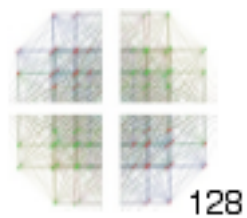
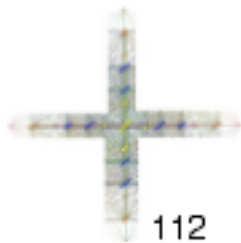
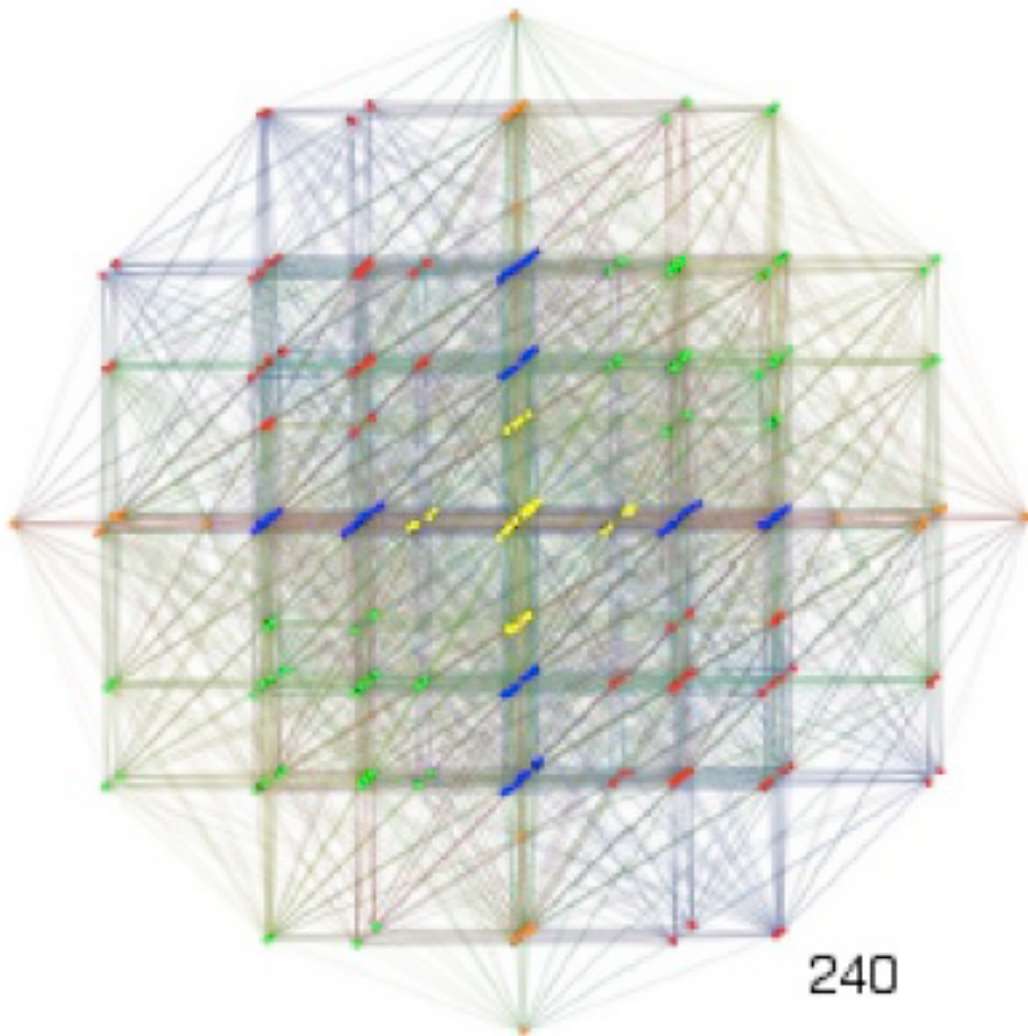


# CI(16) - E8 Lagrangian - AQFT

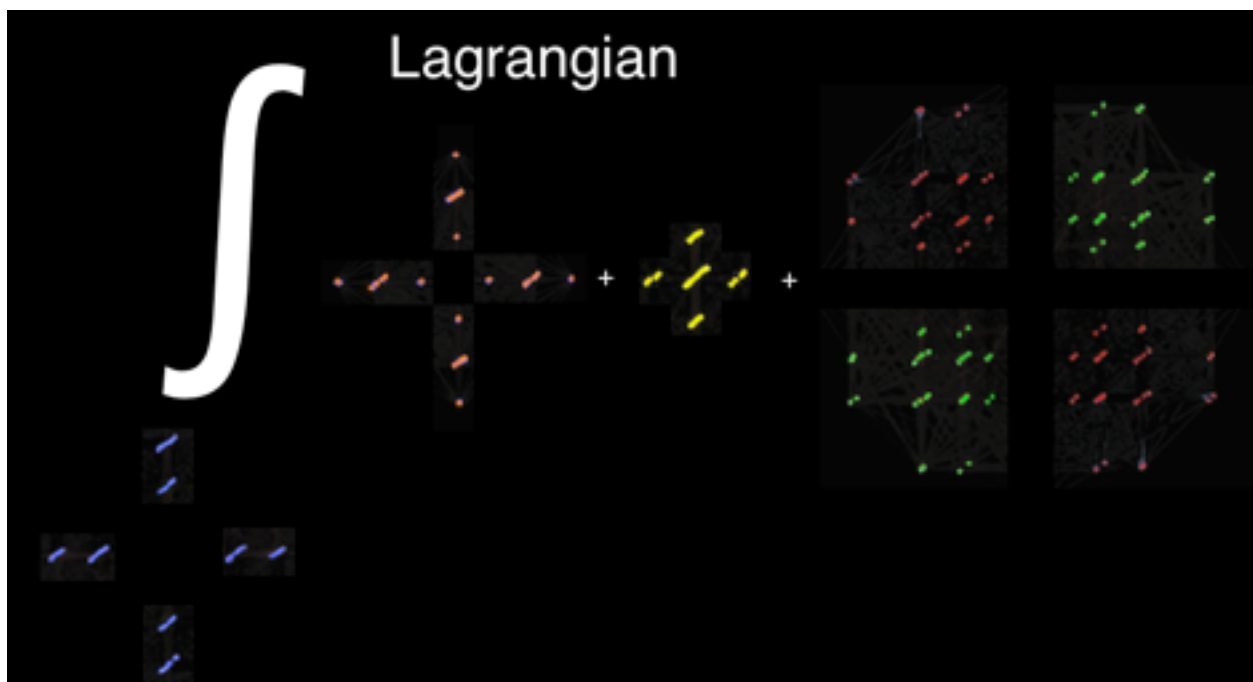


Frank Dodd (Tony) Smith, Jr. - 2014

viXra 1405.0030

## Abstract:

Over the past 30 years or so I have been constructing Physics Models and writing about them as can be seen on my web sites at [www.valdostamuseum.com/hamsmith/](http://www.valdostamuseum.com/hamsmith/) [www.tony5m17h.net/](http://www.tony5m17h.net/) and on viXra - list at [vixra.org/author/frank\\_dodd\\_tony\\_smith\\_jr](http://vixra.org/author/frank_dodd_tony_smith_jr) Due to experimental observations and my learning new techniques over those 30 years my Physics Models have been in a state of evolving flux - for example, 30 years ago their basis was the Lie Algebra Spin(8), then to contain vectors and spinors it was F4, then to contain the geometry of bounded complex domains it was E6, then Real Clifford Algebras were used to describe evolution from a Void Empty Set  $\emptyset$ , then Periodicity showed the importance of Cl(8) and tensor product  $Cl(8) \times Cl(8) = Cl(16)$ , then E8 emerged from Cl(16) to give the structure of a realistic local E8 Lagrangian, then completion of the union of all tensor products of Cl(16) local structures produced a realistic Algebraic Quantum Field Theory (AQFT). Since my works over those 30 years have been written from various points of view it is not easy to navigate among them. This paper is being written from a single point of view (that of May 2014) in the hope that it might be easier for readers to navigate. Although the nice math of my Cl(16)-E8 model is necessary, it is not sufficient. The Cl(16)-E8 model must be consistent with experimental observations. As of now, given that most calculations are tree-level, the model is substantially so consistent. An interesting test over the 2015-2016 time frame will be whether or not the LHC sees two additional Higgs mass states with cross section about 20% of that of a full Standard Model Higgs.



## Preface

Over the past 30 years or so I have been constructing Physics Models and writing about them as can be seen on my web sites at <http://www.valdostamuseum.com/hamsmith/>

<http://www.tony5m17h.net/> and on viXra - list at [http://vixra.org/author/frank\\_dodd\\_tony\\_smith\\_jr](http://vixra.org/author/frank_dodd_tony_smith_jr)

Due to experimental observations and my learning new techniques over those 30 years my Physics Models have been in a state of evolving flux - for example, 30 years ago their basis was the Lie Algebra Spin(8), then to contain vectors and spinors it was F4, then to contain the geometry of bounded complex domains it was E6, then Real Clifford Algebras were used to describe evolution from a Void Empty Set  $\emptyset$ , then Periodicity showed the importance of Cl(8) and tensor product  $Cl(8) \times Cl(8) = Cl(16)$ , then E8 emerged from Cl(16) to give the structure of a realistic local E8 Lagrangian, then completion of the union of all tensor products of Cl(16) local structures produced a realistic Algebraic Quantum Field Theory (AQFT).

Since my works over those 30 years have been written from various points of view it is not easy to navigate among them. This paper is being written from a single point of view (that of May 2014) in the hope that it might be easier for readers to navigate.

A lot of math is used in my Cl(16)-E8 model, some of which may be unfamiliar to many. My efforts to find a single volume for the math of Cl(16) - E8 Lagrangian - AQFT led me to my Princeton University Advanced Calculus text by H. K. Nickerson, D. C. Spencer, and N. E. Steenrod. However, it is over 50 years old, so I have added some Supplementary Material to produce a 21 MB pdf file on the web at

<http://www.valdostamuseum.com/hamsmith/NSS6313.pdf>

TABLE OF CONTENTS OF THE SUPPLEMENTED TEXT:

### **Supplementary Material in Red**

I. THE ALGEBRA OF VECTOR SPACES

II. LINEAR TRANSFORMATIONS OF VECTOR SPACES

### **Lie Groups and Symmetric Spaces**

III. THE SCALAR PRODUCT

IV. VECTOR PRODUCTS IN  $R^3$

### **Vector Products in $R^7$**

V. ENDOMORPHISMS

VI. VECTOR-VALUED FUNCTIONS OF A SCALAR

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VIII. VECTOR-VALUED FUNCTIONS OF A VECTOR

IX. TENSOR PRODUCTS AND THE STANDARD ALGEBRAS

### **Clifford Algebra and Spinors**

X. TOPOLOGY AND ANALYSIS

XI. DIFFERENTIAL CALCULUS OF FORMS

XII. INTEGRAL CALCULUS OF FORMS

XIII. COMPLEX STRUCTURE

### **Potential Theory, Green's Functions, Bergman Kernels, Schwinger Sources**

Although the nice math of my Cl(16)-E8 model is necessary, it is not sufficient. My Cl(16)-E8 model must be, and is, consistent with experimental observations .

Here is a summary of E8 Physics model calculation results. Since ratios are calculated, values for one particle mass and one force strength are assumed. Quark masses are constituent masses. Most of the calculations are tree-level, so more detailed calculations might be even closer to observations.

Dark Energy : Dark Matter : Ordinary Matter = 0.75 : 0.21 : 0.04

Inflationary Gravitational Wave (IGW) tensor-to-scalar ratio  $r = 7/28 = 0.25$

Fermions as Schwinger Sources have geometry of Complex Bounded Domains

with Kerr-Newman Black Hole structure size about  $10^{(-24)}$  cm.

Particle/Force	Tree-Level	Higher-Order
e-neutrino	0	0 for $\nu_1$
mu-neutrino	0	$9 \times 10^{(-3)}$ eV for $\nu_2$
tau-neutrino	0	$5.4 \times 10^{(-2)}$ eV for $\nu_3$
electron	0.5110 MeV	
down quark	312.8 MeV	charged pion = 139 MeV
up quark	312.8 MeV	proton = 938.25 MeV
		neutron - proton = 1.1 MeV
muon	104.8 MeV	106.2 MeV
strange quark	625 MeV	
charm quark	2090 MeV	
tauon	1.88 GeV	
beauty quark	5.63 GeV	
truth quark (low state)	130 GeV	(middle state) 174 GeV (high state) 218 GeV
W+	80.326 GeV	
W-	80.326 GeV	
W0	98.379 GeV	Z0 = 91.862 GeV
Mplanck	$1.217 \times 10^{19}$ GeV	
Higgs VEV (assumed)	252.5 GeV	
Higgs (low state)	126 GeV	(middle state) 182 GeV (high state) 239 GeV
Gravity Gg (assumed)	1	
(Gg)(Mproton <sup>2</sup> / Mplanck <sup>2</sup> )		$5 \times 10^{(-39)}$
EM fine structure	1/137.03608	
Weak Gw	0.2535	
Gw(Mproton <sup>2</sup> / (Mw+ <sup>2</sup> + Mw- <sup>2</sup> + Mz0 <sup>2</sup> ))		$1.05 \times 10^{(-5)}$
Color Force at 0.245 GeV	0.6286	0.106 at 91 GeV

Kobayashi-Maskawa parameters for W+ and W- processes are:

	d	s	b
u	0.975	0.222	0.00249 -0.00388i
c	-0.222 -0.000161i	0.974 -0.0000365i	0.0423
t	0.00698 -0.00378i	-0.0418 -0.00086i	0.999

The phase angle d13 is taken to be 1 radian.

