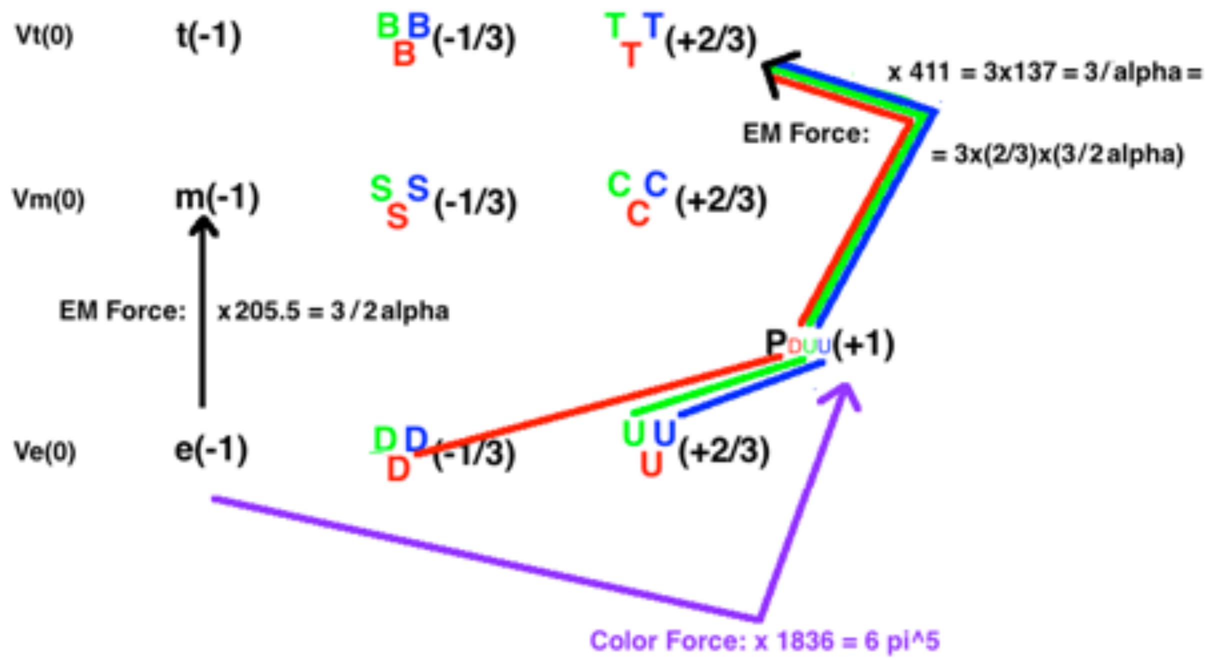
These results when added up give a total mass of second generation fermion particles:

Sigmaf2 = 32.9 GeV

Mendel Sachs and Particle Masses

Frank Dodd (Tony) Smith, Jr. - 2014

Mendel Sachs, in his books “General Relativity and Matter” (1982) and
“Quantum Mechanics from General Relativity” (1986) calculated
electron / muon and Proton / Tquark mass ratios substantially consistent with



Cl(16)-E8 Physics masses e = 0.511 MeV, m = 106 MeV, P = 938 MeV, T = 128.5 GeV
saying (my comments set off by brackets [[ ]]):
“... the inertial mass of an elementary (spinor) particle [i]s
determined by the curvature of space-time in its vicinity, representing
the coupling of this particle to its environment of particle-antiparticle pairs ...
[[In Cl(16)-E8 Physics the particle-antiparticle pairs form a Schwinger Source Kerr-Newman Black Hole]]
Because the coupling of the observed electron to the pairs ... is electromagnetic,
the electron's mass is proportional to the fine structure constant ...

[[In Cl(16)-E8 Physics the gauge symmetry of the force determines the geometry of the Schwinger Source and its Green’s Function.]]

The electron mass is one member of a mass doublet, predicted by this theory.
The other member, the muon, arises because occasionally the observed electron can excite a pair of the background, which in turn changes the features of the geometry of space-time in the vicinity of the electron. ...

[[In Cl(16)-E8 Physics the “excite” producing second and third generations is due to World-Lines traversing CP2 Internal Symmetry Space as well as M4 Physical Spacetime of M4xCP2 Kaluza-Klein]]

Because the excitation of the pair is due to an electromagnetic force,
the new mass ... is 3 / 2 alpha = 206 times greater than the old mass. ...
This theory also predicts that the proton should have a sister member of a doublet ...

To compute the inertial mass of the electron, consider first
the frame of reference whose spatial origin is at the site of the observed electron,
with the pairs of the background in motion relative to this point
...
Using the method of Green's functions ... we see that the quaternion metrical field ... in the linear approximation, reduce to an integral equation with ... solutions ...[that]... are the linear approximation ... to the spin-affine connection field ...
the solutions ... of the integral Equation ... lead directly to the (squared) mass eigenvalues ... the eigenvalues of the mass operator are the absolute values of the squares of the matrix elements above
...
The pairs interact with each other in a way that makes them appear to some 'observed' constituent electron as 'photons'. ... Nevertheless,the pairs do have 'inertia' by virtue of their bound electrons and positrons that are not , in fact, annihilated. ...
From a distance greater than a 'first Bohr orbit' of one of the particle components of a pair, it appears, as a unit , to be an electrically neutral object. But as the (observed) electron comes sufficiently close to the pair so as to interact with its separate components, energy is used up in exciting the pair, thereby decreasing the relative speed between the pair and the observed electron.
If the primary excitation of a pair (as 'seen' by the observed electron) is quadrupolar, and if the ground state of the pair corresponds to n = 1; then the
first excited Bohr orbital with a quadrupolar component is the state with n' = 3.

[[Quadrupolar implies 4+4 Kaluza-Klein of Cl(16)-E8 Physics]]

With these values ... it follows that the ratio of mass eigenvalues is ... 3 / 2 alpha = 206 ... The reason for this is that the curvature of space-time, in the vicinity of the observed electron, that gives rise to its inertia, is a consequence of the electromagnetic coupling between the matter components of the system. ...
Summing up, the inertial mass of an elementary (spinor) particle was determined by the curvature of space-time in its vicinity, representing the coupling of this particle to its environment of particle-antiparticle pairs.

[[Green's functions for each force imply geometric structure of Schwinger Sources]]

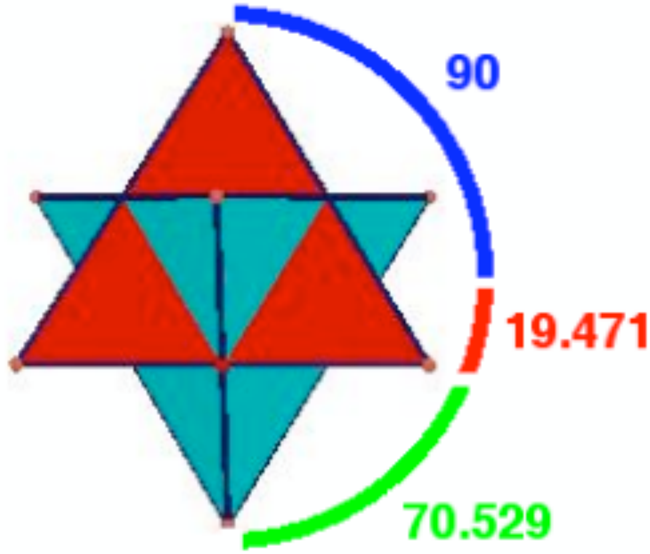
The significant domain of space populated by pairs that contributes to the electron mass is the order of 10^(-15) cm ,,,

[[Schwinger Source size in Cl(16)-E8 Physics is much smaller, about 10^(-24) cm]]

Because the coupling of the observed electron to the pairs - that gives it inertia - is electromagnetic, the electron's mass is proportional to the fine structure constant - which is a measure of the strength of this coupling. ...".

11. Kobayashi-Maskawa Parameters

In E8 Physics the KM Unitarity Triangle angles can be seen on the Stella Octangula



The Kobayashi-Maskawa parameters are determined in terms of
the sum of the masses of the 30 first-generation fermion particles and antiparticles, denoted by

Smf1 = 7.508 GeV,

and the similar sums for second-generation and third-generation fermions, denoted by

Smf2 = 32.94504 GeV and Smf3 = 1,629.2675 GeV.

The resulting KM matrix is:

d		s	b
u	0.975	0.222 0.00249	-0.00388i
c	-0.222 -0.000161i	0.974 -0.0000365i	0.0423
t	0.00698 -0.00378i	-0.0418 -0.00086i	0.999

Below the energy level of ElectroWeak Symmetry Breaking the Higgs mechanism gives mass to particles.

According to a Review on the Kobayashi-Maskawa mixing matrix by Ceccucci, Ligeti, and Sakai in the 2010 Review of Particle Physics (note that I have changed their terminology of CKM matrix to the KM terminology that I prefer because I feel that it was Kobayashi and Maskawa, not Cabibbo, who saw that 3x3 was the proper matrix structure): "... the charged-current W± interactions couple to the ... quarks with couplings given by ...

Vud	Vus	Vub
Vcd	Vcs	Vcb
Vtd	Vts	Vtb

This Kobayashi-Maskawa (KM) matrix is a 3x3 unitary matrix.
It can be parameterized by three mixing angles and the CP-violating KM phase ...
The most commonly used unitarity triangle arises from
Vud Vub∗ + Vcd Vcb∗ + Vtd Vtb∗ = 0,
by dividing each side by the best-known one, Vcd Vcb∗
...
¯ρ + i¯η = –(Vud Vub∗)/(Vcd Vcb∗) is phase-convention- independent ...

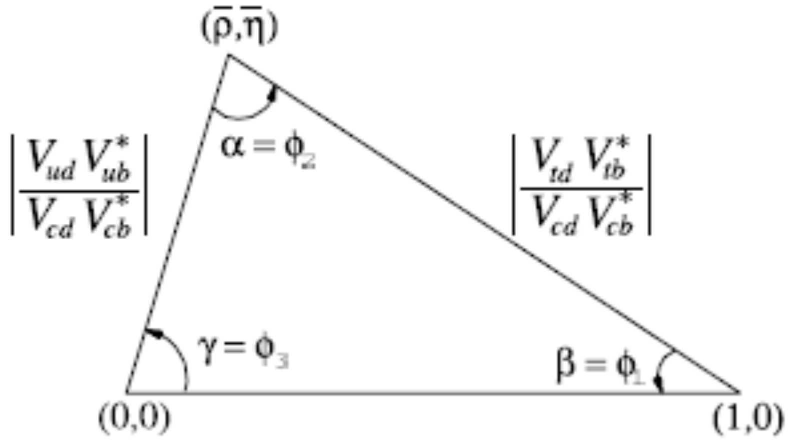


Figure 11.1: Sketch of the unitarity triangle.

... sin 2β = 0.673 ± 0.023 ... α = 89.0 +4.4 −4.2 degrees ... γ = 73 +22 –25 degrees ...
The sum of the three angles of the unitarity triangle, α + β + γ = (183 +22 –25) degrees, is ... consistent with the SM expectation. ...

The area... of ...[the]... triangle...[is]... half of the Jarlskog invariant, J, which is a phase-convention-independent measure of CP violation, defined by Im Vij Vkl Vil∗ Vkj∗ = J SUM(m,n) ε_ikm ε_jln

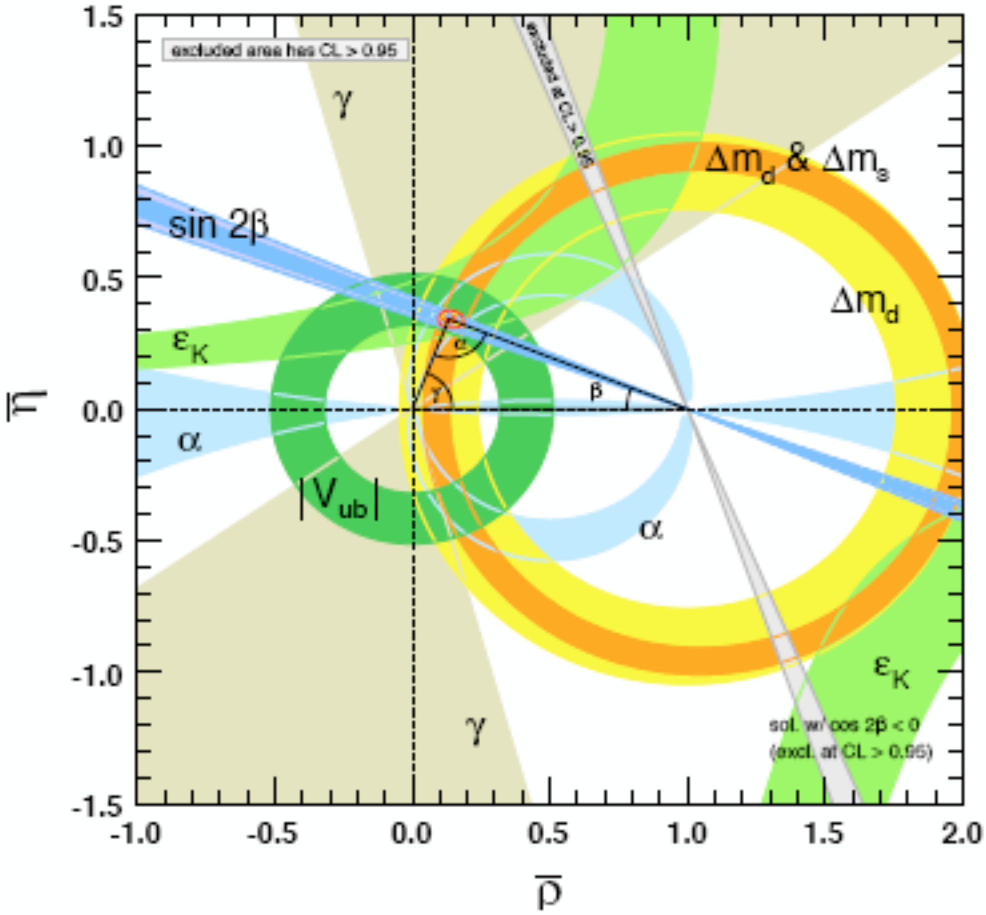


Figure 11.2: Constraints on the $\bar{\rho}, \bar{\eta}$ plane. The shaded areas have 95% CL.

The fit results for the magnitudes of all nine KM elements are ...

0.97428 ± 0.00015	0.2253 ± 0.0007	0.00347 +0.00016 −0.00012
0.2252 ± 0.0007	0.97345 +0.00015 −0.00016	0.0410 +0.0011 −0.0007
0.00862 +0.00026 −0.00020	0.0403 +0.0011−0.0007	0.999152 +0.000030−0.000045

and the Jarlskog invariant is J = (2.91 +0.19-0.11)x10−5. ...".

Above the energy level of ElectroWeak Symmetry Breaking particles are massless.

Kea (Marni Sheppeard) proposed
that in the Massless Realm the mixing matrix might be democratic.
In Z. Phys. C - Particles and Fields 45, 39-41 (1989) Koide said: "...
the mass matrix ... MD ... of the type ... 1/3 x m x

1	1	1
1	1	1
1	1	1

... has name... "democratic" family mixing ...
the ... democratic ... mass matrix can be diagonalized by the transformation matrix A ...

1/sqrt(2)	-1/sqrt(2)	0
1/sqrt(6)	1/sqrt(6)	-2/sqrt(6)
1/sqrt(3)	1/sqrt(3)	1/sqrt(3)

as A MD At =

0	0	0
0	0	0
0	0	m

...".

Up in the Massless Realm you might just say that there is no mass matrix,
just a democratic mixing matrix of the form 1/3 x

1	1	1
1	1	1
1	1	1

with no complex stuff and no CP violation in the Massless Realm.

When go down to our Massive Realm by ElectroWeak Symmetry Breaking
then you might as a first approximation use m = 1
so that all the mass first goes to the third generation as

0	0	0
0	0	0
0	0	1

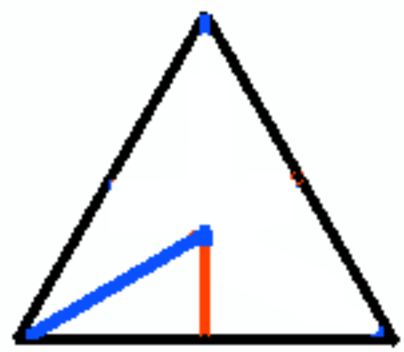
which is physically like the Higgs being a T-Tbar quark condensate.

Consider a 3-dim Euclidean space of generations:

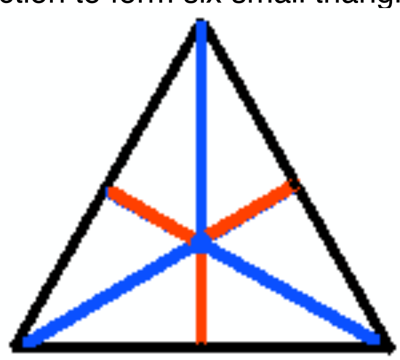
The case of mass only going to one generation
can be represented as a line or 1-dimensional simplex

in which the blue mass-line covers the entire black simplex line.

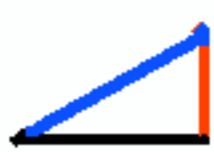
If mass only goes to one other generation
that can be represented by a red line extending to a second dimension
forming a small blue-red-black triangle



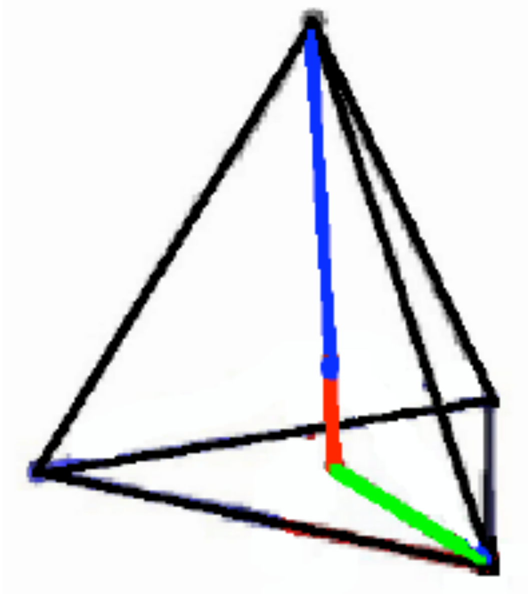
that can be extended by reflection to form six small triangles making up a large triangle



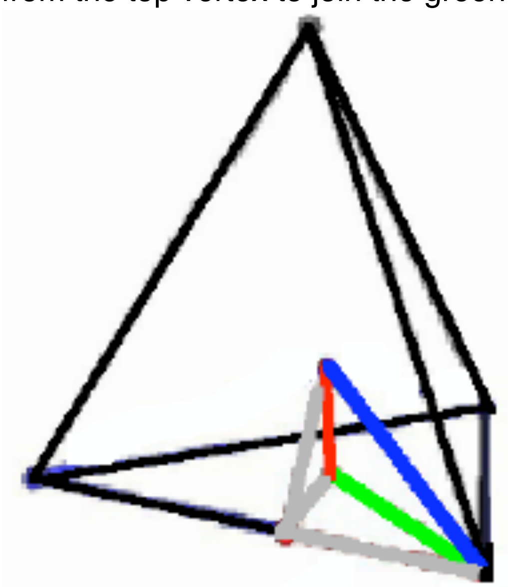
Each of the six component triangles has 30-60-90 angle structure:



If mass goes on further to all three generations
that can be represented by a green line extending to a third dimension



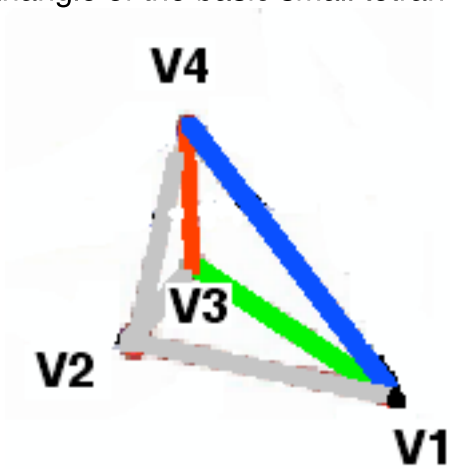
If you move the blue line from the top vertex to join the green vertex



you get a small blue-red-green-gray-gray-gray tetrahedron
that can be extended by reflection to form 24 small tetrahedra
making up a large tetrahedron.

Reflection among the 24 small tetrahedra corresponds
to the 12+12 = 24 elements of the Binary Tetrahedral Group.

The basic blue-red-green triangle of the basic small tetrahedron



has the angle structure of the K-M Unitary Triangle.

Using data from R. W. Gray's "Encyclopedia Polyhedra: A Quantum Module"
with lengths

V1.V2 = (1/2) EL ≡ Half of the regular Tetrahedron's edge length.
V1.V3 = (1 / sqrt(3)) EL ≅ 0.577 350 269 EL
V1.V4 = 3 / (2 sqrt(6)) EL ≅ 0.612 372 436 EL
V2.V3 = 1 / (2 sqrt(3)) EL ≅ 0.288 675 135 EL
V2.V4 = 1 / (2 sqrt(2)) EL ≅ 0.353 553 391 EL
V3.V4 = 1 / (2 sqrt(6)) EL ≅ 0.204 124 145 EL

the Unitarity Triangle angles are:

β = V3.V1.V4 = arccos(2 sqrt(2) / 3) ≅ 19.471 220 634 degrees so sin 2β = 0.6285

α = V1.V3.V4 = 90 degrees

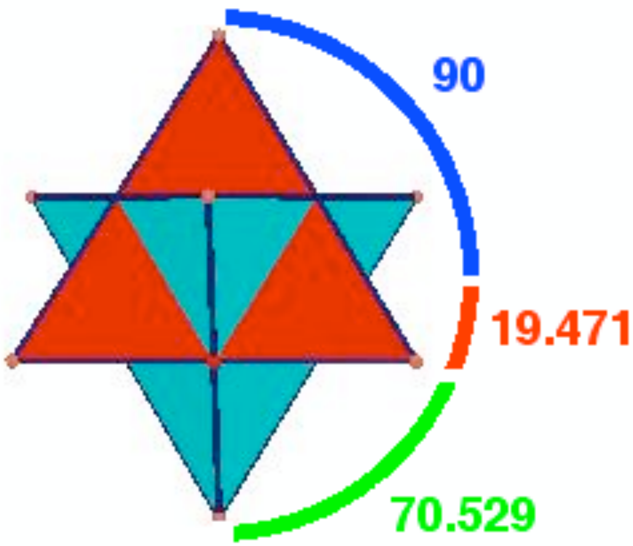
γ = V1.V4.V3 = arcsin(2 sqrt(2) / 3) ≅ 70.528 779 366 degrees

which is substantially consistent with the 2010 Review of Particle Properties

sin 2β = 0.673 ± 0.023 so β = 21.1495 degrees
α = 89.0 +4.4 –4.2 degrees
γ = 73 +22 –25 degrees

and so also consistent with the Standard Model expectation.

The constructed Unitarity Triangle angles can be seen on the Stella Octangula configuration of two dual tetrahedra (image from gauss.math.nthu.edu.tw):



In the Cl(16)-E8 model the Kobayashi-Maskawa parameters are determined in terms of the sum of the masses of the 30 first-generation fermion particles and antiparticles, denoted by
Smf1 = 7.508 GeV,

and the similar sums for second-generation and third-generation fermions, denoted
by Smf2 = 32.94504 GeV and Smf3 = 1,629.2675 GeV.

The reason for using sums of all fermion masses (rather than sums of quark masses only) is that all fermions are in the same spinor representation of Spin(8),
and the Spin(8) representations are considered to be fundamental.

The following formulas use the above masses
to calculate Kobayashi-Maskawa parameters:

phase angle d13 = gamma = 70.529 degrees

sin(theta12) = s12 = [me+3md+3mu]/sqrt([me^2+3md^2+3mu^2]+
+ [mmu^2+3ms^2+3mc^2]) = 0.222198

sin(theta13) = s13 = [me+3md+3mu]/sqrt([me^2+3md^2+3mu^2]+
+ [mtau^2+3mb^2+3mt^2]) = 0.004608

sin(*theta23 = [mmu+3ms+3mc]/sqrt([mtau^2+3mb^2+3mt^2]+
+ [mmu^2+3ms^2+3mc^2])

sin(theta23) = s23 = sin(*theta23) sqrt(Sigmaf2 / Sigmaf1) = 0.04234886

The factor sqrt(Smf2 /Smf1) appears in s23 because an s23 transition is to the second generation and not all the way to the first generation, so that the end product of an s23 transition has a greater available energy than s12 or s13 transitions by a factor of Smf2 / Smf1 .

Since the width of a transition is proportional to the square of the modulus of the relevant KM entry and the width of an s23 transition has greater available energy than the s12 or s13 transitions by a factor of Smf2 / Smf1
the effective magnitude of the s23 terms in the KM entries is increased by the factor sqrt(Smf2 /Smf1) .

The Chau-Keung parameterization is used, as it allows the K-M matrix to be represented as the product of the following three 3x3 matrices:

1	0	0
0	cos(theta23)	sin(theta23)
0	-sin(theta23)	cos(theta23)

cos(theta13)	0	sin(theta13)exp(-i d13)
0	1	0
-sin(theta13)exp(i d13)	0	cos(theta13)

cos(theta12)	sin(theta12)	0
-sin(theta12)	cos(theta12)	0
0	0	1

The resulting Kobayashi-Maskawa parameters
for W+ and W- charged weak boson processes, are:

	d	s	b
u	0.975	0.222	0.00249 -0.00388i
c	-0.222 -0.000161i	0.974 -0.0000365i	0.0423
t	0.00698 -0.00378i	-0.0418 -0.00086i	0.999

The matrix is labelled by either (u c t) input and (d s b) output,
or, as above, (d s b) input and (u c t) output.

For Z0 neutral weak boson processes, which are suppressed by the GIM
mechanism of cancellation of virtual subprocesses, the matrix is labelled by either
(u c t) input and (u'c't') output, or, as below, (d s b) input and (d's'b') output:

	d	s	b
d'	0.975	0.222	0.00249 -0.00388i
s'	-0.222 -0.000161i	0.974 -0.0000365i	0.0423
b'	0.00698 -0.00378i	-0.0418 -0.00086i	0.999

Since neutrinos of all three generations are massless at tree level,
the lepton sector has no tree-level K-M mixing.

In hep-ph/0208080, Yosef Nir says: "... Within the Standard Model,
the only source of CP violation is the Kobayashi-Maskawa (KM) phase ...
The study of CP violation is, at last, experiment driven. ...
The CKM matrix provides a consistent picture
of all the measured flavor and CP violating processes. ...
There is no signal of new flavor physics. ...
Very likely,
the KM mechanism is the dominant source of CP violation in flavor changing processes.
... The result is consistent with the SM predictions. ...".

12. Proton-Neutron Mass Difference

An up valence quark, constituent mass 313 Mev,
does not often swap places with a 2.09 Gev charm sea quark,
but
a 313 Mev down valence quark
can more often swap places with a 625 Mev strange sea quark.

Therefore the Quantum color force
constituent mass of the down valence quark is heavier by about

(ms - md) (md/ms)^2 a(w) lVdsl = 312 x 0.25 x 0.253 x 0.22 Mev = 4.3 Mev,

(where a(w) = 0.253 is the geometric part of the weak force strength
and lVdsl = 0.22 is the magnitude
of the K-M parameter mixing first generation down and second generation strange)

so that the Quantum color force constituent mass Qmd of the down quark is

Qmd = 312.75 + 4.3 = 317.05 MeV.

Similarly, the up quark Quantum color force mass increase is about

(mc - mu) (mu/mc)^2 a(w) lV(uc)l = 1777 x 0.022 x 0.253 x 0.22 Mev = 2.2 Mev,

(where lVucl = 0.22 is the magnitude
of the K-M parameter mixing first generation up and second generation charm)

so that the Quantum color force constituent mass Qmu of the up quark is

Qmu = 312.75 + 2.2 = 314.95 MeV.

Therefore, the Quantum color force Neutron-Proton mass difference is

mN - mP = Qmd - Qmu = 317.05 Mev - 314.95 Mev = 2.1 Mev.

Since the electromagnetic Neutron-Proton mass difference is roughly

mN - mP = -1 MeV

the total theoretical Neutron-Proton mass difference is

mN - mP = 2.1 Mev - 1 Mev = 1.1 Mev,

an estimate that is comparable to the experimental value of 1.3 Mev.

13. Pion as Sine-Gordon Breather

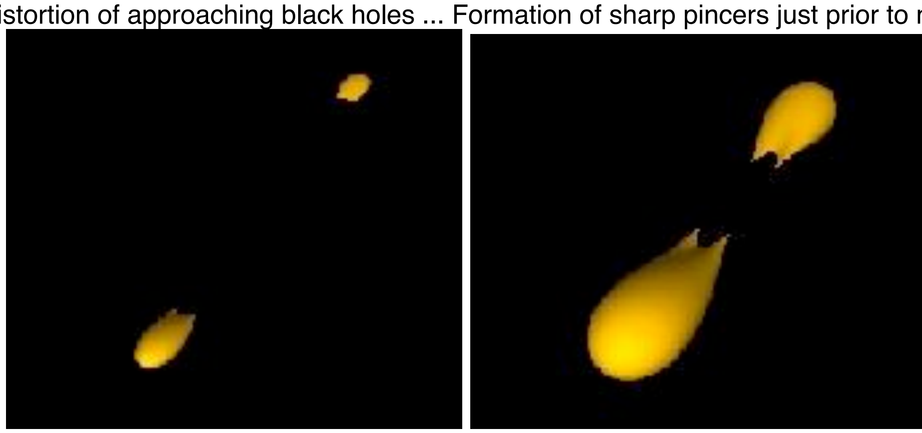
The quark content of a charged pion is a quark - antiquark pair: either Up plus antiDown or Down plus antiUp. Experimentally, its mass is about 139.57 MeV.

The quark is a Schwinger Source Kerr-Newman Black Hole
with constituent mass M 312 MeV.

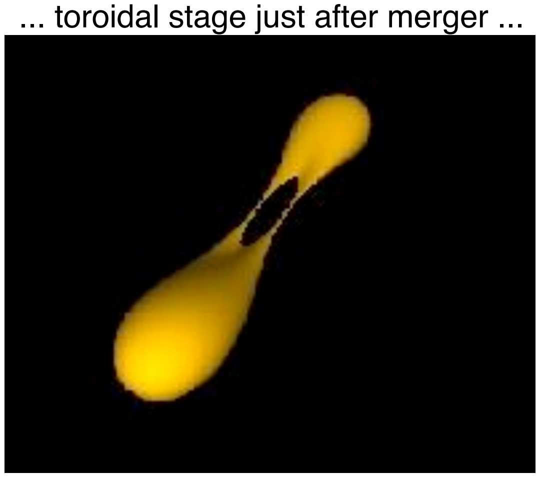
The antiquark is also a Schwinger Source Kerr-Newman Black Hole,
with constituent mass M 312 MeV.

According to section 3.6 of Jeffrey Winicour's 2001 Living Review of the Development of Numerical Evolution Codes for General Relativity (see also a 2005 update):
"... The black hole event horizon associated with ... slightly broken ... degeneracy [of the axisymmetric configuration]... reveals new features not seen in the degenerate case of the head-on collision ... If the degeneracy is slightly broken, the individual black holes form with spherical topology but as they approach, tidal distortion produces two sharp pincers on each black hole just prior to merger. ...

Tidal distortion of approaching black holes ... Formation of sharp pincers just prior to merger ..



... toroidal stage just after merger ...



At merger, the two pincers join to form a single ... toroidal black hole.

The inner hole of the torus subsequently [begins to] close... up (superluminally) ... [If the closing proceeds to completion, it]... produce[s] first a peanut shaped black hole and finally a spherical black hole. ...".

In the physical case of quark and antiquark forming a pion,
the toroidal black hole remains a torus.
The torus is an event horizon and therefore is not a 2-spacelike dimensional torus, but is a (1+1)-dimensional torus with a timelike dimension.

The effect is described in detail in Robert Wald's book General Relativity (Chicago 1984). It can be said to be due to extreme frame dragging, or to timelike translations becoming spacelike as though they had been Wick rotated in Complex SpaceTime.

As Hawking and Ellis say in The LargeScale Structure of Space-Time (Cambridge 1973):
"... The surface r = r+ is ... the event horizon ... and is a null surface ...

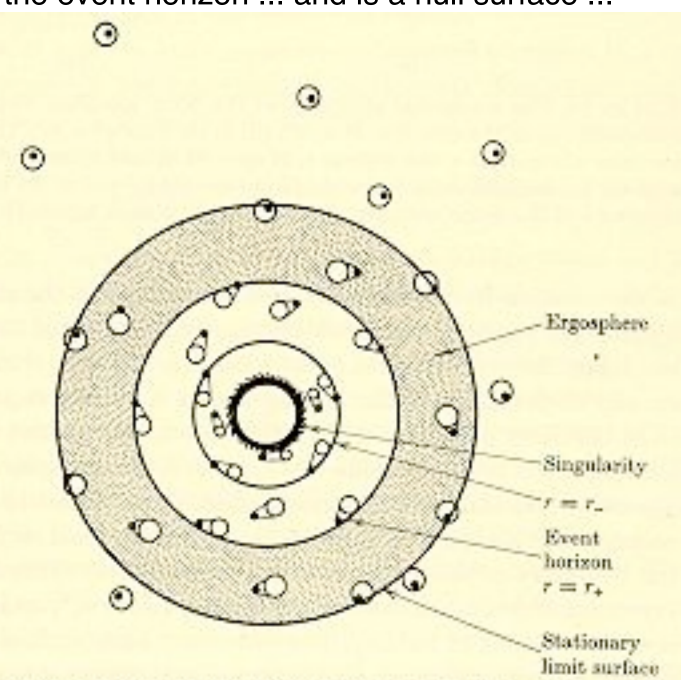


FIGURE 30. The equatorial plane of a Kerr solution with $m^2 > a^2$. The circles represent the position a short time later of flashes of light emitted by the points represented by heavy dots.

... On the surface r = r+ .... the wavefront corresponding to a point on this surface lies entirely within the surface. ...".

A (1+1)-dimensional torus with a timelike dimension can carry a Sine-Gordon Breather.
The soliton and antisoliton of a Sine-Gordon Breather correspond
to the quark and antiquark that make up the pion,
analagous to the Massive Thirring Model.

Sine-Gordon Breathers are described by Sidney Coleman in his Erica lecture paper
Classical Lumps and their Quantum Descendants (1975), reprinted in his book Aspects of Symmetry (Cambridge 1985),
where he writes the Lagrangian for the Sine-Gordon equation as (Coleman's eq. 4.3):

$$L = (1 / B^2) \left((1/2) (df)^2 + A (\cos(f) - 1) \right)$$

Coleman says: "... We see that, in classical physics, B is an irrelevant parameter:
if we can solve the sine-Gordon equation for any non-zero B,
we can solve it for any other B.
The only effect of changing B is the trivial one of changing the energy and momentum assigned to a given solution of the equation. This is not true in quantum physics, because the relevant object for quantum physics is not L but [eq. 4.4]

$$L / \hbar = (1 / (B^2 \hbar)) \left((1/2) (df)^2 + A (\cos(f) - 1) \right)$$

An other way of saying the same thing is to say that in quantum physics we have one more dimensional constant of nature, Planck's constant, than in classical physics. ...
the classical limit, vanishing hbar, is exactly the same as the small-coupling limit, vanishing B ... from now on I will ... set hbar equal to one. ...
... the sine-Gordon equation ...[has]... an exact periodic solution ...[eq. 4.59]...

$$f(x, t) = (4 / B) \arctan\left((n \sin(w t) / \cosh(n w x) \right)$$

where [eq. 4.60] $n = \sqrt{A - w^2} / w$ and w ranges from 0 to A.
This solution has a simple physical interpretation ... a soliton far to the left ...[and]... an antisoliton far to the right. As $\sin(w t)$ increases, the soliton and antisoliton move farther apart from each other. When $\sin(w t)$ passes through one,
they turn around and begin to approach one another. As $\sin(w t)$ comes down to zero ... the soliton and antisoliton are on top of each other ...
when $\sin(w t)$ becomes negative .. the soliton and antisoliton have passed each other.

... Thus, Eq. (4.59) can be thought of as a soliton and an antisoliton oscillation about their common center-of-mass. For this reason, it is called 'the doublet [or Breather] solution'. ... the energy of the doublet ...[eq. 4.64]

$$E = 2 M \sqrt{1 - (w^2 / A)}$$

where [eq. 4.65] $M = 8 \sqrt{A} / B^2$ is the soliton mass.

Note that the mass of the doublet is always less than twice the soliton mass,
as we would expect from a soliton-antisoliton pair. ...

Dashen, Hasslacher, and Neveu ... Phys. Rev. D10, 4114; 4130; 4138 (1974).
...[found that]... there is only a single series of bound states, labeled by the integer N ...
The energies ... are ... [eq. 4.82]

E_N = 2 M sin(B'^2 N / 16)

where N = 0, 1, 2 ... < 8 pi / B'^2 , [eq. 4.83]
B'^2 = B^2 / (1 - (B^2 / 8 pi)) and M is the soliton mass.
M is not given by Eq. (4.65), but is the soliton mass corrected by the DHN formula,
or, equivalently, by the first-order weak coupling expansion. ...
I have written the equation in this form .. to eliminate A,
and thus avoid worries about renormalization conventions.
Note that the DHN formula is identical to the Bohr-Sommerfeld formula,
except that B is replaced by B'. ...
Bohr and Sommerfeld['s] ... quantization formula says that if we have a one-parameter
family of periodic motions, labeled by the period, T,
then an energy eigenstate occurs whenever [eq. 4.66]

[Integral from 0 to T](dt p qdot = 2 pi N,

where N is an integer. ... Eq.(4.66) is cruder than the WKB formula,
but it is much more general;
it is always the leading approximation for any dynamical system ...
Dashen et al speculate that Eq. (4.82) is exact. ...
the sine-Gordon equation is equivalent ... to the massive Thirring model.
This is surprising,
because the massive Thirring model is a canonical field theory
whose Hamiltonian is expressed in terms of fundamental Fermi fields only.
Even more surprising, when B^2 = 4 pi , that sine-Gordon equation is equivalent
to a free massive Dirac theory, in one spatial dimension. ...
Furthermore, we can identify the mass term in the Thirring model
with the sine-Gordon interaction, [eq. 5.13]

M = - (A / B^2) N_m cos(B f)

.. to do this consistently ... we must say [eq. 5.14]

B^2 / (4 pi) = 1 / (1 + g / pi)

....[where]... g is a free parameter, the coupling constant [for the Thirring model]...
Note that if B^2 = 4 pi , g = 0 ,
and the sine-Gordon equation is the theory of a free massive Dirac field. ...
It is a bit surprising to see a fermion appearing as a coherent state of a Bose field.
Certainly this could not happen in three dimensions,
where it would be forbidden by the spin-statistics theorem.
However, there is no spin-statistics theorem in one dimension,
for the excellent reason that there is no spin. ...
the lowest fermion-antifermion bound state of the massive Thirring model
is an obvious candidate for the fundamental meson of sine-Gordon theory. ...
equation (4.82) predicts that
all the doublet bound states disappear when B^2 exceeds 4 pi .

This is precisely the point where
the Thirring model interaction switches from attractive to repulsive. ...
these two theories ... the massive Thirring model .. and ... the sine-Gordon equation ...
define identical physics. ...
I have computed the predictions of ...[various]... approximation methods
for the ration of the soliton mass to the meson mass for three values of B^2 :
4 pi (where the qualitative picture of the soliton as a lump totally breaks down),
2 pi, and pi . At 4 pi we know the exact answer ...
I happen to know the exact answer for 2 pi, so I have included this in the table. ...