**The 3-state system of Higgs and Tquark masses is a property of the CI(16)-E8 model that can be tested at the LHC 2015-2016 run by searching for Higgs middle and high mass states with cross section about 20% of that of a full SM Higgs.**

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# 1. The First Grothendieck Universe is the Empty Set $\emptyset$ which grows by Clifford Iteration to $Cl(16)$ which contains $E8$

$$\begin{array}{l} 1 \\ \emptyset \end{array} = Cl(0) = 1$$

$$\begin{array}{l} 1 & 1 \\ \emptyset & (\emptyset) \end{array} = Cl(1) = 2$$

$$\begin{array}{l} 1 & 2 & 1 \\ \emptyset & (\emptyset) & (\emptyset (\emptyset)) \\ & (\emptyset) & \end{array} = Cl(2) = 4$$

$$\begin{array}{l} 1 & 4 & 6 & 4 & 1 \\ \emptyset & (\emptyset) & (\emptyset (\emptyset)) & ((\emptyset ((\emptyset)) (\emptyset (\emptyset))) & (\emptyset (\emptyset) ((\emptyset)) (\emptyset (\emptyset))) \\ & ((\emptyset)) & (\emptyset ((\emptyset))) & (\emptyset ((\emptyset)) (\emptyset (\emptyset))) & \\ & (((\emptyset))) & (\emptyset (\emptyset (\emptyset))) & (\emptyset (\emptyset) (\emptyset (\emptyset))) & \\ & ((\emptyset (\emptyset))) & ((\emptyset) ((\emptyset))) & ((\emptyset (\emptyset)) ((\emptyset))) & \\ & & ((\emptyset) (\emptyset (\emptyset))) & & \\ & & (((\emptyset)) (\emptyset (\emptyset))) & & \end{array} = Cl(4) = 16$$

$$\begin{array}{cccccccccccccccc} 1 & 16 & 120 & 560 & 1820 & 4368 & 8008 & 11440 & 12870 & 11440 & 8008 & 4368 & 1820 & 560 & 120 & 16 & 1 \end{array}$$

$$= Cl(16) = 2^{16} = 65,536 =$$

$$= ((64+64) + (64+64)) \times ((64+64) + (64+64))$$

$$Cl(16) \text{ BiVectors} = D8 = 120 = 28 + 28 + 64$$

$$Cl(16) \text{ Spinors} = (64+64) + (64+64)$$

$$28 + 28 + 64 + 64 + 64 = E8$$

From  $Cl(1,3) = 16$  to  $Cl(Cl(1,3)) = 65,536$  with  $16 \wedge 16 = 120$

( Color Scheme on this page for  $Cl(1,3)$  is not the same used for  $Cl(16)$  and  $E8$  )

$$1 \ 4 \ 6 \ 4 \ 1 \ \wedge \ 1 \ 4 \ 6 \ 4 \ 1$$

$$1 \wedge 4 = 4$$

$$4 \wedge 6 = 24$$

$$1 \wedge 4 = 4$$

$$6 \wedge 4 = 24$$

$$1 \wedge 6 = 6$$

$$1 \wedge 1 = 1$$

$$6 \wedge 6 = 15$$

$$6 \wedge 1 = 6$$

$$4 \wedge 4 = 6$$

$$4 \wedge 4 = 16$$

$$4 \wedge 4 = 6$$

$$4 \wedge 1 = 4$$

$$4 \wedge 1 = 4$$

- 28 D4 for Gravity +
- 28 D4 for Standard Model +
- 28 AntiSymmetric D4 rotations in 8-dim SpaceTime +
- 28 8x8 Symmetric Off-Diagonal +
- 8 8x8 Symmetric Diagonal for 4 + 4 Klauza-Klein  $M4 \times CP2 = 120$

## E8 structure gives a Fundamental Local Lagrangian

$$E8 \text{ Root Vectors} = 112 + 64 + 64 = 24 + 24 + 64 + 64 + 64$$

Fundamental Local Lagrangian =

$$= \int \text{Standard Model Gauge Gravity} + \text{Fermion Particle-AntiParticle}$$

8-dim SpaceTime

where E8 structure of the Lagrangian Terms is given by:

$$E8 / D8 = 64 + 64$$

64 = 8 Components of 8 Fermion Particles (first generation)

64 = 8 Components of 8 Fermion AntiParticles (first generation)

$$D8 / D4 \times D4 = 64$$

64 = 8-dim SpaceTime Position and Momentum

( Triality Automorphisms:  $64 = 64 = 64$  )

$$D4 \times D4 = 24 + 4 + 24 + 4$$

$$24 + 4 = 28 = D4 \text{ for Gravity Gauge Bosons}$$

$$24 + 4 = 28 = D4 \text{ for Standard Model Gauge Bosons}$$

Standard Model Gauge Gravity term has total weight  $28 \times 1 = 28$

12 generators for SU(3) and U(2) Standard Model

+

16 generators for U(2,2) of Conformal Gravity

=

28 D4 Gauge Bosons

each with 8-dim Lagrangian weight = 1

Fermion Particle-AntiParticle term also has total weight  $8 \times (7/2) = 28$

8 Fermion Particle/Antiparticle types

each with 8-dim Lagrangian weight = 7/2

Since Boson Weight 28 = Fermion Weight 28

**the CI(16)-E8 model has a Subtle SuperSymmetry and is UltraViolet Finite.**

**The CI(16)-E8 model** has 8-dim Lorentz structure satisfying Coleman-Mandula because its fermionic fundamental spinor representations are built with respect to spinor representations for 8-dim Spin(1,7) spacetime.

( See pages 382-384 of Steven Weinberg's book "The Quantum Theory of Fields" Vol. III )

**The CI(16)-E8 model is Chiral** because

E8 contains CI(16) half-spinors (64+64) for a Fermion Generation

but does not contain CI(16) Fermion AntiGeneration half-spinors (64+64).

Fermion +half-spinor Particles with high enough velocity are seen as left-handed.

Fermion -half-spinor AntiParticles with high enough velocity are seen as right-handed.

**The CI(16)-E8 model obeys Spin-Statistics** because

the CP2 part of M4xCP2 Kaluza-Klein has index structure Euler number  $2+1 = 3$  and Atiyah-Singer index  $-1/8$  which is not the net number of generations because

**CP2 has no spin structure but you can use a generalized spin structure**

(Hawking and Pope (Phys. Lett. 73B (1978) 42-44))

to get (for integral m) the generalized CP2 index  $n_R - n_L = (1/2) m (m+1)$

Prior to Dimensional Reduction:  $m = 1$ ,  $n_R - n_L = (1/2) \times 1 \times 2 = 1$  for 1 generation

After Reduction to 4+4 Kaluza-Klein:  $m = 2$ ,  $n_R - n_L = (1/2) \times 2 \times 3 = 1$  for 3 generations

(second and third generations emerge as effective composites of the first)

Hawking and Pope say: "Generalized Spin Structures in Quantum Gravity ...

what happens in CP2 ... is a two-surface K which cannot be shrunk to zero. ...

However, one could replace the electromagnetic field by

a Yang-Mills field whose group G had a double covering  $G \sim$ .

The fermion field would have to occur in representations which changed sign

under the non-trivial element of the kernel of the projection ...  $G \sim \rightarrow G$

while the bosons would have to occur in representations which did not change sign ...".

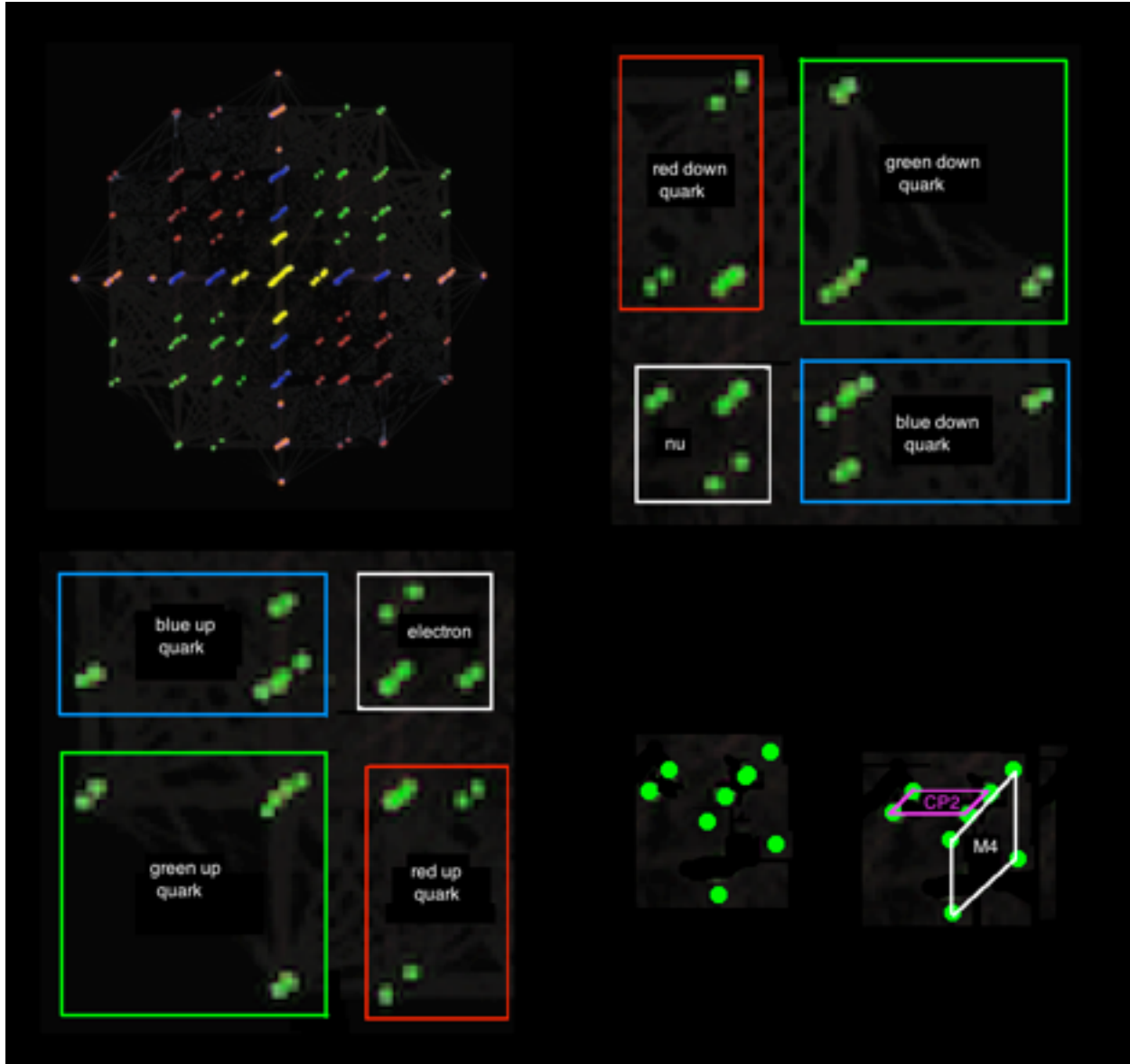
For CI(16)-E8 model gauge bosons are in the  $28+28=56$ -dim  $D4 + D4$  subalgebra of E8.

$D4 = SO(8)$  is the Hawking-Pope G which has double covering  $G \sim = Spin(8)$ .



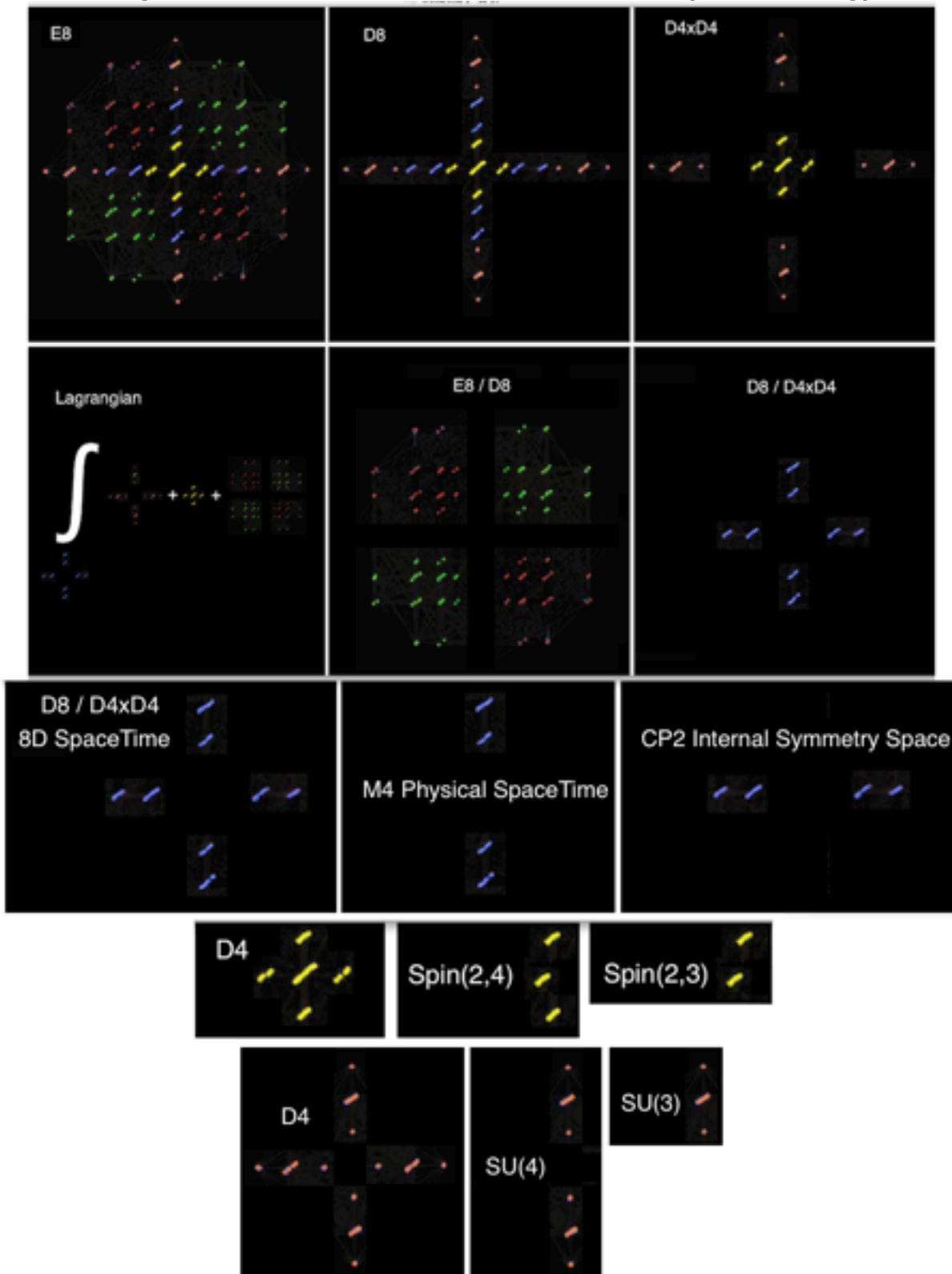
The 8 fermion particles / antiparticles are D4 half-spinors represented within E8 by anti-commutators and so do change sign while the 28 gauge bosons are D4 adjoint represented within E8 by commutators and so do not change sign. Further, **E8 inherits from F4 the property whereby its Spinor Part need not be written as Commutators but can also be written in terms of Fermionic AntiCommutators.** ( vixra 1208.0145 )

**The structure of E8 Spinor Fermions of the First Generation is:**



The Spinor 128 of the 240 E8 Root Vectors are the 64 red and 64 green dots. 64 Green Dots represent Fermion Particles. 64 Red Dots represent AntiParticles.

The structure of E8 Lagrangian and SpaceTime and Gauge Bosons for the Standard Model and Gravity / Dark Energy



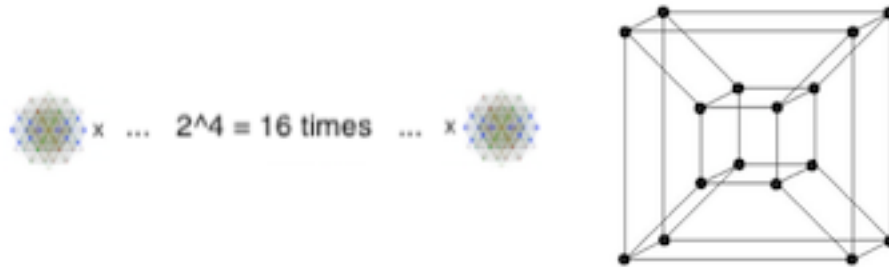
The D8 112 of the 240 E8 Root Vectors are 24 orange and 24 yellow and 64 blue dots.

**2. The Second Grothendieck Universe is Hereditarily Finite Sets such as Discrete Clifford Algebras and Discrete Lattices.**

Each Local Lagrangian with Creation / Annihilation density terms lives in an E8 which in turn lives in a Cl(16) Real Clifford Algebra.

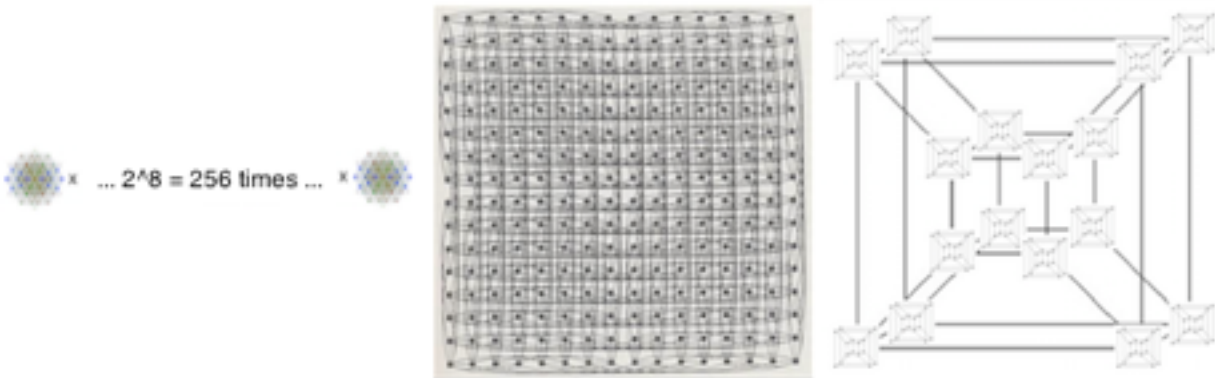
By 8-Periodicity of Real Clifford Algebras tensor products of N copies of Cl(16) form a Clifford Algebra **Cl(16N) = Cl(16) x ... (N times tensor product) ... x Cl(16)** .

**For N = 2^4 = 16** the 16 copies of Cl(16) form E8 Physics of a 4-dim HyperCube



corresponding to 4-dim M4 Physical SpaceTime.

**For N = 2^8 = 256** the 256 copies of Cl(16) form E8 Physics of an 8-dim HyperCube



corresponding to 8-dim E8 SpaceTime (image by Conrad Schneiker in 1987 paper by Hameroff) and to M4 x CP2 Kaluza-Klein SpaceTime with each vertex of the 4-dim M4 HyperCube having a 4-dim CP2 Internal Symmetry Space.

**For N = 2^16 = 65,536 = 4^8** the copies of Cl(16) fill in the 8-dim HyperCube as described by William Gilbert's web page: "... The n-bit reflected binary **Gray code** will describe a path on the edges of an n-dimensional cube that can be used as the initial stage of a Hilbert curve that will fill an n-dimensional cube. ...".

As N grows, the copies of Cl(16) continue to fill the 8-dim HyperCube of E8 SpaceTime using higher Hilbert curve stages from the 8-bit reflected binary Gray code subdividing the initial 8-dim HyperCube into more and more sub-HyperCubes. If the edges of the sub-HyperCubes, equal to the distance between adjacent copies of Cl(16), remain constantly at the Planck Length, then the

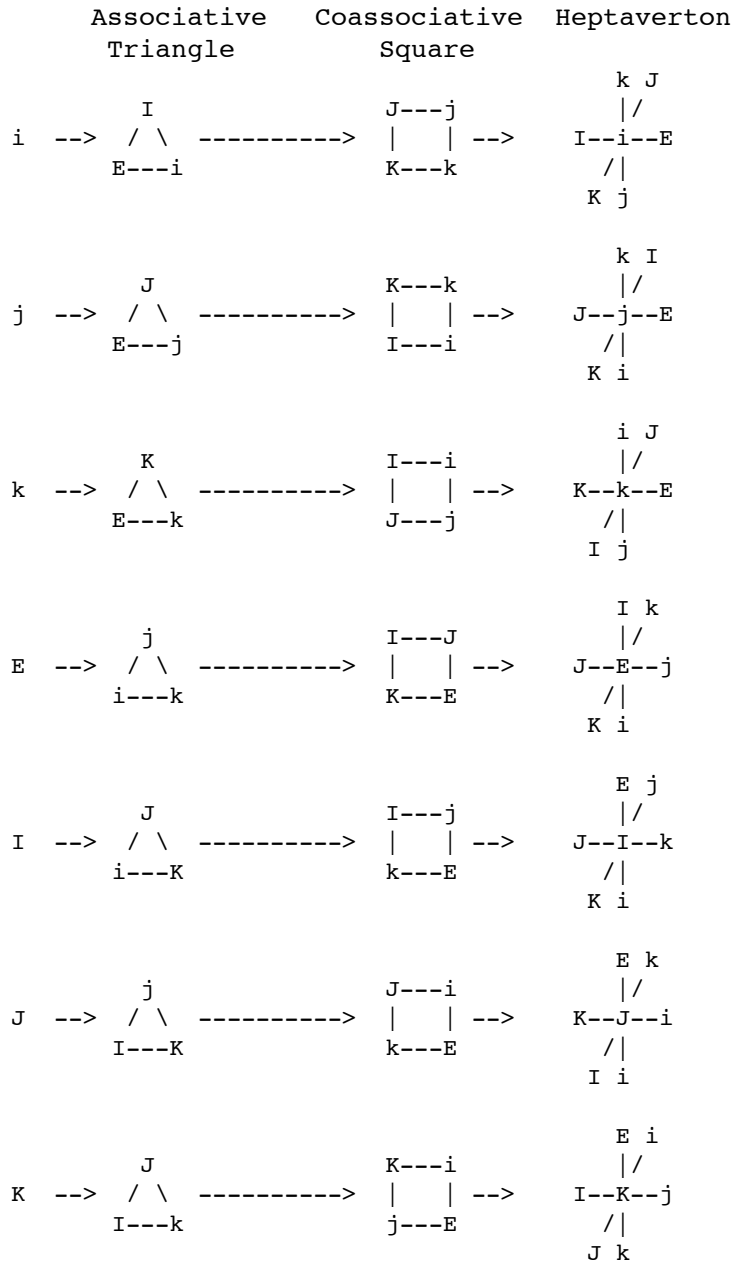
**full 8-dim HyperCube of our Universe expands as N grows to 2^16 and beyond.**

The Union of all Cl(16) tensor products is the Union of all subdivided 8-HyperCubes and their Completion is a huge superposition of 8-HyperCube Continuous Volumes which Completion belongs to the Third Grothendieck Universe.

H. S. M. Coxeter in his paper Regular and Semi-Regular Polytopes III (Math. Z. 200, 3-45, 1988)

about the 240 units of an E8 Integral Domain said: "... "... the 16 + 16 + 16 octaves  $\pm 1, \pm i, \pm j, \pm k, \pm E, \pm I, \pm J, \pm K, (\pm 1 \pm i \pm j \pm k)/2, (\pm E \pm i \pm j \pm k)/2$ , and the 192 others derived from the last two expressions by ... the cyclic permutation ( E, i, j, I, K, k, J ), which preserves the integral domain ... the permutation ( e I J i k K j ), which is an automorphism of the whole ring of octaves (and of the finite [Fano] plane ...) transforms this particular integral domain into another one of R. H. Bruck's cyclic of seven such domains. ...". An 8th E8 Lattice (not a closed Integral Domain, Kirmse's mistake) can be taken to correspond the the 1 Real Element of the Octonion Basis { 1, i, j, k, E, I, J, K}.

There are 7 independent E8 Integral Domain Lattices corresponding to the 7 Octonion Imaginary Basis Elements {i,j,k,E,I,J,K}



## E8 Lattices

E8 Lattices are based on Octonions, which have 480 different multiplication products. E8 Lattices can be combined to form 24-dimensional Leech Lattices and 26-dimensional Bosonic String Theory, which describes E8 Physics when the strings are physically interpreted as World-Lines. A basic String Theory Cell has as its automorphism group the Monster Group whose order is  $2^{46} \cdot 3^{20} \cdot 5^9 \cdot 7^6 \cdot 11^2 \cdot 13^3 \cdot 17 \cdot 19 \cdot 23 \cdot 29 \cdot 31 \cdot 41 \cdot 47 \cdot 59 \cdot 71 = \text{about } 8 \times 10^{53}$ .

For more about the Leech Lattice and the Monster and E8 Physics, see viXra 1210.0072 and 1108.0027 .

E8 Root systems and lattices are discussed by Robert A. Wilson in his 2009 paper "Octonions and the Leech lattice":

"... The (real) octonion algebra is an 8-dimensional (non-division) algebra with an orthonormal basis  $\{ 1 = i_0, i_1, i_2, i_3, i_4, i_5, i_6 \}$  labeled by the projective line  $PL(7) = \{ \infty \} \cup F_7$

...

The E8 root system embeds in this algebra ... take the 240 roots to be ...

**112 octonions** ...  $\pm it \pm iu$  for any distinct  $t, u$

... and ...

**128 octonions**  $(1/2)(\pm 1 \pm i_0 \pm \dots \pm i_6)$  ...[with]... an odd number of minus signs.

**Denote by L the lattice spanned by these 240 octonions**

...

Let  $s = (1/2)(-1 + i_0 + \dots + i_6)$  so  $s$  is in  $L$  ... write  $R$  for  $L$ bar ...

...