Method	B^2 = pi	B^2 = 2 pi	B^2 = 4 pi
Zeroth-order weak coupling expansion eq2.13b	2.55	1.27	0.64
Coherent-state variation	2.55	1.27	0.64
First-order weak coupling expansion	2.23	0.95	0.32
Bohr-Sommerfeld eq4.64	2.56	1.31	0.71
DHN formula eq4.82	2.25	1.00	0.50
Exact	?	1.00	0.50

...[eq. 2.13b]

E = 8 sqrt(A) / B^2

...[is the]... energy of the lump ... of sine-Gordon theory ...
frequently called 'soliton...' in the literature ...
[Zeroth-order is the classical case, or classical limit.] ...
... Coherent-state variation always gives
the same result as the ... Zeroth-order weak coupling expansion ... .
The ... First-order weak-coupling expansion ...
explicit formula ... is (8 / B^2) - (1 / pi). ...".

Using the Cl(16)-E8 model constituent mass of the Up and Down quarks and
antiquarks, about 312.75 MeV, as the soliton and antisoliton masses,
and setting B^2 = pi and using the DHN formula,
the mass of the charged pion is calculated to be (312.75 / 2.25) MeV = 139 MeV
which is close to the experimental value of about 139.57 MeV.

Why is the value B^2 = pi the special value that gives the pion mass ?

(or, using Coleman's eq. (5.14), the Thirring coupling constant g = 3 pi)

Because B^2 = pi is where the First-order weak coupling expansion substantially coincides with the (probably exact) DHN formula. In other words,

The physical quark - antiquark pion lives where the first-order weak coupling expansion is exact.

14. Neutrino Masses Beyond Tree Level

Consider the three generations of neutrinos:
nu_e (electron neutrino); nu_m (muon neutrino); nu_t
and three neutrino mass states: nu_1 ; nu_2 : nu_3
and
the division of 8-dimensional spacetime into
4-dimensional physical Minkowski spacetime
plus
4-dimensional CP2 internal symmetry space.

The heaviest mass state nu_3 corresponds to a neutrino
whose propagation begins and ends in CP2 internal symmetry
space,lying entirely therein. According to the Cl(16)-E8 model
the mass of nu_3 is zero at tree-level
but it picks up a first-order correction
propagating entirely through internal symmetry space by merging
with an electron through the weak and electromagnetic forces,
effectively acting not merely as a point
but
as a point plus an electron loop at beginning and ending points
so
the first-order corrected mass of nu_3 is given by
M_nu_3 x (1/sqrt(2)) = M_e x GW(mproton^2) x alpha_E
where the factor (1/sqrt(2)) comes from the Ut3 component
of the neutrino mixing matrix
so that

M_nu_3 = sqrt(2) x M_e x GW(mproton^2) x alpha_E =
= 1.4 x 5 x 10^5 x 1.05 x 10^(-5) x (1/137) eV =
= 7.35 / 137 = 5.4 x 10^(-2) eV.

The neutrino-plus-electron loop can be anchored by weak force
action through any of the 6 first-generation quarks
at each of the beginning and ending points, and that the
anchor quark at the beginning point can be different from
the anchor quark at the ending point,
so that there are 6x6 = 36 different possible anchorings.

The intermediate mass state nu_2 corresponds to a neutrino
whose propagation begins or ends in CP2 internal symmetry space
and ends or begins in M4 physical Minkowski spacetime,
thus having only one point (either beginning or ending) lying
in CP2 internal symmetry space where it can act not merely
as a point but as a point plus an electron loop.

According to the Cl(16)-E8 model the mass of nu_2 is zero at
tree-level but it picks up a first-order correction at only one
(but not both) of the beginning or ending points
so that so that there are 6 different possible anchorings
for nu_2 first-order corrections, as opposed to the 36 different
possible anchorings for nu_3 first-order corrections,
so that
the first-order corrected mass of nu_2 is less than
the first-order corrected mass of nu_3 by a factor of 6,
so

the first-order corrected mass of nu_2 is
M_nu_2 = M_nu_3 / Vol(CP2) = 5.4 x 10^(-2) / 6
= 9 x 10^(-3)eV.

The low mass state nu_1 corresponds to a neutrino
whose propagation begins and ends in physical Minkowski
spacetime.
thus having only one anchoring to CP2 interna symmetry space.

According to the Cl(16)-E8 model the mass of nu_1 is zero at
tree-level but it has only 1 possible anchoring to CP2
as opposed to the 36 different possible anchorings for nu_3
first-order corrections
or the 6 different possible anchorings for nu_2 first-order
corrections
so that
the first-order corrected mass of nu_1 is less than
the first-order corrected mass of nu_2 by a factor of 6,
so

the first-order corrected mass of nu_1 is
M_nu_1 = M_nu_2 / Vol(CP2) = 9 x 10^(-3) / 6
= 1.5 x 10^(-3)eV.

Therefore:

```
the mass-squared difference D(M23^2) = M_nu_3^2 - M_nu_2^2 =
                            = ( 2916 - 81 ) x 10^(-6) eV^2 =
                            = 2.8 x 10^(-3) eV^2
```

and

```
the mass-squared difference D(M12^2) = M_nu_2^2 - M_nu_1^2 =
                               = ( 81 - 2 ) x 10^(-6) eV^2 =
                               = 7.9 x 10^(-5) eV^2
```

The 3x3 unitary neutrino mixing matrix neutrino mixing matrix U

	nu_1	nu_2	nu_3
nu_e	Ue1	Ue2	Ue3
nu_m	Um1	Um2	Um3
nu_t	Ut1	Ut2	Ut3

can be parameterized (based on the 2010 Particle Data Book)
by 3 angles and 1 Dirac CP violation phase

```
     c12 c13                        s12 c13                        s13 e-id

U = - s12 c23 - c12 s23 s13 eid     c12 c23 - s12 s23 s13 eid      s23 c13

     s12 s23 - c12 c23 s13 eid    - c12 s23 - s12 c23 s13 eid      c23 c13
```

where cij = cos(theta_ij) , sij = sin(theta_ij)

The angles are

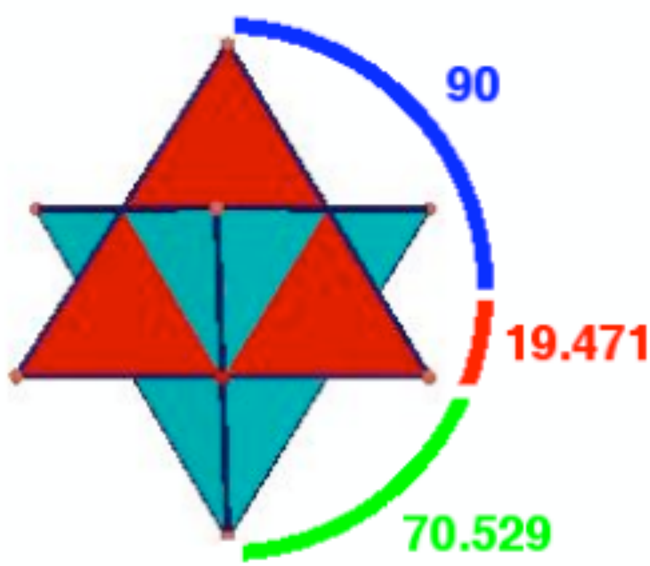
theta_23 = pi/4 = 45 degrees
because
nu_3 has equal components of nu_m and nu_t so
that Um3 = Ut3 = 1/sqrt(2) or, in conventional
notation, mixing angle theta_23 = pi/4
so that cos(theta_23) = 0.707 = sqrt(2)/2 = sin(theta_23)

theta_13 = 9.594 degrees = asin(1/6)
and cos(theta_13) = 0.986
because sin(theta_13) = 1/6 = 0.167 = |Ue3| = fraction of nu_3 that is nu_e

theta_12 = pi/6 = 30 degrees
because
sin(theta_12) = 0.5 = 1/2 = Ue2 = fraction of nu_2 begin/end points
that are in the physical spacetime where massless nu_e lives
so that cos(theta_12) = 0.866 = sqrt(3)/2

d = 70.529 degrees is the Dirac CP violation phase
ei(70.529) = cos(70.529) + i sin(70.529) = 0.333 + 0.943 i
This is because the neutrino mixing matrix has 3-generation structure
and so has the same phase structure as the KM quark mixing matrix
in which the Unitarity Triangle angles are:
β = V3.V1.V4 = arccos(2 sqrt(2) / 3) ≅ 19.471 220 634 degrees so sin 2β = 0.6285
α = V1.V3.V4 = 90 degrees
γ = V1.V4.V3 = arcsin(2 sqrt(2) / 3) ≅ 70.528 779 366 degrees

The constructed Unitarity Triangle angles can be seen on the Stella Octangula
configuration of two dual tetrahedra (image from gauss.math.nthu.edu.tw):



Then we have for the neutrino mixing matrix:

	nu_1	nu_2	nu_3
nu_e	0.866 x 0.986	0.50 x 0.986	0.167 x e-id
nu_m	-0.5 x 0.707	0.866 x 0.707	0.707 x 0.986
	-0.866 x 0.707 x 0.167 x eid	-0.5 x 0.707 x 0.167 x eid	
nu_t	0.5 x 0.707	-0.866 x 0.707	0.707 x 0.986
	-0.866 x 0.707 x 0.167 x eid	-0.5 x 0.707 x 0.167 x eid	

	nu_1	nu_2	nu_3
nu_e	0.853	0.493	0.167 e-id
nu_m	-0.354	0.612	0.697
	-0.102 eid	-0.059 eid	
nu_t	0.354	-0.612	0.697
	-0.102 eid	-0.059 eid	

Since ei(70.529) = cos(70.529) + i sin(70.529) = 0.333 + 0.943 i
and .333e-i(70.529) = cos(70.529) - i sin(70.529) = 0.333 - 0.943 i

	nu_1	nu_2	nu_3
nu_e	0.853	0.493	0.056 - 0.157 i
nu_m	-0.354	0.612	0.697
	-0.034 - 0.096 i	-0.020 - 0.056 i	
nu_t	0.354	-0.612	0.697
	-0.034 - 0.096 i	-0.020 - 0.056 i	

for a result of

	nu_1	nu_2	nu_3
nu_e	0.853	0.493	0.056 - 0.157 i
nu_m	-0.388 - 0.096 i	0.592 - 0.056 i	0.697
nu_t	0.320 - 0.096 i	0.632 - 0.056 i	0.697

which is consistent with the approximate experimental values of mixing angles shown in the Michaelmas Term 2010 Particle Physics handout of Prof Mark Thomson if the matrix is modified by taking into account the March 2012 results from Daya Bay observing non-zero theta_13 = 9.54 degrees.

15. Planck Mass as Superposition Fermion Condensate

At a single spacetime vertex, a Planck-mass black hole is the Many-Worlds quantum superposition of all possible virtual first-generation particle-antiparticle fermion pairs allowed by the Pauli exclusion principle to live on that vertex. (The second generation fermions live on two vertices and the third-generation fermions live on three vertices which pairs or triples of vertices act at our energy levels very much like one vertex.)

Once a Planck-mass black hole is formed, it is stable in the E8 model.
Less mass would not be gravitationally bound at the vertex.
More mass at the vertex would decay by Hawking radiation.

There are 8 fermion particles and 8 fermion antiparticles whose average mass is about
(0 + 0.0005 + 6 x 0.312) / 8 = 0.234 GeV.

There are 8x8 = 64 particle-antiparticle pairs
and 2^64 = 1.8 x 10^19 combinations of pairs, ranging in size from 1 to 64 pairs.

The 64-pair mass is about 64 x 2 x 0.234 = 29.952 GeV
and the 32-pair mass is about 14.976 GeV.

If the 32-pair mass is taken to be typical, then the total mass of all 2^64 combinations would be about 14.976 x 1.8 x 10^19 = 26.957 x 10^19 GeV.

However, the Pauli exclusion principle would prevent participation of pairs of fermions unless the pairs formed a bosonic pion-type state.
Of the 64 pairs, only 12 are bosonic pion-type states,
and
a pion-type state has mass about 139.57 / 625.5 times the mass of its two fermions,
so
the realistic total mass should be about (139.57 / 625.5) (12 / 64) x 26.957 x 10^19 =
= 1.128 x 10^19 GeV.

The value for the Planck mass given by Particle Data Group (2013)
is 1.221 x 10^19 GeV.

16. Force Strength and Boson Mass Calculation

Cl(8) bivector Spin(8) is the D4 Lie algebra two copies of which are in the Cl(16)-E8 model Lagrangian (as the D4xD4 subalgebra of the D8 subalgebra of E8)

$$\int_{\text{4-dim M4}} \text{GG SM} + \text{Fermion Particle-AntiParticle} + \text{Higgs} \; .$$

with the Higgs term coming from integrating over the CP2 Internal Symmetry Space of M4 x CP2 Kaluza-Klein by the Mayer-Trautman Mechanism

This shows that the **Force Strength is made up of two parts**:

the relevant spacetime manifold of gauge group global action
and
the relevant symmetric space manifold of gauge group local action.

The 4-dim spacetime Lagrangian GG SM gauge boson term is:
the integral over spacetime as seen by gauge boson acting globally
of the gauge force term of the gauge boson acting locally
for the gauge bosons of each of the four forces:

U(1) for electromagnetism
SU(2) for weak force
SU(3) for color force
Spin(5) - compact version of antiDeSitter Spin(2,3) subgroup of Conformal Spin(2,4) for gravity by the MacDowell-Mansouri mechanism.

In the conventional picture,
for each gauge force the gauge boson force term contains the force strength,
which in Feynman's picture is the amplitude to emit a gauge boson,
and can also be thought of as the probability = square of amplitude,
in an explicit (like g IFI^2) or an implicit (incorporated into the IFI^2) form.
Either way, the conventional picture is that the force strength g is an ad hoc inclusion.

The Cl(16)-E8 model does not put in force strength g ad hoc,
but constructs the integral such that
the force strength emerges naturally from the geometry of each gauge force.

To do that, for each gauge force:

1 - make the spacetime over which the integral is taken be spacetime as it is seen by that gauge boson, that is, in terms of the symmetric space with global symmetry of the gauge boson:

the U(1) photon sees 4-dim spacetime as T^4 = S1 x S1 X S1 x S1
the SU(2) weak boson sees 4-dim spacetime as S2 x S2
the SU(3) weak boson sees 4-dim spacetime as CP2
the Spin(5) of gravity sees 4-dim spacetime as S4

2 - make the gauge boson force term have the volume of the Shilov boundary corresponding to the symmetric space with local symmetry of the gauge boson.
The nontrivial Shilov boundaries are:

for SU(2) Shilov = RP^1xS^2
for SU(3) Shilov = S^5
for Spin(5) Shilov = RP^1xS^4

The result is (ignoring technicalities for exposition) the geometric factor for force strengths.

Each gauge group is the global symmetry of a symmetric space

S1 for U(1)
S2 = SU(2)/U(1) = Spin(3)/Spin(2) for SU(2)
CP2 = SU(3)/SU(2)xU(1) for SU(3)
S4 = Spin(5)/Spin(4) for Spin(5)

Each gauge group is the local symmetry of a symmetric space

U(1) for itself
SU(2) for Spin(5) / SU(2)xU(1)
SU(3) for SU(4) / SU(3)xU(1)
Spin(5) for Spin(7) / Spin(5)xU(1)

The nontrivial local symmetry symmetric spaces
correspond to bounded complex domains

SU(2) for Spin(5) / SU(2)xU(1) corresponds to IV3
SU(3) for SU(4) / SU(3)xU(1) corresponds to B^6 (ball)
Spin(5) for Spin(7) / Spin(5)xU(1) corresponds to IV5

The nontrivial bounded complex domains have Shilov boundaries

SU(2) for Spin(5) / SU(2)xU(1) corresponds to IV3 Shilov = RP^1xS^2
SU(3) for SU(4) / SU(3)xU(1) corresponds to B^6 (ball) Shilov = S^5
Spin(5) for Spin(7) / Spin(5)xU(1) corresponds to IV5 Shilov = RP^1xS^4

Very roughly, think of the force strength as
integral over global symmetry space of physical (ie Shilov Boundary) volume =
= strength of the force.

That is:
the geometric strength of the force is given by the product of
the volume of a 4-dim thing with global symmetry of the force and
the volume of the Shilov Boundary for the local symmetry of the force.

When you calculate the product volumes (using some tricky normalization stuff),
you see that roughly:

Volume product for gravity is the largest volume
so since (as Feynman says) force strength = probability to emit a gauge boson means
that the highest force strength or probability should be 1
the gravity Volume product is normalized to be 1, and so (approximately):

Volume product for gravity = 1
Volume product for color = 2/3
Volume product for weak = 1/4
Volume product for electromagnetism = 1/137

There are two further main components of a force strength:

1 - for massive gauge bosons, a suppression by a factor of 1 / M^2
2 - renormalization running (important for color force)

Consider Massive Gauge Bosons:

Gravity as curvature deformation of SpaceTime, with SpaceTime as a condensate of
Planck-Mass Black Holes, must be carried by virtual Planck-mass black holes,
so that the geometric strength of gravity should be reduced by 1/Mp^2

The weak force is carried by weak bosons,
so that the geometric strength of the weak force should be reduced by 1/MW^2

That gives the result (approximate):

gravity strength = G (Newton's G)
color strength = 2/3
weak strength = G_F (Fermi's weak force G)
electromagnetism = 1/137

Consider Renormalization Running for the Color Force:: That gives the result:

gravity strength = G (Newton's G)
color strength = 1/10 at weak boson mass scale
weak strength = G_F (Fermi's weak force G)
electromagnetism = 1/137

he use of compact volumes is itself a calculational device,
because it would be more nearly correct,
instead of the integral over the compact global symmetry space of
the compact physical (ie Shilov Boundary) volume=strength of the force
to use
the integral over the hyperbolic spacetime global symmetry space
of the noncompact invariant measure of the gauge force term.

However, since the strongest (gravitation) geometric force strength is to be normalized to 1, the only thing that matters is ratios,
and the compact volumes (finite and easy to look up in the book by Hua)
have the same ratios as the noncompact invariant measures.

In fact, I should go on to say that continuous spacetime and gauge force geometric objects are themselves also calculational devices,
and
that it would be even more nearly correct to do the calculations with respect to a discrete generalized hyperdiamond Feynman checkerboard.

Here are less approximate more detailed force strength calculations:

The force strength of a given force is

alphaforce = (1 / Mforce^2) (Vol(MISforce)) (Vol(Qforce) / Vol(Dforce)^(1 / mforce))

where:

alphaforce represents the force strength;

Mforce represents the effective mass;

MISforce represents the relevant part of the target Internal Symmetry Space;

Vol(MISforce) stands for volume of MISforce and is sometimes also denoted by Vol(M);

Qforce represents the link from the origin to the relevant target for the gauge boson;

Vol(Qforce) stands for volume of Qforce;

Dforce represents the complex bounded homogeneous domain
of which Qforce is the Shilov boundary;

mforce is the dimensionality of Qforce, which is
4 for Gravity and the Color force,
2 for the Weak force (which therefore is considered to have two copies of QW for SpaceTime),
1 for Electromagnetism (which therefore is considered to have four copies of QE for SpaceTime)

Vol(Dforce)^(1 / mforce) stands for a dimensional normalization factor
(to reconcile the dimensionality of the Internal Symmetry Space of the target vertex
with the dimensionality of the link from the origin to the target vertex).

The Qforce, Hermitian symmetric space, and Dforce manifolds for the four forces are:

Spin(5)	Spin(7) / Spin(5)xU(1)	IV5	4	RP^1xS^4
SU(3)	SU(4) / SU(3)xU(1)	B^6(ball)	4	S^5
SU(2)	Spin(5) / SU(2)xU(1)	IV3	2	RP^1xS^2
U(1)	-	-	1	-

The geometric volumes needed for the calculations are mostly taken from the book Harmonic Analysis of Functions of Several Complex Variables in the Classical Domains (AMS 1963, Moskva 1959, Science Press Peking 1958) by L. K. Hua [unit radius scale].

Force	M	Vol(M)
gravity	S^4	8pi^2/3 - S^4 is 4-dimensional
color	CP^2	8pi^2/3 - CP^2 is 4-dimensional
weak	S^2 x S^2	2 x 4pi - S^2 is a 2-dim boundary of 3-dim ball 4-dim S^2 x S^2 = topological boundary of 6-dim 2-polyball Shilov Boundary of 6-dim 2-polyball = S^2 + S^2 = = 2-dim surface frame of 4-dim S^2 x S^
e-mag	T^4	4 x 2pi - S^1 is 1-dim boundary of 2-dim disk

4-dim T^4 = S^1 x S^1 x S^1 x S^1 = topological boundary of 8-dim 4-polydisk
Shilov Boundary of 8-dim 4-polydisk = S^1 + S^1 + S^1 + S^1 =
= 1-dim wire frame of 4-dim T^4

Note (thanks to Carlos Castro for noticing this) also that the volume listed for CP2 is unconventional, but physically justified by noting that S4 and CP2 can be seen as having the same physical volume, with the only difference being structure at infinity.
Note that for U(1) electromagnetism, whose photon carries no charge, the factors Vol(Q) and Vol(D) do not apply and are set equal to 1, and from another point of view, the link manifold to the target vertex is trivial for the abelian neutral U(1) photons of Electromagnetism, so we take QE and DE to be equal to unity.

Force	M	Vol(M)	Q	Vol(Q)	D	Vol(D)
gravity	S^4	8pi^2/3	RP^1xS^4	8pi^3/3	IV5	pi^5/2^4 5!
color	CP^2	8pi^2/3	S^5	4pi^3	B^6(ball)	pi^3/6
Weak	S^2xS^2	2x4pi	RP^1xS^2	4pi^2	IV3	pi^3/24
e-mag	T^4	4x2pi	-	-	-	-

Note (thanks to Carlos Castro for noticing this) that the volume listed for S5 is for a squashed S5, a Shilov boundary of the complex domain corresponding to the symmetric space SU(4) / SU(3) x U(1).

Using the above numbers, the results of the calculations are the relative force strengths at the characteristic energy level of the generalized Bohr radius of each force:

Spin(5)	gravity	approx 10^19 GeV	1	GGmproton^2 approx 5 x 10^-39
SU(3)	color	approx 245 MeV	0.6286	0.6286
SU(2)	weak	approx 100 GeV	0.2535	GWmproton^2 approx 1.05 x 10^-5
U(1)	e-mag	approx 4 KeV	1/137.03608	1/137.03608

The force strengths are given at the characteristic energy levels of their forces, because the force strengths run with changing energy levels.
The effect is particularly pronounced with the color force.
The color force strength was calculated using a simple perturbative QCD renormalization group equation at various energies, with the following results:

Energy Level	Color Force Strength
245 MeV	0.6286
5.3 GeV	0.166
34 GeV	0.121
91 GeV	0.106

Taking other effects, such as Nonperturbative QCD, into account, should give a Color Force Strength of about 0.125 at about 91 GeV

Higgs:

As with forces strengths, the calculations produce ratios of masses,
so that only one mass need be chosen to set the mass scale.

In the Cl(16)-E8 model,
the value of the fundamental mass scale vacuum expectation value v = <PHI>
of the Higgs scalar field is set to be the sum of the physical masses of the weak bosons,
W+, W-, and Z0, whose tree-level masses will then be shown by ratio calculations to be
80.326 GeV, 80.326 GeV, and 91.862 GeV, respectively,
and therefore the electron mass will be 0.5110 MeV.

The relationship between the Higgs mass and v is given
by the Ginzburg-Landau term from the Mayer Mechanism as

(1/4) Tr ([PHI , PHI] - PHI)^2

or, i
n the notation of quant-ph/9806009 by Guang-jiong Ni

(1/4!) lambda PHI^4 - (1/2) sigma PHI^2

where the Higgs mass M_H = sqrt(2 sigma)

Ni says:
"... the invariant meaning of the constant lambda in the Lagrangian is not the coupling constant, the latter will change after quantization ... The invariant meaning of lambda is nothing but the ratio of two mass scales:

lambda = 3 (M_H / PHI)^2

which remains unchanged irrespective of the order ...".

Since <PHI>^2 = v^2, and assuming that lambda = (cos(pi / 6))^2 = 0.866^2
(a value consistent with the Higgs-Tquark condensate model of Michio Hashimoto, Masaharu Tanabashi, and Koichi Yamawaki in their paper at hep-ph/0311165)
we have

M_H^2 / v^2 = (cos(pi / 6))^2 / 3

In the Cl(16)-E8 model, the fundamental mass scale vacuum expectation value v of the Higgs scalar field is the fundamental mass parameter that is to be set to define all other masses by the mass ratio formulas of the model and v is set to be 252.514 GeV
so that

M_H = v cos(pi / 6) / sqrt(1 / 3) = 126.257 GeV

This is the value of the Low Mass State of the Higgs observed by the LHC.
MIddle and High Mass States come from a Higgs-Tquark Condensate System.
The Middle and High Mass States may have been observed by the LHC at 20% of the Low Mass State cross section, and that may be confirmed by the LHC 2015-1016 run.

A Non-Condensate Higgs is represented by a Higgs at a point in M4
that is connected to a Higgs representation in CP2 ISS by a line whose length
represents the Higgs mass

```
Higgs      Higgs in CP2 Internal Symmetry Space
  |            |
  |            |
  |            |
  | mass = 145 | Non-Condensate Higgs Mass = 145
  |            |
  |            |
  |            |
Higgs      Higgs  in M4  spacetime
```

and the value of lambda is 1 = 1^2
so that the Higgs mass would be M_H = v /sqrt(3) = 145.789 GeV

However, in the Cl(16)-E8 model, the Higgs has structure of a Tquark condensate

```
mass = 145
T ---------Tbar     Effective  Higgs  in CP2 Internal Symmetry Space
 \    |    /             |
  \   |   /              |
   \  |  /mass = 145     | Higgs Effective Mass =
    \ | /                | = 145 x cos(pi/6)= 145 x 0.866 = 126
     \|/                 |
   Higgs              Higgs in M4 spacetime
```

in which the Higgs at a point in M4 is connected to a T and Tbar in CP2 ISS
so that the vertices of the Higgs-T-Tbar system are connected
by lines forming an equilateral triangle composed of 2 right triangles
(one from the CP2 origin to the T and to the M4 Higgs and
another from the CP2 origin to the Tbar and to the M4 Higgs).
In the T-quark condensate picture
lambda = 1^2 = lambda(T) + lambda(H) = (sin(pi / 6))^2 + (cos(pi / 6))^2
and
lambda(H) = (cos(pi / 6))^2

Therefore the Effective Higgs mass observed by LHC is:

Higgs Mass = 145.789 x cos(pi/6) = 126.257 GeV.

To get W-boson masses,
denote the 3 SU(2) high-energy weak bosons
(massless at energies higher than the electroweak unification)
by W+, W-, and W0,
corresponding to the massive physical weak bosons W+, W-, and Z0.

The triplet { W+, W-, W0 } couples directly with the T - Tbar quark-antiquark pair,
so that the total mass of the triplet { W+, W-, W0 } at the electroweak unification
is equal to the total mass of a T - Tbar pair, 259.031 GeV.

The triplet { W+, W-, Z0 } couples directly with the Higgs scalar,
which carries the Higgs mechanism by which the W0 becomes the physical Z0,
so that the total mass of the triplet { W+, W-, Z0 }
is equal to the vacuum expectation value v of the Higgs scalar field, v = 252.514 GeV.

What are individual masses of members of the triplet { W+, W-, Z0 } ?

First, look at the triplet { W+, W-, W0 } which can be represented by the 3-sphere S^3.
The Hopf fibration of S^3 as

S^1 --> S^3 --> S^2

gives a decomposition of the W bosons into the neutral W0 corresponding to S^1
and the charged pair W+ and W- corresponding to S^2.

The mass ratio of the sum of the masses of W+ and W- to the mass of W0
should be the volume ratio of the S^2 in S^3 to the S^1 in S3.
The unit sphere S^3 in R^4 is normalized by 1 / 2.
The unit sphere S^2 in R^3 is normalized by 1 / sqrt(3).
The unit sphere S^1 in R^2 is normalized by 1 / sqrt(2).
The ratio of the sum of the W+ and W- masses to the W0 mass should then be
(2 / sqrt3) V(S^2) / (2 / sqrt2) V(S^1) = 1.632993

Since the total mass of the triplet { W+, W-, W0 } is 259.031 GeV,
the total mass of a T - Tbar pair, and the charged weak bosons have equal mass,
we have

M_W+ = M_W- = 80.326 GeV and M_W0 = 98.379 GeV.

The charged W+/- neutrino-electron interchange must be symmetric
with the electron-neutrino interchange, so that the tree-level absence
of right-handed neutrino particles requires that

the charged W+/- SU(2) weak bosons act only on left-handed electrons.

Each gauge boson must act consistently on the entire Dirac fermion particle sector,
so that the
charged W+/- SU(2) weak bosons act only on left-handed fermion particles of all types.

The neutral W0 weak boson does not interchange Weyl neutrinos with Dirac fermions, and so is not restricted to left-handed fermions, but also has a component that acts on both types of fermions, both left-handed and right-handed, conserving parity.

However, the neutral W0 weak bosons are related to the charged W+/- weak bosons by custodial SU(2) symmetry, so that the left-handed component of the neutral W0 must be equal to the left-handed (entire) component of the charged W+/-.

Since the mass of the W0 is greater than the mass of the W+/-,
there remains for the W0 a component acting on both types of fermions.

Therefore the full W0 neutral weak boson interaction is proportional to
(M_W+/-^2 / M_W0^2) acting on left-handed fermions
and
(1 - (M_W+/-^2 / M_W0^2)) acting on both types of fermions.

If (1 - (M_W+/-2 / M_W0^2)) is defined to be sin(theta_w)^2 and denoted by K,
and if the strength of the W+/- charged weak force
(and of the custodial SU(2) symmetry) is denoted by T,
then the W0 neutral weak interaction can be written as W0L = T + K and W0LR = K.

Since the W0 acts as W0L with respect to the parity violating SU(2) weak force
and as W0LR with respect to the parity conserving U(1) electromagnetic force,
the W0 mass mW0 has two components:
the parity violating SU(2) part mW0L that is equal to M_W+/-
and the parity conserving part M_W0LR that acts like a heavy photon.

As M_W0 = 98.379 GeV = M_W0L + M_W0LR,
and as M_W0L = M_W+/- = 80.326 GeV, we have M_W0LR = 18.053 GeV.

Denote by *alphaE = *e^2 the force strength of the weak parity conserving U(1) electromagnetic type force that acts through the U(1) subgroup of SU(2).

The electromagnetic force strength alphaE = e^2 = 1 / 137.03608 was calculated above using the volume V(S^1) of an S^1 in R^2, normalized by 1 / sqrt(2).

The *alphaE force is part of the SU(2) weak force whose strength alphaW = w^2 was calculated above using the volume V(S^2) of an S^2 \subset R^3,
normalized by 1 / sqrt(3).

Also, the electromagnetic force strength alphaE = e^2 was calculated above using a 4-dimensional spacetime with global structure of the 4-torus T^4 made up of four S^1 1-spheres,
while the SU(2) weak force strength alphaW = w^2 was calculated above using two 2-spheres S^2 x S^2,
each of which contains one 1-sphere of the *alphaE force.

Therefore

*alphaE = alphaE (sqrt(2) / sqrt(3))(2 / 4) = alphaE / sqrt(6),

*e = e / (4th root of 6) = e / 1.565 ,

and
the mass mW0LR must be reduced to an effective value

M_W0LReff = M_W0LR / 1.565 = 18.053/1.565 = 11.536 GeV

for the *alphaE force to act like an electromagnetic force in the E8 model:

*e M_W0LR = e (1/5.65) M_W0LR = e M_Z0,

where the physical effective neutral weak boson is denoted by Z0.

Therefore, the correct Cl(16)-E8 model values for weak boson masses
and the Weinberg angle theta_w are:

M_W+ = M_W- = 80.326 GeV;

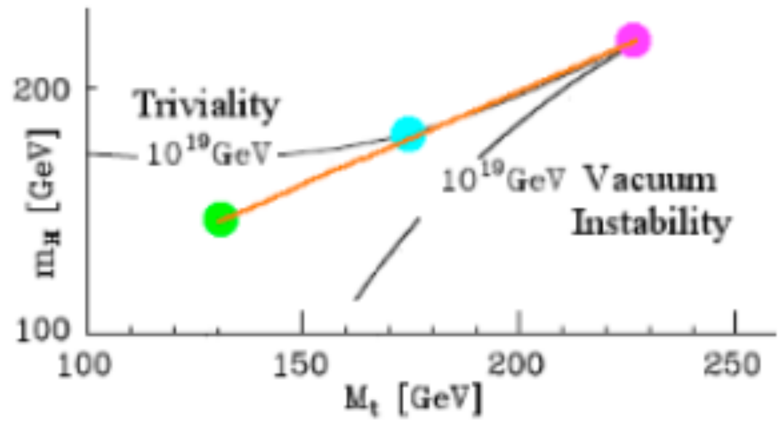
M_Z0 = 80.326 + 11.536 = 91.862 GeV;

Sin(theta_w)^2 = 1 - (M_W+/- / M_Z0)^2 = 1 - (6452.2663 / 8438.6270) = 0.235.

Radiative corrections are not taken into account here, and may change these tree- level values somewhat.

17. Higgs - Truth Quark Condensate System with 3 Mass States

The Cl(16)-E8 model identifies the Higgs with Primitive Idempotents of the Cl(8) real Clifford algebra, whereby the Higgs is not seen as a simple-minded single fundamental scalar particle, but rather the Higgs is seen as a quantum process that creates a fermionic condensate and effectively a 3-state Higgs-Tquark System.



(from hep-ph/0307138)

The Magenta Dot  is the high-mass state of a 220 GeV Truth Quark and a 240 GeV Higgs. It is at the critical point of the Higgs-Tquark System with respect to Vacuum Instability and Triviality. It corresponds to the description in hep-ph/9603293 by Koichi Yamawaki of the Bardeen-Hill-Lindner model. That high-mass Higgs is around 250 GeV in the range of the Higgs Vacuum Instability Boundary which range includes the Higgs VEV.

The Gold Line leading down from the Critical Point roughly along the Triviality Boundary line is based on Renormalization Group calculations with the result that MH / MT = 1.1 as described by Koichi Yamawaki in hep-ph/9603293 .

The Cyan Dot  where the Gold Line leaves the Triviality Boundary to go into our Ordinary Phase is the middle-mass state of a 174 GeV Truth Quark and Higgs around 200 GeV. It corresponds to the Higgs mass calculated by Hashimoto, Tanabashi, and Yamawaki in hep-ph/0311165 where they show that for 8-dimensional Kaluza-Klein spacetime with the Higgs as a Truth Quark condensate 172 < MT < 175 GeV and 178 < MH < 188 GeV.
That mid-mass Higgs is around the 200 GeV range of the Higgs Triviality Boundary at which the composite nature of the Higgs as T-Tbar condensate in (4+4)-dim Kaluza-Klein becomes manifest.

The Green Dot  where the Gold Line terminates in our Ordinary Phase is the low-mass state of a 130 GeV Truth Quark and a 126 GeV Higgs.

The conventional Standard Model has structure:

spacetime is a base manifold
particles are representations of gauge groups
gauge bosons are in the adjoint representation
fermions are in other representations (analagous to spinor)
Higgs boson is in scalar representation

The Cl(16)-E8 model has structure
(from 248-dim E8 = 120-dim adjoint D8 + 128-dim half-spinor D8):

spacetime is in the adjoint D8 part of E8 (64 of 120 D8 adjoints)
gauge bosons are in the adjoint D8 part of E8 (28+28 = 56 of the 120 D8 adjoints)
fermions are in the half-spinor D8 part of E8 (64+64 of the 128 D8 half-spinors.

There is no room for a fundamental Higgs directly appearing in the E8,
rather, it emerges from the Mayer-Trautman Mechanism
with formation of Quaternionic (4+4)-dim M4 x CP2 Kaluza-Klein SpaceTime.
To see how that Higgs works in terms of the Cl(16) = Cl(8)xCl(8) Clifford Algebra,
embed 248-dim E8 into the 256-dim real Clifford algebra Cl(8):

Cl(8)	256 =	1	+ 8	+ 28	+ 56	+ 70	+ 56	+ 28	+ 8	+ 1
Primitive Idempotent	16 =	1				+ 6 + 8				+ 1
E8 Root Vectors	240 =		8	+ 28	+ 56	+ 56	+ 56	+ 28	+ 8	
E8	248 =		8	+ 28	+ 56	+ 64	+ 56	+ 28	+ 8	

The Cl(8) Primitive Idempotent is 16-dimensional and can be decomposed
into two 8-dimensional half-spinor parts each of which is related by Triality
to 8-dimensional spacetime and has Octonionic structure.

In that decomposition: the 1+6+1 = (1+3)+(3+1) is related to two copies of
a 4-dimensional Associative Quaternionic subspace of the Octonionic structure
and
the 8 = 4+4 is related to two copies of
a 4-dimensional Co-Associative subspace of the Octonionic structure

(see the book “Spinors and Calibrations” by F. Reese Harvey)

The 8 = 4+4 Co-Associative part of the Cl(8) Primitive Idempotent
when combined with the 240 E8 Root Vectors forms the full 248-dimensional E8.
It represents a Cartan subalgebra of the E8 Lie algebra.

The (1+3)+(3+1) Associative part of the Cl(8) Primitive Idempotent
corresponds to the Higgs of the Cl(16)-E8 model.

The half-spinors generated by the Higgs part of the Cl(8) Primitive Idempotent represent

neutrino; red, green, blue down quarks; red, green, blue up quarks; electron

so the E8 Higgs effectively creates/annihilates the fundamental fermions
and
the E8 Higgs is effectively a condensate of fundamental fermions.

In the Cl(16)-E8 model the high-energy 8-dimensional Octonionic spacetime reduces,
by freezing out a preferred 4-dim Associative Quaternionic subspace,
to a 4+4 -dimensional Batakis Kaluza-Klein of the form M4 x CP2
with 4-dim M4 physical spacetime.

The (1+3)+(3+1) part of the Cl(8) Primitive Idempotent includes

the 1of Cl(8) grade-0 scalar (that determines M4 transformation properties)
and 3+3 = 6 of the Cl(8) grade-4
and the 1 of Cl(8) grade-8

so the Cl(16)-E8 model Higgs transforms as a scalar
with respect to 4-dim M4 Physical SpaceTime
and is consistent with LHC observations (see arXiv 1307.1432).

Not only does the Cl(16)-E8 model Higgs fermion condensate transform
with respect to 4-dim physical spacetime like the Standard Model Higgs
but
the geometry of the reduction from 8-dim Octonionic spacetime
to (4+4)-dimensional Batakis Kaluza-Klein,
by the Mayer-Trautman Mechanism,
gives the Cl(16)-E8 Higgs ElectroWeak Symmetry-Breaking Ginzburg-Landau structure.

Since the second and third fermion generations emerge dynamically
from the reduction from 8-dim to 4+4 -dim Kaluza-Klein,
they are also created/annihilated by the Primitive Idempotent Cl(16)-E8 Higgs
and are present in the fermion condensate.

**Since the Truth Quark is so much more massive that the other fermions,
the Cl(16)-E8 model Higgs is effectively a Truth Quark condensate.**

When Triviality and Vacuum Stability are taken into account,
the Cl(16)-E8 model Higgs and Truth Quark system has 3 mass states.

As to composite Higgs and the Triviality boundary, Pierre Ramond says in his book Journeys Beyond the Standard Model (Perseus Books 1999) at pages 175-176:
"... The Higgs quartic coupling has a complicated scale dependence. It evolves according to   d lambda / d t = (1 / 16 pi^2) beta_lambda
where the one loop contribution is given by
beta_lambda = 12 lambda^2 - ... - 4 H ...
The value of lambda at low energies is related [to] the physical value of the Higgs mass according to the tree level formula
m_H = v sqrt(2 lambda)
while the vacuum value is determined by the Fermi constant
...
for a fixed vacuum value v, let us assume that the Higgs mass and therefore lambda is large. In that case, beta_lambda is dominated by the lambda^2 term, which drives the coupling towards its Landau pole at higher energies.
Hence the higher the Higgs mass, the higher lambda is and the close[r] the Landau pole to experimentally accessible regions.
This means that for a given (large) Higgs mass,
we expect the standard model to enter a strong coupling regime
at relatively low energies, losing in the process our ability to calculate.
This does not necessarily mean that the theory is incomplete,
only that we can no longer handle it ...
it is natural to think that this effect is caused by new strong interactions,
and that the Higgs actually is a composite ...
The resulting bound on lambda is sometimes called the triviality bound.
The reason for this unfortunate name (the theory is anything but trivial)
stems from lattice studies where the coupling is assumed to be finite everywhere;
in that case the coupling is driven to zero, yielding in fact a trivial theory.
In the standard model lambda is certainly not zero. ...".