$(1/2)(1 + i_0) L = (1/2) R (1 + i_0)$  is closed under multiplication ... Denote this ...by  $A$

... Writing  $B = (1/2)(1 + i_0) A (1 + i_0)$  ...from ... Moufang laws ... we have

$LR = 2B$ , and ...  $BL = L$  and  $RB = R$  ...[also]...  $2B = L$  sbar

...

**the roots of B are**

[ **16 octonions** ]...  $\pm it$  for  $t$  in  $PL(7)$

... together with

[ **112 octonions** ]...  $(1/2)(\pm 1 \pm it \pm i(t+1) \pm i(t+3))$  ...for  $t$  in  $F_7$

... and ...

[ **112 octonions** ]...  $(1/2)(\pm i(t+2) \pm i(t+4) \pm i(t+5) \pm i(t+6))$  ...for  $t$  in  $F_7$

...

**B is not closed under multiplication** ... Kirmse's mistake

...[ but ]... as Coxeter ... pointed out ...

... **there are seven non-associative rings**  $A_t = (1/2)(1 + it) B (1 + it)$ , obtained from  $B$  by swapping 1 with  $it$  ... for  $t$  in  $F_7$

...

$LR = 2B$  and  $BL = L$  ...[which]... appear[s] not to have been noticed before ... some work ... by Geoffrey Dixon ...".

Geoffrey Dixon says in his book "Division Algebras, Lattices, Physics, Windmill Tilting" using notation  $\{e_0, e_1, e_2, e_3, e_4, e_5, e_6, e_7\}$  for the Octonion basis elements that Robert A. Wilson denotes by  $\{1=i_0, i_1, i_2, i_3, i_4, i_5, i_6\}$  and I sometimes denote by  $\{1, i, j, k, E, I, J, K\}$ : "...

$$\begin{aligned}\Xi_0 &= \{\pm e_a\}, \\ \Xi_2 &= \{(\pm e_a \pm e_b \pm e_c \pm e_d)/2 : a, b, c, d \text{ distinct}, \\ &\quad e_a(e_b(e_c e_d)) = \pm 1\},\end{aligned}$$

$$\begin{aligned}\Xi^{\text{even}} &= \Xi_0 \cup \Xi_2, \\ \mathcal{E}_8^{\text{even}} &= \text{span}\{\Xi^{\text{even}}\},\end{aligned}$$

$$\begin{aligned}\Xi_1 &= \{(\pm e_a \pm e_b)/\sqrt{2} : a, b \text{ distinct}\}, \\ \Xi_3 &= \{(\sum_{a=0}^7 \pm e_a)/\sqrt{8} : \text{even number of '+'s}\},\end{aligned}$$

$$\begin{aligned}\Xi^{\text{odd}} &= \Xi_1 \cup \Xi_3, \\ \mathcal{E}_8^{\text{odd}} &= \text{span}\{\Xi^{\text{odd}}\}\end{aligned}$$

(spans over integers)

$\Xi^{\text{even}}$  has  $16+224 = 240$  elements ...  $\Xi^{\text{odd}}$  has  $112+128 = 240$  elements ...

$\mathcal{E}_8^{\text{even}}$  does not close with respect to our given octonion multiplication ...[but]...

the set  $\Xi^{\text{even}}[0-a]$ , derived from  $\Xi^{\text{even}}$  by replacing each occurrence of  $e_0$  ... with  $e_a$ , and vice versa, is multiplicatively closed. ...".

Geoffrey Dixon's  $\Xi^{\text{even}}$  corresponds to Wilson's B which I denote as  $1E_8$ .

Geoffrey Dixon's  $\Xi^{\text{even}}[0-a]$  correspond to Wilson's seven  $A_t$  which I denote as  $iE_8, jE_8, kE_8, EE_8, IE_8, JE_8, KE_8$ .

Geoffrey Dixon's  $\Xi^{\text{odd}}$  corresponds to Wilson's L.

My view is that **the  $E_8$  domains  $1E_8 = \Xi^{\text{even}} = B$  is fundamental** because

$E_8$  domains  $iE_8, jE_8, kE_8, EE_8, IE_8, JE_8, KE_8 = \Xi^{\text{even}}[0-a]$  are derived from  $1E_8$  and L and L s are also derived from  $1E_8 = \Xi^{\text{even}} = B$ .

Using the notation  $\{1, i, j, k, E, I, J, K\}$  for Octonion basis  
 notice that in the  $Cl(16)$ -E8 model introduction of Quaternionic substructure  
 to produce  $(4+4)$ -dim  $M4 \times CP2$  Kaluza-Klein SpaceTime  
 requires breaking Octonionic light-cone elements

$(\pm 1 \pm i \pm j \pm k \pm E \pm I \pm J \pm K) / 2$   
 into Quaternionic 4-term forms like  $(\pm A \pm B \pm C \pm D) / 2$ .

To do that, consider that there are  $(8!4) = 70$  ways to choose 4-term subsets  
 of the 8 Octonionic basis element terms. Using all of them produces  
 224 4-term subsets in each of the 7 Octonion Imaginary E8 lattices  
 $iE8, jE8, kE8, EE8, IE8, JE8, KE8$  each of which also has 16 1-term first-shell vertices.

56 of the 70 4-term subsets appear as 8 in each of the 7 Octonion Imaginary E8 lattices.

The other  $70 - 56 = 14$  4-term subsets occur in sets of 3 among  $7 \times 6 = 42$  4-term subsets  
 as indicated in the following detailed list of the 7 Octonion Imaginary E8 lattices:

**EE8:**

112 of D8 Root Vectors

16 appear in all 7 of  $iE8, jE8, kE8, EE8, IE8, JE8, KE8$

$\pm 1, \pm i, \pm j, \pm k, \pm E, \pm I, \pm J, \pm K$

96 appear in 3 of  $iE8, jE8, kE8, EE8, IE8, JE8, KE8$

$(\pm 1 \pm K \pm E \pm k) / 2$   $(\pm i \pm j \pm I \pm J) / 2$   $kE8, EE8, KE8$

$(\pm 1 \pm J \pm j \pm E) / 2$   $(\pm I \pm K \pm k \pm i) / 2$   $jE8, EE8, JE8$

$(\pm 1 \pm E \pm I \pm i) / 2$   $(\pm K \pm k \pm J \pm j) / 2$   $iE8, EE8, IE8$

128 of D8 half-spinors appear only in EE8

$(\pm 1 \pm I \pm J \pm K) / 2$   $(\pm E \pm i \pm j \pm k) / 2$

$(\pm 1 \pm k \pm i \pm J) / 2$   $(\pm j \pm I \pm K \pm E) / 2$

$(\pm 1 \pm i \pm K \pm j) / 2$   $(\pm k \pm J \pm E \pm I) / 2$

$(\pm 1 \pm j \pm k \pm I) / 2$   $(\pm J \pm E \pm i \pm K) / 2$

**iE8:**

112 of D8 Root Vectors

16 appear in all 7 of iE8, jE8, kE8, EE8, IE8, JE8, KE8

$\pm 1, \pm i, \pm j, \pm k, \pm E, \pm I, \pm J, \pm K$

96 appear in 3 of iE8, jE8, kE8, EE8, IE8, JE8, KE8

$(\pm 1 \pm I \pm i \pm E)/2 (\pm j \pm k \pm J \pm K)/2$  iE8 , EE8 , IE8

$(\pm 1 \pm K \pm J \pm i)/2 (\pm j \pm k \pm E \pm I)/2$  iE8 , JE8 , KE8

$(\pm 1 \pm i \pm k \pm j)/2 (\pm E \pm I \pm J \pm K)/2$  iE8 , jE8 , kE8

128 of D8 half-spinors appear only in iE8

$(\pm 1 \pm k \pm K \pm I)/2 (\pm i \pm j \pm E \pm J)/2$

$(\pm 1 \pm E \pm j \pm K)/2 (\pm i \pm k \pm I \pm J)/2$

$(\pm 1 \pm j \pm I \pm J)/2 (\pm i \pm k \pm E \pm K)/2$

$(\pm 1 \pm J \pm E \pm k)/2 (\pm i \pm j \pm I \pm K)/2$

**jE8:**

112 of D8 Root Vectors

16 appear in all 7 of iE8, jE8, kE8, EE8, IE8, JE8, KE8

$\pm 1, \pm i, \pm j, \pm k, \pm E, \pm I, \pm J, \pm K$

96 appear in 3 of iE8, jE8, kE8, EE8, IE8, JE8, KE8

$(\pm 1 \pm k \pm j \pm i)/2 (\pm E \pm I \pm J \pm K)/2$  iE8 , jE8 , kE8

$(\pm 1 \pm I \pm K \pm j)/2 (\pm i \pm k \pm E \pm J)/2$  jE8 , IE8 , KE8

$(\pm 1 \pm j \pm E \pm J)/2 (\pm i \pm k \pm I \pm K)/2$  jE8 , EE8 , JE8

128 of D8 half-spinors appear only in jE8

$(\pm 1 \pm E \pm I \pm k)/2 (\pm i \pm j \pm J \pm K)/2$

$(\pm 1 \pm i \pm J \pm I)/2 (\pm j \pm k \pm E \pm K)/2$

$(\pm 1 \pm J \pm k \pm K)/2 (\pm i \pm j \pm E \pm I)/2$

$(\pm 1 \pm K \pm i \pm E)/2 (\pm j \pm k \pm I \pm J)/2$

**kE8:**

112 of D8 Root Vectors

16 appear in all 7 of iE8, jE8, kE8, EE8, IE8, JE8, KE8

$\pm 1, \pm i, \pm j, \pm k, \pm E, \pm I, \pm J, \pm K$

96 appear in 3 of iE8, jE8, kE8, EE8, IE8, JE8, KE8

$(\pm 1 \pm J \pm k \pm I)/2 (\pm i \pm j \pm E \pm K)/2$  kE8 , IE8 , JE8

$(\pm 1 \pm j \pm i \pm k)/2 (\pm E \pm I \pm J \pm K)/2$  iE8 , jE8 , kE8

$(\pm 1 \pm k \pm K \pm E)/2 (\pm i \pm j \pm I \pm J)/2$  kE8 , EE8 , KE8

128 of D8 half-spinors appear only in kE8

$(\pm 1 \pm K \pm j \pm J)/2 (\pm i \pm k \pm E \pm I)/2$

$(\pm 1 \pm I \pm E \pm j)/2 (\pm i \pm k \pm J \pm K)/2$

$(\pm 1 \pm E \pm J \pm i)/2 (\pm j \pm k \pm I \pm K)/2$

$(\pm 1 \pm i \pm I \pm K)/2 (\pm j \pm k \pm E \pm J)/2$



**IE8:**

112 of D8 Root Vectors

16 appear in all 7 of  $iE8, jE8, kE8, EE8, IE8, JE8, KE8$

$\pm 1, \pm i, \pm j, \pm k, \pm E, \pm I, \pm J, \pm K$

96 appear in 3 of  $iE8, jE8, kE8, EE8, IE8, JE8, KE8$

$(\pm 1 \pm j \pm I \pm K)/2 (\pm i \pm k \pm E \pm J)/2$   $jE8, IE8, KE8$

$(\pm 1 \pm i \pm E \pm I)/2 (\pm j \pm k \pm J \pm K)/2$   $iE8, EE8, IE8$

$(\pm 1 \pm I \pm J \pm k)/2 (\pm i \pm j \pm E \pm K)/2$   $kE8, IE8, JE8$

128 of D8 half-spinors appear only in  $IE8$

$(\pm 1 \pm J \pm i \pm j)/2 (\pm k \pm E \pm I \pm K)/2$

$(\pm 1 \pm K \pm k \pm i)/2 (\pm j \pm E \pm I \pm J)/2$

$(\pm 1 \pm k \pm j \pm E)/2 (\pm i \pm I \pm J \pm K)/2$

$(\pm 1 \pm E \pm K \pm J)/2 (\pm i \pm j \pm k \pm I)/2$

**JE8:**

112 of D8 Root Vectors

16 appear in all 7 of  $iE8, jE8, kE8, EE8, IE8, JE8, KE8$

$\pm 1, \pm i, \pm j, \pm k, \pm E, \pm I, \pm J, \pm K$

96 appear in 3 of  $iE8, jE8, kE8, EE8, IE8, JE8, KE8$

$(\pm 1 \pm E \pm J \pm j)/2 (\pm i \pm k \pm I \pm K)/2$   $jE8, EE8, JE8$

$(\pm 1 \pm k \pm I \pm J)/2 (\pm i \pm j \pm E \pm I)/2$   $kE8, IE8, JE8$

$(\pm 1 \pm J \pm i \pm K)/2 (\pm j \pm k \pm E \pm I)/2$   $iE8, JE8, KE8$

128 of D8 half-spinors appear only in  $JE8$

$(\pm 1 \pm i \pm k \pm E)/2 (\pm j \pm I \pm J \pm K)/2$

$(\pm 1 \pm j \pm K \pm k)/2 (\pm i \pm E \pm I \pm J)/2$

$(\pm 1 \pm K \pm E \pm I)/2 (\pm i \pm j \pm k \pm J)/2$

$(\pm 1 \pm I \pm j \pm i)/2 (\pm k \pm E \pm J \pm K)/2$

**KE8:**

112 of D8 Root Vectors

16 appear in all 7 of  $iE8, jE8, kE8, EE8, IE8, JE8, KE8$

$\pm 1, \pm i, \pm j, \pm k, \pm E, \pm I, \pm J, \pm K$

96 appear in 3 of  $iE8, jE8, kE8, EE8, IE8, JE8, KE8$

$(\pm 1 \pm i \pm k \pm J)/2 (\pm j \pm k \pm E \pm I)/2$   $iE8, JE8, KE8$

$(\pm 1 \pm E \pm k \pm K)/2 (\pm i \pm j \pm I \pm J)/2$   $kE8, EE8, KE8$

$(\pm 1 \pm K \pm j \pm I)/2 (\pm i \pm k \pm E \pm J)/2$   $jE8, IE8, KE8$

128 of D8 half-spinors appear only in  $KE8$

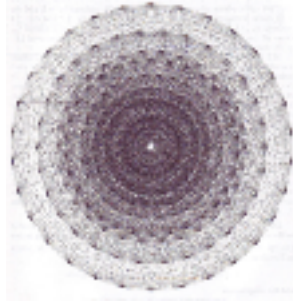
$(\pm 1 \pm j \pm E \pm i)/2 (\pm k \pm I \pm J \pm K)/2$

$(\pm 1 \pm J \pm I \pm E)/2 (\pm i \pm j \pm k \pm K)/2$

$(\pm 1 \pm I \pm i \pm k)/2 (\pm j \pm E \pm J \pm K)/2$

$(\pm 1 \pm k \pm J \pm j)/2 (\pm i \pm E \pm I \pm K)/2$

Coxeter said in "Integral Cayley Numbers" (Duke Math. J. 13 (1946) 561-578 and in "Regular and Semi-Regular Polytopes III" (Math. Z. 200 (1988) 3-45):  
 "... the 240 integral Cayley numbers of norm 1 ... are the vertices of 4\_21

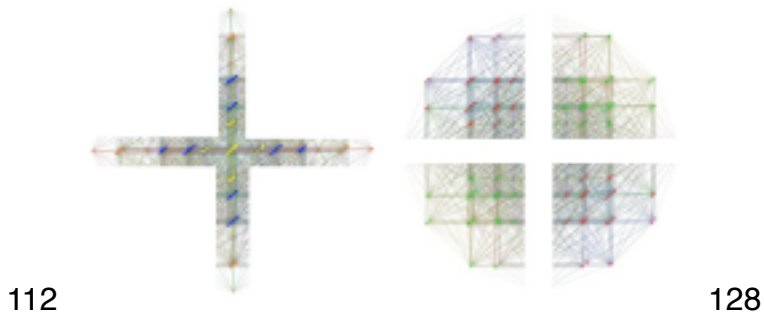


The polytope 4\_21 ... has cells of two kinds ...  
 a seven-dimensional "cross polytope" (or octahedron-analogue) B\_7  
 ... there are ... 2160 B\_7's ...  
 and ...  
 a seven-dimensional regular simplex A\_7  
 ... there are 17280 A\_7's  
 ...  
 the 2160 integral Cayley numbers of norm 2 are  
 the centers of the 2160 B\_7's of a 4\_21 of edge 2  
 ...  
 the 17280 integral Cayley numbers of norm 4 (other than the doubles  
 of those of norm 1) are the centers of the 17280 A\_7's of a 4\_21 of edge 8/3 ...  
 [ Using notation of {a1,a2,a3,a4,a5,a6,a7,a8} for Octonion basis elements we have ]

**norm 1**

**112** like ( +/- a1 +/- a2 )  
 [which correspond to 112 = 16 + 96 = 16 + 6x16 in each of the 7 E8 lattices]

**128** like (1/2) ( - a1 + a2 + a3 + ... + a8 ) with an odd number of minus signs  
 [which correspond to 128 = 8x16 in each of the 7 E8 lattices]



## norm 2

**16** like  $\pm 2 a_1$

[which correspond to 16 for the 112 in each of the 7 E8 lattices]

**1120** like  $\pm a_1 \pm a_2 \pm a_3 \pm a_4$

[which correspond to  $70 \times 16 = (56+14) \times 16$  that appear in the 7 E8 lattices

with each of the 14 appearing in three of the 7 E8 lattices so that  
the 14 account for  $(14/7) \times 3 \times 16 = 6 \times 16 = 96$  in each of the 7 E8 lattices  
and for  $14 \times 16 = \mathbf{224}$  of the **1120**

and

with each of the 56 appearing in only one of the 7 E8 lattices so that  
the 56 account for  $(56/7) \times 16 = 128$  in each of the 7 E8 lattices  
and for  $56 \times 16 = \mathbf{896} = \mathbf{7 \times 128}$  of the **1120** ]

**1024** like  $(1/2)(3a_1 + 3a_2 + a_3 + a_4 + \dots + a_8)$  with an even number of minus signs  
[which correspond to  $\mathbf{8 \times 128} = 8$  copies of the 128-dim Mirror D8 half-spinors that  
are not used in the 7 E8 lattices. ...] ...".

One of the 128-dimensional Mirror D8 half-spinors from the 1024  
combines with

the 128 from the 1120 corresponding to the one of the 7 E8 lattices that corresponds  
to the central norm  $1240 = 112 + 128$

and

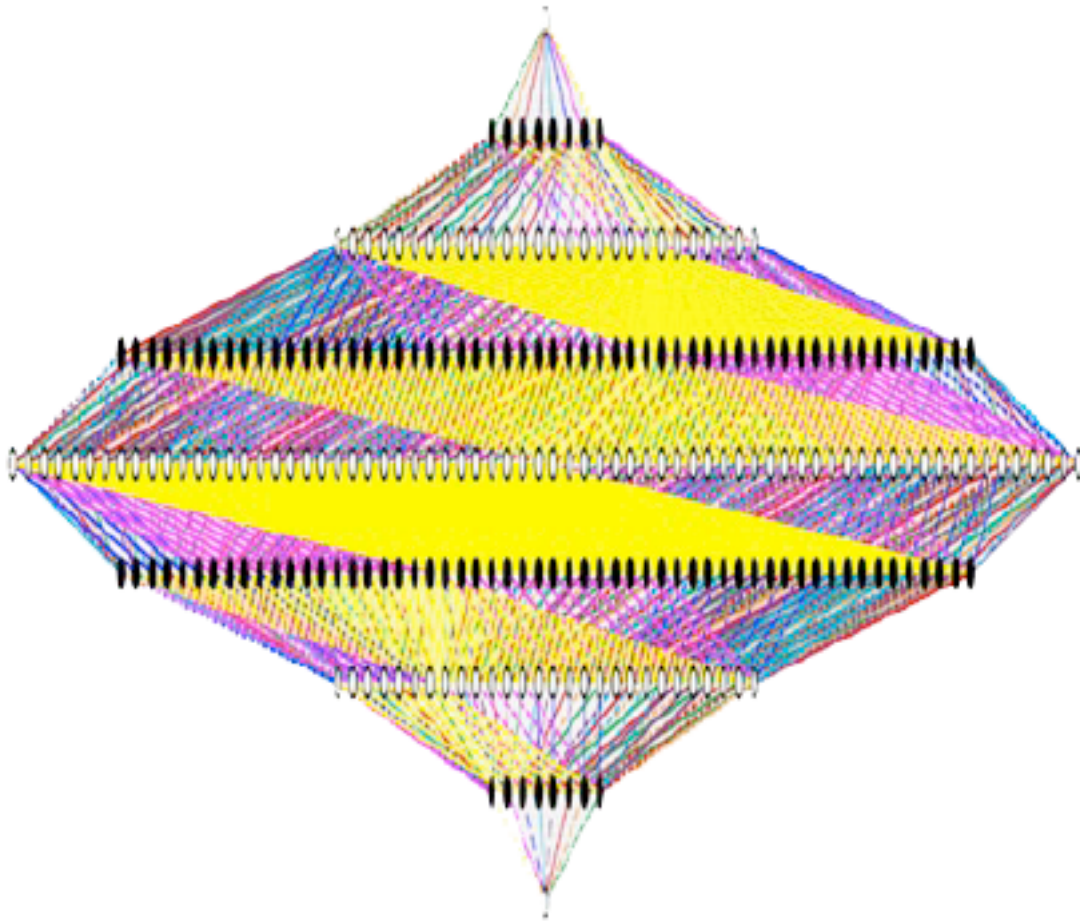
the result is formation of a  $128 + 128 = 256$  corresponding to the Clifford Algebra  $Cl(8)$   
so that

the norm 2 second layer contains 7 copies of 256-dimensional  $Cl(8)$

so the 2160 norm 2 vertices can be seen as

$$\mathbf{7(128+128) + 128 + 16 + 224 = 2160 \text{ vertices.}}$$

The 256 vertices of each pair 128+128 form an 8-cube with 1024 edges, 1792 square faces, 1792 cubic cells, 1120 tesseract 4-faces, 448 5-cube 5-faces, 112 6-cube 6-faces, and 16 7-cube 7-faces. The image format of African Adinkra for 256 Odu of IFA



shows  $Cl(8)$  graded structure  $1 + 8 + 28 + 56 + 70 + 56 + 28 + 8 + 1$  of 8-cube vertices. Physically they represent **Operators in  $H_{92} \times SL(8)$  Generalized Heisenberg Algebra** that is the Maximal Contraction of  $E_8$ :

**Odd-Grade Parts of  $Cl(8)$  =**

= **128 D8 half-spinors** of one of  $iE_8, jE_8, kE_8, EE_8, IE_8, JE_8, KE_8$   
 $8+56$  grades-1,3 = Fermion Particle 8-Component Creation (AntiParticle Annihilation)  
 $56+8$  grades-5,7 = Fermion AntiParticle 8-Component Creation (Particle Annihilation)

**Even-Grade Subalgebra of  $Cl(8)$  = 128 Mirror D8 half-spinors =**

28 grade-2 = Gauge Boson Creation (16 for Gravity, 12 for Standard Model)  
 28 grade-6 = Gauge Boson Annihilation (16 for Gravity, 12 for Standard Model)  
 (each 28 = 24 Root Vectors + 4 of Cartan Subalgebra)  
 64 of grade-4 = 8-dim Position x Momentum  
 $1+(3+3)+1$  grades-0,4,8 = Primitive Idempotent:

(1+3) = Higgs Creation; (3+1) = Higgs Annihilation  
 = **112 D8 Root Vectors + 8 of  $E_8$  Cartan Subalgebra + 8 Higgs Operators**

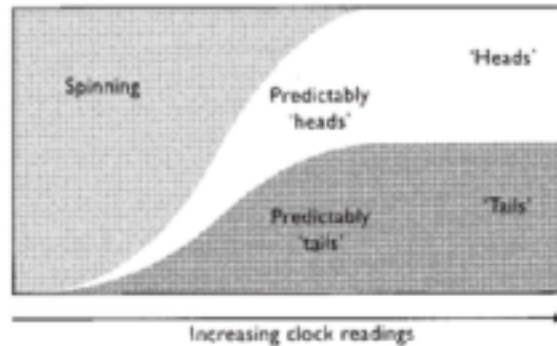
### 3. The Third Grothendieck Universe is the Completion of Union of all tensor products of $Cl(16)$ Real Clifford algebra

Since the  $Cl(16)$ -E8 Lagrangian is Local and Classical, it is necessary to patch together Local Lagrangian Regions to form a Global Structure describing a Global  $Cl(16)$ -E8 Algebraic Quantum Field Theory (AQFT).

The usual Hyperfinite II1 von Neumann factor for creation and annihilation operators on Fermionic Fock Space over  $C^{(2n)}$  is constructed by completion of the union of all tensor products of  $2 \times 2$  Complex Clifford algebra matrices, which have Periodicity 2, so

for the  $Cl(16)$ -E8 model based on Real Clifford Algebras with Periodicity 8, whereby any Real Clifford Algebra, no matter how large, can be embedded in a tensor product of factors of  $Cl(8)$  and of  $Cl(8) \times Cl(8) = Cl(16)$ , the completion of the union of all tensor products of  $Cl(16) = Cl(8) \times Cl(8)$  produces a generalized Hyperfinite II1 von Neumann factor that gives the  $Cl(16)$ -E8 model a natural Algebraic Quantum Field Theory.

The overall structure of  $Cl(16)$ -E8 AQFT is similar to the Many-Worlds picture described by David Deutsch in his 1997 book "The Fabric of Reality" said (pages 276-283): "... there is no fundamental demarcation between snapshots of other times and snapshots of other universes ... Other times are just special cases of other universes ... Suppose ... we toss a coin ... Each point in the diagram represents one snapshot



... in the multiverse there are far too many snapshots for clock readings alone to locate a snapshot relative to the others. To do that, we need to consider the intricate detail of which snapshots determine which others. ...

in some regions of the multiverse, and in some places in space, the snapshots of some physical objects do fall, for a period, into chains, each of whose members determines all the others to a good approximation ...".

The Real Clifford Algebra  $Cl(16)$  containing E8 for the Local Lagrangian of a Region is equivalent to a "snapshot" of the Deutsch "multiverse".

The completion of the union of all tensor products of all  $Cl(16)$ -E8 Local Lagrangian Regions forms a generalized hyperfinite II1 von Neumann factor AQFT and emergently self-assembles into a structure = Deutsch multiverse.