Composite Higgs as Tquark condensate studies by Yamawaki et al have produced realistic models that are consistent with the Cl(16)-E8 model with a 3-State System:

1 - The basic Cl(16)-E8 model state
with **Tquark mass = 130 GeV and Higgs mass = 126 GeV**

2 - Triviality boundary 8-dim Kaluza-Klein state described by Hashimoto, Tanabashi, and Yamawaki in hep-ph/0311165 where they say:
“... "... We perform the most attractive channel (MAC) analysis in the top mode standard model with TeV-scale extra dimensions, where the standard model gauge bosons and the third generation of quarks and leptons are put in D(=6,8,10,...) dimensions. In such a model, bulk gauge couplings rapidly grow in the ultraviolet region. In order to make the scenario viable, only the attractive force of the top condensate should exceed the critical coupling, while other channels such as the bottom and tau condensates should not. We then find that the top condensate can be the MAC for D=8 ... We predict masses of the top (m_t) and the Higgs (m_H) ...

based on the renormalization group for the top Yukawa and Higgs quartic couplings with the compositeness conditions at the scale where the bulk top condenses ... for ...[Kaluza-Klein type]... dimension... D=8 ...
m_t = 172-175 GeV and m_H=176-188 GeV ...".

3 - Critical point BHL state
with **Tquark mass = 218 +/- 3 GeV and Higgs mass = 239 +/- 3 GeV**
As Yamawaki said in hep-ph/9603293: "... the BHL formulation of the top quark condensate ... is based on the RG equation combined with the compositeness condition ... start[s] with the SM Lagrangian which includes explicit Higgs field at the Lagrangian level ... BHL is crucially based on the perturbative picture ...[which]... breaks down at high energy near the compositeness scale Λ ...[10^19 GeV]... there must be a certain matching scale Λ_Matching such that the perturbative picture (BHL) is valid for mu < Λ_Matching, while only the nonperturbative picture (MTY) becomes consistent for mu > Λ_Matching ...
However, thanks to the presence of a quasi-infrared fixed point,
BHL prediction is numerically quite stable against ambiguity at high energy region, namely, rather independent of whether this high energy region is replaced by MTY or something else. ... Then we expect mt = mt(BHL) = ... = 1/(sqrt(2)) ybart v within 1-2%, where ybart is the quasi-infrared fixed point given by Beta(ybart) = 0 in ... the one-loop RG equation ...
The composite Higgs loop changes ybart^2 by roughly the factor Nc/(Nc +3/2) = 2/3 compared with the MTY value, i.e., 250 GeV -> 250 x sqrt(2/3) = 204 GeV, while the electroweak gauge boson loop with opposite sign pulls it back a little bit to a higher value. The BHL value is then given by mt = 218 +/- 3 GeV, at Λ = 10^19 GeV.
The Higgs boson was predicted as a tbar-t bound state
with a mass MH = 2mt based on the pure NJL model calculation.
Its mass was also calculated by BHL through the full RG equation ...
the result being ... MH / mt = 1.1) at /.\ = 10^19 GeV ...
... the top quark condensate proposed by Miransky, Tanabashi and Yamawaki (MTY) and by Nambu independently ... entirely replaces the standard Higgs doublet by a composite one formed by a strongly coupled short range dynamics (four-fermion interaction) which triggers the top quark condensate.
The Higgs boson emerges as a tbar-t bound state and hence is deeply connected with the top quark itself. ... MTY introduced explicit four-fermion interactions responsible for the top quark condensate in addition to the standard gauge couplings. Based on the explicit solution of the ladder SD equation, MTY found that even if all the dimensionless four-fermion couplings are of O(1), only the coupling larger than the critical coupling yields non-zero (large) mass ... The model was further formulated in an elegant fashion by Bardeen, Hill and Lindner (BHL) in the SM language, based on the RG equation and the compositenes condition.
BHL essentially incorporates 1/Nc sub-leading effects such as those of the composite Higgs loops and ... gauge boson loops which were disregarded by the MTY formulation. We can explicitly see that BHL is in fact equivalent to MTY at 1/Nc-leading order. Such effects turned out to reduce the above MTY value 250 GeV down to 220 GeV ...".

8-dim Kaluza-Klein spacetime physics as required by Hashimoto, Tanabashi, and Yamawaki for the Middle State of the 3-State System was described by N. A. Batakis in Class. Quantum Grav. 3 (1986) L99-Ll05 in terms a M4xCP2 structure similar to that of the Cl(16)-E8 model. Although spacetime and Standard Model gauge bosons worked well for Batakis, he became discouraged by difficulties with fermions, perhaps because he did not use Clifford Algebras with natural spinor structures for fermions.

Calculations of the Low-Mass State of Higgs and Truth Quark have been given in Chapters 10 and 16 of this paper. Here are similar details for Middle and High Mass:

Middle Mass State:

In the Cl(16)-E8 model, the Middle-Mass Higgs has structure that is not restricted to Effective M4 Spacetime as is the case with the Low-Mass Higgs Ground State

but extends to the full 4+4 = 8-dim structure of M4xCP2 Kaluza-Klein.

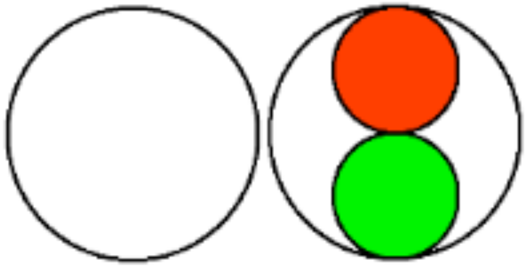
```
T ----------- Tbar        in CP2 Internal Symmetry Space
 \           /
  \         /
   \       /
    \     /
     \   /
    Higgs                 in M4 Physical SpaceTime
```

Therefore the Mid-Mass Higgs looks like a 3-particle system of Higgs + T + Tbar.

The T and Tbar form a Pion-like state.
Since Tquark Mid-Mass State is 174 GeV
the Middle-Mass T-Tbar that lives in the CP2 part of (4+4)-dim Kaluza-Klein
has mass (174+174) x (135 / (312+312) = 75 GeV.

The Higgs that lives in the M4 part of (4+4)-dim Kaluza-Klein
has, by itself, its Low-Mass Ground State Effective Mass of 125 GeV.
So, the total Mid-Mass Higgs lives in full 8-dim Kaluza-Klein
with mass 75+125 = 200 GeV.
This is consistent with the Mid-Mass States of the Higgs and Tquark
being on the Triviality Boundary of the Higgs - Tquark System
and with the 8-dim Kaluza-Klein model in hep-ph/0311165 by Hashimoto, Tanabashi, and Yamawaki.

As to the cross-section of the Middle-Mass Higgs



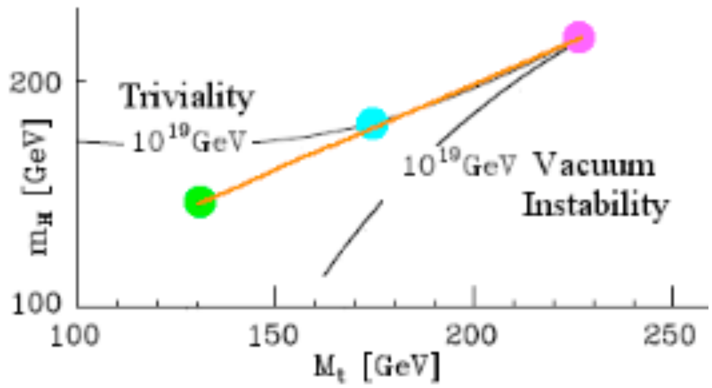
consider that the entire Ground State cross-section lives only in 4-dim M4 spacetime
(left white circle)
while the Middle-Mass Higgs cross-section lives in full 4+4 = 8-dim Kaluza-Klein
(right circle with red area only in CP2 ISS and white area partly in CP2 ISS
with only green area effectively living in 4-dim M4 spacetime)
so that
our 4-dim M4 Physical Spacetime experiments only see for the Middle-Mass Higgs
a cross-section that is 25% of the full Ground State cross-section.
The 25% may also be visualized in terms of 8-dim coordinates {1,i,j,k,E,I,J,K}

	1	i	j	k	E	I	J	K
1	11	1i	1j	1k	1E	1I	1J	1K
i	i1	ii	ij	ik	iE	iI	iJ	iK
j	j1	ji	jj	jk	jE	jI	jJ	jK
k	k1	ki	kj	kk	kE	kI	kJ	kK
E	E1	Ei	Ej	Ek	EE	EI	EJ	EK
I	I1	Ii	Ij	Ik	IE	II	IJ	IK
J	J1	Ji	Jj	Jk	JE	JI	JJ	JK
K	K1	Ki	Kj	Kk	KE	KI	KJ	KK

in which {1,i,j,k} represent M4 and {E,I,J,K} represent CP2.

High Mass State:

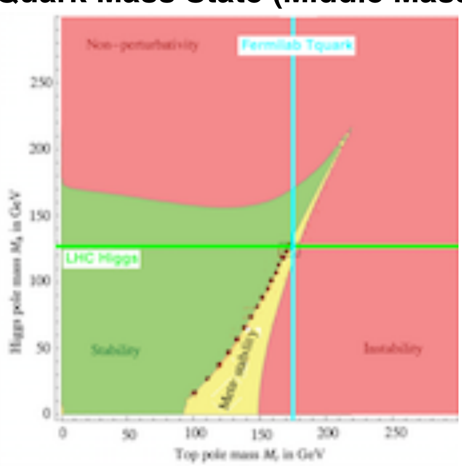
In the Cl(16)-E8 model, the the High-Mass Higgs State is at the Critical Point of the Higgs-Tquark System



(from hep-ph/0307138)

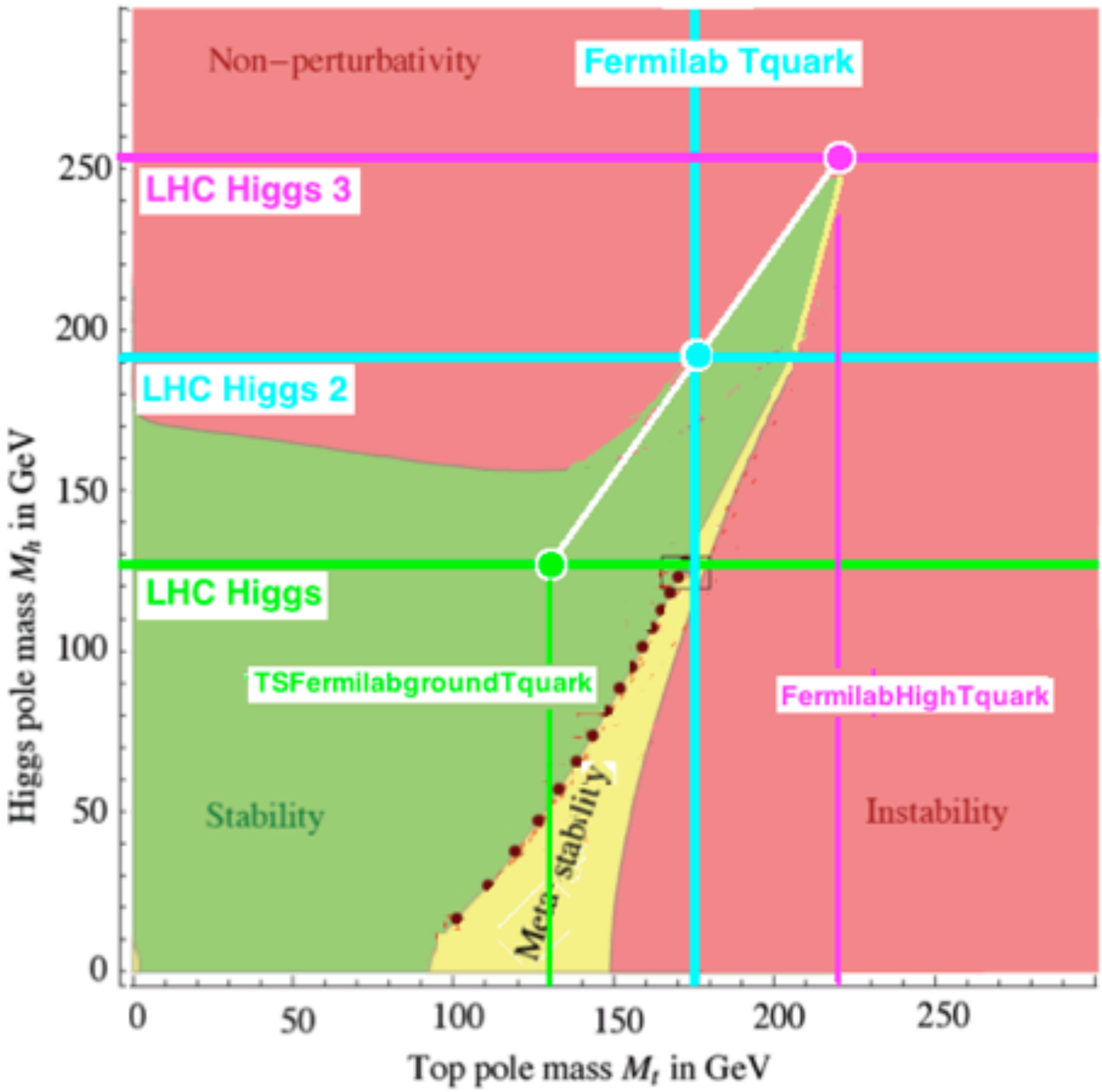
where the Triviality Boundary intersects the Vacuum Instability Boundary which is also at the Higgs Vacuum Expectation Value VEV around 250 GeV.
As with the Middle-Mass Higgs,
the High-Mass Higgs lives in all 4+4 = 8 Kaluza-Klein dimensions
and so has a cross-section that is about 25% of the Higgs Ground State cross-section.

The Cl(16)-E8 model view is 3 Mass States for Higgs and Truth Quark.
Opposed to the Cl(16)-E8 view is the Fermilab / CERN / Establishment view
that there is only one Higgs Mass State (Low Mass around 126 GeV)
and only one Truth Quark Mass State (Middle Mass around 174 GeV).



Their view is represented in the above Mh-Mt diagram adapted from arXiv 1307.3536 by Buttazzo, Degrassi, Giardino, Giudice, Sala, Salvio, and Strumia who say "... from data ... of the Higgs ... and the ... [Tquark] Yukawa coupling ... we extrapolate ... SM parameters up to large energies ... Then we study the phase diagram of the Standard Model in term of high-energy parameters, finding that
the measured Higgs mass roughly corresponds to ... vacuum metastability ...
the SM Higgs vacuum is not the true vacuum ... our universe is potentially unstable ...”.

The Cl(16)-E8 model has no vacuum metastability problem
because it has 3 sets of Higgs-Tquark mass states which modify the phase diagram



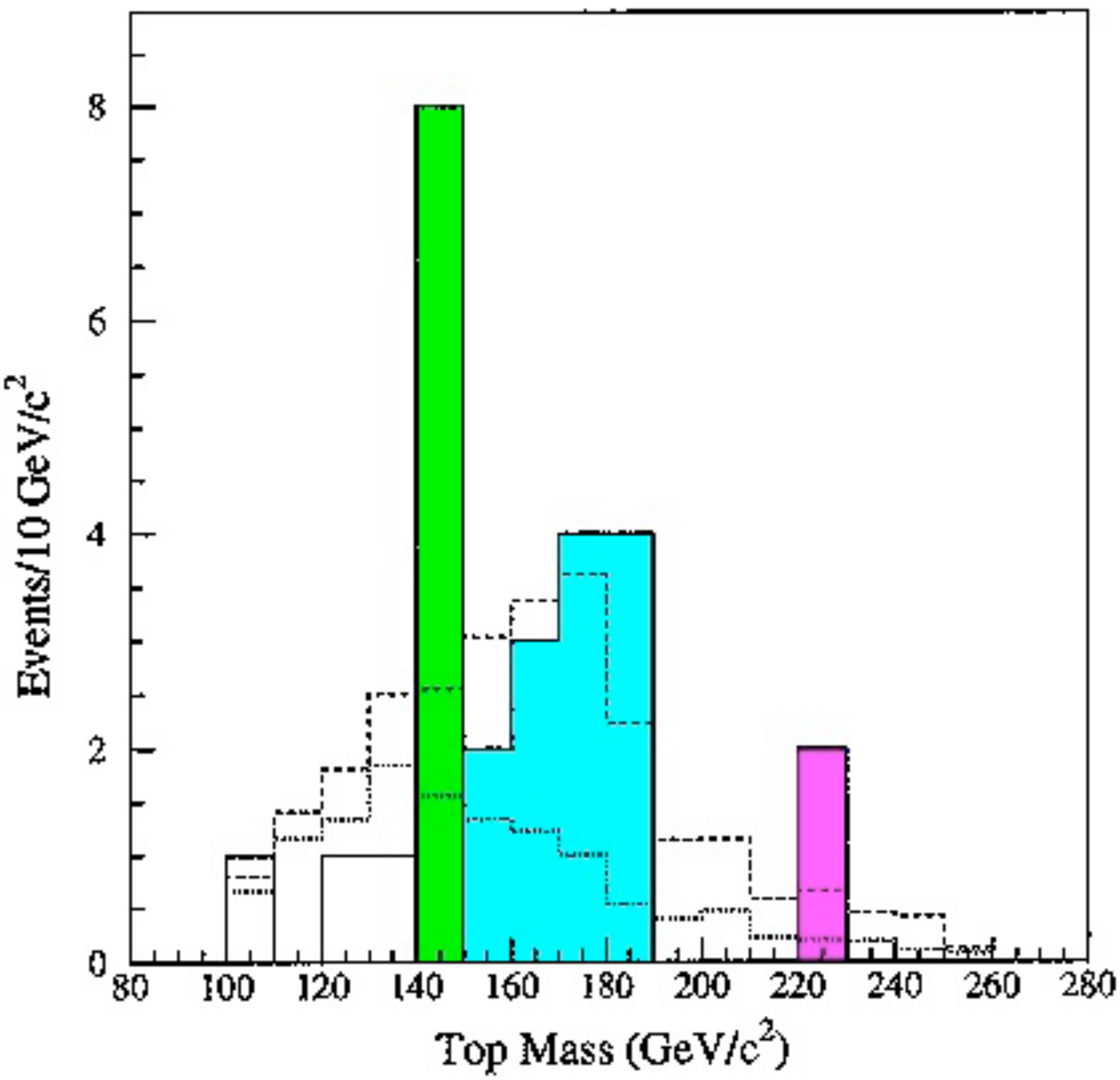
so that the Low-Mass Ground State is in the region of Stability
and the Middle-Mass State is at the boundary of Non-Perturbativity
and the High-Mass State at the Critical Point has Higgs mass = Higgs VEV.

The two additional Tquark mass states and Higgs mass states are
not recognized by the Fermilab/CERN/Establishment.

The two Tquark states (TSFermilabgroundTquark and FermilabHighTquark) have been
seen at Fermilab and
the LHC has seen indications of the Two Higgs states (LHC Higgs 2 and LHC Higgs 3)
whose status should be clarified by the 2015-2016 LHC Run.

Here are details of those additional Fermilab and LHC states:

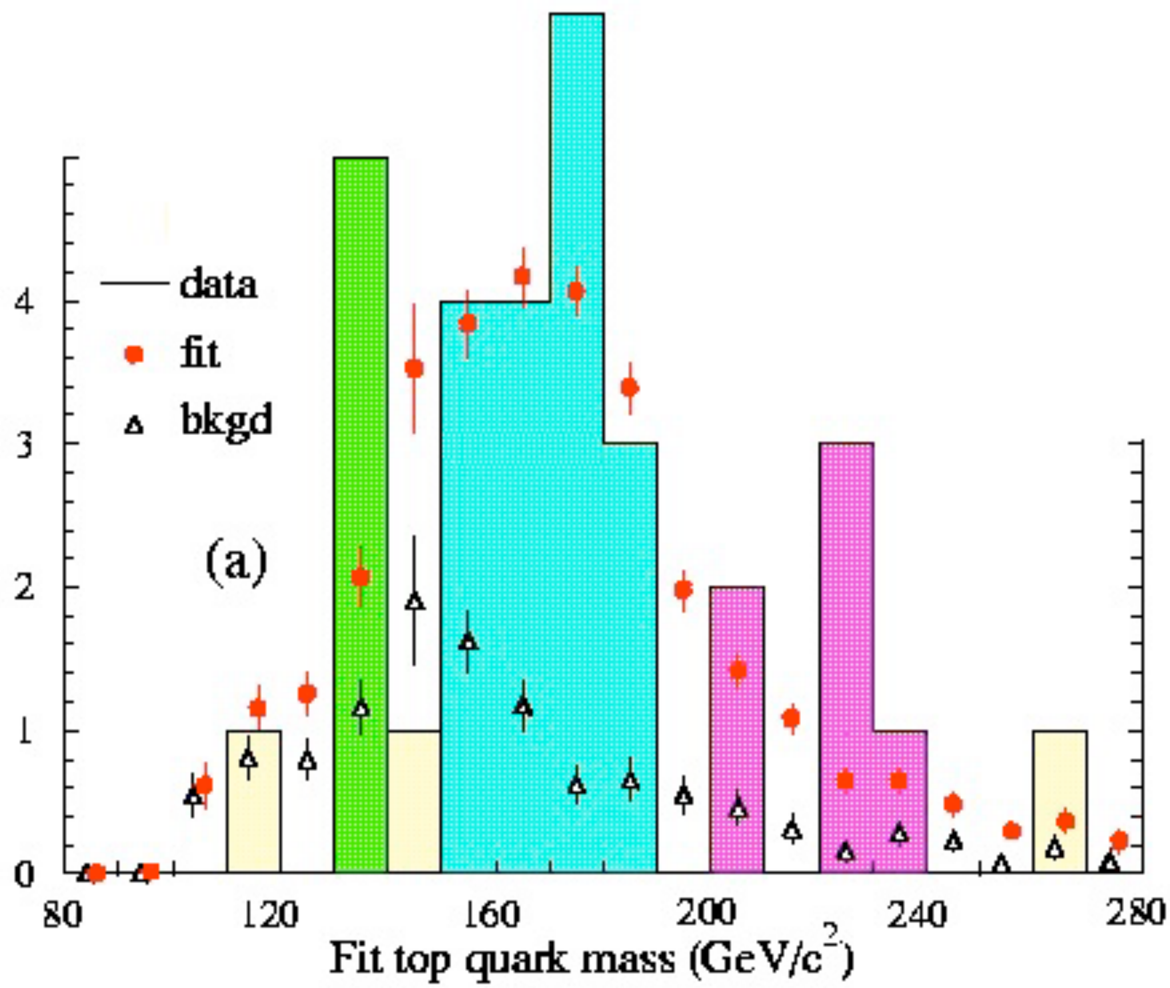
In 1994 a semileptonic histogram from CDF



(from FERMILAB-PUB-94/097-E)

seems to me to show all three states of the T-quark.

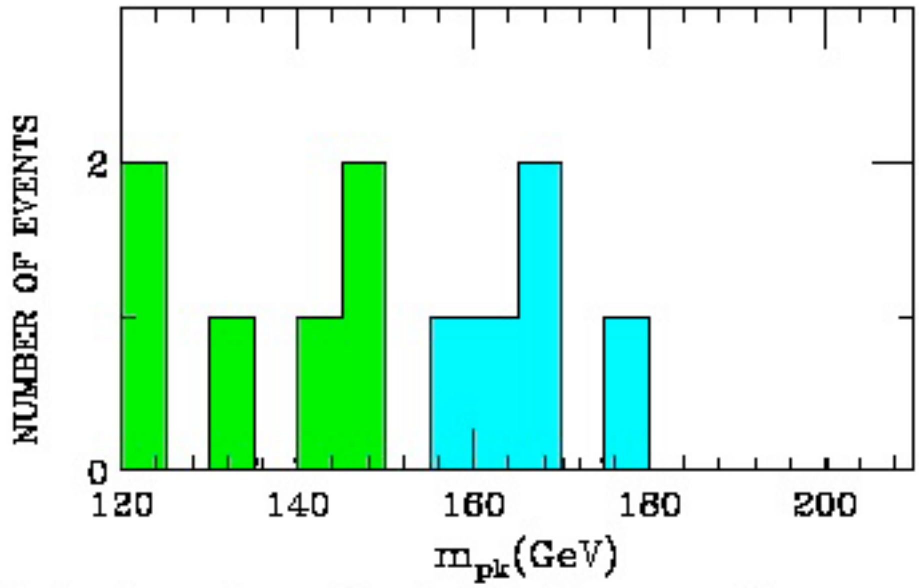
In 1997 a semileptonic histogram from D0



(from hep-ex/9703008)

also seems to me to show all three states of the T-quark.
The fact that the low (green) state showed up in both independent detectors indicates
a significance of 4 sigma.
Some object that the low (green) state peak should be as wide as the peak for the middle (cyan) state,
but
my opinion is that the middle (cyan) state should be wide because it is on the Triviality boundary where the composite nature of the Higgs as T-Tbar condensate becomes manifest and
the low (cyan) state should be narrow because it is in the usual non-trivial region where the T-quark acts more nearly as a single individual particle.

In 1998 a dilepton histogram from CDF

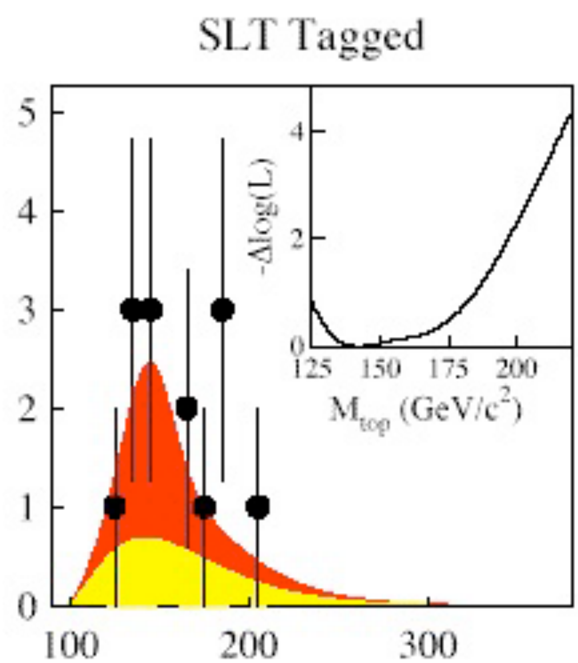


The distribution of m_{pk} values determined from 11 CDF dilepton events available empirically.

(from hep-ex/9802017)

seems to me to show both the low (green) state and the middle (cyan) T-quark state.

In 1998 an analysis of 14 SLT tagged lepton + 4 jet events by CDF

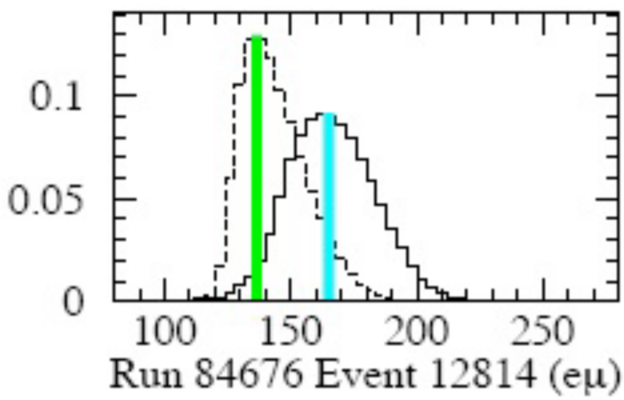


(from hep-ex/9801014)

showed a T-quark mass of 142 GeV (+33,-14) that seems to me to be consistent with the low (green) state of the T-quark.

In 1997 the Ph.D. thesis of Erich Ward Varnes (Varnes-fermilab-thesis-1997-28) at page 159 said:

"... distributions for the dilepton candidates. For events with more than two jets, the dashed curves show the results of considering only the two highest ET jets in the reconstruction ...



...” (colored bars added by me)

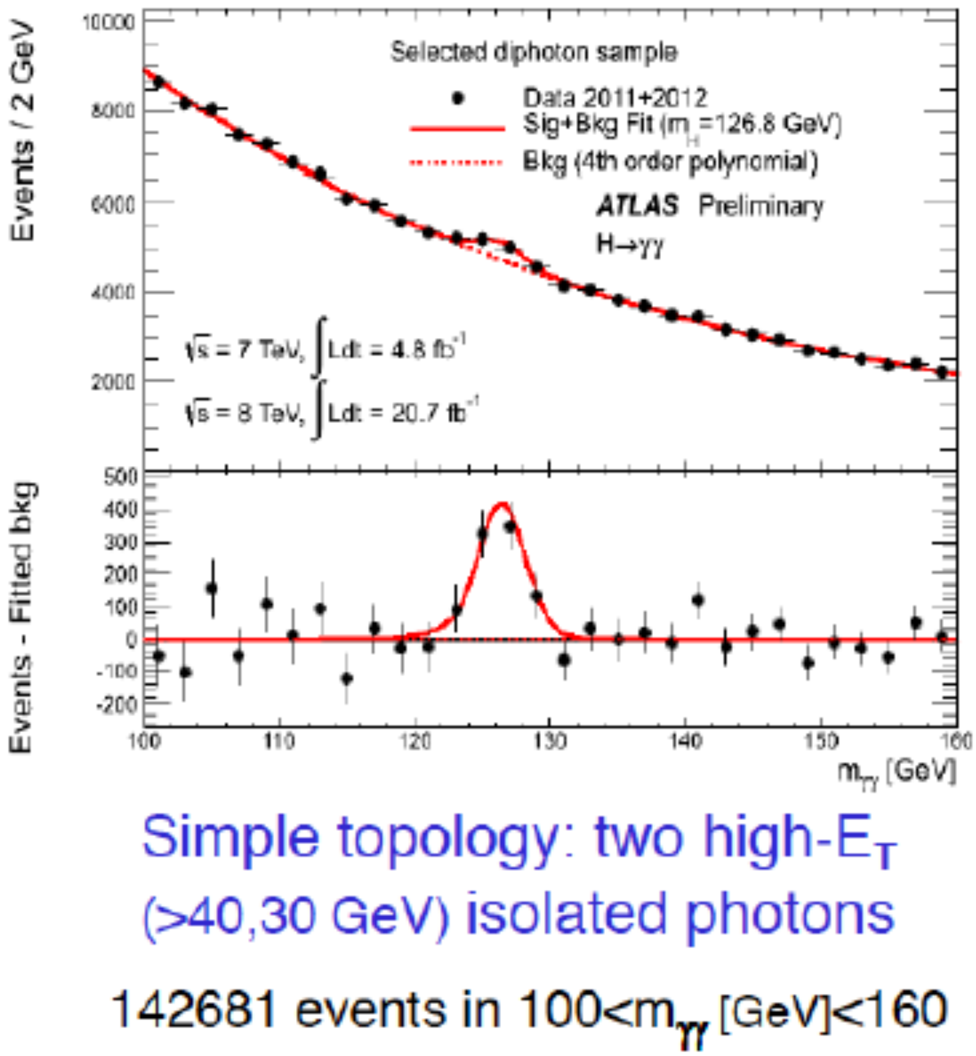
The event for all 3 jets (solid curve) seens to me to correspond to
decay of a middle (cyan) T-quark state
with one of the 3 jets corresponding to
decay from the Triviality boundary down to the low (green) T-quark state,
whose immediately subsequent decay is corresponds to the 2-jet (dashed curve) event
at the low (green) energy level.

After 1998 Fermilab and CERN have focussed attention on detailed analysis
of the middle (cyan) T-quark state, getting much valuable detailed information
about it but not producing much information about the low or high Tquark states.

In the 25/fb of data collected through the run ending with the long shutdown at the end of 2012,
the LHC has observed a 126 GeV state of the Standard Model Higgs boson.

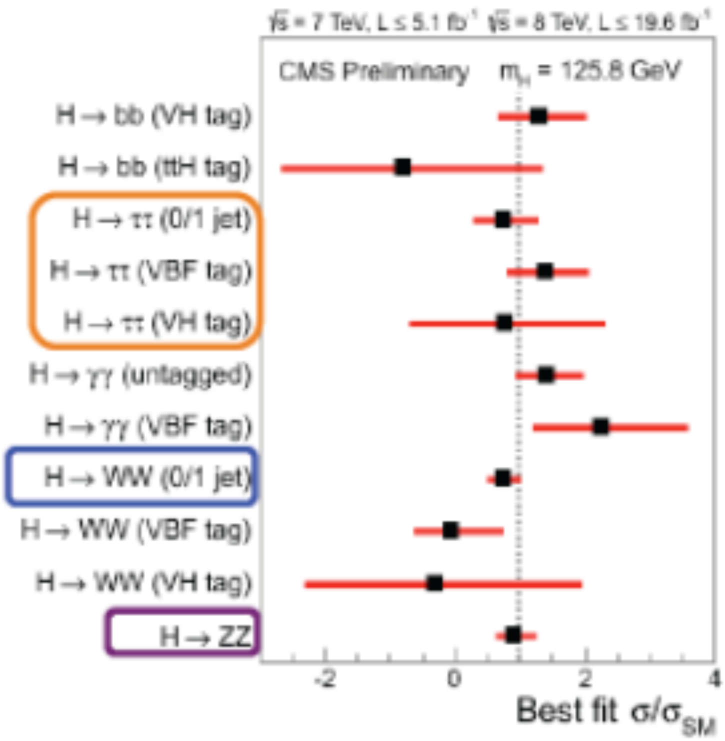
Here are some details about the LHC observation at 126 GeV and related results shown at Moriond 2013:

The digamma histogram for ATLAS



shows only one peak below 160 GeV and it is around 126 GeV.

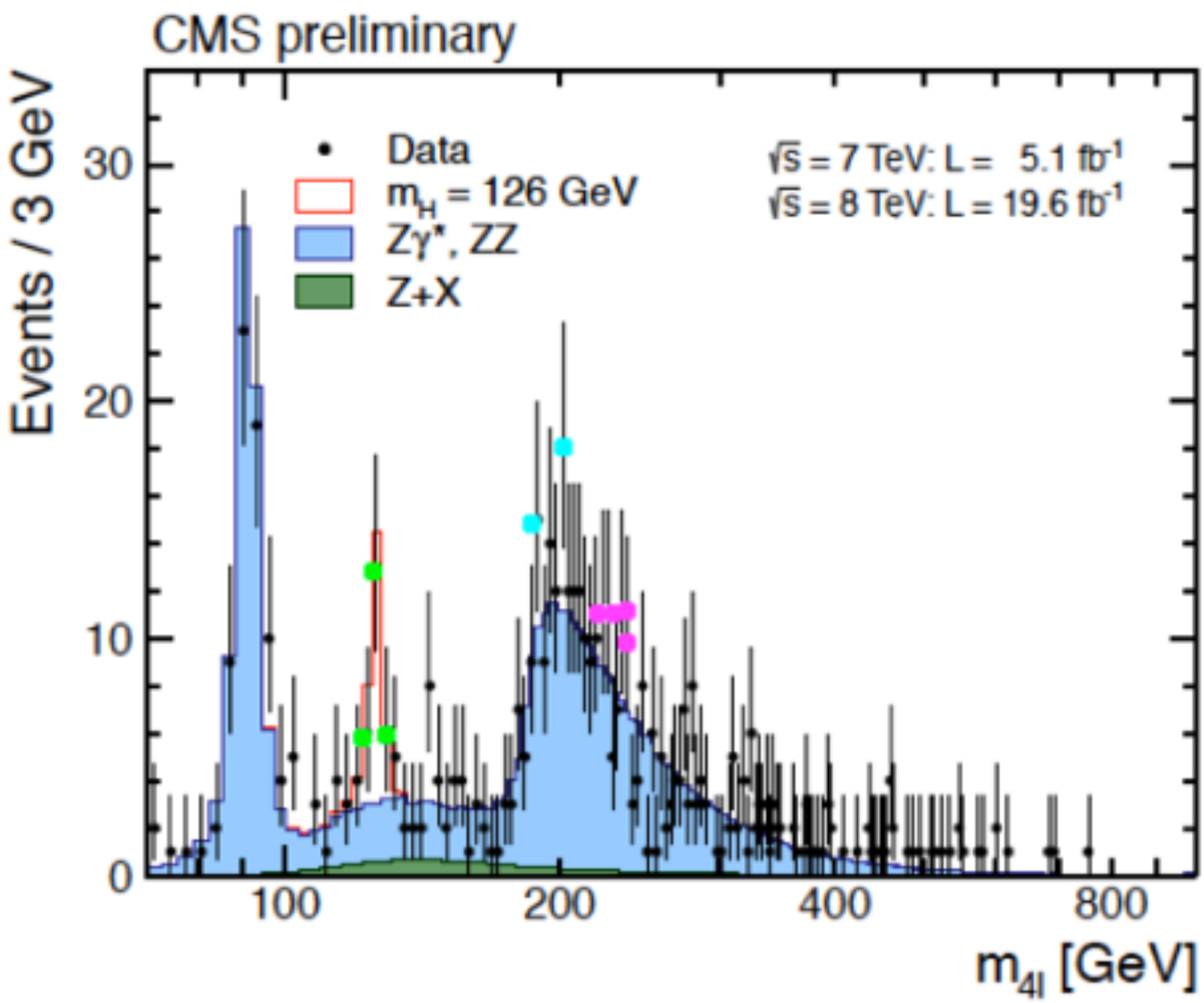
CMS shows the cross sections for Higgs at 125.8 GeV

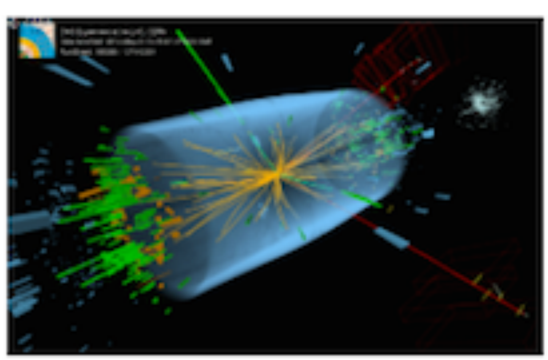


$H \rightarrow ZZ$(0/1 jet) : $0.84^{+0.32}_{-0.26}$
$H \rightarrow ZZ$(dijet tag): $1.22^{+0.84}_{-0.57}$

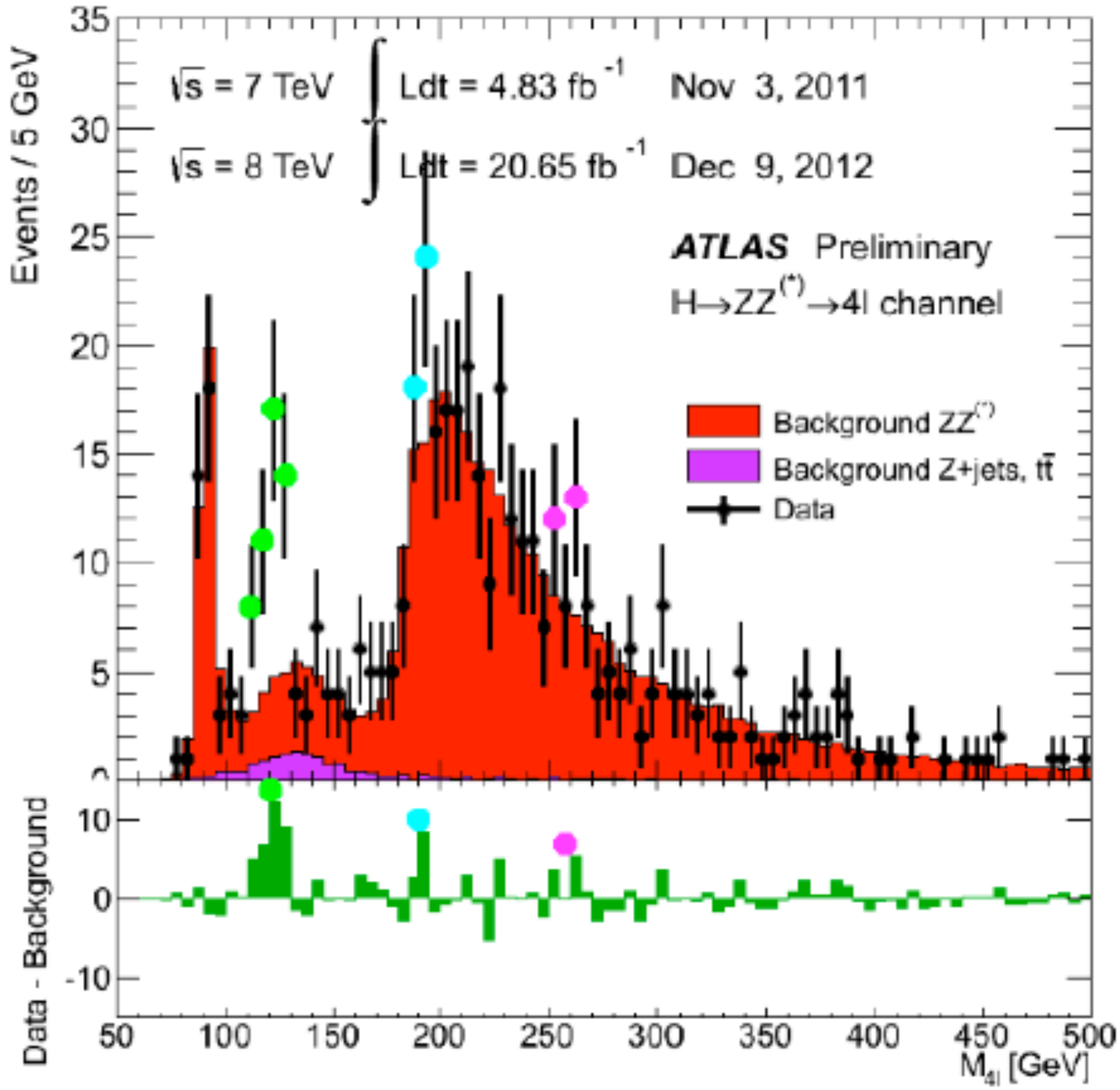
to be substantially consistent with the Standard Model for the WW and ZZ channels, a bit low for tau-tau and bb channels (but that is likely due to very low statistics there), and a bit high for the digamma channel (but that may be due to phenomena related to the Higgs as a Tquark condensate).

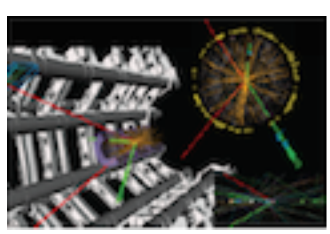
A CMS histogram (some colors added by me) for the Golden Channel Higgs to ZZ to 4l shows the peak around 126 GeV (green dots - lowHiggs mass state).
The CMS histogram also indicates other excesses
around 200 GeV (cyan dots - midHiggs mass state)
and around 250 GeV (magenta dots - highHiggs mass state).
An image of one of the events is shown below the histogram.



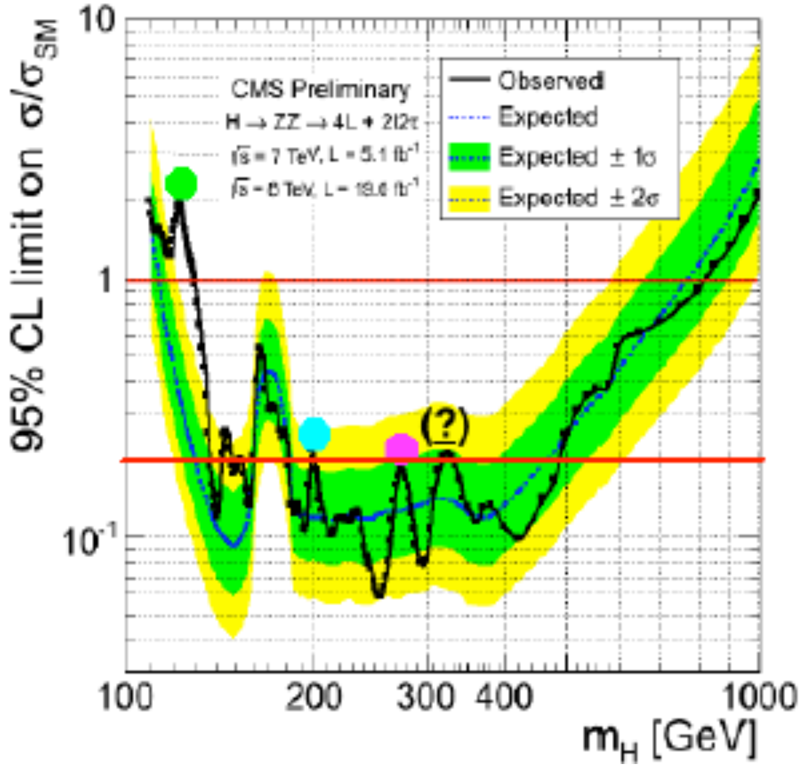


An ATLAS ZZ to 4l histogram (some colors added by me)
show the peak around 126 GeV (green dots - low Higgs mass state.
The ATLAS histogram also indicates other excesses
around 200 GeV (cyan dots - middle Higgs mass state)
and around 250 GeV (magenta dots - high Higgs mass state) .
An image of one of the events is shown below the histogram.





CMS showed a Brazil Band Plot for the High Mass Higgs to ZZ to 4l/2l2tau channel where
the top red line represents the expected cross section of a single Standard Model Higgs and the lower red line represents about 20% of the expected Higgs SM cross section.



The green dot peak is at the 126 GeV Low Mass Higgs state
with expected Standard Model cross section.
The cyan dot peak is around the 200 GeV Mid Mass Higgs state
expected to have about 25% of the SM cross section.
The magenta dot peak is around the 250 (+/- 20 or so) GeV High Mass Higgs state
expected to have about 25% of the SM cross section.

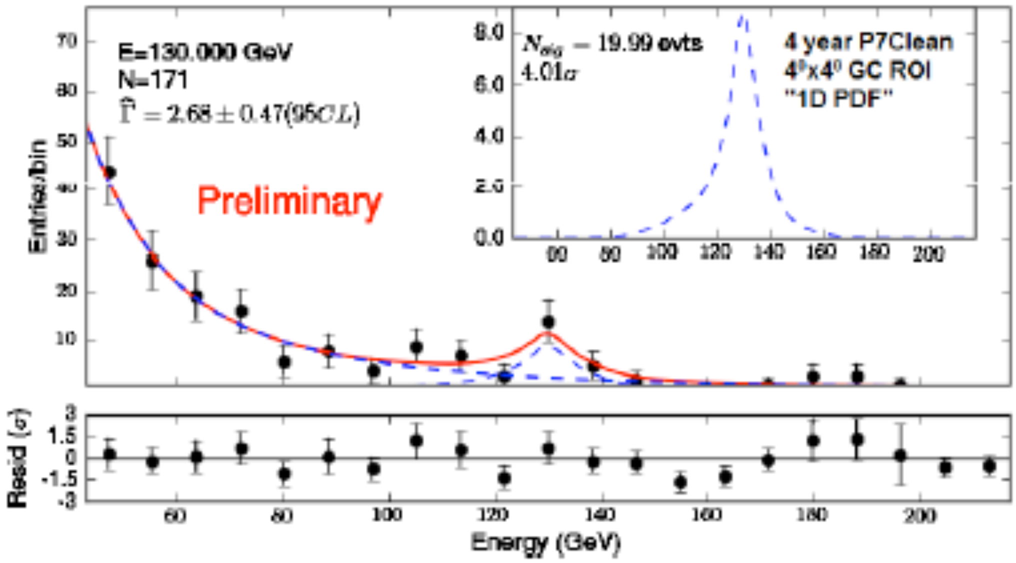
The (?) peak is around 320 GeV where I would not expect a Higgs Mass state
and I note that in fact it seems to have gone away in the full ATLAS ZZ to 4l histogram shown above because between 300 and 350 GeV the two sort-of-high excess bins are adjacent to deficient bins .
It will probably be no earlier than 2015 (after the long shutdown) that the LHC will produce substantially more data than the 25/fb available at Moriond 2013 and therefore no earlier than 2016 for the green and yellow Brazil Bands to be pushed down (throughout the 170 GeV to 500 GeV region) below 10 per cent (the 10^(-1) line) of the SM cross section as is needed to show whether or not the cyan dot, magenta dot, and/or (?) peaks are real or statistical fluctuations.

My guess (based on the Cl(16)-E8 model) is that the cyan dot and magenta dot peaks will prove to be real and that the (?) peak will go away as a statistical fluctuation.

Sgr A* and Higgs = Tquark-Tantiquark Condensate

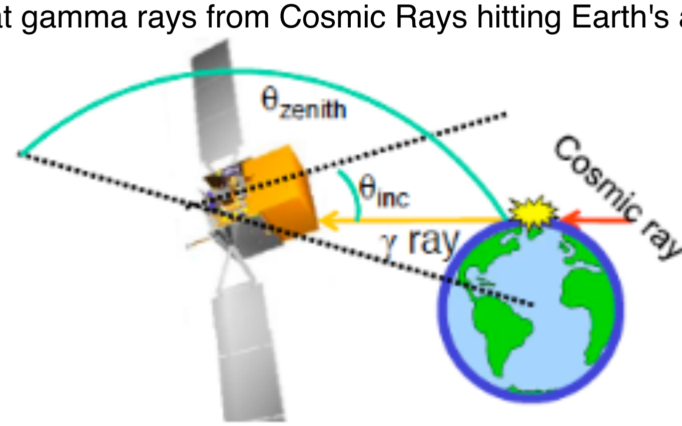
Sagittarius A* (Sgr A*) is a very massive black hole in the center of our Galaxy into which large amounts of Hydrogen fall. As the Hydrogen approaches Sgr A* it increases in energy, ionizing into protons and electrons, and eventually producing a fairly dense cloud of infalling energetic protons whose collisions with ambient protons are at energies similar to the proton-proton collisions at the LHC.

Andrea Albert at The Fermi Symposium 11/2/2012 said: "... gamma rays detectable by the Fermi Large Area Telescope [FLAT] ...



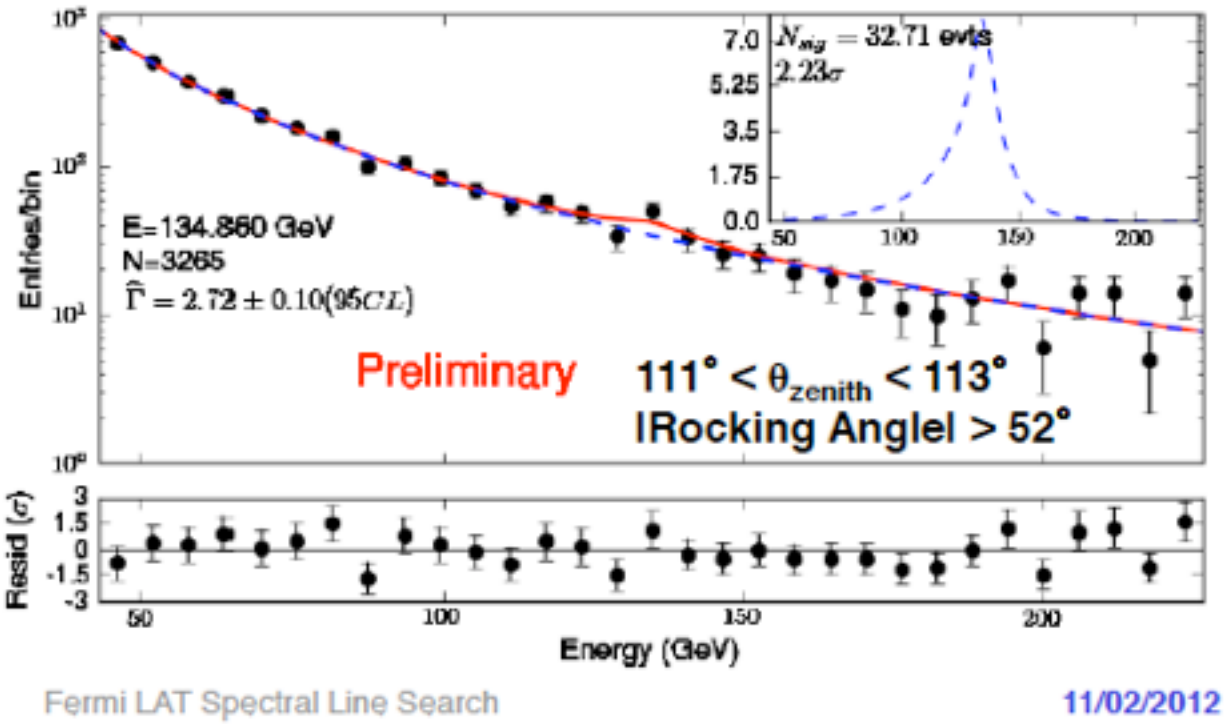
... Line-like Feature near 135 GeV ... localized in the galactic center ...".

In addition to the Galactic Center observations,
Fermi LAT looked at gamma rays from Cosmic Rays hitting Earth's atmosphere



by looking at the Earth Limb.

Andrea Albert at The Fermi Symposium 11/2/2012 also said: "... Earth Limb is a bright gamma-ray source ... From cosmic-ray interactions in the atmosphere ...



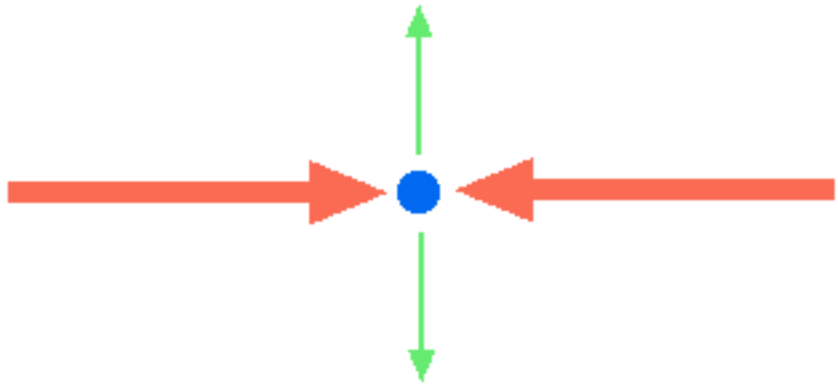
... Line-like feature ... at 135 GeV .. Appears when LAT is pointing at the Limb ...".

Since 90% of high-energy Cosmic Rays are Protons and since their collisions with Protons and other nuclei in Earth's atmosphere produce gamma rays,
the 135 GeV Earth Limb Line seen by Fermi LAT is also likely to be the Higgs produced by collisions analagous to those at the LHC.

Olivier K. in a comment in Jester's blog on 10 November 2012 said:
"... Could the 135 GeV bump be related ... to current Higgs ... properties ? ...
The coincidence between GeV figures ...[for LHC] Higgs mass and
this [Fermi LAT] bump is thrilling for an amateur like me...".

Jester in his resonaances blog on 17 April 2012 said, about Fermi LAT:
"... the plot shows the energy of *single* photons as measured by Fermi,
not the invariant mass of photon pairs ...".
Since the LHC 125 GeV peak is for "invariant mass of photon pairs"
and the Fermi LAT 135 GeV peak is for ""single" photons"
how could both correspond to a Higgs mass state around 130 GeV ?

The LHC sees collisions of high-energy protons (red arrows) forming Higgs (blue dot)



with the Higgs at rest decaying into a photon pair (green arrows)
giving the observed Higgs peak (around 130 GeV) as invariant mass of photon pairs.

Fermi LAT at Galactic Center and Earth Limb sees collisions of one high-energy proton
with a low-energy (relatively at rest) proton forming Higgs

with Higgs moving fast from momentum inherited from the high-enrgy proton decaying
into two photons: one with low energy not observed by Fermi LAT
and the other being observed by Fermi LAT as a high-energy gamma ray carrying
almost all of the Higgs decay energy (around 130 GeV) as a "single" photon.

Therefore, the coincidence noted by Olivier K. is probably a realistic phenomenon.

18. Segal-type Conformal gravity with conformal generator structure giving Dark Energy, Dark Matter, and Ordinary Matter ratio

MacDowell-Mansouri Gravity is described by Rabindra Mohapatra in section 14.6 of his book "Unification and Supersymmetry":

§14.6. Local Conformal Symmetry and Gravity

Before we study supergravity, with the new algebraic approach developed, we would like to discuss how gravitational theory can emerge from the gauging of conformal symmetry. For this purpose we briefly present the general notation for constructing gauge covariant fields. The general procedure is to start with the Lie algebra of generators X_A of a group

$$[X_A, X_B] = f_{AB}^C X_C, \tag{14.6.1}$$

where f_{AB}^C are structure constants of the group. We can then introduce a gauge field connection h_μ^A as follows:

$$h_\mu = h_\mu^A X_A. \tag{14.6.2}$$

Let us denote the parameter associated with X_A by ε^A. The gauge transformations on the fields h_μ^A are given as follows:

$$\delta h_\mu^A = \partial_\mu \varepsilon^A + h_\mu^B \varepsilon^C f_{CB}^A \equiv (D_\mu \varepsilon)^A. \tag{14.6.3}$$

We can then define a covariant curvature

$$R_{\mu\nu}^A = \partial_\nu h_\mu^A - \partial_\mu h_\nu^A + h_\nu^B h_\mu^C f_{CB}^A. \tag{14.6.4}$$

Under a gauge transformation

$$\delta_{\text{gauge}} R_{\mu\nu}^A = R_{\mu\nu}^B \varepsilon^C f_{CB}^A. \tag{14.6.5}$$

We can then write the general gauge invariant action as follows:

$$I = \int d^4x\, Q_{AB}^{\mu\nu\rho\sigma} R_{\mu\nu}^A R_{\rho\sigma}^B. \tag{14.6.6}$$

Let us now apply this formalism to conformal gravity. In this case

$$h_\mu = P_m e_\mu^m + M_{mn} \omega_\mu^{mn} + K_m f_\mu^m + D b_\mu. \tag{14.6.7}$$

The various $R_{\mu\nu}$ are

$$R_{\mu\nu}(P) = \partial_\nu e_\mu^m - \partial_\mu e_\nu^m + \omega_\mu^{mn} e_\nu^n - \omega_\nu^{mn} e_\mu^n - b_\mu e_\nu^m + b_\nu e_\mu^m, \tag{14.6.8}$$

$$R_{\mu\nu}(M) = \partial_\nu \omega_\mu^{mn} - \partial_\mu \omega_\nu^{mn} - \omega_\nu^{mp} \omega_{\mu,p}^n - \omega_\mu^{mp} \omega_{\nu,p}^n - 4(e_\mu^m f_\nu^n - e_\nu^m f_\mu^n), \tag{14.6.9}$$

$$R_{\mu\nu}(K) = \partial_\nu f_\mu^m - \partial_\mu f_\nu^m - b_\mu f_\nu^m + b_\nu f_\mu^m + \omega_\mu^{mn} f_\nu^n - \omega_\nu^{mn} f_\mu^n, \tag{14.6.10}$$

$$R_{\mu\nu}(D) = \partial_\nu b_\mu - \partial_\mu b_\nu + 2e_\mu^m f_\nu^m - 2e_\nu^m f_\mu^m. \tag{14.6.11}$$

The gauge invariant Lagrangian for the gravitational field can now be written down, using eqn. (14.6.6), as

$$S = \int d^4x\, \varepsilon_{mnrs} \varepsilon^{\mu\nu\rho\sigma} R_{\mu\nu}^{mn}(M) R_{\rho\sigma}^{rs}(M). \tag{14.6.12}$$

We also impose the constraint that

$$R_{\mu\nu}(P) = 0, \tag{14.6.13}$$

which expresses ω_μ^{mn} as a function of (e, b). The reason for imposing this constraint has to do with the fact that P_m transformations must be eventually identified with coordinate transformation. To see this point more explicitly let us consider the vierbein e_μ^m. Under coordinate transformations

$$\delta_{GC}(\xi^\nu)e_\mu^m = \partial_\mu \xi^\lambda e_\lambda^m + \xi^\lambda \partial_\lambda e_\mu^m. \tag{14.6.14}$$

Using eqn. (14.6.8) we can rewrite

$$\delta_{GC}(\xi^\nu)e_\mu^m = \delta_P(\xi e^n)e_\mu^m + \delta_M(\xi\omega^{mn})e_\mu^m + \delta_D(\xi b)e_\mu^m + \xi^\nu R_{\mu\nu}^m(P),$$

where

$$\delta_P(\xi^n)e_\mu^m = \partial_\mu \xi^m + \xi^n \omega_\mu^{mn} + \xi^m b_\mu. \tag{14.6.15}$$

If $R^{\mu\nu}(P) = 0$, the general coordinate transformation becomes related to a set of gauge transformations via eqn. (14.6.15).

At this point we also wish to point out how we can define the covariant derivative. In the case of internal symmetries $D_\mu = \partial_\mu - iX_A h_\mu^A$; now since momentum is treated as an internal symmetry we have to give a rule. This follows from eqn. (14.6.15) by writing a redefined translation generator $\bar{P}$ such that

$$\delta_{\bar{P}}(\xi) = \delta_{GC}(\xi^\nu) - \sum_{A'} \delta_{A'}(\xi^m h_m^A), \tag{14.6.16}$$

where A' goes over all gauge transformations excluding translation. The rule is

$$\delta_{\bar{P}}(\xi^m)\phi = \xi^m D_m^C \phi. \tag{14.6.17}$$

We also wish to point out that for fields which carry spin or conformal charge, only the intrinsic parts contribute to D_m^C and the orbital parts do not play any rule.

Coming back to the constraints we can then vary the action with respect to f_μ^m to get an expression for it, i.e.,

$$e_\nu^m f_{\mu m} = -\tfrac{1}{4}[e_m^\lambda e_{m\nu} R_{\mu\lambda}^{mn} - \tfrac{1}{6} g_{\mu\nu} R], \tag{14.6.18}$$

where f_μ^m has been set to zero in R written in the right-hand side.

This eliminates (from the theory the degrees of freedom) ω_μ^{mn} and f_μ^m and we are left with e_μ^m and b_μ. Furthermore, these constraints will change the transformation laws for the dependent fields so that the constraints do not change.

Let us now look at the matter coupling to see how the familiar gravity theory emerges from this version. Consider a scalar field ϕ. It has conformal weight $\lambda = 1$. So we can write a convariant derivative for it, eqn. (14.6.17)

$$D_\mu^C \phi = \partial_\mu \phi - \phi b_\mu. \tag{14.6.19}$$

We note that the conformal charge of ϕ can be assumed to be zero since $K_m = x^2 \partial$ and is the dimension of inverse mass. In order to calculate $\Box^c \phi$ we start with the expression for d'Alambertian in general relativity

$$\frac{1}{e}\partial_\nu(g^{\mu\nu} e D_\mu^C \phi). \tag{14.6.20}$$

The only transformations we have to compensate for are the conformal transformations and the scale transformations. Since

$$\delta b_\mu = -2\xi_K^m e_{m\mu}, \qquad \delta(\phi b_\mu) = \phi \delta b_\mu = -2\phi f_\mu^m e_m^\mu = +\tfrac{2}{12}\phi R, \tag{14.6.21}$$

where, in the last step, we have used the constraint equation (14.6.18). Putting all these together we find

$$\Box^C \phi = \frac{1}{e}\partial_\nu(g^{\mu\nu} e D_\mu^c \phi) + b_\mu D_\mu^c \phi + \tfrac{2}{12}\phi R. \tag{14.6.22}$$

Thus, the Lagrangian for conformal gravity coupled to matter fields can be written as

$$S = \int e\, d^4x\, \tfrac{1}{2}\phi \Box^C \phi. \tag{14.6.23}$$

Now we can use conformal transformation to gauge $b_\mu = 0$ and local scale transformation to set $\phi = \kappa^{-1}$ leading to the usual Hilbert action for gravity. To summarize, we start with a Lagrangian invariant under full local conformal symmetry and fix conformal and scale gauge to obtain the usual action for gravity. We will adopt the same procedure for supergravity. An important technical point to remember is that, $\Box^C$, the conformal d'Alambertian contains R, which for constant ϕ, leads to gravity. We may call ϕ the auxiliary field.

After the scale and conformal gauges have been fixed,
the conformal Lagrangian becomes a de Sitter Lagrangian.

Einstein-Hilbert gravity can be derived from the de Sitter Lagrangian,
as was first shown by MacDowell and Mansouri (Phys. Rev. Lett. 38 (1977) 739).
(Frank Wilczek, in hep-th/9801184 says that the MacDowell-Mansouri "... approach
to casting gravity as a gauge theory was initiated by MacDowell and Mansouri ...
S. MacDowell and F. Mansouri, Phys. Rev. Lett. 38 739 (1977) ... ,
and independently Chamseddine and West ... A. Chamseddine and P. West Nucl. Phys. B 129, 39 (1977);
also quite relevant is A. Chamseddine, Ann. Phys. 113, 219 (1978). ...".)

The minimal group required to produce Gravity,
and therefore **the group that is used in calculating Force Strengths**,
is the [anti] de Sitter group, as is described by
Freund in chapter 21 of his book Supersymmetry (Cambridge 1986) (chapter 21 is a Non-Supersymmetry chapter leading up to a Supergravity description in the following chapter 22):
"... Einstein gravity as a gauge theory ... we expect a set of gauge fields w^ab_u for the Lorentz group and a further set e^a_u for the translations, ...
Everybody knows though, that Einstein's theory contains but one spin two field,
originally chosen by Einstein as g_uv = e^a_u e^b_v n_ab
(n_ab = Minkowski metric).
What happened to the w^ab_u ?
The field equations obtained from the Hilbert-Einstein action by varying the
w^ab_u are algebraic in the w^ab_u ... permitting us to express the w^ab_u in
terms of the e^a_u ... The w do not propagate ...
We start from the four-dimensional de-Sitter algebra ... so(3,2).
Technically this is the anti-de-Sitter algebra ...
We envision space-time as a four-dimensional manifold M.
At each point of M we have a copy of SO(3,2) (a fibre ...) ...
and we introduce the gauge potentials (the connection) h^A_mu(x)
A = 1,..., 10 , mu = 1,...,4. Here x are local coordinates on M.
From these potentials h^A_mu we calculate the field-strengths
(curvature components) [let @ denote partial derivative]
R^A_munu = @_mu h^A_nu - @_nu h^A_mu + f^A_BC h^B_mu h^C_nu
...[where]... the structure constants f^C_AB ...[are for]... the anti-de-Sitter algebra ....
We now wish to write down the action S as an integral over
the four-manifold M ... S(Q) = INTEGRAL_M R^A /\ R^B Q_AB
where Q_AB are constants ... to be chosen ... we require
... the invariance of S(Q) under local Lorentz transformations
... the invariance of S(Q) under space inversions ...
...[AFTER A LOT OF ALGEBRA NOT SHOWN IN THIS QUOTE]...
we shall see ...[that]... the action becomes invariant
under all local [anti]de-Sitter transformations ...[and]... we recognize ... t
he familiar Hilbert-Einstein action with cosmological term in vierbein notation ...
Variation of the vierbein leads to the Einstein equations with cosmological term.
Variation of the spin-connection ... in turn ... yield the torsionless Christoffel
connection ... the torsion components ... now vanish.
So at this level full sp(4) invariance has been checked.
... Were it not for the assumed space-inversion invariance ...
we could have had a parity violating gravity. ...
Unlike Einstein's theory ...[MacDowell-Mansouri].... does not require Riemannian
invertibility of the metric. ... the solution has torsion ... produced by an interference
between parity violating and parity conserving amplitudes.
Parity violation and torsion go hand-in-hand.
Independently of any more realistic parity violating solution of the gravity
equations this raises the cosmological question whether
the universe as a whole is in a space-inversion symmetric configuration. ...".

According to gr-qc/9809061 by R. Aldrovandi and J. G. Peireira:
"... If the fundamental spacetime symmetry of the laws of Physics is that given by the de Sitter instead of the Poincare group, the P-symmetry of the weak cosmological-constant limit and the Q-symmetry of the strong cosmological constant limit can be considered as limiting cases of the fundamental symmetry. ...
... N ...[is the space]... whose geometry is gravitationally related to an infinite cosmological constant ...[and]... is a 4-dimensional cone-space in which ds = 0, and whose group of motion is Q. Analogously to the Minkowski case, N is also a homogeneous space, but now under the kinematical group Q, that is, N = Q/L [where L is the Lorentz Group of Rotations and Boosts]. In other words, the point-set of N is the point-set of the special conformal transformations. Furthermore, the manifold of Q is a principal bundle P(Q/L,L), with Q/L = N as base space and L as the typical fiber. The kinematical group Q, like the Poincare group, has the Lorentz group L as the subgroup accounting for both the isotropy and the equivalence of inertial frames in this space. However, the special conformal transformations introduce a new kind of homogeneity. Instead of ordinary translations, all the points of N are equivalent through special conformal transformations. ...
... Minkowski and the cone-space can be considered as dual to each other, in the sense that their geometries are determined respectively by a vanishing and an infinite cosmological constants. The same can be said of their kinematical group of motions: P is associated to a vanishing cosmological constant and Q to an infinite cosmological constant.
The dual transformation connecting these two geometries is the spacetime inversion x^u -> x^u / sigma^2 . Under such a transformation, the Poincare group P is transformed into the group Q, and the Minkowski space M becomes the conespace N. The points at infinity of M are concentrated in the vertex of the conespace N, and those on the light-cone of M becomes the infinity of N. It is concepts of space isotropy and equivalence between inertial frames in the conespace N are those of special relativity. The difference lies in the concept of uniformity as it is the special conformal transformations, and not ordinary translations, which act transitively on N. ..."

Gravity and the Cosmological Constant come from the MacDowell-Mansouri Mechanism and the 15-dimensional Spin(2,4) = SU(2,2) Conformal Group, which is made up of:

3 Rotations
3 Boosts
4 Translations
4 Special Conformal transformations
1 Dilatation

The **Cosmological Constant / Dark Energy** comes from the **10 Rotation, Boost, and Special Conformal generators** of the Conformal Group Spin(2,4) = SU(2,2), so the fractional part of our Universe of the Cosmological Constant should be **about 10 / 15 = 67% for tree level**.

Black Holes, including **Dark Matter Primordial Black Holes**, are curvature singularities in our 4-dimensional physical spacetime, and since Einstein-Hilbert curvature comes from the **4 Translations** of the 15-dimensional Conformal Group Spin(2,4) = SU(2,2) through the MacDowell-Mansouri Mechanism (in which the generators corresponding to the 3 Rotations and 3 Boosts do not propagate), the fractional part of our Universe of Dark Matter Primordial Black Holes should be **about 4 / 15 = 27% at tree level**.

Since **Ordinary Matter** gets mass from the Higgs mechanism which is related to the **1 Scale Dilatation** of the 15-dimensional Conformal Group Spin(2,4) = SU(2,2), the fractional part of our universe of Ordinary Matter should be **about 1 / 15 = 6% at tree level**.

However,
as Our Universe evolves the Dark Energy, Dark Matter, and Ordinary Matter densities evolve at different rates, so that the differences in evolution must be taken into account from the initial End of Inflation to the Present Time.

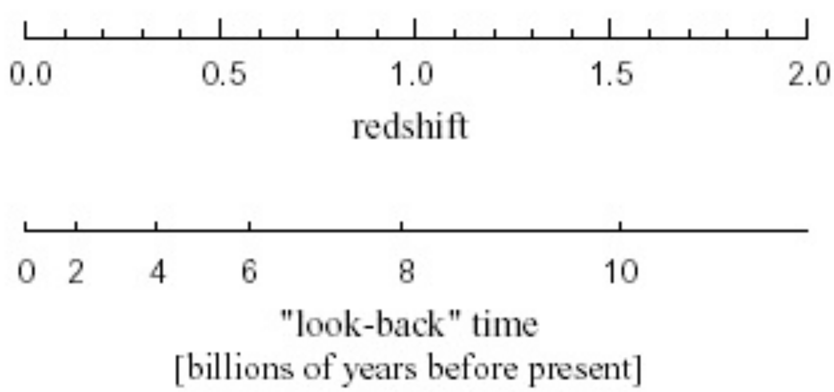
Without taking into account any evolutionary changes with time, our Flat Expanding Universe should have roughly:

67% Cosmological Constant
27% Dark Matter - possilbly primordial stable Planck mass black holes
6% Ordinary Matter

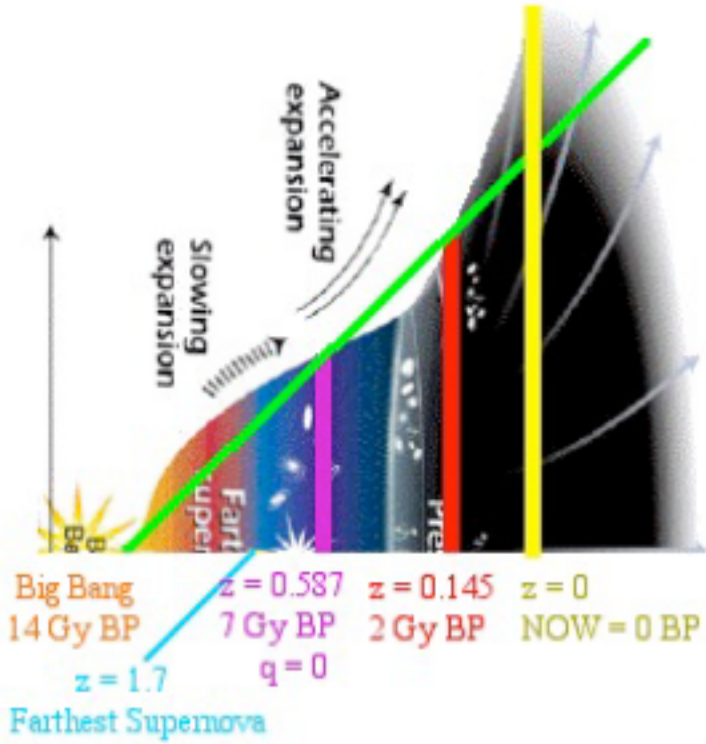
As Dennnis Marks pointed out to me,
since density rho is proportional to (1+z)^3(1+w) for red-shift factor z
and a constant equation of state w:
w = -1 for Λ and the average overall density of Λ Dark Energy remains constant with time and the expansion of our Universe;
and
w = 0 for nonrelativistic matter so that the overall average density of Ordinary Matter declines as 1 / R^3 as our Universe expands;
and
w = 0 for primordial black hole dark matter - stable Planck mass black holes - so that Dark Matter also has density that declines as 1 / R^3 as our Universe expands;
so that the ratio of their overall average densities must vary with time, or scale factor R of our Universe, as it expands.
Therefore,
the above calculated ratio 0.67 : 0.27 : 0.06 is valid
only for a particular time, or scale factor, of our Universe.

When is that time ? Further, what is the value of the ratio now ?

Since WMAP observes Ordinary Matter at 4% NOW,
the time when Ordinary Matter was 6% would be
at redshift z such that
1 / (1+z)^3 = 0.04 / 0.06 = 2/3 , or (1+z)^3 = 1.5 , or 1+z = 1.145 , or z = 0.145.
To translate redshift into time,
in billions of years before present, or Gy BP, use this chart



from a www.supernova.lbl.gov file SNAPoverview.pdf to see that
the time when Ordinary Matter was 6%
would have been a bit over 2 billion years ago, or 2 Gy BP.



In the diagram, there are four Special Times in the history of our Universe:
the Big Bang Beginning of Inflation (about 13.7 Gy BP);

1 - the End of Inflation = Beginning of Decelerating Expansion
(beginning of green line also about 13.7 Gy BP);

2 - the End of Deceleration (q=0) = Inflection Point =
= Beginning of Accelerating Expansion
(purple vertical line at about z = 0.587 and about 7 Gy BP).
According to a hubblesite web page credited to Ann Feild, the above diagram "... reveals changes in the rate of expansion since the universe's birth 15 billion years ago. The more shallow the curve, the faster the rate of expansion. The curve changes noticeably about 7.5 billion years ago, when objects in the universe began flying apart as a faster rate. ...".
According to a CERN Courier web page: "... Saul Perlmutter, who is head of the Supernova Cosmology Project ... and his team have studied altogether some 80 high red-shift type Ia supernovae. Their results imply that the universe was decelerating for the first half of its existence, and then began accelerating approximately 7 billion years ago. ...".
According to astro-ph/0106051 by Michael S. Turner and Adam G. Riess: "... current supernova data ... favor deceleration at z > 0.5 ... SN 1997ff at z = 1.7 provides direct evidence for an early phase of slowing expansion if the dark energy is a cosmological constant ...".

3 - the Last Intersection of the Accelerating Expansion of our Universe of Linear Expansion (green line) with the Third Intersection (at red vertical line at z = 0.145 and about 2 Gy BP), which is also around the times of the beginning of the Proterozoic Era and Eukaryotic Life, Fe_2O_3 Hematite ferric iron Red Bed formations, a Snowball Earth, and the start of the Oklo fission reactor. 2 Gy is also about 10 Galactic Years for our Milky Way Galaxy and is on the order of the time for the process of a collision of galaxies.

4 - Now.

Those four Special Times define four Special Epochs:

The Inflation Epoch, beginning with the Big Bang and ending with the End of Inflation. The Inflation Epoch is described by Zizzi Quantum Inflation ending with Self-Decoherence of our Universe (see gr-qc/0007006).

The Decelerating Expansion Epoch, beginning with the Self-Decoherence of our Universe at the End of Inflation. During the Decelerating Expansion Epoch, the Radiation Era is succeeded by the Matter Era, and the Matter Components (Dark and Ordinary) remain more prominent than they would be under the "standard norm" conditions of Linear Expansion.

The Early Accelerating Expansion Epoch, beginning with the End of Deceleration and ending with the Last Intersection of Accelerating Expansion with Linear Expansion. During Accelerating Expansion, the prominence of Matter Components (Dark and Ordinary) declines, reaching the "standard norm" condition of Linear Expansion at the end of the Early Accelerating Expansion Epoch at the Last Intersection with the Line of Linear Expansion.

The Late Accelerating Expansion Epoch, beginning with the Last Intersection of Accelerating Expansion and continuing forever, with New Universe creation happening many times at Many Times. During the Late Accelerating Expansion Epoch, the Cosmological Constant Λ is more prominent than it would be under the "standard norm" conditions of Linear Expansion.

Now happens to be about 2 billion years into the Late Accelerating Expansion Epoch.

What about Dark Energy : Dark Matter : Ordinary Matter now ?

As to how the Dark Energy Λ and Cold Dark Matter terms have evolved during the past 2 Gy, a rough estimate analysis would be:

Λ and CDM would be effectively created during expansion in their natural ratio 67 : 27 = 2.48 = 5 / 2, each having proportionate fraction 5 / 7 and 2 / 7, respectively;
CDM Black Hole decay would be ignored; and
pre-existing CDM Black Hole density would decline by the same 1 / R^3 factor as Ordinary Matter, from 0.27 to 0.27 / 1.5 = 0.18.