For the  $Cl(16)$ -E8 model AQFT to be realistic, it must be consistent with **EPR entanglement relations**. Joy Christian in arXiv 0904.4259 said: "... a [geometrically] correct local-realistic framework ... provides exact, deterministic, and local underpinnings ... The alleged non-localities ... result from misidentified [geometries] of the EPR elements of reality. ... The correlations are ... the classical correlations [ such as those ] among the points of a 3 or 7-sphere ...  $S^3$  and  $S^7$  ... are ... parallelizable ... The correlations ... can be seen most transparently in the elegant language of Clifford algebra ...". Since E8 is a Lie Group and therefore parallelizable and lives in Clifford Algebra  $Cl(16)$ , **the  $Cl(16)$ -E8 model is consistent with EPR.**

**The Creation-Annihilation Operator structure of  $Cl(16)$ -E8 AQFT** is given by the

Maximal Contraction of E8 = semidirect product  $A_7 \times h_{92}$

where  $h_{92} = 92+1+92 = 185$ -dim Heisenberg algebra and  $A_7 = 63$ -dim  $SL(8)$

The Maximal E8 Contraction  $A_7 \times h_{92}$  can be written as a 5-Graded Lie Algebra

$$28 + 64 + (SL(8, \mathbb{R}) + 1) + 64 + 28$$

$$\text{Central Even Grade } 0 = SL(8, \mathbb{R}) + 1$$

The 1 is a scalar and  $SL(8, \mathbb{R}) = Spin(8) + \text{Traceless Symmetric } 8 \times 8 \text{ Matrices}$ ,

so  $SL(8, \mathbb{R})$  represents a local 8-dim SpaceTime in Polar Coordinates.

$$\text{Odd Grades } -1 \text{ and } +1 = 64 + 64$$

Each =  $64 = 8 \times 8 =$  Creation/Annihilation Operators for 8 components of 8 Fundamental Fermions.

$$\text{Even Grades } -2 \text{ and } +2 = 28 + 28$$

Each = Creation/Annihilation Operators for 28 Gauge Bosons of Gravity + Standard Model.

The  $Cl(16)$ -E8 AQFT inherits structure from the  $Cl(16)$ -E8 Local Lagrangian

$$\int \text{Standard Model Gauge Gravity} + \text{Fermion Particle-AntiParticle} \\ \text{8-dim SpaceTime}$$

The  $Cl(16)$ -E8 generalized Hyperfinite II<sub>1</sub> von Neumann factor Algebraic Quantum Field Theory is based on the Completion of the Union of all Tensor Products of the form

$$Cl(16) \times \dots (N \text{ times tensor product}) \dots \times Cl(16) = Cl(16N)$$

For  $N = 2^8 = 256$  the copies of  $Cl(16)$  are on the 256 vertices of the 8-dim HyperCube



**For  $N = 2^{16} = 65,536 = 4^8$  the copies of  $CI(16)$  fill in the 8-dim HyperCube** as described by William Gilbert's web page: "... The n-bit reflected binary **Gray code** will describe a path on the edges of an n-dimensional cube that can be used as the initial stage of a Hilbert curve that will fill an n-dimensional cube. ...".

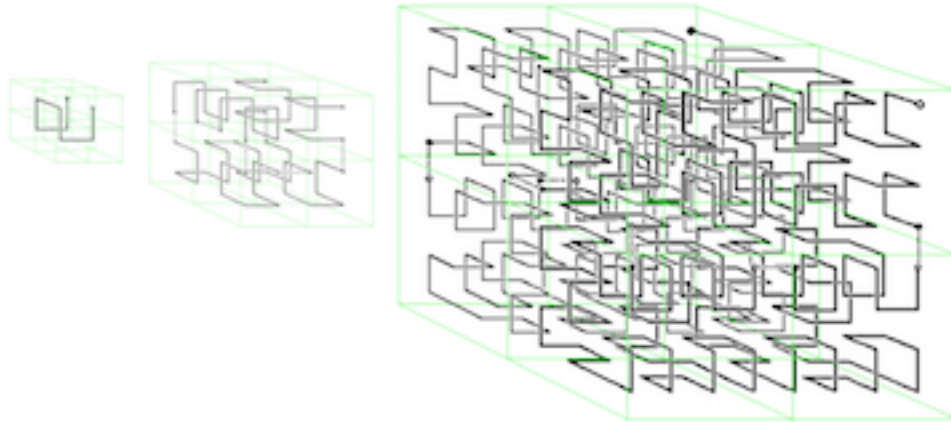
The vertices of the Hilbert curve are at the centers of the  $2^8$  sub-8-HyperCubes whose edge lengths are  $1/2$  of the edge lengths of the original 8-dim HyperCube

**As  $N$  grows**, the copies of  $CI(16)$  continue to fill the 8-dim HyperCube of E8 SpaceTime using higher Hilbert curve stages from the 8-bit reflected binary Gray code subdividing the initial 8-dim HyperCube into more and more sub-HyperCubes.

If edges of sub-HyperCubes, equal to the distance between adjacent copies of  $CI(16)$ , remain constantly at the Planck Length, then the

**full 8-dim HyperCube of our Universe expands as  $N$  grows to  $2^{16}$  and beyond** similarly to the way shown by this 3-HyperCube example for  $N = 2^3, 4^3, 8^3$

from Wiliam Gilbert's web page:



**The Union of all  $CI(16)$  tensor products is  
the Union of all subdivided 8-HyperCubes  
and  
their Completion is a huge superposition of 8-HyperCube Continuous Volumes  
which Completion belongs to the Third Grothendieck Universe.**

## 4. World-Line String Bohm Quantum Potential and Quantum Consciousness

The CI(16)-E8 AQFT inherits structure from the CI(16)-E8 Local Lagrangian

$$\int \text{Standard Model Gauge Gravity} + \text{Fermion Particle-AntiParticle} \\ \text{8-dim SpaceTime}$$

whereby World-Lines of Particles are represented by Strings moving in a space whose dimensionality includes  $8v = 8\text{-dim SpaceTime Dimensions} + 8s+ = 8 \text{ Fermion Particle Types} + 8s- = 8 \text{ Fermion AntiParticle Types}$  combined in the traceless part  $J(3,0)_o$  of the  $3 \times 3$  Octonion Hermitian Jordan Algebra

$a$	$8s+$	$8v$
$8s+^*$	$b$	$8s-$
$8v^*$	$8s-^*$	$-a-b$

which has total dimension  $8v + 8s+ + 8s- + 2 = 26$  and is the space of a 26D String Theory with Strings seen as World-Lines.

Slices of  $8v \text{ SpaceTime}$  are represented as D8 branes. Each D8 brane has Planck-Scale Lattice Structure superpositions of 8 types of E8 Lattice denoted by  $1E8, iE8, jE8, kE8, EE8, IE8, JE8, KE8$

Stack D8 branes to get SpaceTime with Strings = World-Lines  
with  
a and b representing  
ordering of D8 brane stacks and Bohm-type Quantum Potential

Let  $Oct_{16}$  = discrete multiplicative group  $\{ +/-1, +/-i, +/-j, +/-k, +/-E, +/-I, +/-J, +/-K \}$ .  
Orbifold by  $Oct_{16}$  the  $8s+$  to get  $8 \text{ Fermion Particle Types}$   
Orbifold by  $Oct_{16}$  the  $8s-$  to get  $8 \text{ Fermion AntiParticle Types}$

Gauge Bosons from  $1E8$  and  $EE8$  parts of a D8 give  $U(2) \text{ Electroweak Force}$   
Gauge Bosons from  $IE8, JE8,$  and  $KE8$  parts of a D8 give  $SU(3) \text{ Color Force}$   
Gauge Bosons from  $1E8, iE8, jE8,$  and  $kE8$  parts of a D8 give  $U(2,2) \text{ Conformal Gravity}$

The  $8 \times 8$  matrices for collective coordinates linking one D8 to the next D8 give  $\text{Position} \times \text{Momentum}$



Green, Schwartz, and Witten say in their book "Superstring Theory" vol. 1 (Cambridge 1986)  
 "... For the ... closed ... bosonic string .... The first excited level ... consists of ...  
 the ground state ... tachyon ... and ... a scalar ... 'dilaton' ... and ...  
 SO(24) ... little group of a ...[26-dim]... massless particle ... and ...  
 a ... massless ... spin two state ...".  
 Closed string tachyons localized at orbifolds of fermions produce virtual clouds of  
 particles / antiparticles that dress fermions.

Dilatons are Goldstone bosons of spontaneously broken scale invariance that  
 (analagous to Higgs) go from mediating a long-range scalar gravity-type force  
 to the nonlocality of the Bohm-Sarfatti Quantum Potential.

The SO(24) little group is related to the Monster automorphism group that is  
 the symmetry of each cell of Planck-scale local lattice structure.

The massless spin two state is what I call the Bohmion:  
 the carrier of the Bohm Force of the Bohm-Sarfatti Quantum Potential.  
 Peter R. Holland says in his book "The Quantum Theory of Motion" (Cambridge 1993)  
 "... the total force ... from the quantum potential ... does not ... fall off with distance ...  
 because ... the quantum potential ... depends on the form of ...[the quantum state]...  
 rather than ... its ... magnitude ...".

Quantum Consciousness is due to Resonant Quantum Potential Connections among  
 Quantum State Forms. The Quantum State Form of a Conscious Brain is determined by  
 the configuration of a subset of its  $10^{18}$  to  $10^{19}$  Tubulin Dimers  
 with math description in terms of a large Real Clifford Algebra.

First consider Superposition of States involving one tubulin  
 with one electron of mass  $m$  and two different position states separated by a .  
 The Superposition Separation Energy Difference is the gravitational energy  

$$E_{\text{electron}} = G m^2 / a$$

For any single given tubulin  $a = 1$  nanometer =  $10^{-7}$  cm so that for a single Electron  

$$T = h / E_{\text{electron}} = ( \text{Compton} / \text{Schwarzschild} ) ( a / c ) = 10^{26} \text{ sec} = 10^{19} \text{ years}$$

Now consider the case of  $N$  Tubulin Electrons in Coherent Superposition  
 Jack Sarfatti defines coherence length  $L$  by  $L^3 = N a^3$  so that  
 the Superposition Energy  $E_N$  of  $N$  superposed Conformation Electrons is  

$$E_N = G M^2 / L = N^{5/3} E_{\text{electron}}$$

The decoherence time for the system of  $N$  Tubulin Electrons is  

$$T_N = h / E_N = h / N^{5/3} E_{\text{electron}} = N^{-5/3} 10^{26} \text{ sec}$$

So we have the following rough approximate Decoherence Times  $T_N$

Time $T_N$	Number of Involved Tubulins
$10^{-5}$ sec	$10^{18}$
$25 \times 10^{-3}$ sec (40 Hz)	$10^{16}$

# Quantum Resonant States in Superposition

A Quantum Resonant Consciousness (QRC) Superposition State is a Tubulin Configuration of up to  $2^{64} = 10^{19}$  Tubulins (each Tubulin = 1 qubit) with each QRC State in the Superposition being organized with respect to the E8 inside Cl(16) Clifford Algebras.

Each QRC State, analagous to a Possible Conscious Thought, is represented by a Chain of Local E8-Cl(16) Deutsch-type Multiverse Snapshots in which each Link in the Chain is a Central Local E8-Cl(16) Multiverse Shapshot connected to a Past Local E8-Cl(16) Multiverse Snapshot and a Future Local E8-Cl(16) Multiverse Snapshot.

Since Cl(16) is  $2^{16} = 65,536$ -dimensional each Link in the QRC State Chain requires the information of  $2^{16} \times 2^{16} \times 2^{16} = 2^{48}$  Tubulin qubits.

The remaining  $2^{(64-48)} = 2^{16} = 2^6 \times 2^{10} = 64 \times 1024$  Tubulin qubits represent: 64 Links in each Chain of a Possible Conscious Thought and 1024 Possible Conscious Thoughts in the QRC Superposition.

After Decoherence of the QRC Superposition there emerges the One Actual Thought.

Each of the Local E8-Cl(16) Multiverse Snapshots is described by an E8 State.

Since E8 has 240 Root Vectors and

the 240 Root Vectors correspond to the 240-Polytope (see "Geometric Frustration" by Sadoc and Mosseri (Cambridge 2006) where they say "The polytope 240 ...[is]... not a regular polytope ... but ... an ordered structure on a hypersphere ... S3 ... which is chiral ... generated by adding two replicas of the {3,3,5}, displaced along a screw axis of S3 ...".)

each Local E8-Cl(16) Multiverse Snapshot is represented by a pair of {3,3,5} 600-cells.

Each of the 600-cells has 120 vertices corresponding to the 120-dimensional Icosahedral Double (ID) group which in turn corresponds to E8 (John McKay said on usenet sci.math in 1993:

"... For each finite subgroup of SU2, we get an affine Dynkin diagram ...

$$\begin{array}{c}
 3 \\
 | \\
 E[8] \quad 1-2-3-4-5-6-4-2
 \end{array}$$

... The [ McKay ] correspondence is ...

E[8] ...[ corresponds to ] 2.Alt[5] = SL(2,5) binary icosahedral [ ID group ] ...