The Ordinary Matter excess 0.06 - 0.04 = 0.02 plus the first-order CDM excess 0.27 - 0.18 = 0.09 should be summed to get a total first-order excess of 0.11, which in turn should be distributed to the Λ and CDM factors in their natural ratio 67 : 27, producing, for NOW after 2 Gy of expansion:

CDM Black Hole factor = 0.18 + 0.11 x 2/7 = 0.18 + 0.03 = 0.21
for a total calculated Dark Energy : Dark Matter : Ordinary Matter ratio for now of

0.75 : 0.21 : 0.04

so that the present ratio of 0.73 : 0.23 : 0.04 observed by WMAP seems to me to be substantially consistent with the cosmology of the E8 model.

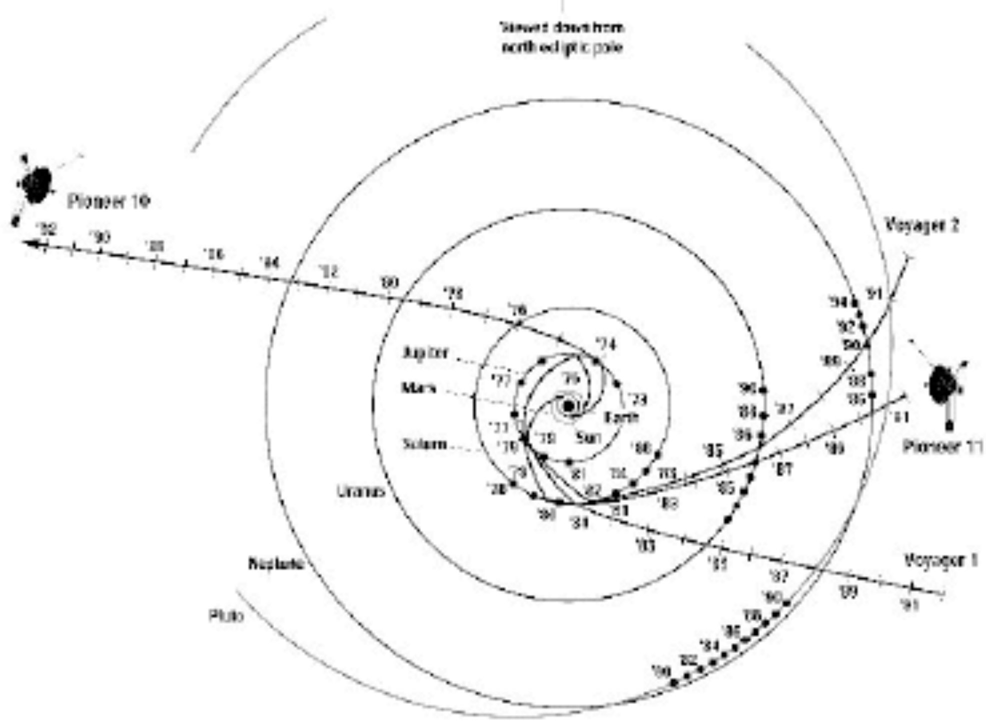
2013 Planck Data (arxiv 1303.5062) showed "... anomalies ... previously observed in the WMAP data ... alignment between the quadrupole and octopole moments ... asymmetry of power between two ... hemispheres ... Cold Spot ... are now confirmed at ... 3 sigma ... but a higher level of confidence ...".

Now the Cl(16)-E8 model rough evolution calculation is: DE : DM : OM = 75 : 20 : 05
WMAP: DE : DM : OM = 73 : 23 : 04
Planck: DE : DM : OM = 69 : 26 : 05
basic E8 Conformal calculation: DE : DM : OM = 67 : 27 : 06

Since uncertainties are substantial, I think that there is reasonable consistency.

19. Dark Energy explanations for Pioneer Anomaly and Uranus spin-axis tilt

After the Inflation Era and our Universe began its current phase of expansion, some regions of our Universe become Gravitationally Bound Domains (such as, for example, Galaxies) in which the 4 Conformal GraviPhoton generators are frozen out, forming domains within our Universe like IceBergs in an Ocean of Water. On the scale of our Earth-Sun Solar System, the region of our Earth, where we do our local experiments, is in a Gravitationally Bound Domain.



Pioneer spacecraft are not bound to our Solar System and are experiments beyond the Gravitationally Bound Domain of our Earth-Sun Solar System.
In their Study of the anomalous acceleration of Pioneer 10 and 11 gr-qc/0104064 John D. Anderson, Philip A. Laing, Eunice L. Lau, Anthony S. Liu, Michael Martin Nieto, and Slava G. Turyshev say: "... The latest successful precession maneuver to point ...[Pioneer 10]... to Earth was accomplished on 11 February 2000, when Pioneer 10 was at a distance from the Sun of 75 AU. [The distance from the Earth was [about] 76 AU with a corresponding round-trip light time of about 21 hour.] ... The next attempt at a maneuver, on 8 July 2000, was unsuccessful ... conditions will again be favorable for an attempt around July, 2001. ... At a now nearly constant velocity relative to the Sun of 12.24 km/s, Pioneer 10 will continue its motion into interstellar space, heading generally for the red star Aldebaran ... about 68 light years away ... it should take Pioneer 10 over 2 million years to reach its neighborhood....
[the above image is] Ecliptic pole view of Pioneer 10, Pioneer 11, and Voyager

trajectories. Digital artwork by T. Esposito. NASA ARC Image # AC97-0036-3.

... on 1 October 1990 ... Pioneer 11 ... was [about] 30 AU away from the Sun ...

The last communication from Pioneer 11 was received in November 1995, when the spacecraft was at distance of [about] 40 AU from the Sun. ... Pioneer 11 should pass close to the nearest star in the constellation Aquila in about 4 million years ...

... Calculations of the motion of a spacecraft are made on the basis of the range time-delay and/or the Doppler shift in the signals. This type of data was used to determine the positions, the velocities, and the magnitudes of the orientation maneuvers for the Pioneer, Galileo, and Ulysses spacecraft considered in this study. ... The Pioneer spacecraft only have two- and three-way S-band Doppler. ... analyses of radio Doppler ... data ... indicated that an apparent anomalous acceleration is acting on Pioneer 10 and 11 ... The data implied an anomalous, constant acceleration with a magnitude a_P = 8 x 10^(-8) cm/cm/s^2, directed towards the Sun ...

... the size of the anomalous acceleration is of the order c H, where H is the Hubble constant ...

... Without using the apparent acceleration, CHASMP shows a steady frequency drift of about -6 x 10^(-9) Hz / s, or 1.5 Hz over 8 years (one-way only). ... This equates to a clock acceleration, -a_t, of -2.8 x 10^(-18) s / s^2 . The identity with the apparent Pioneer acceleration is a_P = a_t c. ...

... Having noted the relationships

a_P = c a_t

and that of ...

a_H = c H -> 8 x 10^(-8) cm / s^2

if H = 82 km / s / Mpc ...

we were motivated to try to think of any ... "time" distortions that might ... fit the CHASMP Pioneer results ... In other words ...

Is there any evidence that some kind of "time acceleration" is being seen?

... In particular we considered ... Quadratic Time Augmentation. This model adds a quadratic-in-time augmentation to the TAI-ET (International Atomic Time - Ephemeris Time) time transformation, as follows

ET -> ET + (1/2) a_ET ET^2

The model fits Doppler fairly well

...

There was one [other] model of the ...[time acceleration]... type that was especially fascinating. This model adds a quadratic in time term to the light time as seen by the DSN station:

delta_TAI = TAI_received - TAI_sent ->

-> delta_TAI + (1/2) a_quad (TAI_received^2 - TAI_sent^2)

It mimics a line of sight acceleration of the spacecraft, and could be thought of as an expanding space model.

Note that a_quad affects only the data. This is in contrast to the a_t ... that affects both the data and the trajectory. ... This model fit both Doppler and range very well. Pioneers 10 and 11 ... the numerical relationship between the Hubble constant and a_P ... remains an interesting conjecture. ...".

In his book “Mathematical Cosmology and Extragalactic Astronomy” (Academic Press 1976) (pages 61-62 and 72), Irving Ezra Segal says:
"... Temporal evolution in ... Minkowski space ... is
H -> H + s I
... unispace temporal evolution ... is ...
H -> (H + 2 tan(a/2)) / (1 - (1/2) H tan(a/2)) = H + a I + (1/4) a H^2 + O(s^2)
...".

Therefore,
the Pioneer Doppler anomalous acceleration is an experimental observation of a system that is not gravitationally bound in the Earth-Sun Solar System, and its results are consistent with Segal's Conformal Theory.

Rosales and Sanchez-Gomez say, at gr-qc/9810085:
"... the recently reported anomalous acceleration acting on the Pioneers spacecrafts should be a consequence of the existence of some local curvature in light geodesics when using the coordinate speed of light in an expanding spacetime. This suggests that the Pioneer effect is nothing else but the detection of cosmological expansion in the solar system. ... the ... problem of the detected misfit between the calculated and the measured position in the spacecrafts ... this quantity differs from the expected ... just in a systematic "bias" consisting on an effective residual acceleration directed toward the center of coordinates;
its constant value is ... H c ...
This is the acceleration observed in Pioneer 10/11 spacecrafts. ... a periodic orbit does not experience the systematic bias but only a very small correction ... which is not detectable ... in the old Foucault pendulum experiment ... the motion of the pendulum experiences the effect of the Earth based reference system being not an inertial frame relatively to the "distant stars". ... Pioneer effect is a kind of a new cosmological Foucault experiment, the solar system based coordinates, being not the true inertial frame with respect to the expansion of the universe, mimics the role that the rotating Earth plays in Foucault's experiment ...".

The Rosales and Sanchez-Gomez idea of a 2-phase system in which objects bound to the solar system (in a "periodic orbit") are in one phase (non-expanding pennies-on-a-balloon) while unbound (escape velocity) objects are in another phase (expanding balloon) that "feels" expansion of our universe is very similar to my view of such things as described on this page.
The Rosales and Sanchez-Gomez paper very nicely unites:

the physical 2-phase (bounded and unbounded orbits) view;
the Foucault pendulum idea; and the cosmological value H c.

My view, which is consistent with that of Rosales and Sanchez-Gomez,
can be summarized as a 2-phase model based on Segal's work
which has two phases with different metrics:

a metric for outside the inner solar system, a dark energy phase in which gravity is described in which all 15 generators of the conformal group are effective, some of which are related to the dark energy by which our universe expands;
and
a metric for where we are, in regions dominated by ordinary matter, in which the 4 special conformal and 1 dilation degrees of freedom of the conformal group are suppressed and the remaining 10 generators (antideSitter or Poincare, etc) are effective, thus describing ordinary matter phenomena.

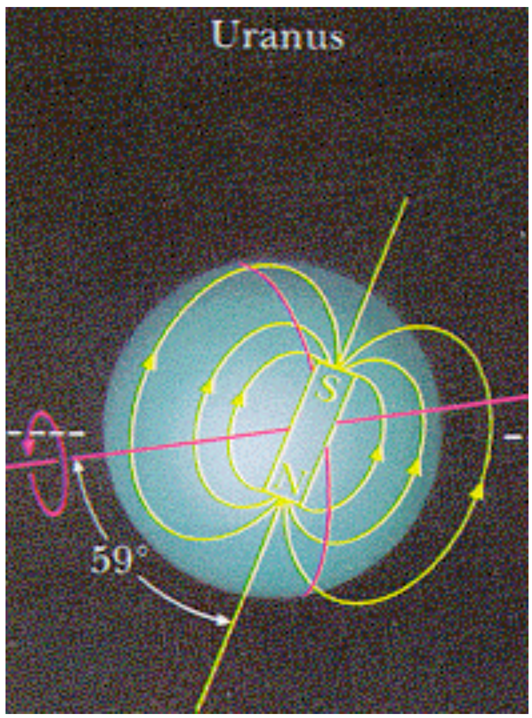
If you look closely at the difference between the metrics in those two regions, you see that the full conformal dark energy region gives an "extra acceleration" that acts as a "quadratic in time term" that has been considered as an explanation of the Pioneer effect by John D. Anderson, Philip A. Laing, Eunice L. Lau, Anthony S. Liu, Michael Martin Nieto, and Slava G. Turyshev in their paper at gr-qc/0104064.

Jack Sarfatti has a 2-phase dark energy / dark matter model that can give a similar anomalous acceleration in regions where c^2 /\ dark energy / dark matter is effectively present. If there is a phase transition (around Uranus at 20 AU)
whereby ordinary matter dominates inside that distance from the sun
and exotic dark energy / dark matter appears at greater distances,
then Jack's model could also explain the Pioneer anomaly
and it may be that Jack's model with ordinary and exotic phases
and my model with deSitter/Poincare and Conformal phases
may be two ways of looking at the same thing.
As to what might be the physical mechanism of the phase transition, Jack says "... Rest masses of [ordinary matter] particles ... require the smooth non-random Higgs Ocean ... which soaks up the choppy random troublesome zero point energy ...".
In other words in a region in which ordinary matter is dominant, such as the Sun and our solar system, the mass-giving action of the Higgs mechanism "soaks up" the Dark Energy zero point conformal degrees of freedom that are dominant in low-ordinary mass regions of our universe (which are roughly the intergalactic voids that occupy most of the volume of our universe).
That physical interpretation is consistent with my view.

Transition at Orbit of Uranus:
It may be that the observation of the Pioneer phase transition at Uranus from ordinary to anomalous acceleration is an experimental result that gives us a first look at dark energy / dark matter phenomena that could lead to energy sources that could be even more important than nuclear energy.

In gr-qc/0104064 Anderson et al say:
"... Beginning in 1980 ... at a distance of 20 astronomical units (AU) from the Sun ... we found that the largest systematic error in the acceleration residuals was a constant bias, aP, directed toward the Sun. Such anomalous data have been continuously received ever since. ...",

so that the transition from inner solar system Minkowski acceleration to outer Segal Conformal acceleration occurs at about 20 AU, which is about the radius of the orbit of Uranus. That phase transition may account for the unique rotational axis of Uranus,



which lies almost in its orbital plane.
The most stable state of Uranus may be with its rotational axis pointed toward the Sun, so that the Solar hemisphere would be entirely in the inner solar system Minkowski acceleration phase and the anti-Solar hemisphere would be in entirely in the outer Segal Conformal acceleration phase.

Then the rotation of Uranus would not take any material from one phase to the other, and there would be no drag on the rotation due to material going from phase to phase.

Of course, as Uranus orbits the Sun, it will only be in that most stable configuration twice in each orbit, but an orbit in the ecliptic containing that most stable configuration twice (such as its present orbit) would be in the set of the most stable ground states, although such an effect would be very small now.
However, such an effect may have been been more significant on the large gas/dust cloud that was condensing into Uranus and therefore it may have caused Uranus to form initially with its rotational axis pointed toward the Sun.
In the pre-Uranus gas/dust cloud, any component of rotation that carried material from one phase to another would be suppressed by the drag of undergoing phase transition, so that, after Uranus condensed out of the gas/dust cloud, the only remaining component of Uranus rotation would be on an axis pointing close to the Sun, which is what we now observe.
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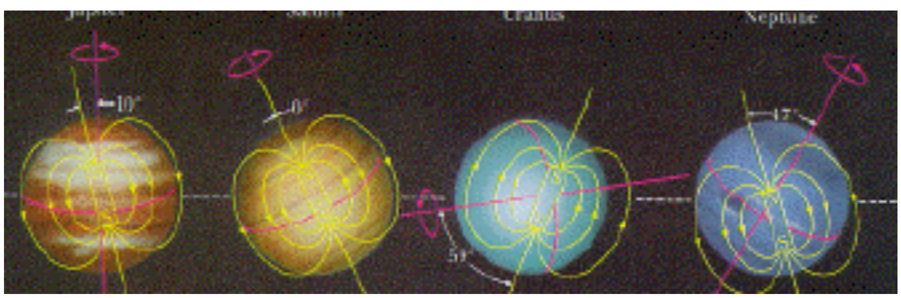
Much of the perpendicular (to Uranus orbital plane) angular momentum from the original gas/dust cloud may have been transferred (via particles "bouncing" off the phase boundary) to the clouds forming Saturn (inside the phase boundary) or Neptune (outside the phase boundary, thus accounting for the substantial (relative to Jupiter) deviation of their rotation axes from exact perpendicularity (see images above and below from “Universe”, 4th ed, by William Kaufmann, Freeman 1994).



According to Utilizing Minor Planets to Assess the Gravitational Field in the Outer Solar System, astro-ph/0504367, by Gary L. Page, David S. Dixon, and John F. Wallin:
"... the great distances of the outer planets from the Sun and the nearly circular orbits of Uranus and Neptune makes it very difficult to use them to detect the Pioneer Effect. ... The ratio of the Pioneer acceleration to that produced by the Sun at a distance equal to the semimajor axis of the planets is 0.005, 0.013, and 0.023 percent for Uranus, Neptune, and Pluto, respectively. ... Uranus' period shortens by 5.8 days and Neptune's by 24.1, while Pluto's period drops by 79.7 days. ... an equivalent change in aphelion distance of 3.8 x 10^10, 1.2 x 10^11, and 4.3 x 10^11 cm for Uranus, Neptune, and Pluto. In the first two cases, this is less than the accepted uncertainty in range of 2 x 10^6 km [or 2 x 10^11 cm] (Seidelmann 1992). ... Pluto['s] ... orbit is even less well-determined ... than the other outer planets. ... . [C]omets ... suffer ... from outgassing ... [and their nuclei are hard to locate precisely] ...".

According to a google cache of an Independent UK 23 September 2002 article by Marcus Chown:
"... The Pioneers are "spin-stabilised", making them a particularly simple platform to understand. Later probes ... such as the Voyagers and the Cassini probe ... were stabilised about three axes by intermittent rocket boosts. The unpredictable accelerations caused by these are at least 10 times bigger than a small effect like the Pioneer acceleration, so they completely cloak it. ...".

20. Dark Energy experiment by BSCCO Josephson Junctions and geometry of 600-cell

I. E. Segal proposed a MInkowski-Conformal 2-phase Universe
and
Beck and Mackey proposed 2 Photon-GraviPhoton phases:
Minkowski/Photon phase locally Minkowski with ordinary Photons and
Gravity weakened by $1 / (M_{Planck})^2 = 5 \times 10^{-39}$.
so that we see Dark Energy as only 3.9 GeV/m^3
Conformal/GraviPhoton phase with GraviPhotons and Conformal symmetry
(like the massless phase of energies above Higgs EW symmetry breaking)
With massless Planck the $1 / M_{Planck}^2$ Gravity weakening goes away
and the Gravity Force Strength becomes the strongest possible = 1
so Conformal Gravity Dark Energy should be enhanced by M_{Planck}^2
from the Minkowski/Photon phase value of 3.9 GeV/m^3.

The Energy Gap of our Universe as superconductor condensate spacetime
is from 3×10^{-18} Hz (radius of universe) to 3×10^{43} Hz (Planck length).
Its RMS amplitude is 10^{13} Hz = 10 THz = energy of neutrino masses =
= critical temperature Tc of BSCCO superconducting crystals.
Neutrino masses are involved because their mass is zero at tree level
and their masses that we observe come from virtual graviphotons becoming virtual neutrino-antineutrino pairs.

BSCCO superconducting crystals are by their structure natural Josephson Junctions. Dark Energy accumulates (through graviphotons) in the superconducting layers of BSCCO.
Josephson Junction control voltage acts as a valve for access to the BSCCO Dark Energy, an idea due to Jack Sarfatti.

Christian Beck and Michael C. Mackey in astro-ph/0703364 said: "... Electromagnetic dark energy .... is based on a Ginzburg-Landau ... phase transition for the gravitational activity of virtual photons ... in two different phases:
gravitationally active [GraviPhotons] ...
and gravitationally inactive [Photons]
...
Let $|P|^2$ be the number density of gravitationally active photons ...
start from a Ginzburg-Landau free energy density ...

$$F = a |P|^2 + (1/2) b |P|^4$$

... The equilibrium state Peq is ... a minimum of F ... for T > Tc ...

$$Peq = 0 \text{ [and] } Feq = 0$$

... for T< Tc

$$|Peq|^2 = - a / b \text{ [and] } Fdeq = -(1/2) a^2 / b$$

... temperature T [of] virtual photons underlying dark energy ... is ..

$$h v = \ln 3 \, k T$$

... dark energy density ...[is]...

rho_dark = (1/2) (pi h / c^3) (v_c)^4

... The currently observed dark energy density in the universe of about 3.9 GeV/m^3 implies that the critical frequency v_c is ...

v_c = 2.01 THz

... BCS Theory yields ... for Fermi energy ... in copper ... 7.0 eV and the critical temperature of ... YBCO ... around 90 K ...

h v_c = 8 x 10^(-3) eV

... Solar neutrino measurements provide evidence for a neutrino mass of about m_v c^2 = 9 x 10^-3 eV ...

[the Cl(16)-E8 model has first-order masses for the 3 generations of neutrinos as
1 x 10^(-3) and 9 x 10^(-3) and 5.4 x 10(-2) eV]

... in solid state physics the critical temperature is essentially determined by the energy gap of the superconductor ... (i.e. the energy obtained when a Cooper pair forms out of two electrons) ...
for [graviphotons] ... at low temperatures (frequencies) Cooper-pair like states [of neutrino-antineutrino pairs] can form in the vacuum ... the ... energy gap would be of the order of typical neutrino mass differences ...".

Clovis Jacinto de Matos and Christian Beck in arXiv 0707.1797 said: "...
Tajmar's experiments ... at Austrian Research Centers Gmbh-ARC ...
with ... rotating superconducting rings ... demonstrated ...
a clear azimuthal acceleration ... directly proportional to the superconductive ring angular acceleration, and
an angular velocity orthogonal to the ring's equatorial plane ...
In 1989 Cabrera and Tate, through the measurement of the London moment magnetic trapped flux, rekported an anomalous Cooper pair mass excess in thin rotating Niobium supeconductive rings ...
A non-vanishing cosmological constant (CC) /\ can be interpreted in terms of a non-vanishing vacuum energy density

rho_vac = (c^4 / 8 pi G) /\

which corresponds to dark energy with equation of state w = -1.
The ... astronomically observed value [is]... /\ = 1.29 x 10^(-52) [1/m^2] ...
Graviphotons can form weakly bounded states with Cooper pairs, increasing their mass slightly from m to m' .
The binding energy is Ec = u c^2 :

m' = m + my - u

... Since the graviphotons are bounded to the Cooper pairs, their zeropoint energies form a condensate capable of the gravitoelectrodynamic properties of superconductive cavities. ...
Beck and Mackey's Ginzburg-Landau-like theory leads to a finite dark energy density dependent on the frequency cutoff v_c of vacuum fluctuations:

rho* = (1/2) (pi h / c^3) (v_c)^4

in vacuum one may put $\rho^* = \rho_{vac}$ from which the cosmological cutoff frequency v_{cc} is estimated as

$$v_{cc} = 2.01 \text{ THz}$$

The corresponding "cosmological" quantum of energy is:

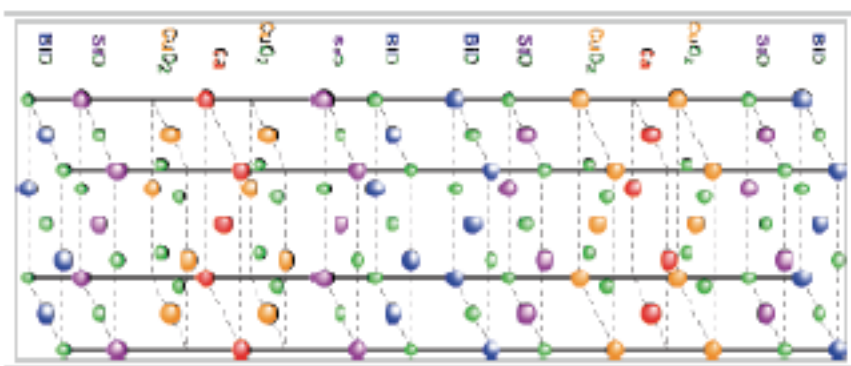
$$E_{cc} = h\, v_{cc} = 8.32 \text{ MeV}$$

... In the interior of superconductors ... the effective cutoff frequency can be different ... $h\, v = \ln 3\, k\, T$ ... we find the cosmological critical temprature T_{cc}

$$T_{cc} = 87.49 \text{ K}$$

This temperature is characteristic of the BSCCO High-Tc superconductor. ...".

Xiao Hu and Shi-Zeng Lin in arXiv 0911.5371 said: "... The Josephson effect is a phenomenon of current flow across two weakly linked superconductors separated by a thin barrier, i.e. Josephson junction, associated with coherent quantum tunneling of Cooper pairs. ... The Josephson effect also provides a unique way to generate high-frequency electromagnetic (EM) radiation by dc bias voltage ... The discovery of cuprate high-Tc superconductors accelerated the effort to develop novel source of EM waves based on a stack of atomically dense-packed intrinsic Josephson junctions (IJJs), since the large superconductivity gap covers the whole terahertz (THz) frequency band. Very recently, strong and coherent THz radiations have been successfully generated from a mesa structure of $Bi_2Sr_2CaCu_2O_{8+d}$ single crystal ...[BSCCO image from Wikipedia



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which works both as the source of energy gain and as the cavity for resonance. This experimental breakthrough posed a challenge to theoretical study on the phase dynamics of stacked IJJs, since the phenomenon cannot be explained by the known solutions of the sine-Gordon equation so far. It is then found theoretically that, due to huge inductive coupling of IJJs produced by the nanometer junction separation and the large London penetration depth ... of the material,
a novel dynamic state is stabilized in the coupled sine-Gordon system, in which +/- pi kinks in phase differences are developed responding to the standing wave of Josephson plasma and are

stacked alternately in the c-axis. This novel solution of the inductively coupled sine-Gordon equations captures the important features of experimental observations.
The theory predicts an optimal radiation power larger than the one observed in recent experiments by orders of magnitude ...".

What are some interesting BSCCO JJ Array configurations ?

Christian Beck and Michael C. Mackey in astro-ph/0605418 describe
"... the AC Josephson effect ...
a Josephson junction consists of two superconductors with an insulator sandwiched in between. In the Ginzburg-Landau theory each superconductor is described by a complex wave function whose absolute value squared yields the density of superconducting electrons. Denote the phase difference between the two wave functions ... by P(t).
...
at zero external voltage a superconductive current given by Is = Ic sin(P) flows between the two superconducting electrodes ... Ic is the maximum superconducting current the junction can support.
...
if a voltage difference V is maintained across the junction, then the phase difference P evolves according to

d P / dt = 2 e V / hbar

i.e. the current ... becomes an oscillating curent
with amplitude Ic and frequency v = 2 e V / h
This frequency is the ... Josephson frequency ... The quantum energy h v ... can be interpreted as the energy change of a Cooper pair that is transferred across the junction ...".

Xiao Hu and Shi-Zeng Lin in arXiv 1206.516 said:
"... to enhance the radiation power in teraherz band based on the intrinsic Josephson Junctions of Bi2Sr2CaCu2O8+d single crystal ...
we focus on the case that the Josephson plasma is uniform along a long crystal as established by the cavity formed by the dielectric material. ...
A ... pi kink state ... is characterized by static +/- pi phase kinks in the lateral directions of the mesa, which align themselves alternatingly along the c -axis. The pi phase kinks provide a strong coupling between the uniform dc current and the cavity modes, which permits large supercurrent flow into the system at the cavity resonances, thus enhances the plasma oscillation and radiates strong EM wave ...
The maximal radiation power ... is achieved when the length of BSCCO single crystal at c-axis equals the EM wave length. ...".

Each long BSCCO single crystal looks geometrically like a line so configure the JJ Array using BSCCO crystals as edges.

The simplest polytope, the **Tetrahedron, is made of 6 edges**:
Feigelman, Ioffe, Geshkenbein, Dayal, and Blatter in cond-mat/0407663 said:
"... Superconducting tetrahedral quantum bits ...

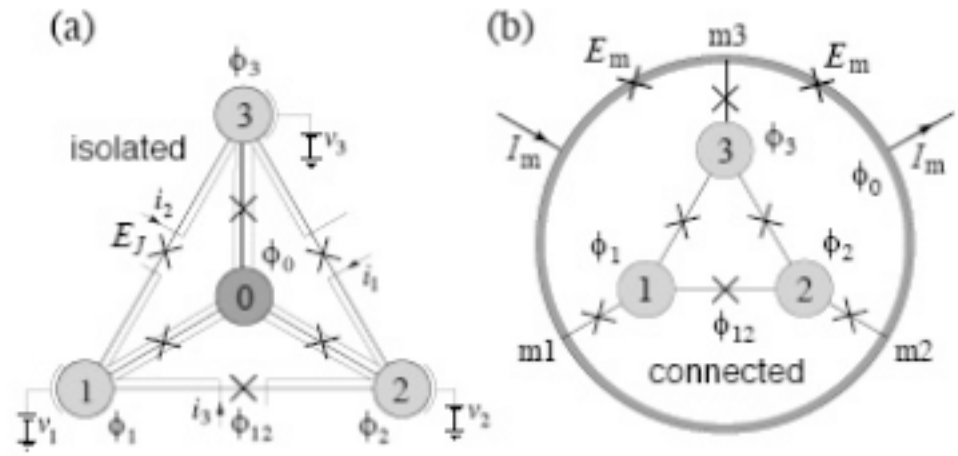
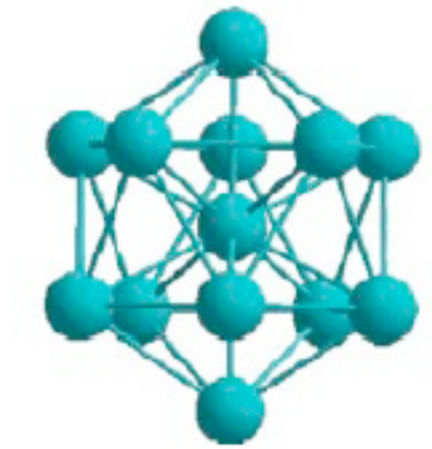


FIG. 1: (a) Tetrahedral superconducting qubit involving four islands and six junctions (with Josephson coupling E_J and charging energy E_C); all islands and junctions are assumed to be equal and arranged in a symmetric way. The islands are attributed phases ϕ_i, $i = 0, \ldots, 3$. The qubit is manipulated via bias voltages v_i and bias currents i_i. In order to measure the qubit's state it is convenient to invert the tetrahedron as shown in (b) — we refer to this version as the 'connected' tetrahedron with the inner dark-grey island in (a) transformed into the outer ring in (b). The measurement involves additional measurement junctions with couplings $E_m \gg E_J$ on the outer ring which are driven by external currents I_m (schematic, see Fig. 6 for details); the large coupling E_m effectively binds the ring segments into one island.

... tetrahedral qubit design ... emulates a spin-1/2 system in a vanishing magnetic field, the ideal starting point for the construction of a qubit. Manipulation of the tetrahedral qubit through external bias signals translates into application of magnetic fields on the spin;
the application of the bias to different elements of the tetrahedral qubit corresponds to rotated operations in spin space. ...".

42 edges make an Icosahedron plus its center
(image from Physical Review B 72 (2005) 115421 by Rogan et al)

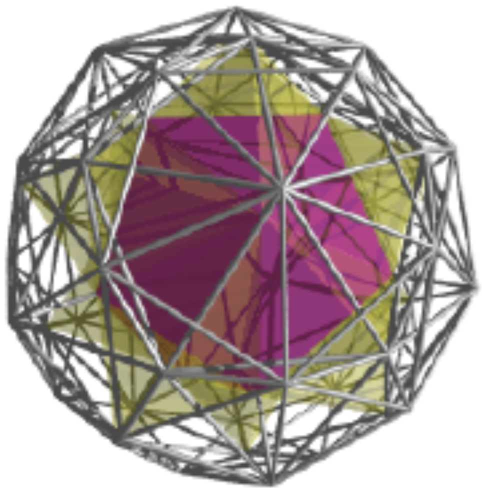


with 30 exterior edges and 12 edges from center to vertices.
It has 20 cells which are approximate Tetrahedra in flat 3-space but become exact regular Tetrahedra in curved 3-space.

Could an approximate-20Tetrahedra-Icosahedron configuration of 42 BSCCO JJ tap into Dark Energy so that the Dark Energy might regularize the configuration to exact Tetrahedra and so curve/warp spacetime from flat 3-space to curved 3-space ?

720 edges make a 4-dimensional 600-cell

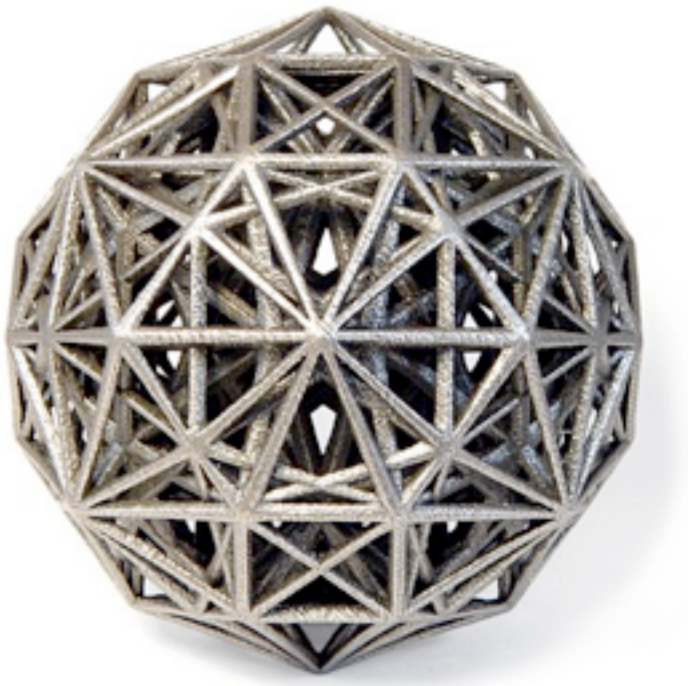
(image from Wikipedia)



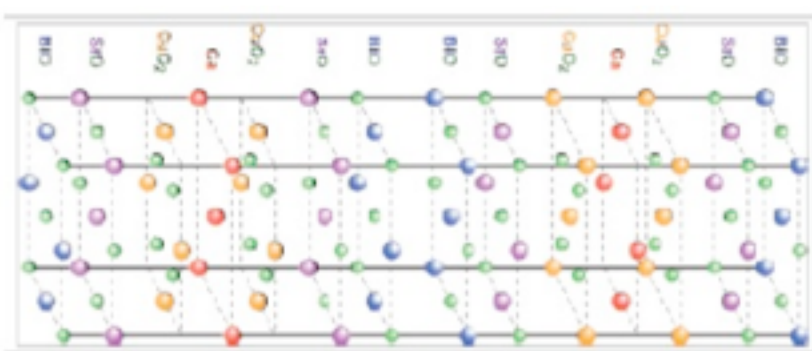
At each vertex 20 Tetrahedral faces meet forming an Icosahedron
which is exact because the 600-cell lives on a curved 3-shere in 4-space.
It has 600 Tetrahedral 3-dim faces and 120 vertices

Could a 600 approximate-Tetrahedra configuration of 720 BSCCO JJ
approximating projection of a 600-cell into 3-space
tap into Dark Energy so that the Dark Energy might regularize
the configuration to exact Tetrahedra and an exact 600-cell
and so curve/warp spacetime from flat 3-space to curved 3-space ?

The basic idea of Dark Energy from BSCCO Josephson Junctions is based on the 600-cell as follows: Consider 3-dim models of 600-cell such as metal sculpture from Bathsheba Grossman who says:
"… for it I used an orthogonal projection rather than the Schlegel diagrams of the other polytopes I build.
… In this projection all cells are identical, as there is no perspective distortion. …".



For the Dark Energy experiment each of the 720 lines would be made of a single BSCCO crystal



whose layers act naturally to make the BSCCO crystal an intrinsic Josephson Junction. (see Wikipedia and arXiv 0911.5371)

Each of the 600 tetrahedral cells of the 600-cell has 6 BSCCO crystal JJ edges.

Since the 600-cell is in flat 3D space the tetrahedra are distorted.

According to the ideas of Beck and Mackey (astro--ph/0703364) and
of Clovis Jacinto de Matos (arXiv 0707.1797) the superconducting
Josephson Junction layers of the 720 BSCCO crystals will bond with
Dark Energy GraviPhotons that are pushing our Universe to expand.

My idea is that the Dark Energy GraviPhotons will not like being
configured as edges of tetrahedra that are distorted in our flat 3D space
and
they will use their Dark Energy to make all 600 tetrahedra to be exact and
regular by curving our flat space (and space-time).

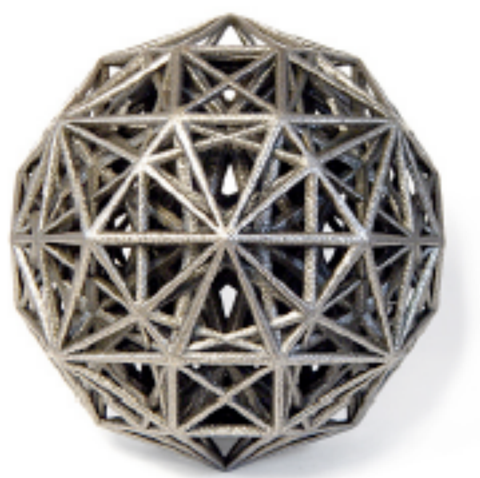
My view is that the Dark Energy Graviphotons will have enough strength
to do that because their strength will NOT be weakened by
the (1 / M_Planck)^2 factor that makes ordinary gravity so weak.

It seems to me to be a clearly designed experiment that will either

1- not work and show my ideas to be wrong or

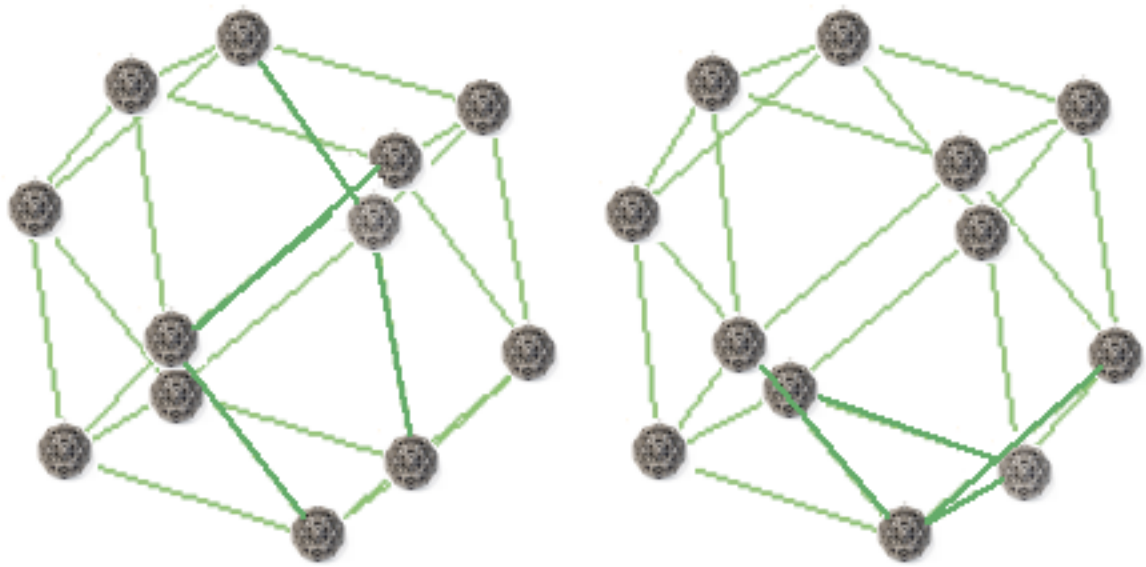
2 - work and open the door for humans to work with Dark Energy.

Consider BSCCO JJ 600-cells

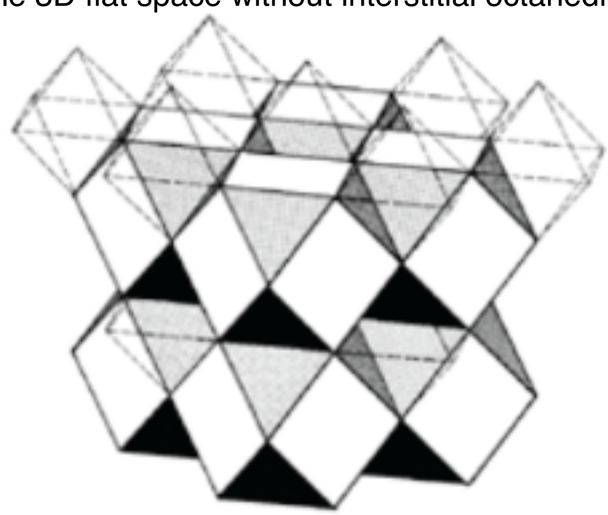


in this configuration:

First put 12 of the BSCCO JJ 600-cells at the vertices of a cuboctahedron
shown here as a 3D stereo pair:

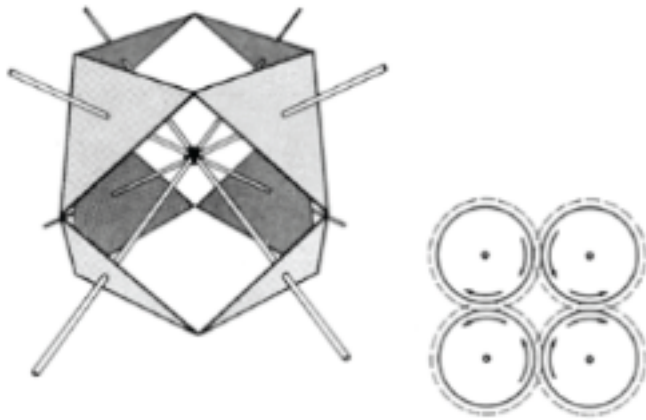


Cuboctahedra do not tile 3D flat space without interstitial octahedra

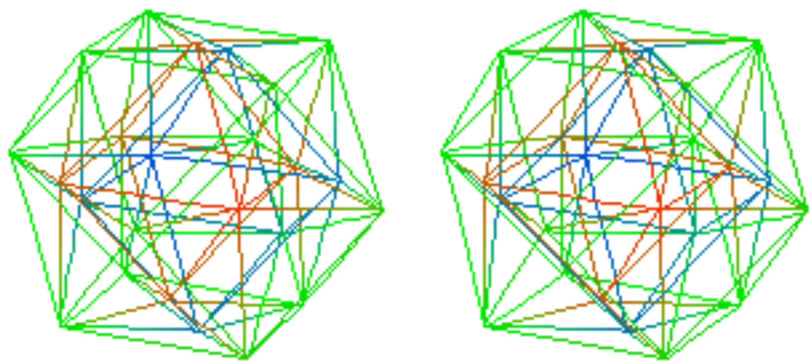


but BSCCO JJ 600-cell cuboctahedra can be put together
square-face-to-square-face in flat 3D configurations including flat sheets.

As Buckminster Fuller described, the 8 triangle faces of a cuboctahedron



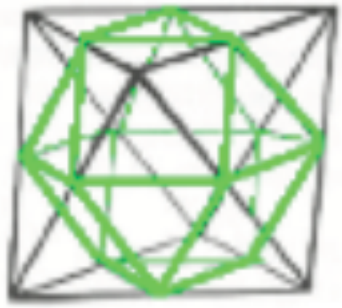
give it an inherently 4D structure consistent with the green cuboctahedron



central figure of a 24-cell (3D stereo 4thD blue-green-red color)
that tiles flat Euclidean 4D space.

So, cuboctahedral BSCCO JJ 600-cell structure likes flat 3D and 4D space
but
if BSCCO JJ Dark Energy act to transform flat space into curved space
like a 720-edge 600-cell with 600 regular tetrahedra
then
Dark Energy should transform cuboctahedral BSCCO JJ 600-cell structure
into
a 720-edge BSCCO JJ 600-cell structure that likes curved space.

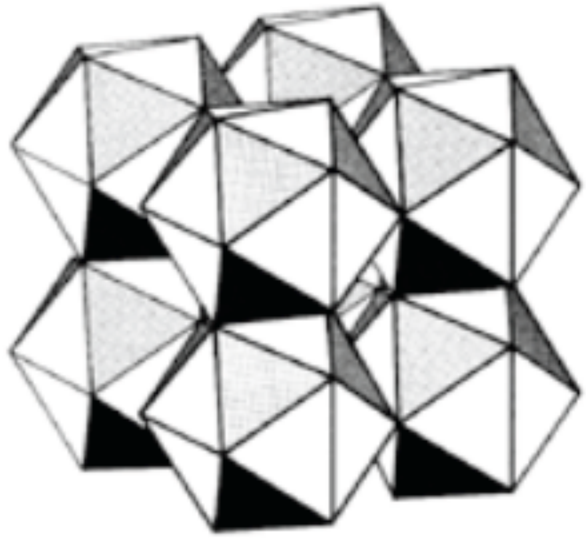
There is a direct JItterbug transformation of
the 12-vertex cuboctahedron to the 12-vertex icosahedron



whereby the 12 cuboctahedron vertices as midpoints of octahedral edges
are mapped to
12 icosahedron vertices as Golden Ratio points of octahedral edges.
There are two ways to map a midpoint to a Golden Ratio point.
For the Dark Energy experiment the same choice of mapping
should be made consistently throughout the BSCCO JJ 600-cell structure.

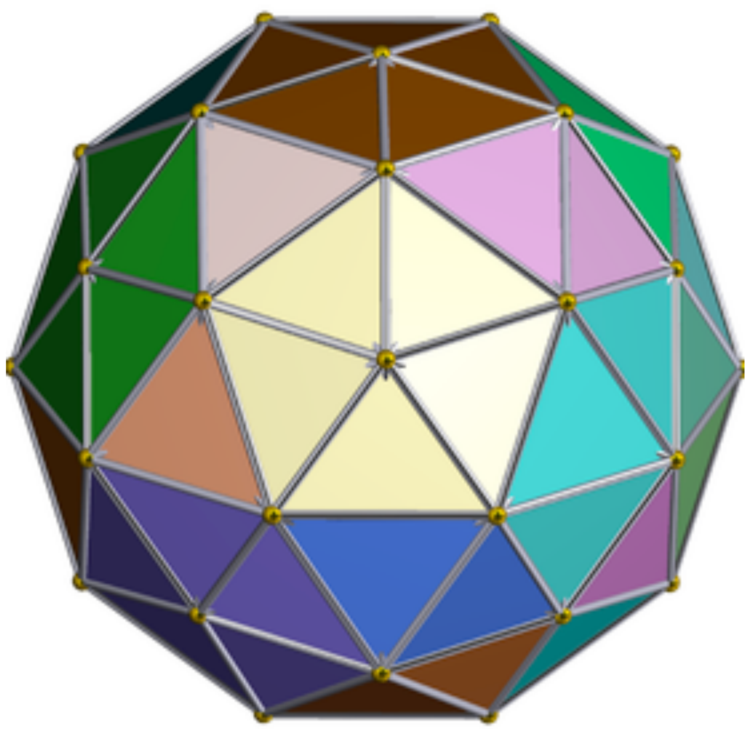
The result of the Jitterbug mapping is that
each cuboctahedron in the BSCCO JJ 600-cell structure
with its 12 little BSCCO JJ 600-cells at its 12 vertices
is mapped to
an icosahedron with 12 little BSCCO JJ 600-cells at its 2 vertices

and the overall cuboctahedral BSCCO JJ 600-cell structure
is transformed into
an overall icosahedral BSCCO JJ 600-cell structure



does not fit in flat 3D space in a naturally characteristic way
(This is why icosahedral QuasiCrystal structures do not extend as simply throughout flat 3D space as do cuboctahedral structures).

However, the BSCCO JJ 600-cell structure JItterbug icosahedra
do live happily in 3-sphere curved space within the icosahedral 120-cell



which has the same 720-edge arrangement as the 600-cell (see Wikipedia).
The icosahedral 120-cell is constructed by 5 icosahedra around each edge.
It has:

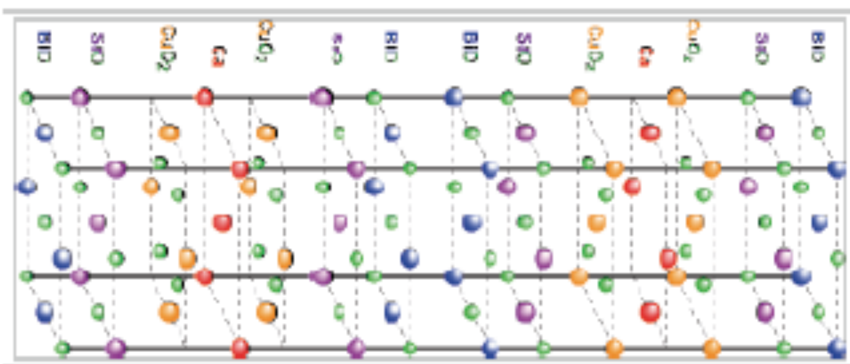
cells - 120 {3,5}
faces - 1200 {3}
edges - 720
vertices - 120
vertex figure - {5,5/2}
symmetry group H4,[3,3,5]
dual - small stellated 120-cell

In summary,

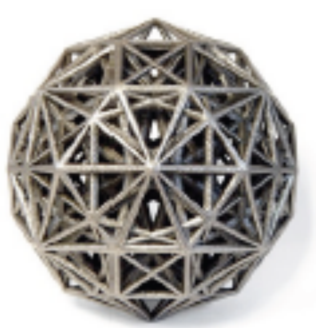
Jitterbug transformations and BSCCO Josephson Junctions may be the Geometric Key to controlling Dark Energy

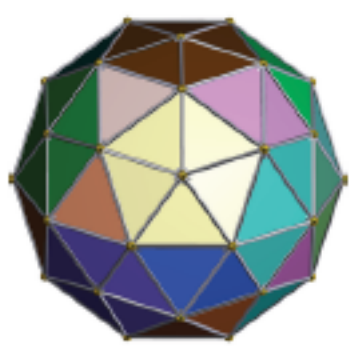
(as were Chain Reactions for Nuclear Fission and Ellipsoidal Focussing for H-Bombs)

The Energy Gap of our Universe as superconductor condensate spacetime is from 3 x 10^(-18) Hz (radius of universe) to 3 x 10^43 Hz (Planck length).
Its RMS amplitude is 10^13 Hz = 10 THz = energy of neutrino masses =
= critical temperature Tc of BSCCO superconducting crystals.



BSCCO superconducting crystals are natural Josephson Junctions.
Dark Energy accumulates in the superconducting layers of BSCCO.
The basic idea of Dark Energy from BSCCO Josephson Junctions is based on the 600-cell each of whose 720 edge-lines would be made of a single BSCCO crystal. It may be useful to use a Jitterbug-type transformation between a 600-cell configuration and a configuration based on icosahedral 120-cells which also have 720 edge-lines:



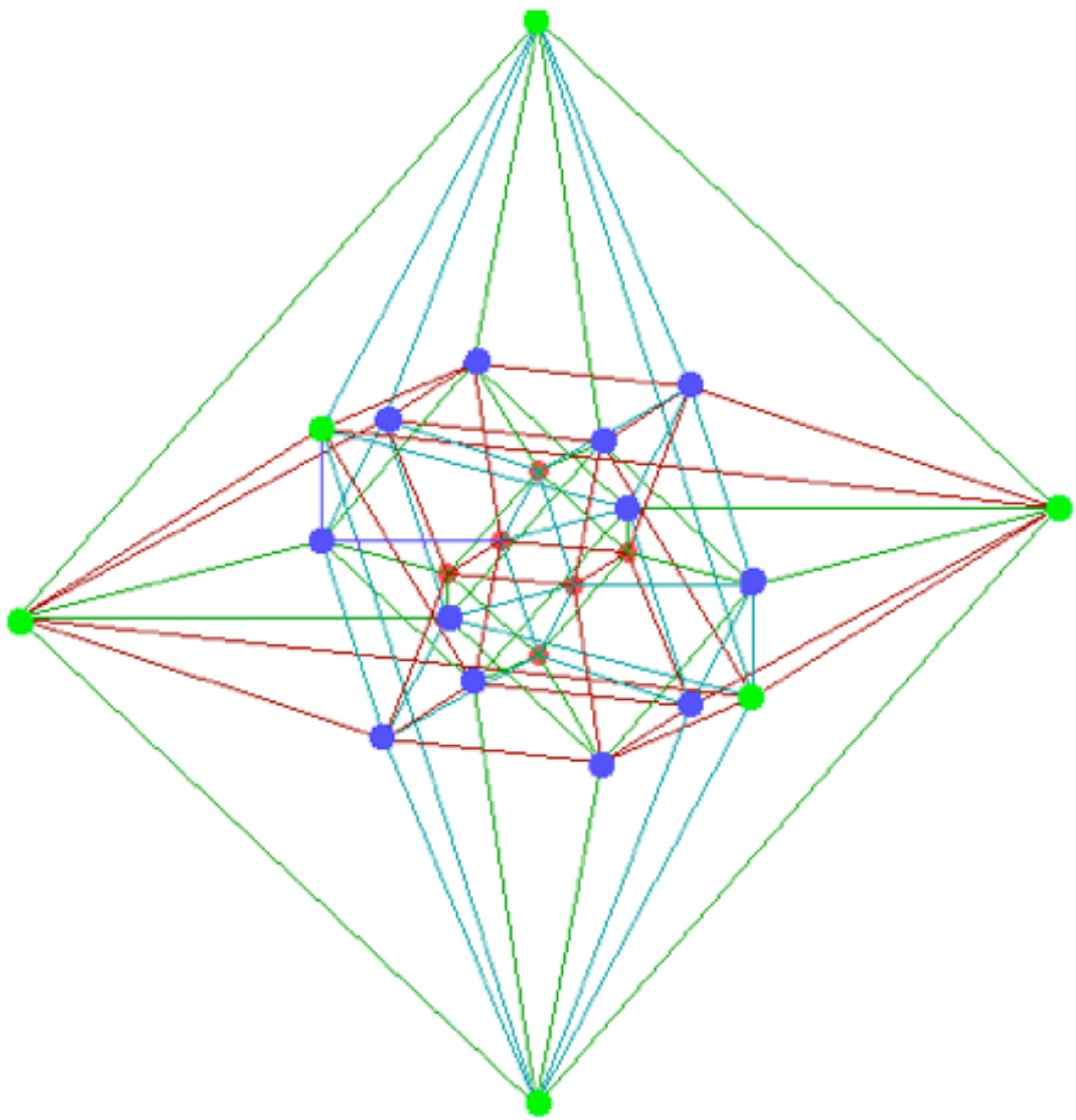


21. 600-cell Geometry of Cl(16)-E8 Physics

Start by building a 600-cell from a 24-cell.

24-cell diagrams here are adapted from those of Frans Marcelis at
http://members.home.nl/fg.marcelis/24-cell.htm#stereographic%20projection

The 24-cell is made up of an Outer Octahedron (green),
a Central Cuboctahedron (blue), and an Inner Octahedron (red).
Physically, it corresponds to the 24 Root Vectors of a D4 Gauge Group
that can represent either Gravity + Dark Energy or the Standard Model.

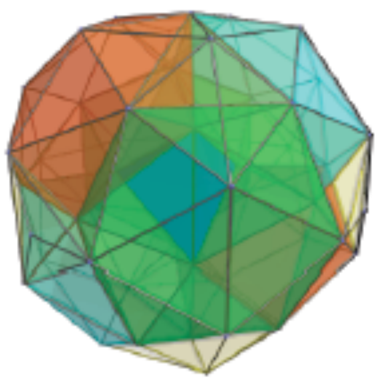


To build a 600-cell, first surround each of the 24 vertices with 5 Tetrahedra
which gives you 120 of its 600 Tetrahedra.

Next, look at the 24 Octahedra that fill up the volume of the 24-cell.
Each Octahedron contains an Icosahedron

(image from wolfram mathworld)

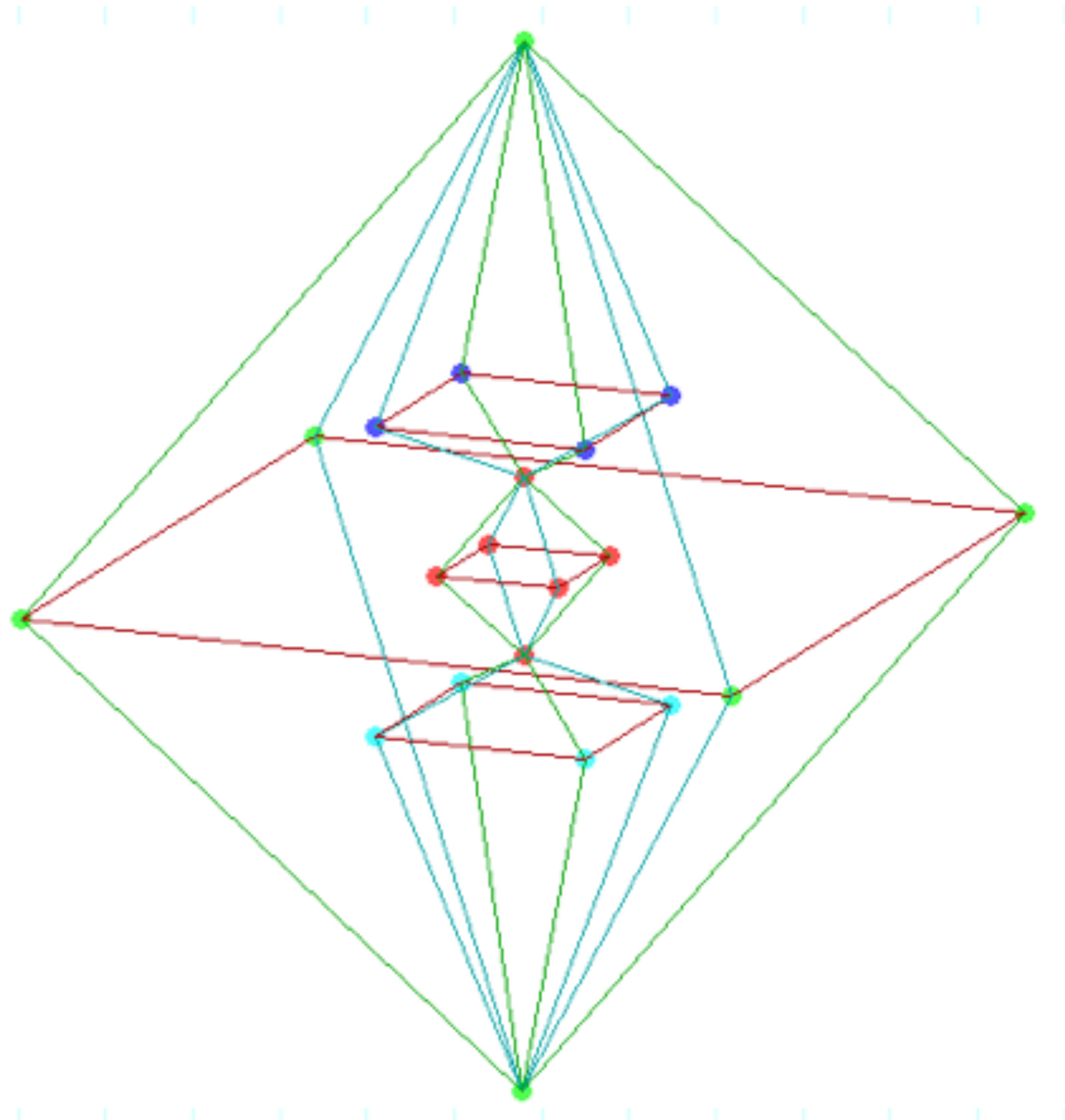
plus some extra volume in each Octahedron.
The extra volume can be divided into 24 vertex Tetrahedra + 96 edge Tetrahedra
so the 24-cell becomes a Snub 24-cell with 24 Icosahedral and 120 Tetrahedral cells

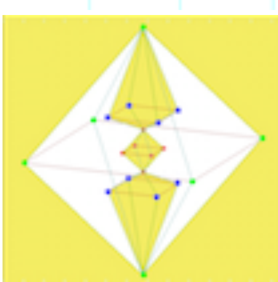


(image from eusebia.dyndns.org)

Each of the 24 Icosahedra contains 20 Tetrahedra for a total of 480 Tetrahedra
which when added to the 24+96 = 120 Tetrahedra outside the Icosahedra
(Tetrahedra are only approximately regular in 3D space but become regular in 4D)
give you the 480+120 = 600 Tetrahedra of the 600-cell.

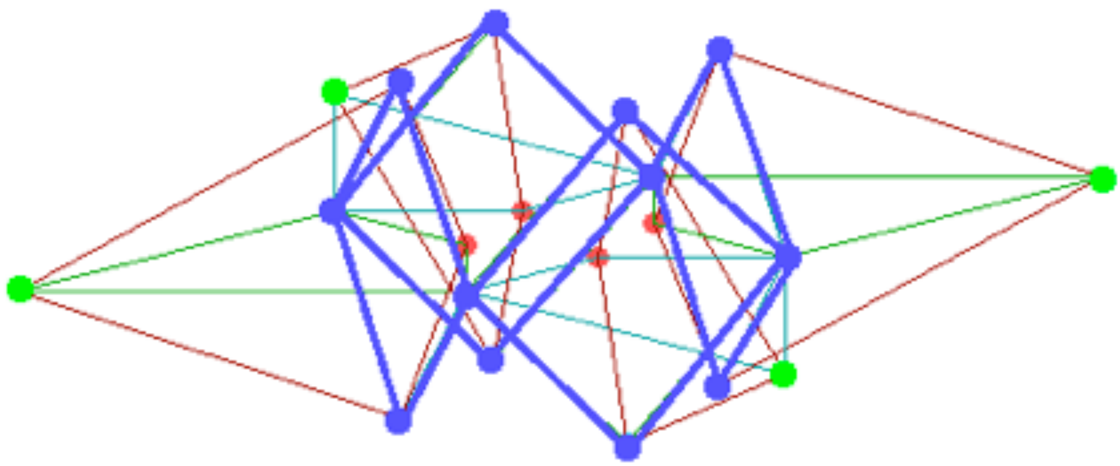
These are 4 of the Octahedra corresponding to 4-dim M4 physical spacetime
within (4+4)-dim M4 x CP2 Kaluza-Klein of B4 / D4
They account for 4x20 = 80 of the 600-cell Tetrahedra.

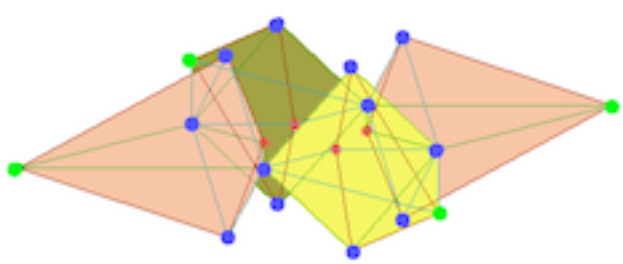




These are 4 of the Octahedra corresponding to 4-dim CP2 internal symmetry space within the (4+4)-dim M4 x CP2 Kaluza-Klein of B4 / D4

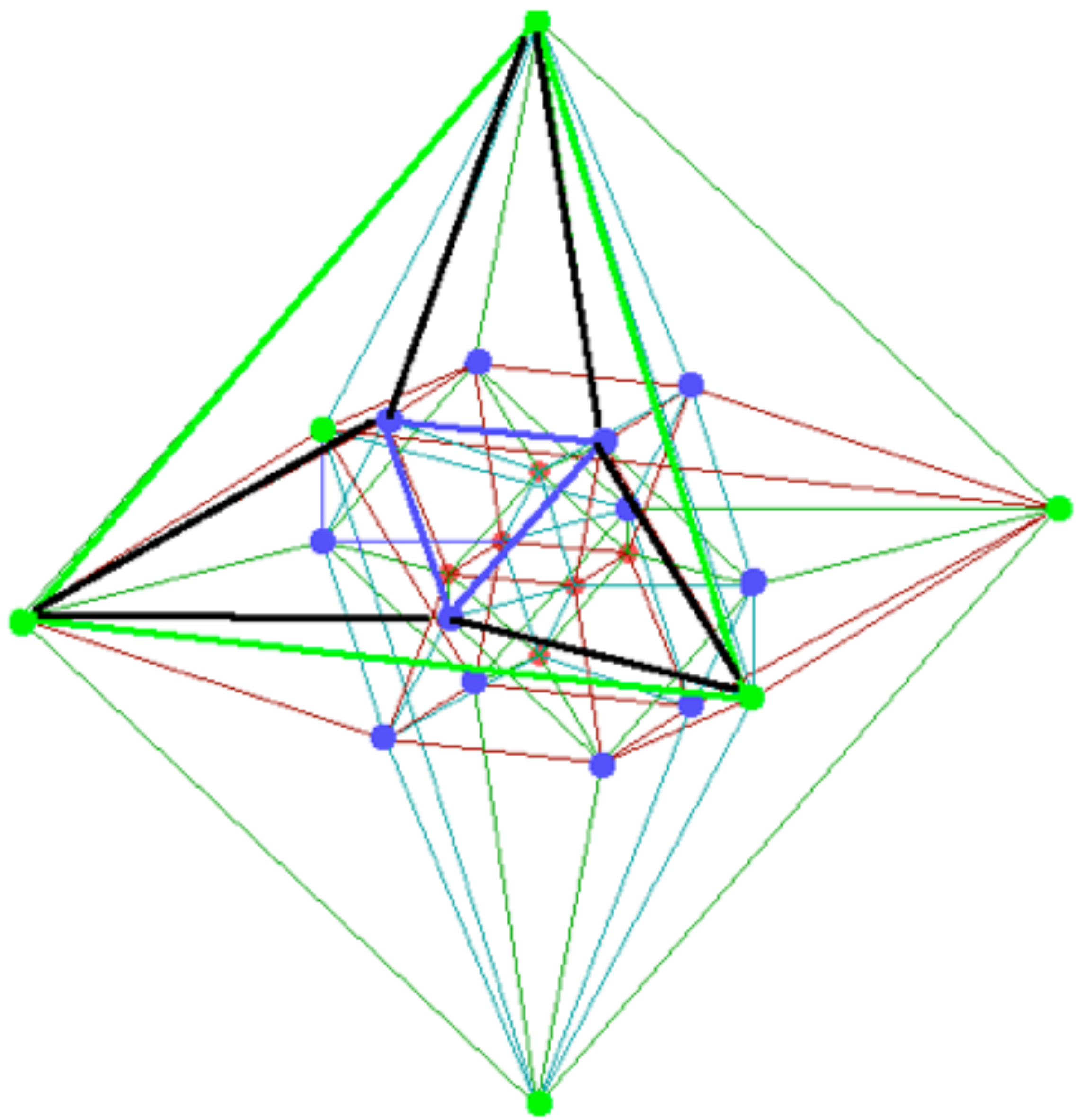
They account for 4x20 = 80 of the 600-cell Tetrahedra.





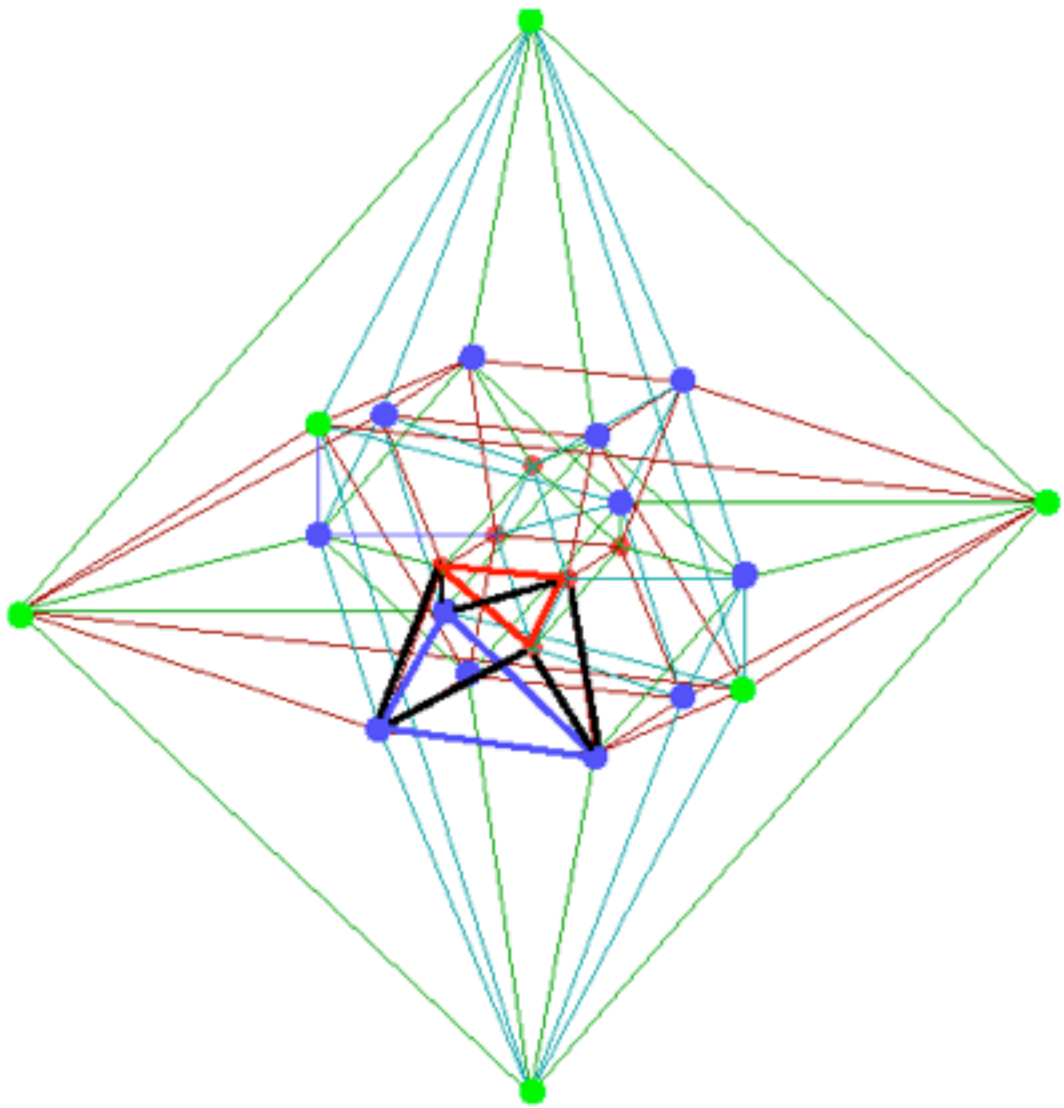
This is one of the 8 Octahedra corresponding to 8 fundamental Fermion Particles
within the (8+8)-dim spinor-type Octonionic Projective Plane of F4 / B4.
The other 7 are similarly configured
on each of the other 7 faces of the outer (green) Octahedron.

These 8 account for 8x20 = 160 of the 600-cell Tetrahedra.



This is one of the 8 Octahedra corresponding to 8 fundamental Fermion AntiParticles
within the (8+8)-dim spinor-type Octonionic Projective Plane of F4 / B4.
The other 7 are similarly configured
on each of the other 7 faces of the inner (red) Octahedron.

These 8 account for 8x20 = 160 of the 600-cell Tetrahedra.

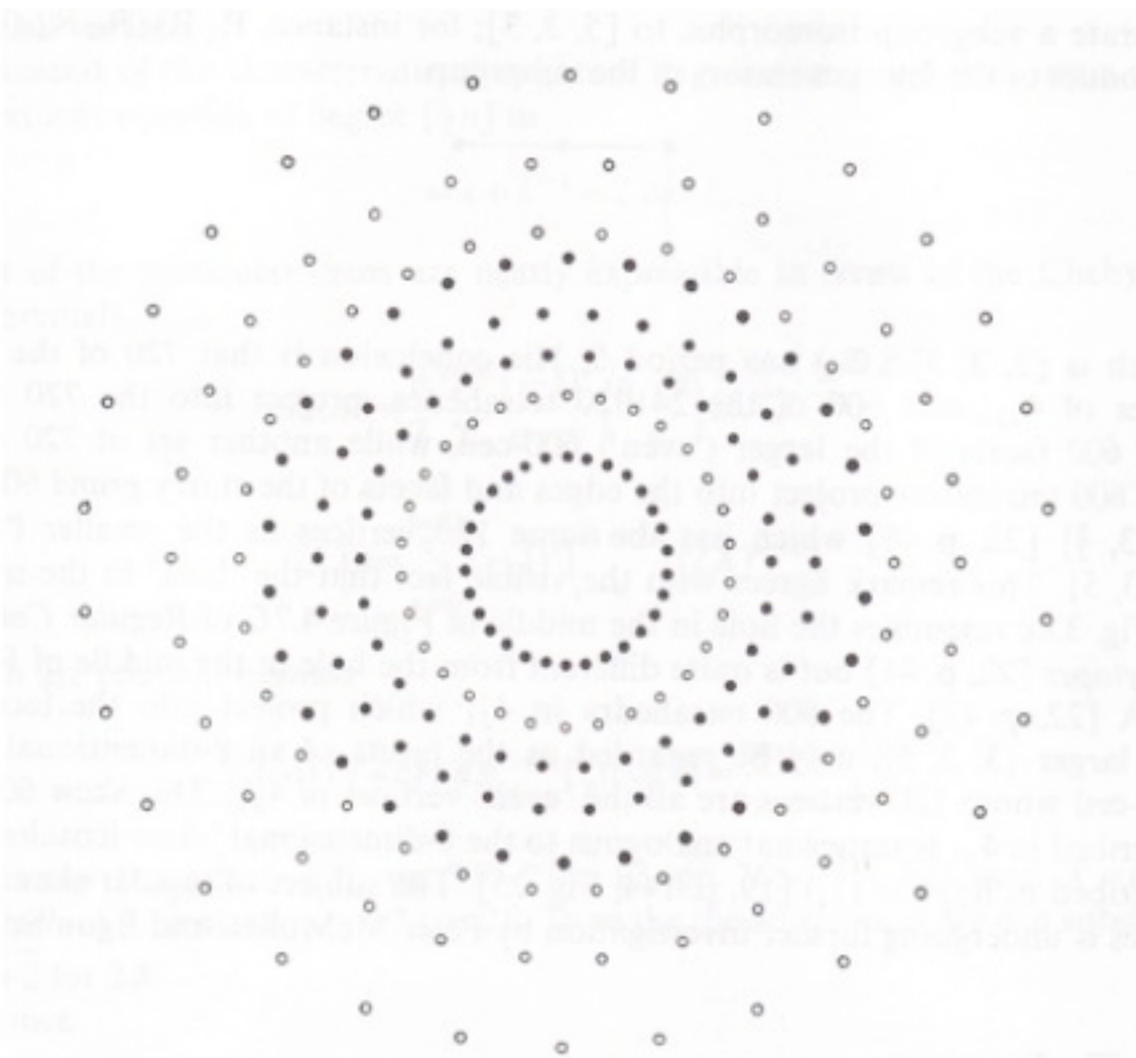


That is all 600 = 120 + 80 + 80 + 160 + 160 = 280 + 320 Tetrahedra of the 600-cell
corresponding to Cl(16)-E8 Physics through the structure of F4 as follows:

120 to 28-dim D4 for SU(2,2) Gauge Gravity or 28-dim D4 for Standard Model SU(3)
80 + 80 to B4 / D4 for (4+4)-dim Kaluza-Klein M4 x CP2
160 +160 to F4 / B4 for (8+8)-dim OP2 spinor fermions

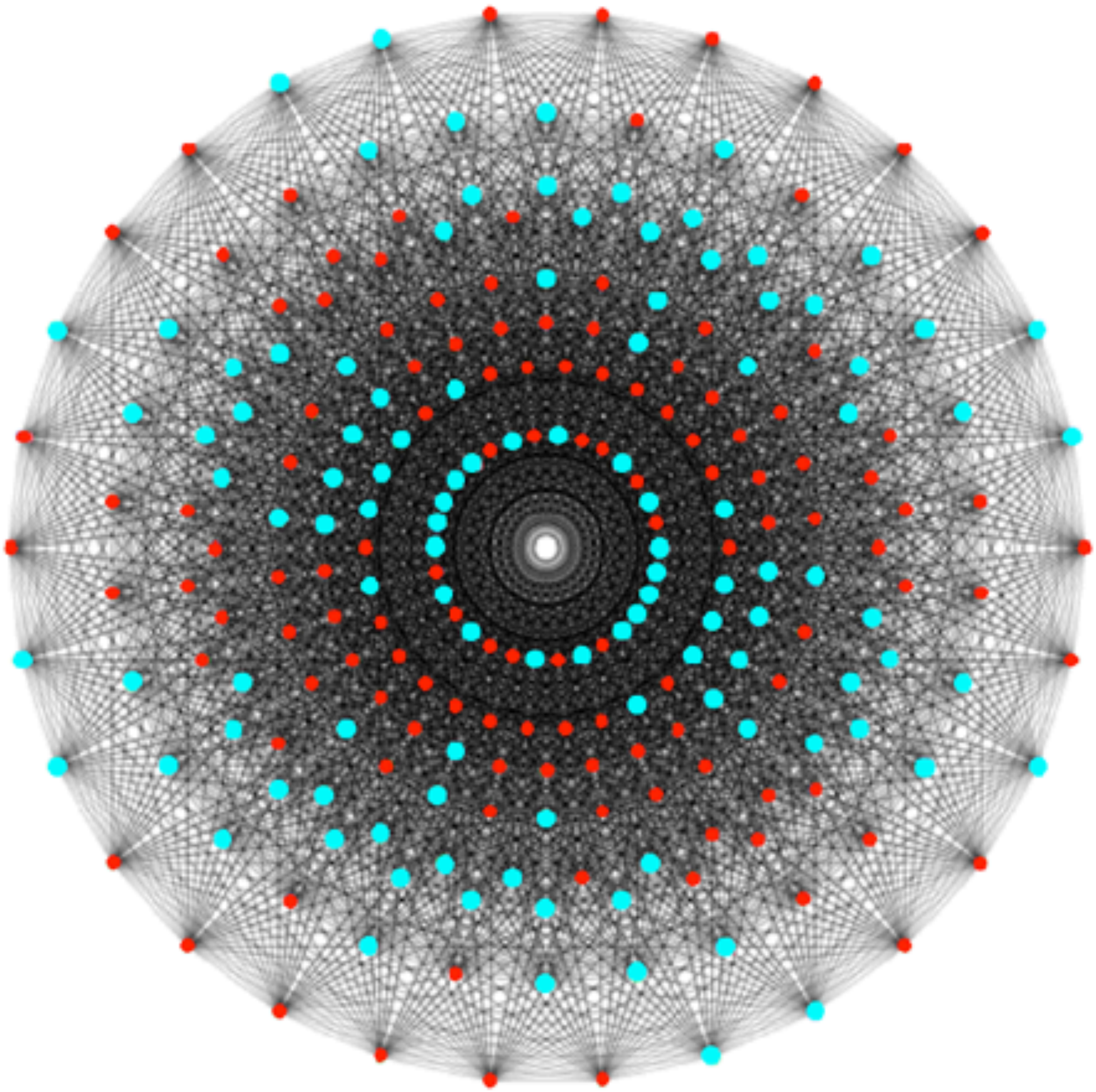
Here is another way to look at 600-cells with respect to Cl(16)-E8 Physics:
The 240 Root Vectors of E8 can be represented by two 120-vertex 600-cells.
One 600 cell corresponds to 4-dim Minkowski SpaceTime and to Gravity + Dark Energy
and to 4 of the 8 coordinates of each Fermion Particle and AntiParticle
while
the other 600 cell corresponds to CP2 Internal Symmetry Space and to Standard Model Gauge Bosons
and to the other 4 of the 8 coordinates of each Fermion Particle and AntiParticle.

Here is a more detailed discussion:
The 240 Root Vectors of E8 can be projected onto 2D as 8 circles of 30 vertices each
as shown in this diagram from Regular and Semi-Regular Polytopes III by Coxeter



in which there are 4 circles of white dots and 4 circles of black dots
with the 120 white-dots being like the 120 black dots expanded by the Golden Ratio.