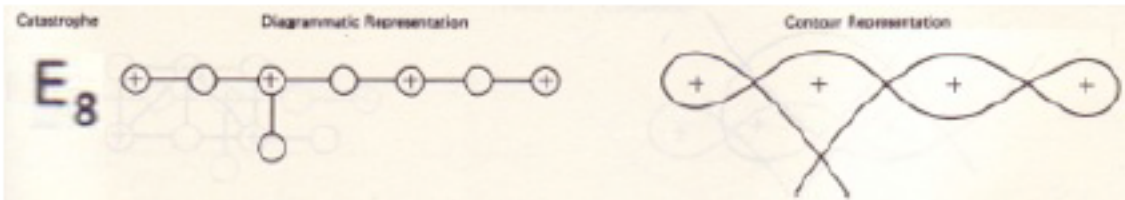
There are [ 8+1 = 9 balance numbers for E8 ]...

The sum of the numbers [ 1+2+3+4+5+6+4+2+3 = 30 is ] h = Coxeter number.

The sum of the squares is the order of ...[ 120-element ID for E8 ]...

They are the periods of products of pairs of Fischer involutions mod centre ... E[8] ...[ for ]... Monster ... for the E8 - icosahedral ... case, the singularity is  $x^2+y^3+z^5=0$  ...".

Robert Gilmore, in his book "Catastrophe Theory" (Dover 1981) said:  
 "...[ The Icosahedral Double Group Catastrophe ]... E8 ...[ has ]... Catastrophe Germ ...  $X^3 + Y^5$   
 ...[ with ]... Perturbation ...  $a_1 Y + a_2 Y^2 + a_3 Y^3 + a_4 X + a_5 X Y + a_6 X Y^2 + a_7 X Y^3$



The germs  $E_6, E_8$  are

$$E_6: f(x, y) = x^3 + y^4$$

$$E_8: f(x, y) = x^3 + y^5$$

The rules for determinacy and unfolding are particularly easy to carry out for  $E_6$  and  $E_8$  because both  $\partial f / \partial x$  and  $\partial f / \partial y$  are monomials. These calculations are summarized diagrammatically in Fig. 23.4.

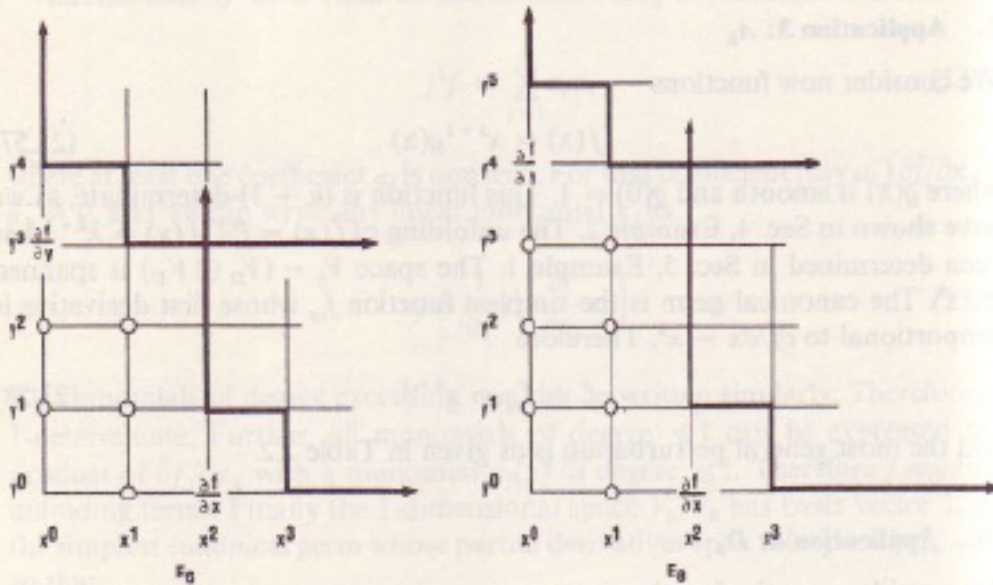


Figure 23.4 For  $E_6$  and  $E_8$  all monomials of degree 4 and 5 can be expressed in the form  $(\partial f / \partial x)_m$ . The unfolding terms are represented by open circles. We exclude the constant term.

for E8 ...[ with ]... control parameter space  $R^7$  ...[ basis  $\{a_1, a_2, a_3, a_4, a_5, a_6, a_7\}$  ]... the maximum number ... of isolated critical points ...[ is ]... 8 ...".

In his Appendix to Jeffrey Mishlove's book "Roots of Consciousness", Saul-Paul Sirag did not "exclude the constant term" as Robert Gilmore did, so, if we add a control parameter  $a_0$ , we see that the ID E8 Catastrophe Control Parameter Space is  $R^8$  with basis  $\{a_0, a_1, a_2, a_3, a_4, a_5, a_6, a_7\}$ . Adding two basis elements  $\{X, Y\}$  of ID Catastrophe Germ space whose polynomials are invariant under the Icosahedral Double Group ID results in  $8+2 = 10$  dimensions.

David Ford and John McKay wrote in the book "The Geometric Vein" (Springer-Verlag 1981):

"... The columns of the character tables of ... the binary icosahedral group ... [ ID Icosahedral Double Group]... of order 120

are the (suitably normalized) eigenvectors of the Cartan matrices of type ... E8 ...

[ Let  $gr = (1/2) (-1 - \sqrt{5})$  and  $GR = (1/2) (-1 + \sqrt{5})$  and note that  $gr + GR = -1$  ]

...  
 1 1 1 1 1 1 1 1 1  
 2 -2 0 -1 1 GR gr -gr -GR  
 2 -2 0 -1 1 gr GR -GR -gr  
 3 3 -1 0 0 -gr -GR -GR -gr  
 3 3 -1 0 0 -GR -gr -gr -GR  
 4 4 0 1 1 -1 -1 -1 -1  
 4 -4 0 1 -1 -1 -1 1 1  
 5 5 1 -1 -1 0 0 0 0  
 6 -6 0 0 0 1 1 -1 -1  
 ..."

A Cartan matrix for E8 is

```

  2  0 -1  0  0  0  0  0
  0  2  0 -1  0  0  0  0
-1  0  2 -1  0  0  0  0
  0 -1 -1  2 -1  0  0  0
  0  0  0 -1  2 -1  0  0
  0  0  0  0 -1  2 -1  0
  0  0  0  0  0 -1  2 -1
  0  0  0  0  0  0 -1  2
  
```

Note that E8 can be constructed from the representations of E6 and D8.

The grade-1 vector representation of D8 is 120-dimensional.

The half-spinor representation of D8 is 128-dimensional.

The adjoint representation of E8 is 120 + 128 = 248-dimensional.

E6 has 27-dimensional and 78-dimensional representations.

E8 Dynkin representations are:

147,250  
|

248 - 30,380 - 2,450,240 - 146,325,270 - 6,899,079,264 - 6,696,000 - 3,875

To construct them:

First, construct the exterior/wedge products of the E8 adjoint 248:

The grade-1 part has dimension 248.

The grade-2 part has dimension  $248 \wedge 248 = 30,628$ .

The grade-3 part has dimension  $248 \wedge 248 \wedge 248 = 2,511,496$ .

The grade-4 part has dimension  $248 \wedge 248 \wedge 248 \wedge 248 = 153,829,130$ .

The grade-5 part has dimension  $248 \wedge 248 \wedge 248 \wedge 248 \wedge 248 = 7,506,861,544$ .

Now:

Keep the grade-1 part of dimension 248.

Subtract off 248 from  $248 \wedge 248 = 30,628$  to get 30,380.

Subtract off  $2 \times 248 \wedge 248 = 2 \times 30,628$  from  $248 \wedge 248 \wedge 248 = 2,511,496$  to get 2,450,240.

Subtract off  $2 \times 2,511,496$  and 2,450,240 and 30,628 from  $248 \wedge 248 \wedge 248 \wedge 248 = 153,829,130$  to get 146,325,270.

Subtract off  $2 \times 153,829,130$  and  $2 \times 146,325,270$  and  $2 \times 2,511,496$  and 2,450,240 and 248 from  $248 \wedge 248 \wedge 248 \wedge 248 \wedge 248 = 7,506,861,544$  to get 6,899,079,264.

These are 5 of the 8 fundamental representations of E8.

They, like the D(N) and A(N) series constructions,  
are all in the same exterior algebra (of  $\Lambda^{248}$ ),  
and so can be represented as the vertices of a pentagon

What about the 6th and 7th fundamental representations of E8?

Consider the 27-dimensional E6 representation space. Add 32 copies of the 128-dimensional D8 half-spinor space, and subtract off one copy of the 248-dimensional E8 representation space to get a  $27 + 32 \times 128 - 248 = 3,875$ -dimensional representation space.

Now, consider the antisymmetric exterior wedge algebra of that 3,875-dimensional space.

The grade-1 part has dimension 3,875. The grade-2 part has dimension  $3,875 \wedge 3,875 = 7,505,875$ .

Now:

Keep the grade-1 part of dimension 3,875.

From the grade-2 part, subtract off  $5 \times 147,250$  and  $2 \times 30,628$  and  $3 \times 3,875$  and  $3 \times 248$  from  $3,875 \wedge 3,875 = 7,505,875$  to get 6,696,000. They are the 6th and 7th fundamental representations.

Since they are not in the same  $\Lambda^{248}$  exterior algebra as the 5 pentagon-vertex fundamental representations of E8, they should not be vertices in the same plane as the pentagon.

However, since they are in the same  $\Lambda^{3,875}$  exterior algebra, they should be collinear, one above and one below the pentagon, thus forming a pentagonal bipyramid.

What about the 8th fundamental representation of E8?

Consider  $2 \times 24 \times 24 - 1 = 2 \times 576 - 1 = 1,151$  copies of the 128-dimensional D8 half-spinor space, and subtract off one copy of the 78-dimensional E6 representation space

to get a representation space of dimension  $1,151 \times 128 - 78 = 147,328 - 78 = 147,250$ .

Now, consider the antisymmetric exterior wedge algebra of that 147,250-dimensional space.

The grade-1 part has dimension 147,250. It is the 8th fundamental representation of E8.

Since it is not in the same  $\Lambda^{248}$  exterior algebra as the 5 pentagon-vertex fundamental representations of E8, it should not be a vertex in the same plane as the pentagon.

Also, since it is not in the same  $\Lambda^{3,875}$  exterior algebra as the two bipyramid-peak-vertex fundamental representations of E8, it should not be a vertex on the same line as the pentagonal bipyramid axis.

It should represent a vertex creating a triangle whose base is one of the sides of the pentagon and whose top is near one of the bipyramid-peak-vertices, to which it is connected by a line.

To produce a symmetric figure, the vertex must be reproduced in 5 copies, one over each of the 5 sides of the pentagon.

Then, for the entire figure to be symmetric, it must form an icosahedron.

The binary icosahedral group  $\{2,3,5\}$  is of order 120.

Another way to look at it is:

The graded sequence  $248 \quad 248 \wedge 248 \quad 248 \wedge 248 \wedge 248 \quad 248 \wedge 248 \wedge 248 \wedge 248 \quad 248 \wedge 248 \wedge 248 \wedge 248 \wedge 248$

has symmetry  $Cy(5)$  of order 5 for cyclic permutations, but do not use Hodge duality

since  $248 \wedge 248 \wedge 248 \wedge 248 \wedge 248$  is fixed by its relation to  $3,875 \wedge 3,875 \wedge 3,875$ .

The graded sequence  $3,875 \quad 3,875 \wedge 3,875 \quad 3,875 \wedge 3,875 \wedge 3,875$

has symmetry  $Cy(3)$  of order 3 for cyclic permutations, but do not use Hodge duality

since  $3,875 \wedge 3,875 \wedge 3,875$  is fixed by its relation to  $248 \wedge 248 \wedge 248 \wedge 248 \wedge 248$ .

The graded sequence  $147,250 \quad 147,250 \wedge 147,250$

has symmetry  $Cy(2)$  of order 2 for cyclic permutations, but do not use Hodge duality

since  $147,250 \wedge 147,250$  is fixed by its relation to  $248 \wedge 248 \wedge 248 \wedge 248 \wedge 248$ .

The +/- signs for the D5 half-spinors inherited from E6 through E7 have symmetry of order 2.

Since E8 is the sum of the 120-dimensional adjoint representation of D8

plus ONE of the 128-dimensional half-spinor representations of D8,

there is a choice to be made as to which of the two half-spinor representations of D8 are used.

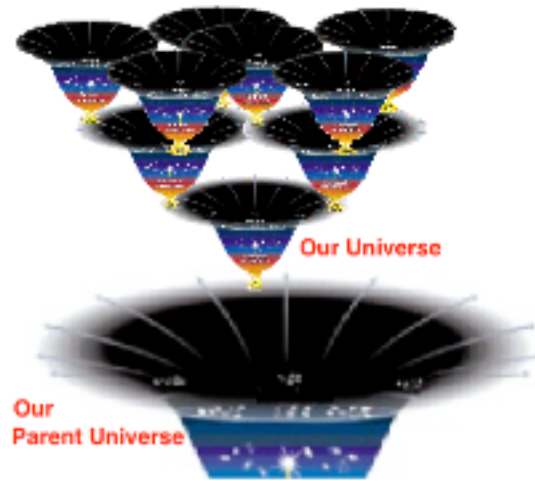
As they are mirror images of each other, that choice has a symmetry of order 2.

Therefore:

the total symmetry group is of order  $5 \times 3 \times 2 \times 2 \times 2 = 120$ ,

the symmetry of the binary icosahedral group  $\{2,3,5\}$  corresponding by McKay to the E8 Lie Algebra.