The 120 + 120 division of the 240 is not the division into spacetime + particles.
That division is shown by
the 240 Root Vectors of E8 being projected onto 2D as 8 circles of 30 vertices each in which there are 112 large dots (colored cyan) and 128 small dots (colored red)



as shown in this diagram adapted from http://www.madore.org/~david/math/e8w.html where Madore says: "... E8 roots can be described, in the coordinate system we have chosen, as the (112) points having coordinates (±1,±1,0,0,0,0,0,0) (where both signs can be chosen independently and the two non-zero coordinates can be anywhere) together with those (128) having coordinates (±½,±½,±½,±½,±½,±½,±½,±½) (where all signs can be chosen independently except that there must be an even number of minuses) ...

the E_8 root system ... can be described as a remarkable polytope in 8 dimensions (also known as the Gosset 4_{21} polytope) having 240 vertices (known, in this context, as "*roots*"), and 6720 edges ...

all vertices are on a sphere with the origin as center; this is specific to E_8 ...
the opposite of each root is again a root, and each one is orthogonal to 126 others, while forming an angle of π/3 with 56 others (those that are connected to it by an edge): the only possible angles between two roots are 0, π/3, π/2, 2π/3 and π.
The group of symmetries of this object is the group, known as the Weyl group of E_8, generated by the (orthogonal) reflections about the hyperplane orthogonal to each root: this is a group of order 696729600 which can also be described as $O_8^+(2)$.
It is also the group of automorphism of the adjacency graph of the polytope.

Those 112 roots which have coordinates of the form (±1,±1,0,0,0,0,0,0) are shown as larger dots, and constitute a so-called D_8 root system inside the E_8 root system, which, as a polytope, is a rectified octacross; the reflections determined by those vertices generate a subgroup of order 5160960 (the Weyl group of D_8, a subgroup of index 2 in $\{\pm 1\} \wr S8$) of the full Weyl group of E_8.

The 128 remaining vertices (forming a demiocteract) are shown as smaller dots; alone, they are not a root system because the reflection determined by one of them does not fix that subset.
Note that this division of the 240 vertices as 112+128 is particular to the chosen coordinate system and is not preserved by symmetries of the whole (except, precisely, by those living in the smaller Weyl group of D_8;
so there are 135 ways of making this decomposition).

One can further divide the roots in two by calling half of them "*positive*" in such a way that the sum of two positive roots, if it is a root, is always positive, and that for every root either it or its opposite is positive; there are many ways to do this (in fact, precisely as many as there are elements in the Weyl group), and we have chosen the division given by a lexicographic order on the coordinates: we call positive those roots such that the leftmost nonzero coordinate is positive (or, by numbering the roots lexicographically from 0 to 239, the positive ones are those numbered 120 through 239). A choice of positive roots is equivalent to a choice of *fundamental* (or *simple*) roots: these are the positive roots which cannot be written as a sum of two positive roots, and it then turns out that these form a basis of the ambient 8-space and, remarkably, that every positive root can be written as a linear combination of fundamental roots with nonnegative integer coefficients (equivalently, the fundamental roots form a non-orthogonal basis in which the coordinates of every root are either all nonnegative or all nonpositive; there is a uniquely defined *greatest root*, whose coordinates in terms of fundamental roots dominates that of every other root, and which happens to be one half the sum of *all* positive roots, fundamental or not: for E_8, it is $\langle 4,3,6,5,4,3,2,2 \rangle$ and, for our choices, it is root number 239, or (1,1,0,0,0,0,0,0)). Any choice of positive/fundamental roots can be brought to any other choice by a unique element of the Weyl group.

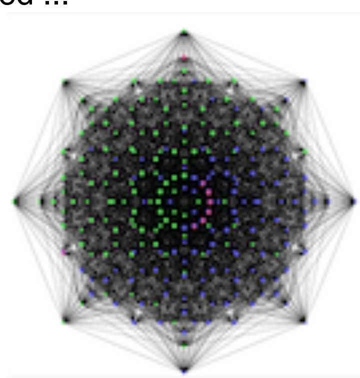
If we represent the eight fundamental roots and connect two by a line whenever they form an angle of $2\pi/3$ (the only other possibility being that they are orthogonal: in the case of E_8, the angles of $3\pi/4$ and $5\pi/6$ do not occur), we obtain the so-called *Dynkin diagram*, which in the case of E_8 has seven nodes in a simple chain and an eighth branching from the third. Here, we number the fundamental roots in the same total order as chosen to define the positive roots (i.e., lexicographic order on the coordinates; then the fundamental roots 1 through 8 are the roots numbered 120, 121, 122, 126, 132, 140, 150 and 162), and the Dynkin diagram has fundamental roots 8–1–3–4–5–6–7 in a chain and fundamental root number 2 branching off from 3.

The fundamental roots are important because the reflection with respect to them suffice to generate the Weyl group. Furthermore, the minimal length of an expression of a given element of the Weyl group as such a product of fundamental reflections (the *length* relative to the given element for the chosen system of fundamental roots) is equal to the number of positive roots whose image is a negative root; and composing by a fundamental reflection will always increase or decrease by 1 the length of the Weyl group element. ...

reflection with respect ...[a]... root ... permutes that root with its opposite,
fixes 126 others, and exchanges the 112 remaining roots as 56 pairs ...
The 696729600 elements of the Weyl group are generated by such reflections ...

Each element of the Weyl group can be written as a product (of a uniquely defined length) of reflections by eight fundamental roots ...

The default ... projection ... in which positive roots (for the particular order chosen) are represented in blue and negative roots in green, and the eight fundamental roots (relative to that order) are labeled ...



... is related to the chosen coordinate system in that it can be described by linearly combining the coordinates with coefficients given by eight consecutive complex sixteenth roots of unity. ...".

The 112 large cyan dots correspond to the D8 subalgebra of E8
which represents SpaceTime and Gauge Bosons

The 128 small red dots correspond to Fermion Particles and AntiParticles

The circles break down like this:
inner - black dots - 18 SpaceTime Gauge Boson and 12 Fermion
second from center - white dots - 10 SpaceTime Gauge Boson and 20 Fermion
third from center - black dots - 10 SpaceTime Gauge Boson and 20 Fermion
fourth from center - black dots - 10 SpaceTime Gauge Boson and 20 Fermion
fifth from center - black dots - 18 SpaceTime Gauge Boson and 12 Fermion
sixth from center - white dots - 18 SpaceTime Gauge Boson and 12 Fermion
seventh from center - white dots - 18 SpaceTime Gauge Boson and 12 Fermion
eighth from center (outer) - white dots - 10 SpaceTime Gauge Boson and 20 Fermion

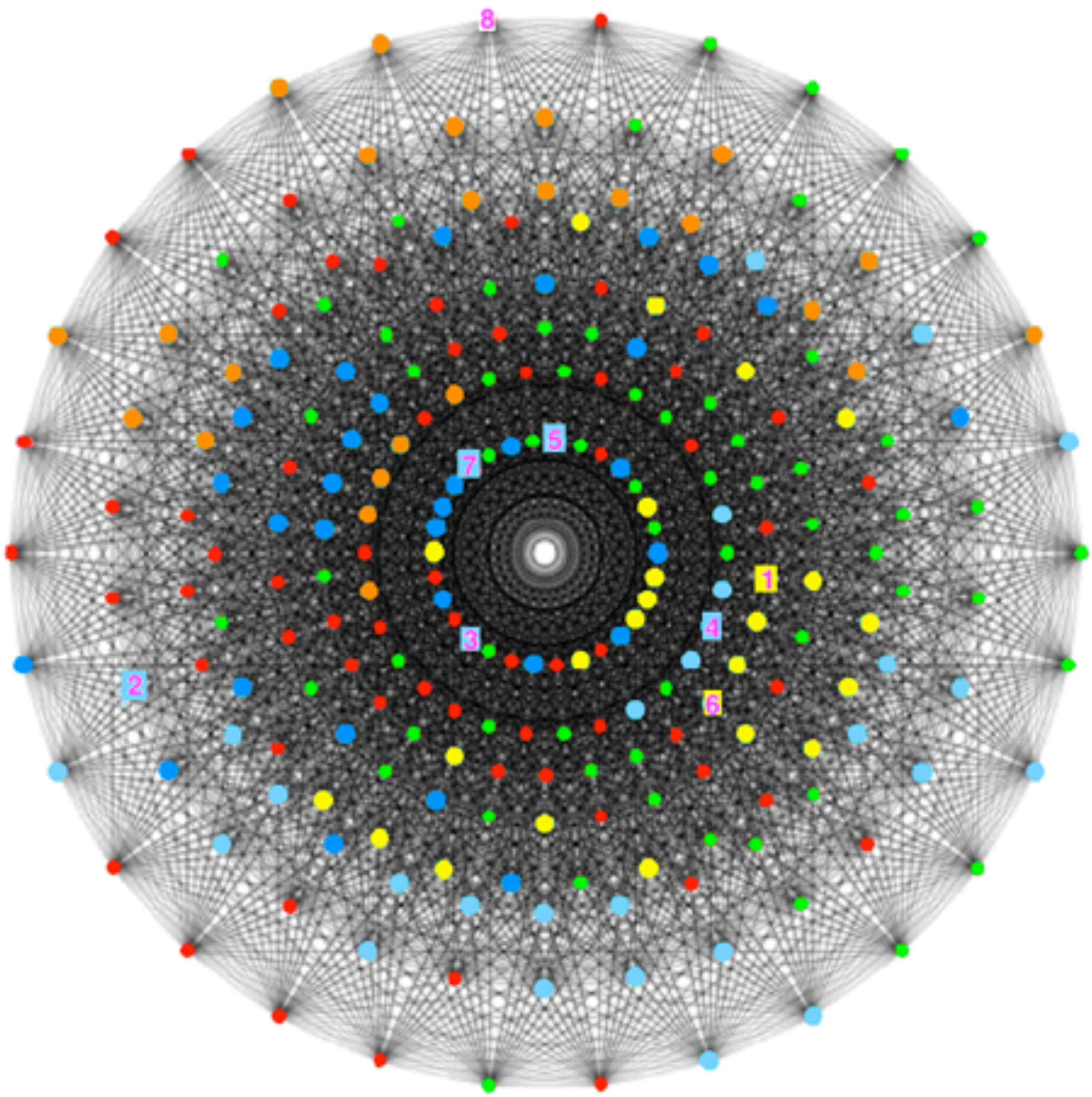
There are 4x12 + 4x20 = 128 Fermion small red dots
and
4x18 + 4x10 = 112 SpaceTime Gauge Boson large cyan dots

The black-dot 4 circles of the small 600-cell contain
56 SpaceTime Gauge Boson cyan dots and 64 Fermion red dots.
You can take the small 600-cell to correspond to M4 4D physical spacetime
so that
24 of the 56 give Gauge Bosons for Gravity + Dark Energy
and
32 of the 56 give 4 M4 spacetime components of 8-dim Momentum
and
32 of the 64 give 4 M4 spacetime components of 8 fundamental Fermion Particles
and
32 of the 64 give 4 M4 spacetime components of 8 fundamental Fermion AntiParticles

The white-dot 4 circles of the large (by Golden ratio) 600-cell also contain
56 SpaceTime Gauge Boson cyan dots and 64 Fermion red dots.
You can take the small 600-cell to correspond to CP2 4D Internal Symmetry Space
so that
24 of the 56 give Gauge Bosons for Standard Model SU(3)
and
32 of the 56 give 4 CP2 internal symmetry space components of 8-dim Momentum
and
32 of the 64 give 4 CP2 internal symmetry components of 8 Fermion Particles
and
32 of the 64 give 4 CP2 internal symmetry components of Fermion AntiParticles

In terms of the Madore 8 circles of 30 version of the 240 E8 Root Vectors:

8 Fundamental Root Vectors 1-8 of which 5 are in the 64 representing SpaceTime
and 2 are in the 24 representing Conformal Gravity
and the 8th is in the 64 representing Fermion Particles.



E8 / D8 = 128 = 64 + 64:
63 green representing 63 of the 64 representing Fermion Particles
64 red representing Fermion AntiParticles
59 blue (light and dark) representing 59 of the 64 representing D8 / D4xD4 SpaceTime
24 orange representing D4 containing the Standard Model SU(3)
22 yellow of the 24 representing D4 containing Conformal SU(2,2) = Spin(2,4) Gravity

There are:
22 yellow dots + 2 Fundamental Root Vector (nos. 1,6 of 8) = 24
(+ 4 Cartan Elements) for Gravity + Dark Energy:
5 in black circle 1 (inner)
5 in black circle 3
5 in black circle 4
9 in black circle 5
3 + 4+2 + 3 = 12 Conformal SU(2,2)=Spin(2,4) Root Vectors

24 orange dots (+ 4 Cartan Elements) for the Standard Model:
5 in white circle 2
8 in white circle 6
7 in white circle 7
4 in white circle 8 (outer)
1+2+2+1 = 6 Standard Model SU(3) Root Vectors

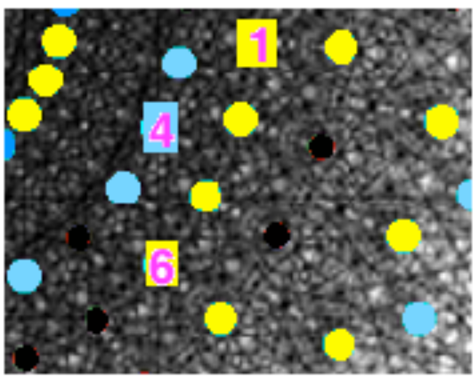
63 green dots + 1 Fundamental Root Vector (no. 8 of 8) = 64 for Fermion Particles:
6 in black circle 1 (inner)
10 in white circle 2
10 in black circle 3
10 in black circle 4
6 in black circle 5
6 in white circle 6
6 in white circle 7
10 in white circle 8 (outer)

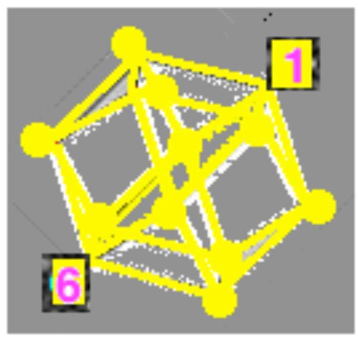
64 red dots for Fermion AntiParticles:
6 in black circle 1 (inner)
10 in white circle 2
10 in black circle 3
10 in black circle 4
6 in black circle 5
6 in white circle 6
6 in white circle 7
10 in white circle 8 (outer)

59 blue dots + 5 Fundamental Root Vector (nos. 2,3,4,5,7 of 8) =
= (28+4)+32 (positive+negative) = 64 for SpaceTime:
4+9 = 13 in black circle 1 (inner)
5+0 = 5 in white circle 2
0+5 = 5 in black circle 3
0+5 = 5 in black circle 4
0+9 = 9 in black circle 5
9+1 = 10 in white circle 6
9+2 = 11 in white circle 7
5+1 = 6 in white circle 8 (outer)

E8 / D8 Fermion Particles and AntiParticles are distributed through all 8 circles
D8 / D4xD4 SpaceTime is distributed through all 8 circles
32 dark blue Negative Root Vectors (28 of them in the 4 circles of the inner 600-cell) correspond to CP2 internal symmetry space of M4xCP2 (4+4)-dim Kaluza-Klein that is directly related to
the D4 (orange) of Standard Model in 24 Negative Root Vectors in the outer 600-cell and
27 light blue Positive Root Vectors in the outer Golden Ratio 600-cell 4 circles
plus 2 Fundamental Root Vectors 4 and 2 in the outer Ratio 600-cell
plus 3 Fundamental Root Vectors 3-5-7 in the inner 600-cell
correspond (27+2+3 = 32) to M4 physical spacetime of M4xCP2 Kaluza-Klein

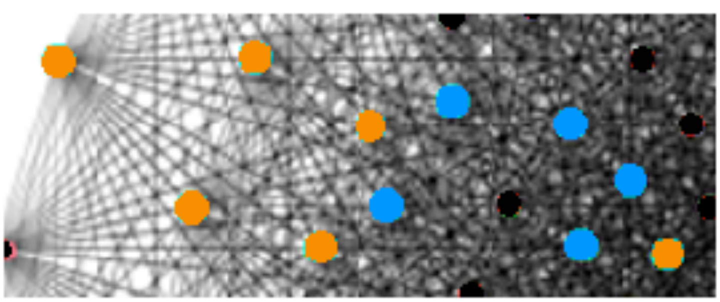
D4 (yellow) of Conformal Gravity is in 22 Positive Root Vectors in the inner 600-cell and 2 Fundamental Root Vectors 1 and 6 in the inner 600-cell for 22+2 = 24
These 12 D4 Conformal Gravity Root Vectors = cuboctahedron polytope

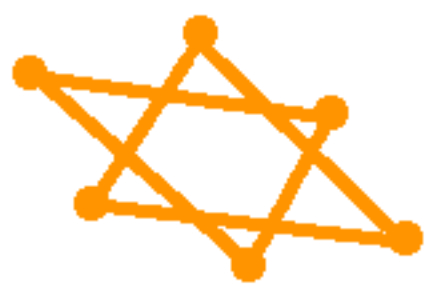




represent the Conformal D3 = SU(2,2) = Spin(2,4) subgroup of that D4

D4 (orange) of Standard Model SU(3) is in 24 Negative Root Vectors in the outer 600-cell
These 6 D4 Standard Model Root Vectors = Star of David polytope

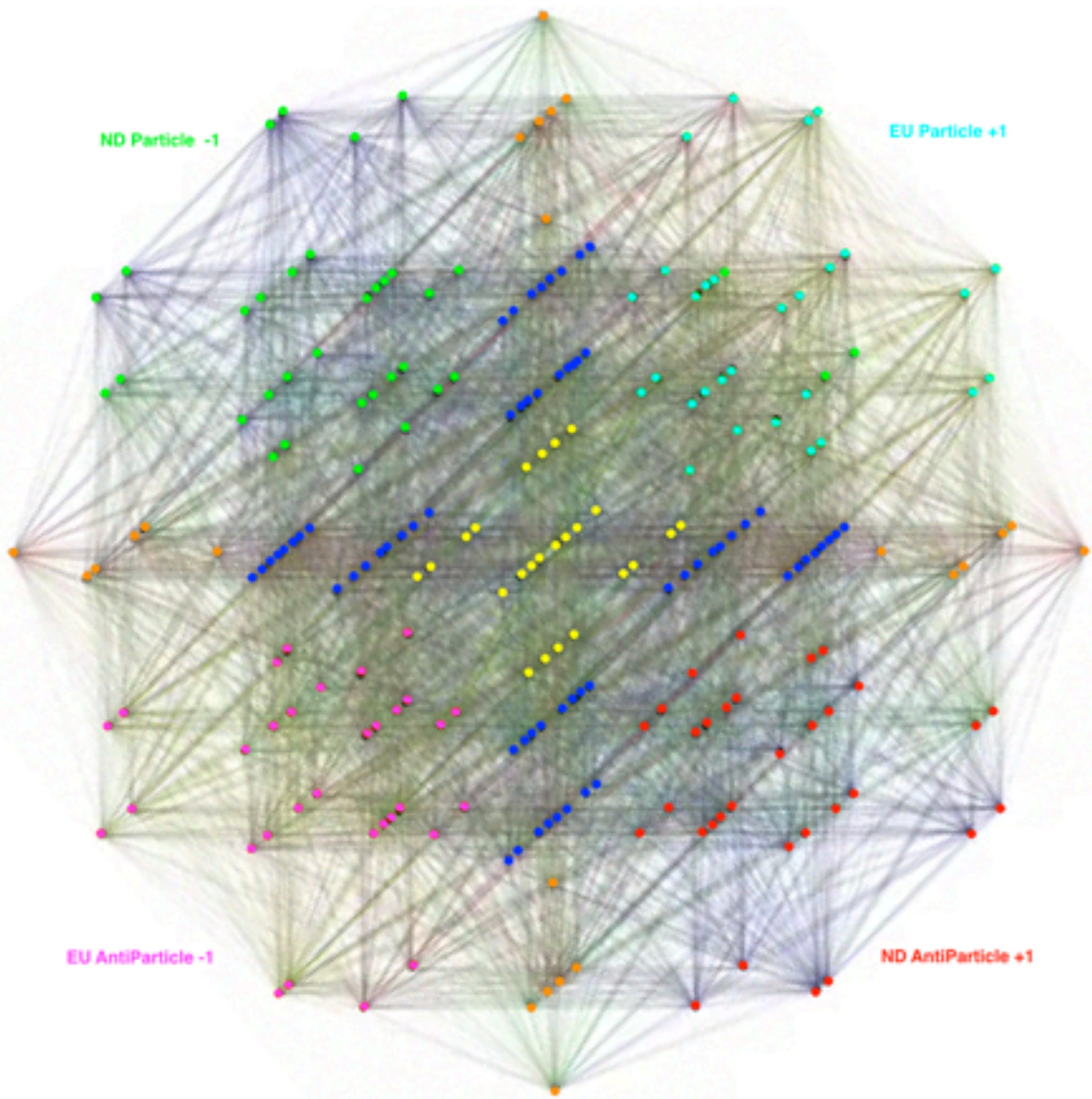




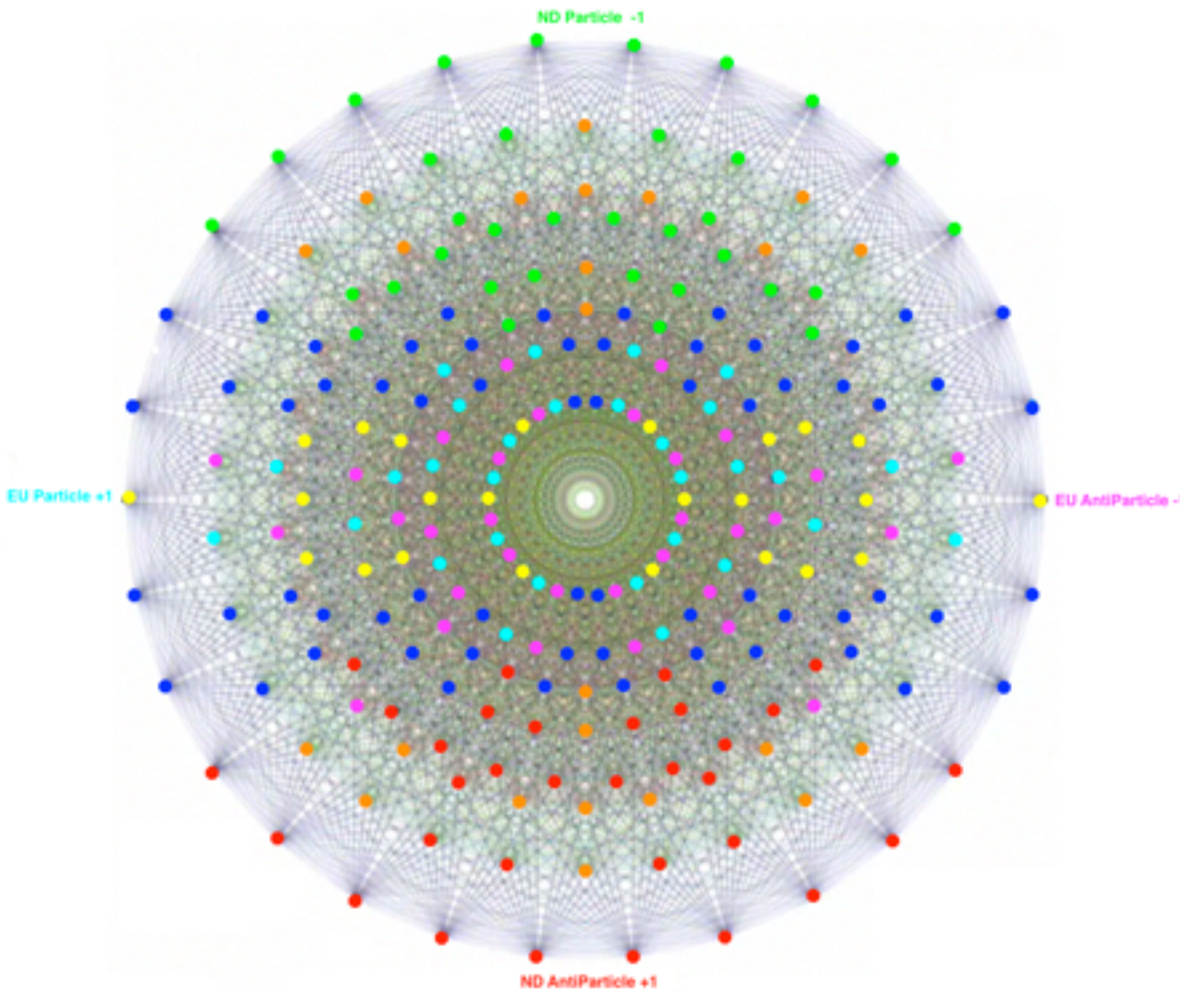
represent the SU(3) subgroup of that D4

The Madore 8 circles of 30 version gives realistic physics
but the physics interpretation of the vertices is not clear and obvious to me.

As Madore says there are many versions of 8 circles of 30 and
as to clearly visualizing how to build a realistic Lagrangian
I prefer a version of 8 circles of 30 that is derived from the square/cube type projection



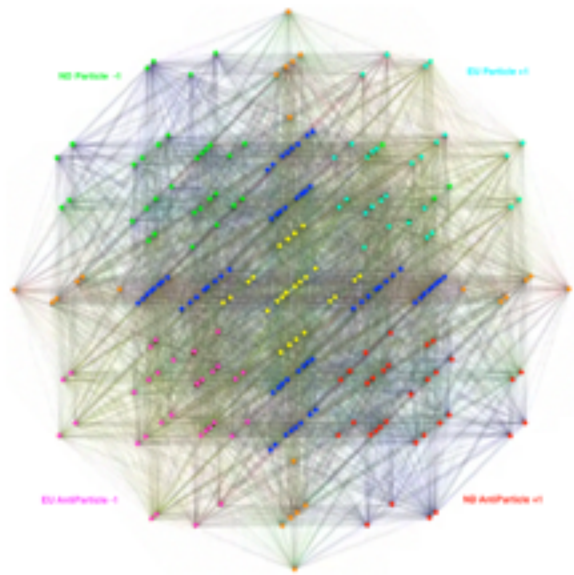
that I use in my paper at http://vixra.org/pdf/1405.0030v9.pdf
in which it the physical interpretations of the root vectors are clear:
green / cyan and red / magenta for fermion particles and antiparticles (E8 / D8)
blue for M4 x CP2 Kaluza-Klein SpaceTime (D8 / D4xD4)
yellow for Gravity + Dark Energy (one of the D4)
orange for Standard Model SU(3) (the other D4)
The square/cube version transforms into this 8x30 version

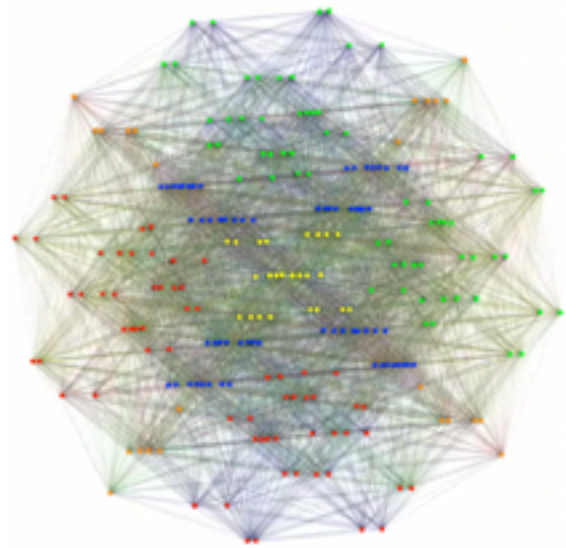


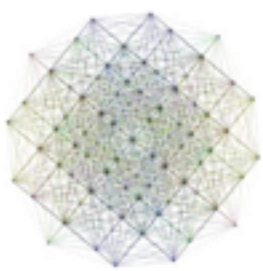
by means a video from mathematica code by Garrett Lisi ca 2007.

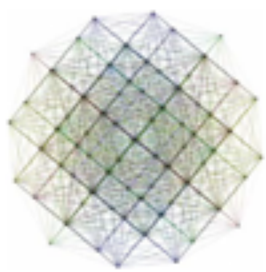
I have converted the video into a pdf slide sequence
and added vertex-colored square images at the beginning
and vertex-colored circle images at the end which pdf file is at
http://tony5m17h.net/E8squarecirclepdf.pdf

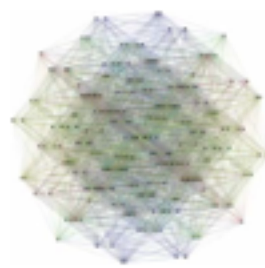
Here are some small images from that pdf file:

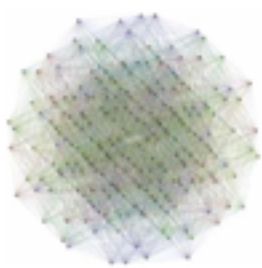


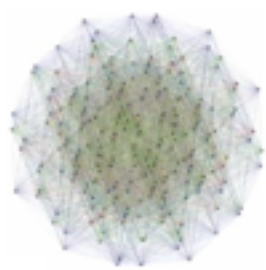


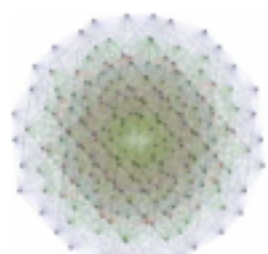


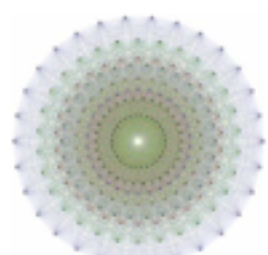


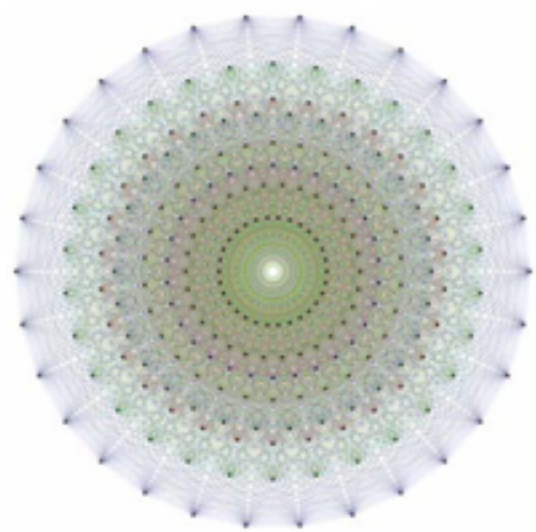


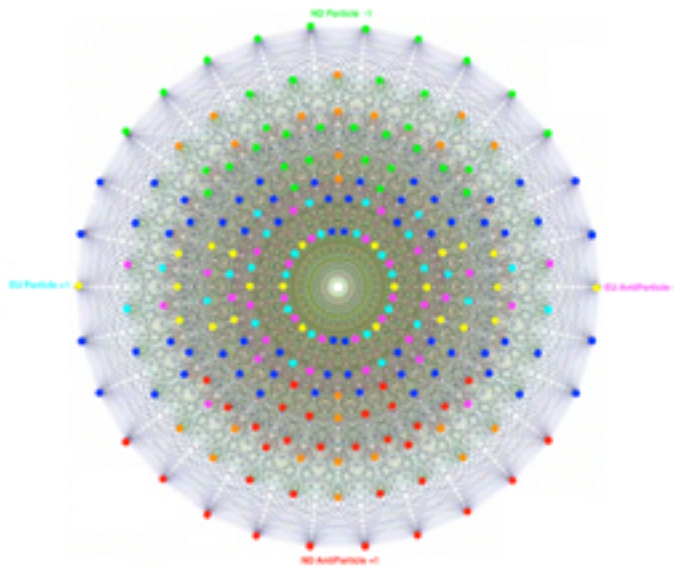




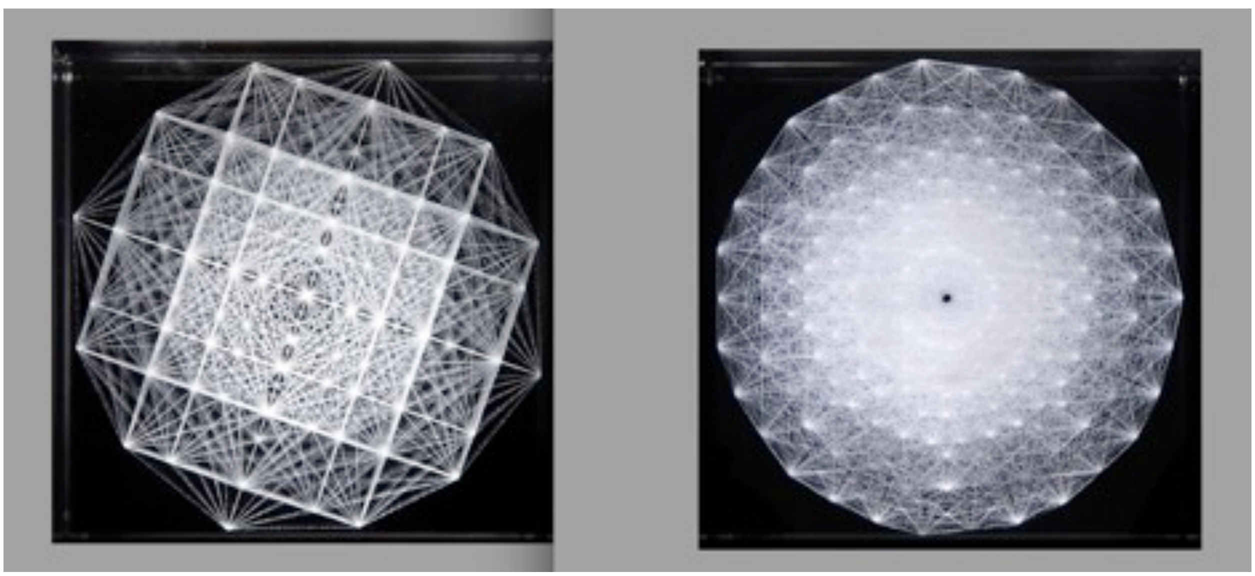








Similar code was used by Bathsheba Grossman in making her E8 cystal cube
two of whose faces



show that the square and circle projections are of the same E8.

22. From SU(2) to E8 for Cl(16)-E8 Physics

Frank Dodd (Tony) Smith, Jr. - 2014

Cl(16)-E8 Physics is described in viXra 1405.0030 from a top-down point of view of fundamental Clifford Algebra structure containing E8 leading to Lagrangian based on starting with E8 (the top Lie algebra) and then looking down at its substructures:

E8 / D8 = half-spinor Fermions (8 components of 8 Particles and 8 AntiParticles)
D8 / D4sm x D4g = (4+4)-dim M4 x CP2 Kaluza-Klein position x momentum
D4g = Conformal Gravity + Dark Energy and ghosts for Standard Model
D4sm = Standard Model Gauge Groups and ghosts for Gravity + Dark Energy

This paper takes a complementary bottom-up point of view to show that you can
start with the simplest Non-Abelian Gauge Group SU(2)
and
add the next step SU(3)
and
then go to SU(4) with cuboctahedron root vector polytope
and
then go to the D4sm Lie algebra with 24-cell root vector polytope
and
then go to a 600-cell whose 120 vertices give half of the 240 of E8

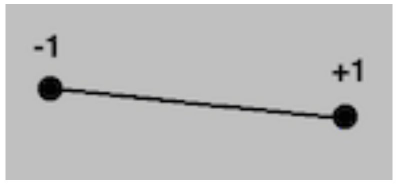
The you can get the other half of E8 by
starting with another cuboctahedron root vector polytope,
for U(2,2) of Conformal Gravity + Dark Energy
and
then go to the D4g Lie algebra with 24-cell root vector polytope
and
then go to a second 600-cell whose 120 vertices give the other half of E8

The two approaches, top-down of viXra 1405.0030 and bottom-up here,
give the same physics results
but
I think that you can get a deeper intuitive understanding of the physics
by looking at Cl(16)-E8 Physics from both points of view.

With that in mind, I have written this paper with heavy emphasis on intuitive graphics bearing in mind that technical issues have already been covered in viXra 1405.0030 .

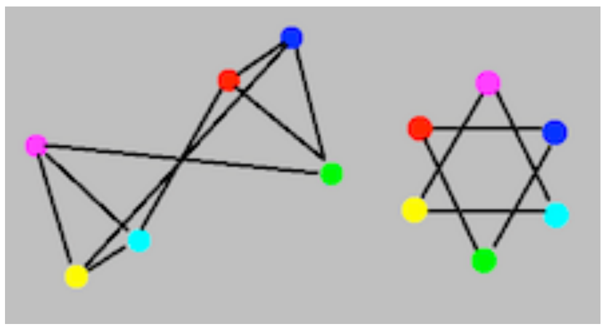
Standard Model

dipole



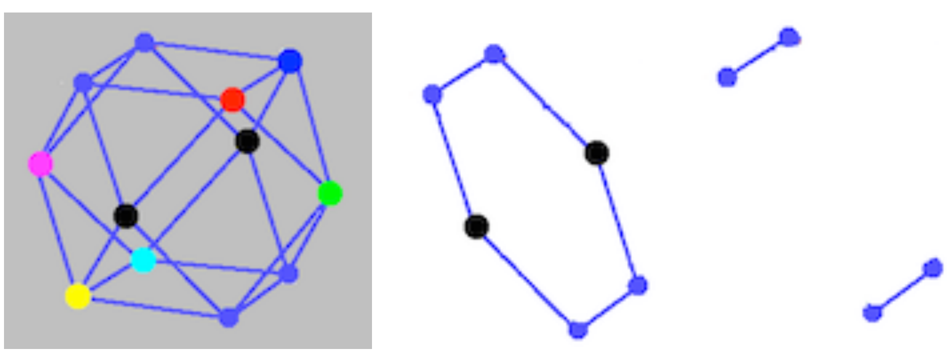
The 2 vertices of the dipole correspond
to
the 2 electric-charged gauge bosons W+ and W-
of the SU(2) Weak Force Gauge Group.

color



The 6 vertices of the Star of David correspond
to
the 6 color-charged gauge bosons Gluons (RY, RM, BM, BC, GC, GY)
of the SU(3) Color Force Gauge Group.

cuboctahedron

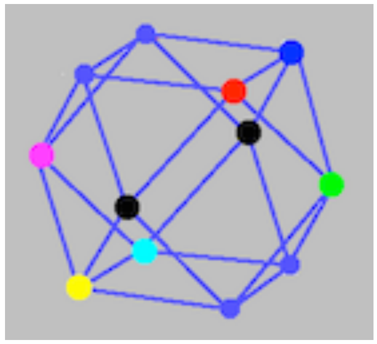
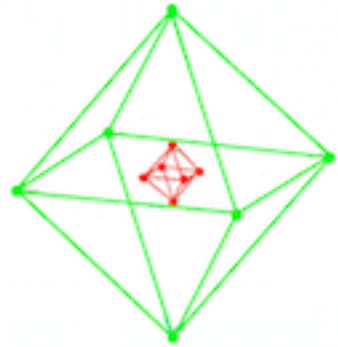


The 12 vertices of the cuboctahedron correspond to
the 6 charged gauge boson Gluons of the SU(3) Color Force Gauge Group
and 6-dim CP3 = SU(4) / U(3) Projective Twistors which include

2 charged gauge bosons W+ and W- of the Chiral SU(2) Weak Force Gauge Group

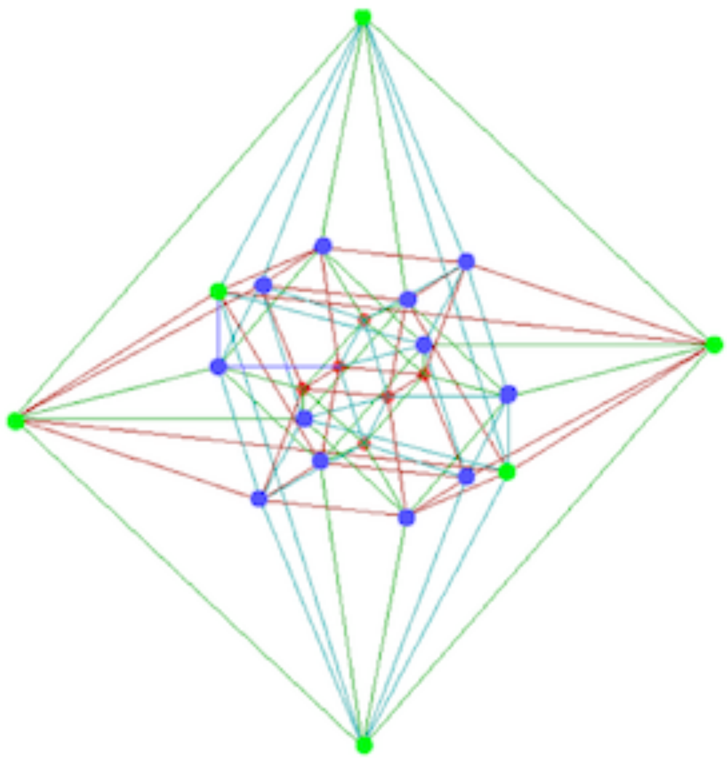
and

4-dim CP2 subspace of CP3 as ghosts for Special Conformal Transformations.

The 12 vertices of the cuboctahedron plus the 6+6 = 12 vertices of two octahedra
make the 24 vertices of the 24-cell, the root vectors of the D4 Lie Algebra

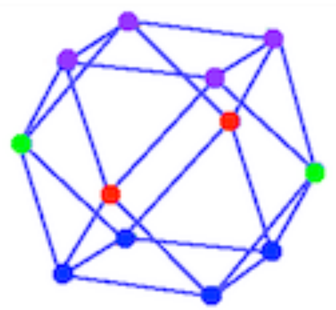
D4sm for the Standard Model



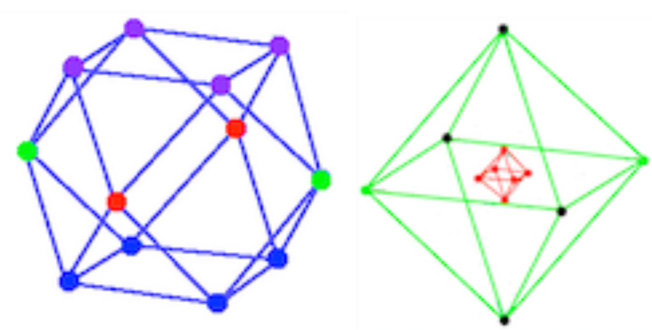
The 6+6 = 12 vertices of the two octahedra (red+green) represent ghosts
for 4 SpaceTime Translations
and 6 Lorentz Group Generators (including 2 Cartan elements)
and the other 2 Cartan elements of Conformal Gravity + Dark Energy U(2,2).

Conformal Gravity + Dark Energy

cuboctahedron



The 12 vertices of the cuboctahedron correspond to
the 2 generators of 3-dim space rotations (red) represented by Quaternions {i,j} (ij = k)
the 2 generators of space-time boosts (green) also represented by {i,j}
the 4 spacetime translations (blue)
the 4 spacetime Special Conformal Transformations (purple)

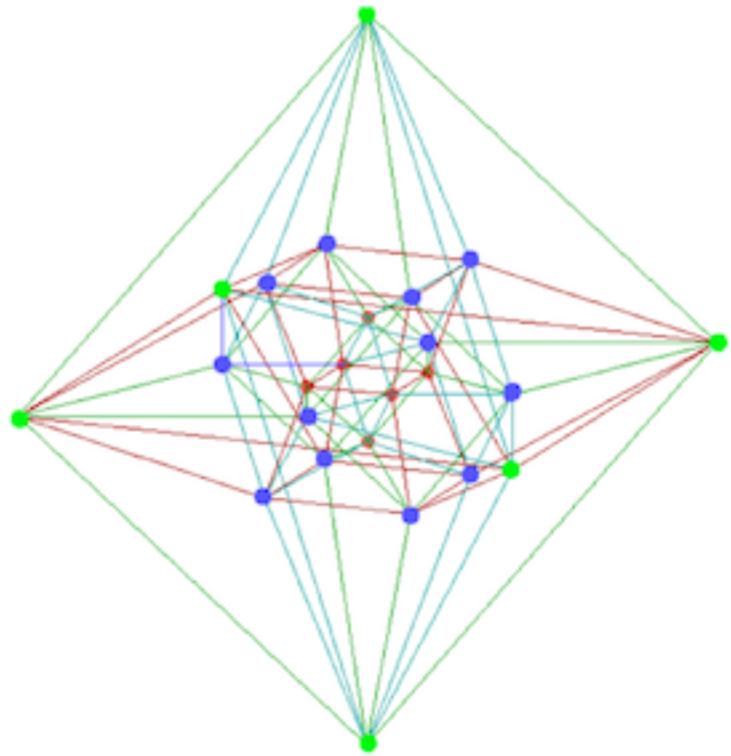


The 12 vertices of the cuboctahedron plus the 6+6 = 12 vertices of two octahedra make the 24 vertices of the 24-cell, the root vectors of the D4 Lie Algebra

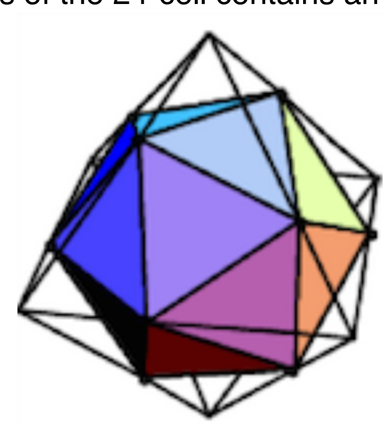
D4g for Conformal Gravity + Dark Energy

The 6+6 = 12 vertices of the two octahedra represent ghosts
for the 12 generators of the Standard Model Gauge Groups:
SU(3) (red 6 charged Gluons) x SU(2) (green 2 charged W-bosons x U(1) (black)
plus 3 Cartan elements (2 for SU(3) and 1 for SU(2)) (black)

D4sm and D4g are each represented by the 24-cell



Each of the 24 octahedral cells of the 24-cell contains an icosahedron.

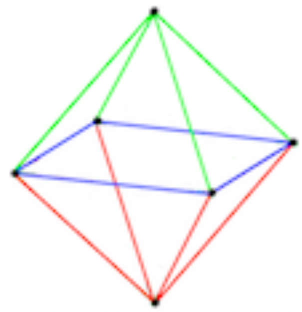


Each of the 24 icosahedra contains 20 tetrahedra for a total of 480 tetrahedra.
Each of the 24 vertices of the 24-cell is surrounded
by 5 tetrahedra that fill up the space of the 24 octahedra not in the 24 icosahedra,
for a total of 24x5 = 120 more tetrahedra so that
the 24-cell has been mapped into a 480+120 = 600-cell with 600 tetrahedral cells.
However,
it is not the tetrahedral cells that correspond to fundamental physics entities,
but rather it is the vertices. The 600-cell has 120 vertices:

24 from the 24-cell corresponding to gauge bosons and ghosts
and
96 from icosahedral vertices on each of the 96 edges of the 24-cell.

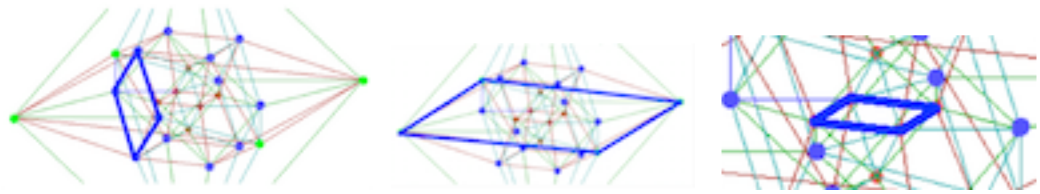
What fundamental physics entities correspond to the 96 vertices ?

Each of the 96 vertices lives on one of the 96 edges of the 24-cell which are edges of the octahedral cells of the 24-cell, so look at their relative positions in the octahedra.



4 square edges (blue) correspond to spacetime

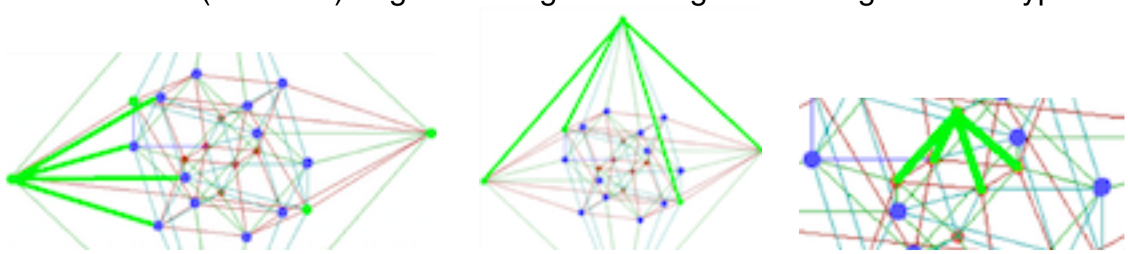
There are (6x4 = 24) edges + 4 edges + 4 edges = 32 edges of this type:



The D4sm 600-cell has 32 spacetime vertex-entities
and the D4g 600-cell also has 32 spacetime vertex-entities.
The total 32+32 = 64 = 8x8 position x momentum for (4+4)-dim M4 x CP2 Kaluza-Klein
The D4sm entities act on CP2 = SU(3) / U(2) Internal Symmetry Space.
The D4g entitites act on M4 Minkowski Physical Spacetime.

4 edges (green) going down from a common vertex correspond to Fermion Particles

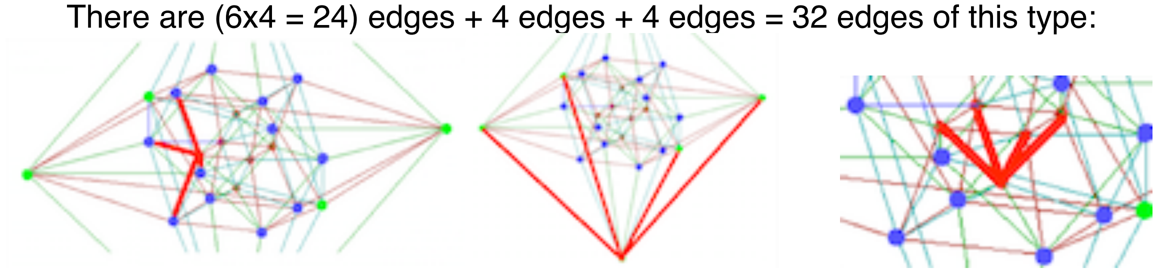
There are (6x4 = 24) edges + 4 edges + 4 edges = 32 edges of this type:



The D4sm particles are 32 = 4x8 = 4 CP2 coordinates of 8 Fundamental Particles.
The D4g particles are 32 = 4x8 = 4 M4 coordinates of 8 Fundamental Particles.

4 edges (red) going up from a common vertex correspond to Fermion AntiParticles

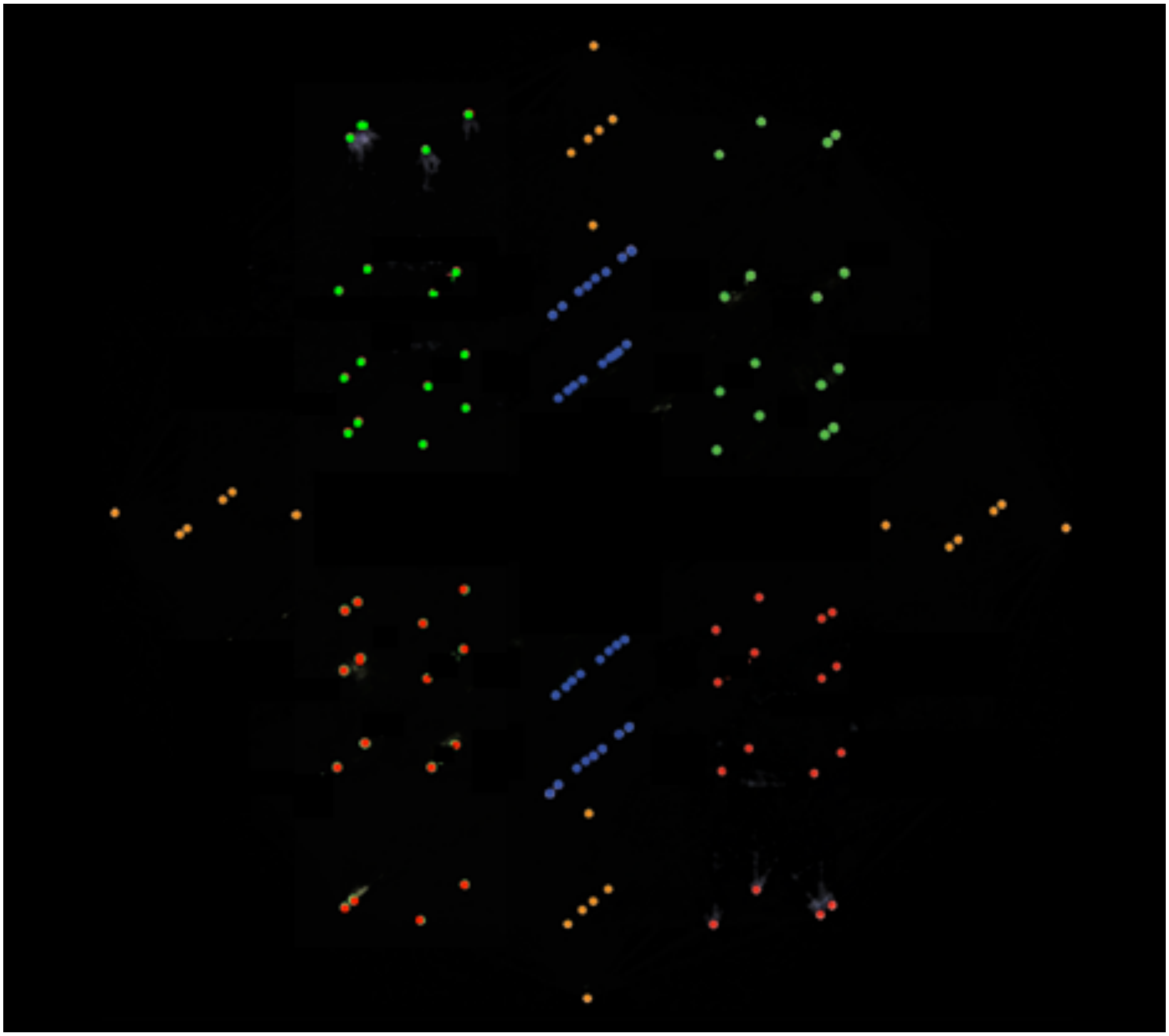
There are (6x4 = 24) edges + 4 edges + 4 edges = 32 edges of this type:



The D4sm particles are 32 = 4x8 = 4 CP2 coordinates of 8 Fundamental AntiParticles.
The D4g particles are 32 = 4x8 = 4 M4 coordinates of 8 Fundamental AntiParticles.

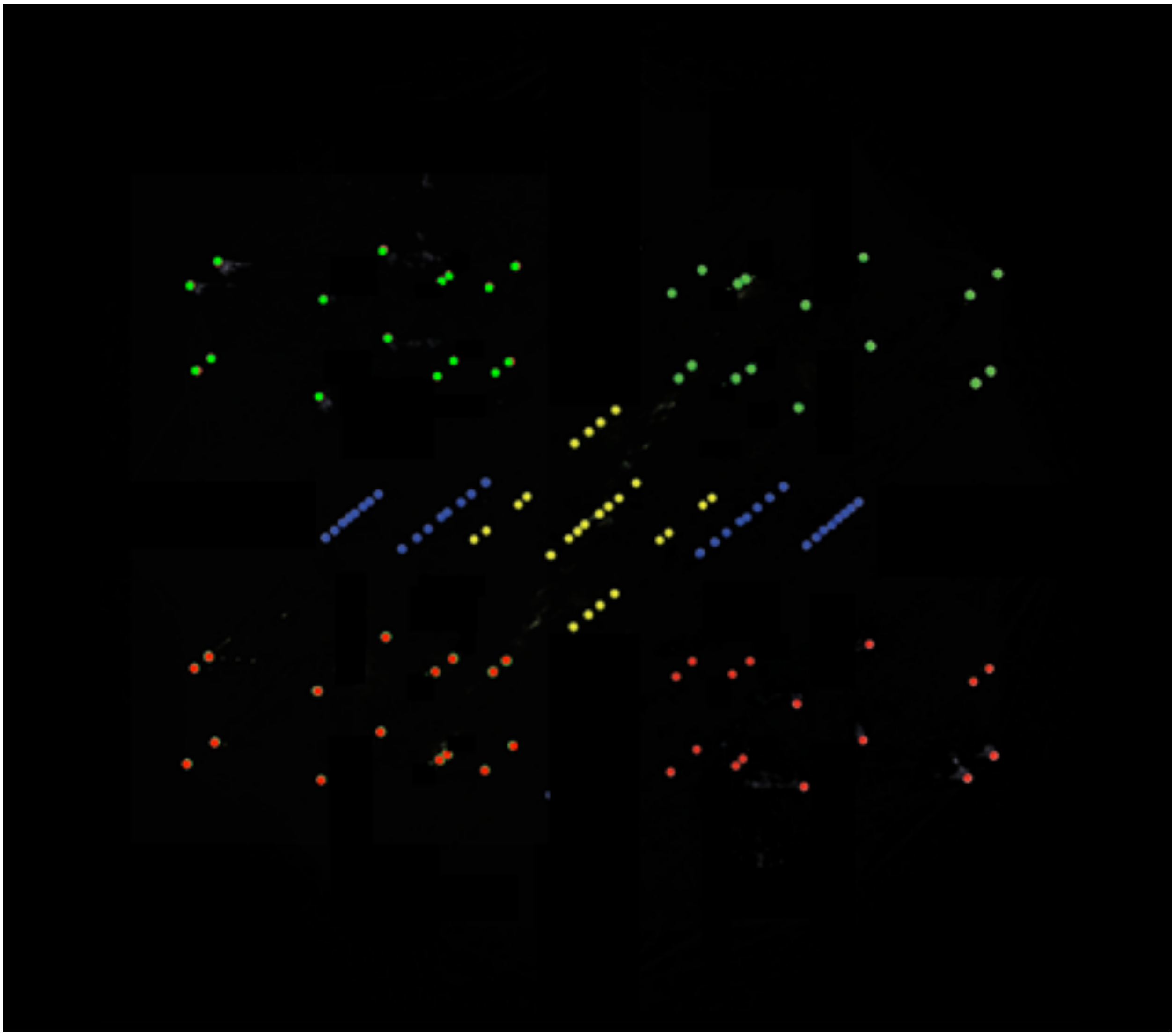
The 120 + 120 vertices of

the Standard Model D4sm 600-cell

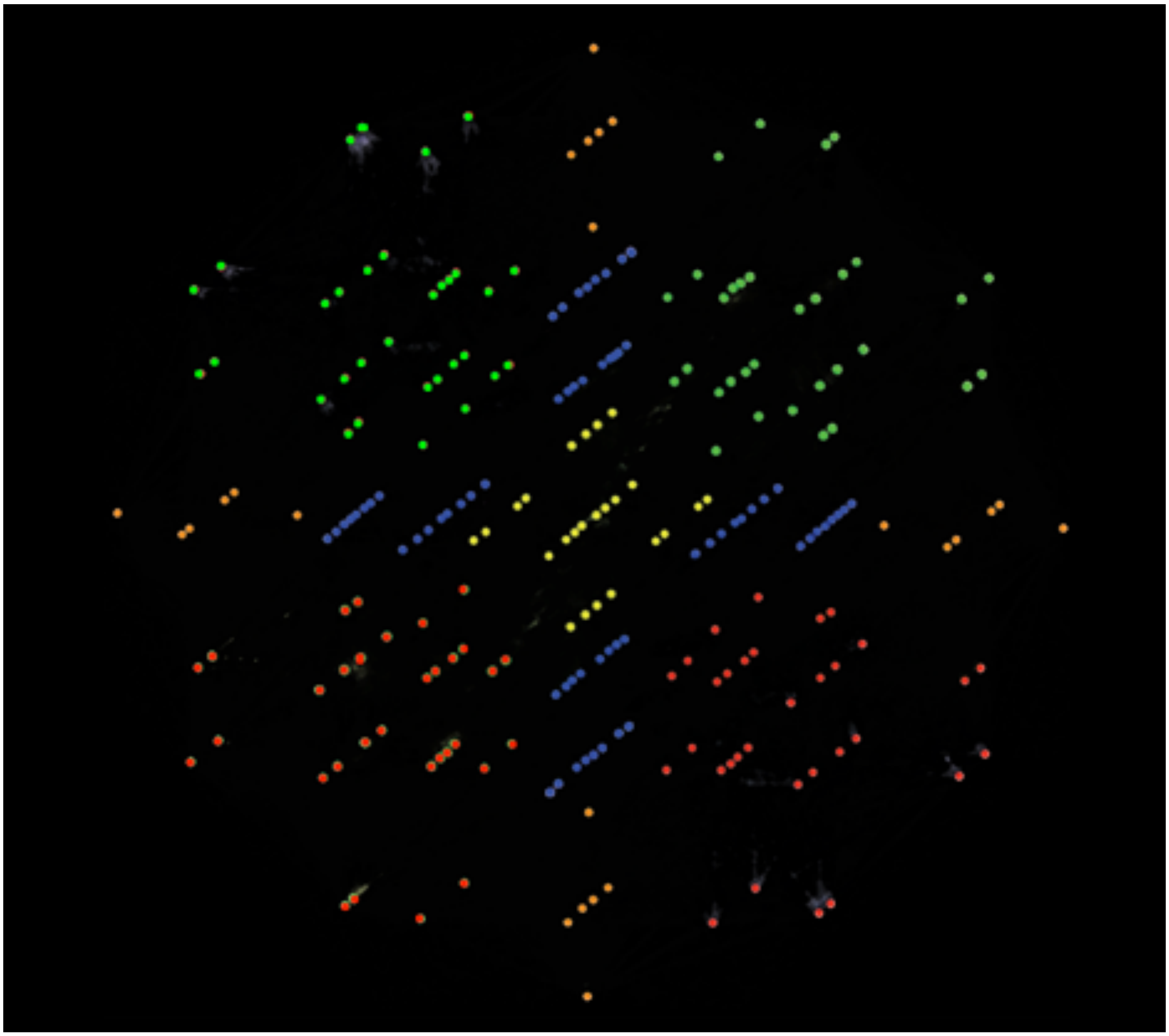


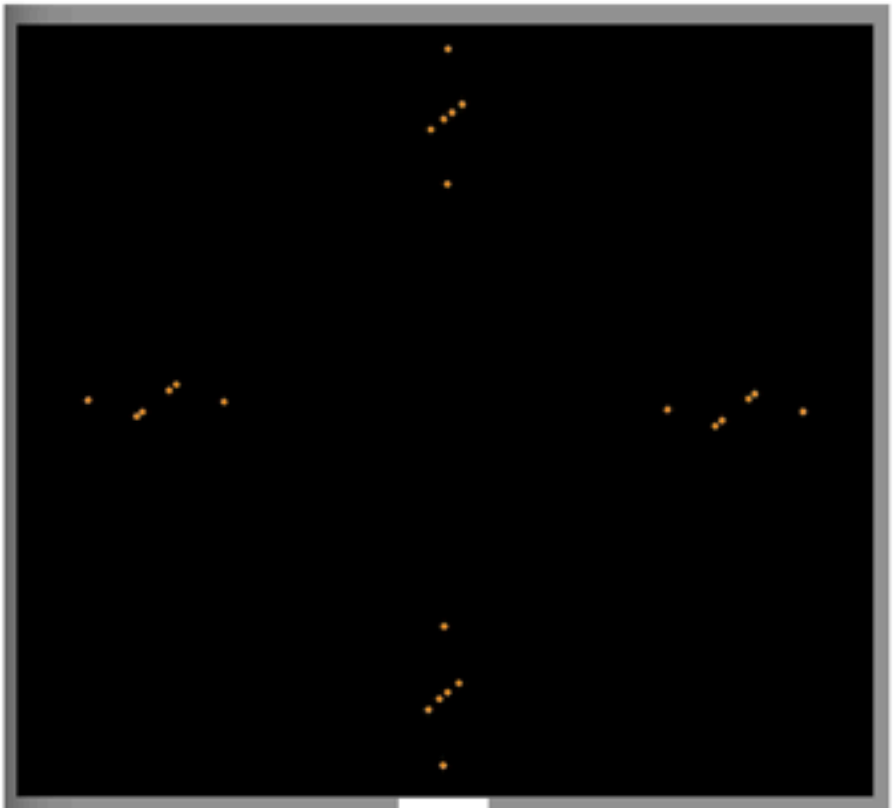
and

the Gravity + Dark Energy D4g 600-cell



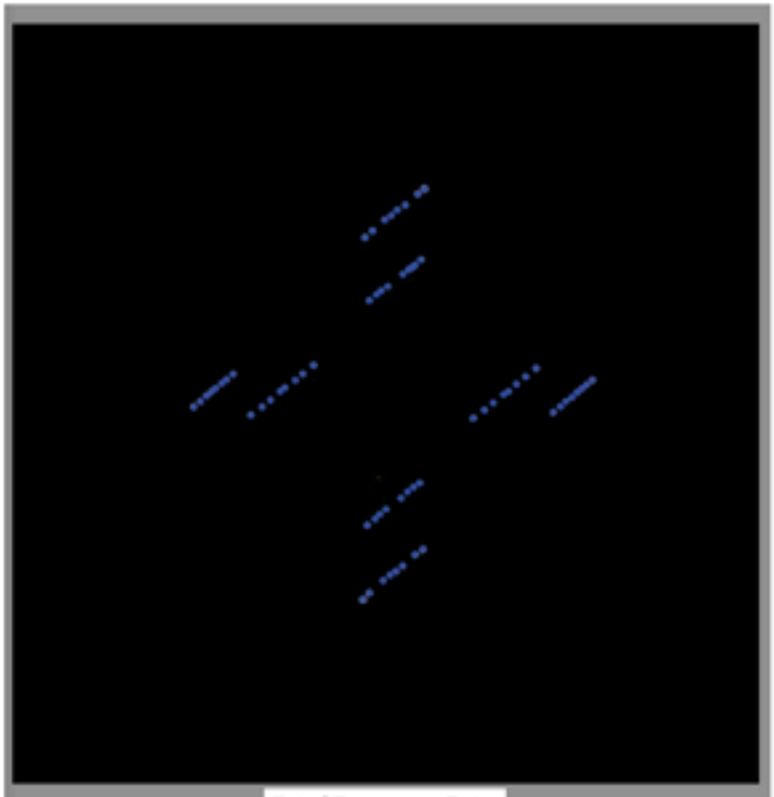
combine
to form the 240 root vector vertices of E8



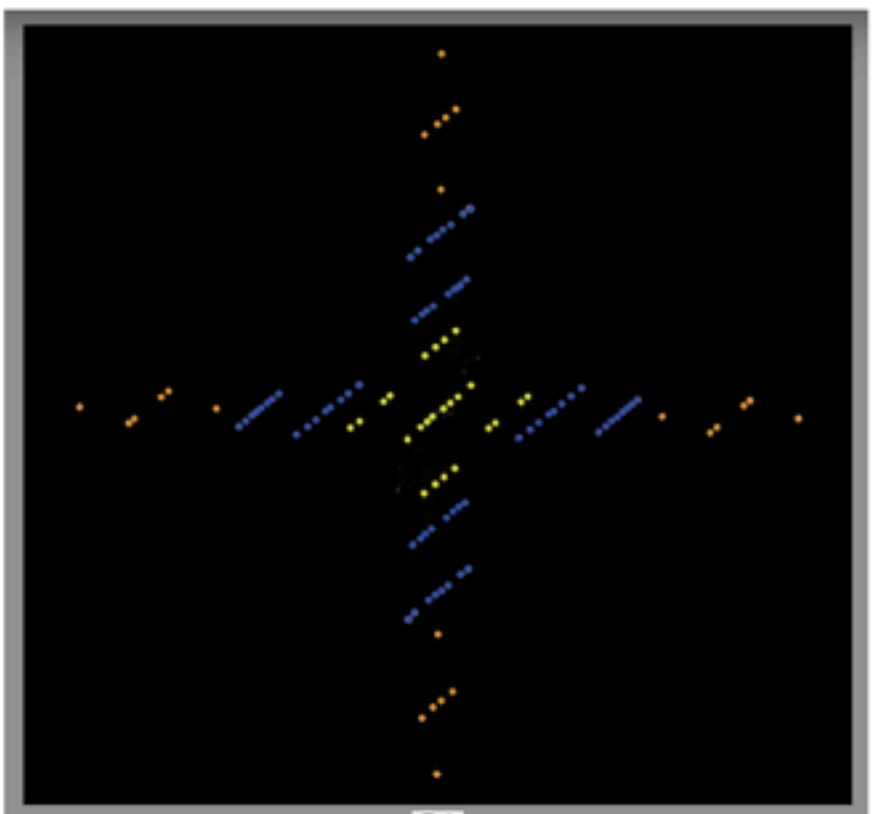


D4sm

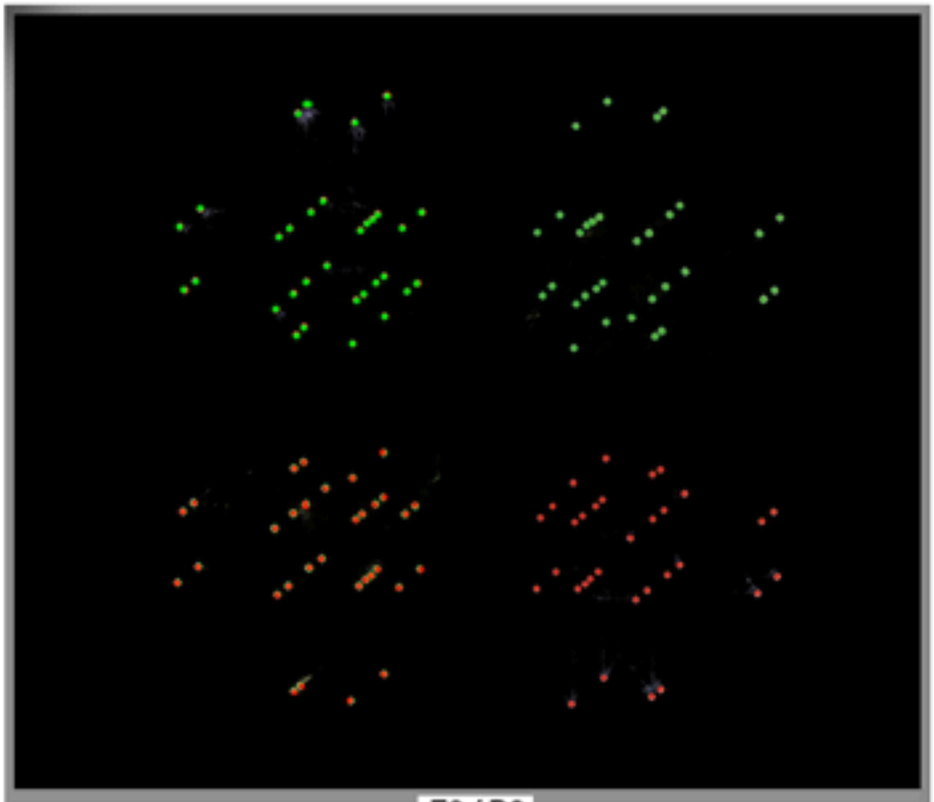
D4g



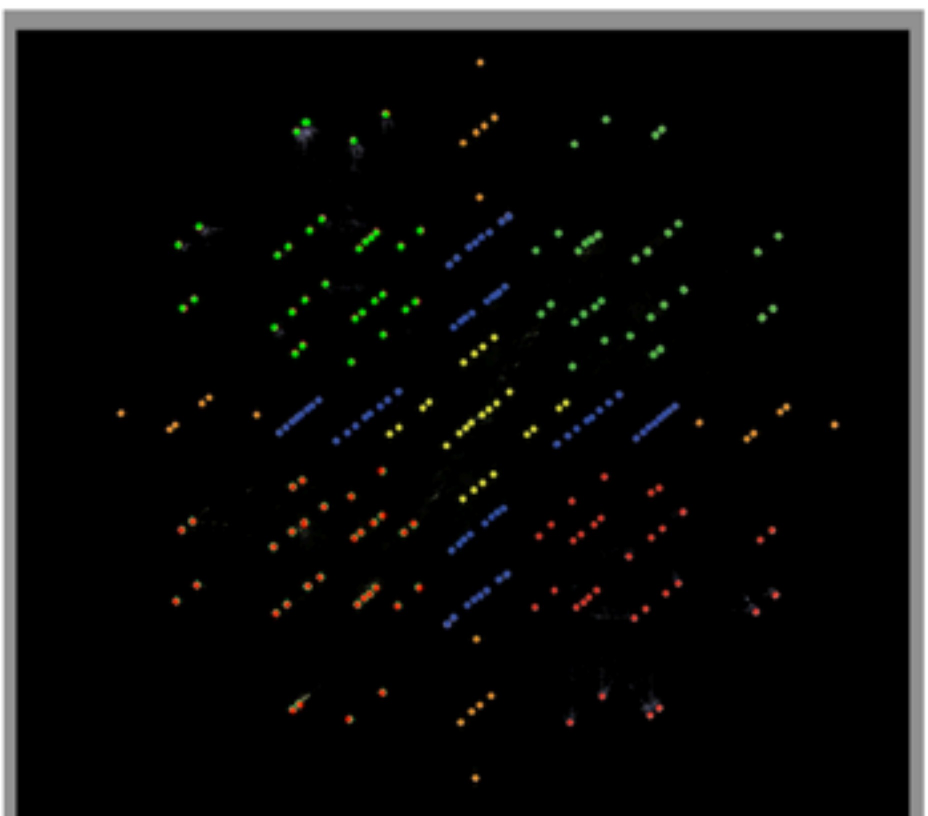
D8 / D4sm x D4g



D8



E8 / D8



E8

23. How Garrett Lisi's E8
differs
from my Clifford Algebra based Cl(16)-E8 Physics model:

Frank Dodd (Tony) Smith< Jr. - 2014 msg to Ben Goertzel

Garrett Lisi uses E8 as a gauge group over a separate 4-dim spacetime
while
my E8 is NOT just a gauge group
but includes (4+4)-dim Kaluza-Klein spacetime as part of E8

I see E8 not as merely a gauge group but
as an algebraic structure that tells you how build a Lagrangian
with (in terms of Lie algebras E8, D8, D4g, D4sm as described below)

E8 / D8 gives Densities for Fermions
and
D4g and D4sm give Densities for Gravity and the Standard Model
Densities are integrated over
Kaluza-Klein spacetime represented by D8 / D4g x D4sm

==

The bottom line is:

Garrett's E8 does not contain his external 4-dim spacetime
but
does contain 166 unobserved things

My Cl(16)-E8 model contains all things that have been observed
and nothing that is not observed
and (4+4)-dim Kaluza-Klein spacetime is included in my E8.

Here are details:

Most recently Lisi's E8 has been described by Douglas and Repka in
http://arxiv.org/pdf/1305.6946v3.pdf
and Garrett himself tweeted on 25 Sep 2014 that the paper was "Nice."
so I will use it as well as Garrett's paper
http://arxiv.org/pdf/1006.4908v1.pdf
as a basis for comparison.

First I will describe Garrett's 2010 paper at arXiv 1006.4908

248-dim E8 = 120-dim D8 + 128-dim half-spinor of D8
where D8 represents spin(12,4)
so that

Lisi breaks down E8 into:

120 = 91-dim spin(11,3) + 29-dim D8 / spin(11,3)
+
128 = 64-dim positive chiral half-spinor of spin(11,3) + 64-dim negative half-spinor of spin(11,3)

In Lisi's scheme
91-dim spin(11,3) breaks down into:

6 for Gravity spin(1,3) = Lorentz + boosts
40 for a frame-Higgs + 45 for spin(10) GUT gauge bosons which 45 break down into:
1 photon
8 gluons
3 weak bosons
3 new weak bosons
30 X-bosons

29-dim D8 / spin(11,3) breaks down into:

20 more X-bosons
1 Peccei-Quinn w-boson
8 more frame Higgs including two axions

64-dim positive chiral spin(3,11) half-spinor = one generation of fermions
based on
2 chirality staes x 2 charge states of 8 fermion particles (e; R,G,B up quarks : Neu; R,G,B down quarks
andt
2 chirality states x 2 charge states of 8 fermion antiparticles

64-dim negative chiral spin(3,11) half-spinor = one generation of mirror fermions
These are not observed now.

===

Now I will describe the more recent arXiv 1305.6946v3
that uses a complex so(14)C instead of Spin(11,3). Both are 91-dimensional
and
instead of adding just a 64-dim fermion thing to Spin(11,3)
there is added to 91-dim so(14)C a 78-dim thing that breaks down into 14-dim + 64-dim

The 64-dim thing is a half-spinor of so(14) and has the same physics interpretation
as in Lisi's 2010 model.

In Lisi's new expanded scheme scheme
91-dim so(14)C lives in 120-dim D8 and breaks down into:

6 for Gravity spin(1,3) = Lorentz + boosts
40 for a frame-Higgs + 45 for spin(10) GUT gauge bosons which 45 break down into:
1 photon
8 gluons
3 weak bosons
3 new weak bosons
30 X-bosons

14-dim thing is translations in 14-dim vector space.

The 120 - 91 - 14 = 15 things are

6 more X-bosons = 20 - 14
1 Peccei-Quinn w-boson
8 more frame Higgs including two axions

64-dim positive chiral so(14)C half-spinor = one generation of fermions
based on
2 chirality staes x 2 charge states of 8 fermion particles (e; R,G,B up quarks : Neu; R,G,B down quarks
andt
2 chirality states x 2 charge states of 8 fermion antiparticles

64-dim negative chiral so(14)C half-spinor = one generation of mirror fermions
These are not observed now.

==

Compare the Garrett Lisi breakdown with what I do:

I also break down E8 into:
120 = D8
+
128 = chiral half-spinor of D8 =
= 64-dim positive chiral half-spinor of spin(14) + 64-dim negative half-spinor of spin(14)

but in my scheme
E8 / D8 = 64 + 64 =
8 Kaluza-Klein 8-dim components of 8 fermion particles
and
8 Kaluza-Klein 8-dim components of 8 fermion antiparticles

D8 contains two copies of 28-dim D4,
one D4g for Gravity + Dark Energy and the other D4sm for the Standard Model

D4g = 16 generators for Gravity + 12 ghosts for the Standard Model

D4sm = (1+8+3) = 12 generators for the Standard Model +16 ghosts for Gravity

D8 / D4g x D4sm = 8x8 = 64-dim representation of 8-dim Kaluza-Klein 8-position x 8-momentum

===

Both Garrett and I have as fundamental only the first generation of fermions.
In my model, the second and third generations come from the geometry of (4+4) -dim Kaluza-Klein which geometry also produces the Higgs.

Garrett's E8 does not contain his external 4-dim spacetime.

It has 166 unobserved things

40 frame-Higgs
3 new weak bosons
30 X-bosons
14 translations in so(14)C vector pace
6 more X-bosons
1 Peccei-Quinn w-boson
8 more frame Higgs including two axions
64 half-spinors for one generation of mirror fermions

and 82 observed things

6 for Gravity spin(1,3) = Lorentz + boosts
1 photon
8 gluons
3 weak bosons
64 half-spinors for one generation of mirror fermions

My Cl(16)-E8 model contains all things that have been observed
and nothing that is not observed
and (4+4)-dim Kaluza-Klein spacetime is included in my E8.