## 5. Our Universe emerged from its parent in Octonionic Inflation



As Our Parent Universe expanded to a Cold Thin State Quantum Fluctuations occurred. Most of them just appeared and disappeared as Virtual Fluctuations, but at least one Quantum Fluctuation had enough energy to produce 64 Unfoldings and reach Paola Zizzi's State of Decoherence thus making it a Real Fluctuation that became Our Universe.

As Our Universe expands to a Cold Thin State, it will probably give birth to Our Child, GrandChild, etc, Universes.

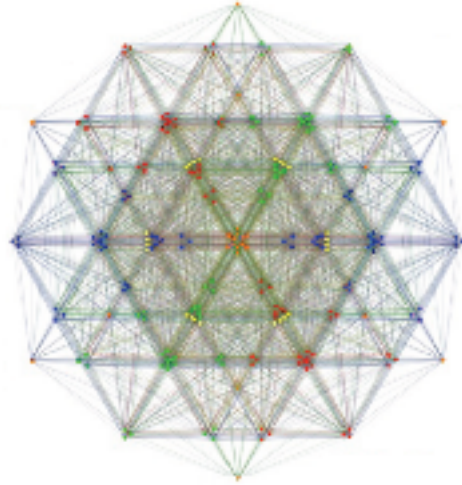
Unlike "the inflationary multiverse" described by Andrei Linde in arXiv 1402.0526 as "a scientific justification of the anthropic principle", in the  $Cl(16)$ -E8 model ALL Universes (Ours, Ancestors, Descendants) have the SAME Physics Structure as E8 Physics ( viXra 1312.0036 and 1310.0182 )

In the  $Cl(16)$ -E8 model, our SpaceTime remains Octonionic 8-dimensional throughout inflation.

Stephen L. Adler in his book Quaternionic Quantum Mechanics and Quantum Fields (1995) said at pages 50-52, 561: "... If the multiplication is associative, as in the complex and quaternionic cases, we can remove parentheses in ... Schroedinger equation dynamics ... to conclude that ... the inner product  $\langle f(t) | g(t) \rangle$  ... is invariant ... this proof fails in the octonionic case, and hence one cannot follow the standard procedure to get a unitary dynamics. ...[so there is a]... **failure of unitarity in octonionic quantum mechanics ...**".

**The NonAssociativity and Non-Unitarity of Octonions accounts for particle creation without the need for a conventional inflaton field.**

**Inflation begins in Octonionic Cl(16)-E8 Physics with a Quantum Fluctuation initially containing only one Cl(16) E8 Local Lagrangian Region**



The Fermion Representation Space for a Cl(16) E8 Local Lagrangian Region is  $E_8 / D_8 =$  the  $64+64 = 128$ -dim  $+half$ -spinor space  $64s_{++} + 64s_{+-}$  of Cl(16)  
 $64s_{++} = 8$  components of 8 Fermion Particles  
 $64s_{+-} = 8$  components of 8 Fermion Antiparticles

By 8-Periodicity of Real Clifford Algebras Cl(16) = tensor product Cl(8) x Cl(8) where the two copies of Cl(8) can be denoted by Cl(8)G and Cl(8)SM ( in E8 Physics Cl(8)G gives Gravity with Dark Energy and Cl(8)SM gives the Standard Model ) Cl(8)G and Cl(8)SM each have 8-dim half-spinor spaces  $8G_{s+}$   $8G_{s-}$  and  $8SM_{s+}$   $8SM_{s-}$   
 $8G_{s+}$  and  $8SM_{s+}$  representing 8 Fermion Particles  
 $8G_{s-}$  and  $8SM_{s-}$  representing 8 Fermion Antiparticles

so that

$64s_{++} = 8G_{s+} \times 8SM_{s+}$  for First Generation Particles of E8 Physics  
 $64s_{+-} = 8G_{s+} \times 8SM_{s-}$  for First Generation AntiParticles of E8 Physics  
 $64s_{-+} = 8G_{s-} \times 8SM_{s+}$  for AntiGeneration Particles ( NOT in E8 Physics )  
 $64s_{--} = 8G_{s-} \times 8SM_{s-}$  for AntiGeneration AntiParticles ( NOT in E8 Physics )

where

$+/-$  half-spinor of Cl(8)G determines  $+/-$  half-spinor of Cl(16) and Generation or AntiGeneration ( only  $+half$ -spinor Generation is in E8 )

$+/-$  half-spinor of Cl(8)SM determines Particle or AntiParticle

E8 Physics has Representation space for 8 Fermion Particles + 8 Fermion Antiparticles on the original  $Cl(16)$  E8 Local Lagrangian Region that is  $64s_{++} + 8$  of  $64s_{+-} =$



where a Fermion Representation slot \_ of the  $8+8 = 16$  slots can be filled

by Real Fermion Particles ● or Real Fermion Antiparticles ●

IF the Quantum Fluctuation( QF ) has enough Energy to produce them as Real and IF the  $Cl(16)$  E8 Local Lagrangian Region has an Effective Path from its QF Energy to that Particular slot. ( see Appendix III for Geoffrey Dixon's ideas and Effective Path of QF Energy )

Since E8 contains only the 128 +half-spinors and none of the 128 -half-spinors of  $Cl(16)$  the only Effective Path of QF Energy to E8 Fermion Representation slots goes to the only Fermion Particle slots that are also of type + that is, to the 8 Fermion Particle Representation slots

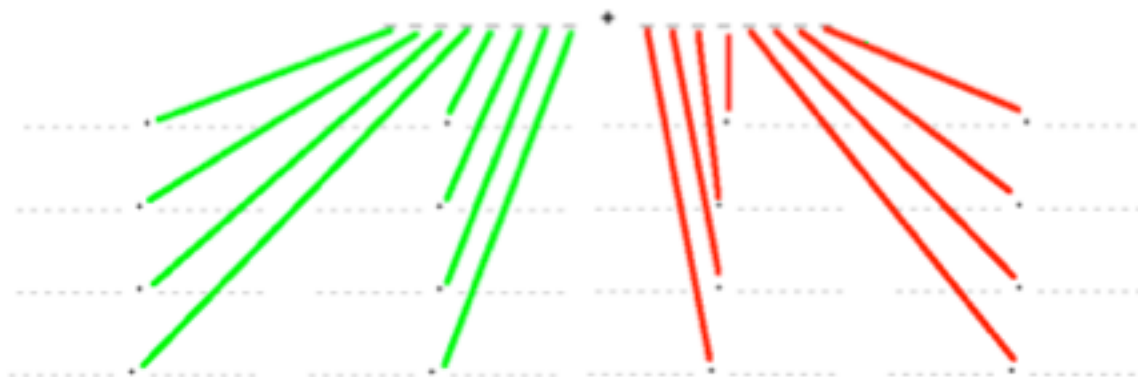


Next, consider **the first Unfolding step of Octonionic Inflation**. It is based on all  $16 = 8$  Fermion Particle slots + 8 Fermion Antiparticle Representation slots whether or not they have been filled by QF Energy.

7 of the 8 Fermion Particle slots correspond to the 7 Imaginary Octonions and therefore to the 7 Independent E8 Integral Domain Lattices and therefore to 7 New  $Cl(16)$  E8 Local Lagrangian Regions.

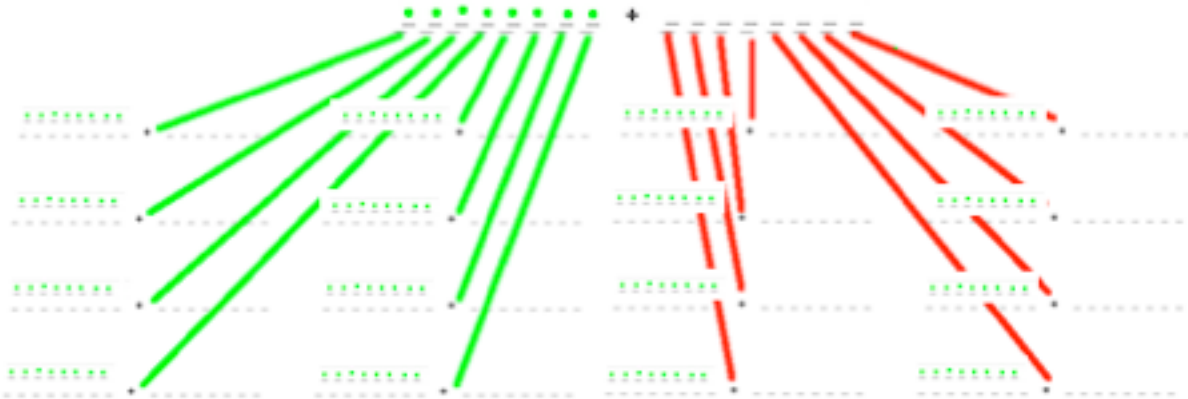
The 8th Fermion Particle slot corresponds to the 1 Real Octonion and therefore to the 8th E8 Integral Domain Lattice ( not independent - see Kirmse's mistake ) and therefore to the 8th New  $Cl(16)$  E8 Local Lagrangian Region.

Similarly, the 8 Fermion Antiparticle slots Unfold into 8 more New New  $Cl(16)$  E8 Local Lagrangian Regions, so that one Unfolding Step is a 16-fold multiplication of  $Cl(16)$  E8 Local Lagrangian Regions:



If the QF Energy is sufficient, the Fermion Particle content after the first Unfolding is is





so it is clear that **the Octonionic Inflation Unfolding Process creates Fermion Particles with no Antiparticles, thus explaining the dominance of Matter over AntiMatter in Our Universe.**

Each Unfolding has duration of the Planck Time  $T_{\text{planck}}$  and none of the components of the Unfolding Process Components are simultaneous, so that **the total duration of N Unfoldings is  $2^N T_{\text{planck}}$ .**

**Paola Zizzi** in gr-qc/0007006 said: "... **during inflation, the universe can be described as a superposed state of quantum ... [ qubits ].** the self-reduction of the superposed quantum state is ... reached at the end of inflation ...[at]... **the decoherence time ... [  $T_{\text{decoh}} = 10^9 T_{\text{planck}} = 10^{(-34)}$  sec ] ...** and **corresponds to a superposed state of ... [  $10^{19} = 2^{64}$  qubits ]. ...".**

**Why decoherence at 64 Unfoldings =  $2^{64}$  qubits ?**

$2^{64}$  qubits corresponds to the Clifford algebra  $Cl(64) = Cl(8 \times 8)$ . By the periodicity-8 theorem of Real Clifford algebras,  $Cl(64)$  is the smallest Real Clifford algebra for which we can reflexively identify each component  $Cl(8)$  with a vector in the  $Cl(8)$  vector space. This reflexive identification/reduction causes our universe to decohere at  $N = 2^{64} = 10^{19}$  which is roughly the number of Quantum Consciousness Tubulins in the Human Brain.

The Real Clifford Algebra  $Cl(8)$  is the basic building block of Real Clifford Algebras due to 8-Periodicity whereby  $Cl(8N) = Cl(8) \times \dots (N \text{ times tensor product}) \dots \times Cl(8)$

An Octonionic basis for the  $Cl(8)$  8-dim vector space is  $\{1, i, j, k, E, I, J, K\}$

NonAssociativity, NonUnitarity, and Reflexivity of Octonions is exemplified by the 1-1 correspondence between Octonion Basis Elements and  $E_8$  Integral Domains

$$\begin{aligned} 1 &\Leftrightarrow 0E_8 \\ i &\Leftrightarrow 1E_8 \\ j &\Leftrightarrow 2E_8 \\ k &\Leftrightarrow 3E_8 \\ E &\Leftrightarrow 4E_8 \\ I &\Leftrightarrow 5E_8 \\ J &\Leftrightarrow 6E_8 \\ K &\Leftrightarrow 7E_8 \end{aligned}$$

where  $1E_8, 2E_8, 3E_8, 4E_8, 5E_8, 6E_8, 7E_8$  are 7 independent Integral Domain  $E_8$  Lattices and  $0E_8$  is an 8th  $E_8$  Lattice (Kirmse's mistake) not closed as an Integral Domain.

Using that correspondence expands the basis  $\{1, i, j, k, E, I, J, K\}$  to  $\{0E_8, 1E_8, 2E_8, 3E_8, 4E_8, 5E_8, 6E_8, 7E_8\}$

Each of the  $E_8$  Lattices has 240 nearest neighbor vectors so the total dimension of the Expanded Space is  $240 \times 240 \times 240 \times 240 \times 240 \times 240 \times 240 \times 240$

Everything in the Expanded Space comes directly from the original  $Cl(8)$  8-dim space so all Quantum States in the Expanded Space can be held in Coherent Superposition. However,

if further expansion is attempted, there is no direct connection to original  $Cl(8)$  space and any Quantum Superposition undergoes Decoherence.

If each 240 is embedded reflexively into the 256 elements of  $Cl(8)$  the total dimension is  $256 \times 256 \times 256 \times 256 \times 256 \times 256 \times 256 \times 256 = 256^8 = 2^{(8 \times 8)} = 2^{64} = Cl(8) \times Cl(8) \times Cl(8) \times Cl(8) \times Cl(8) \times Cl(8) \times Cl(8) \times Cl(8) = Cl(8 \times 8) = Cl(64)$  so the largest Clifford Algebra that can maintain Coherent Superposition is  $Cl(64)$  which is why Zizzi Quantum Inflation ends at the  $Cl(64)$  level.

**At the end of 64 Unfoldings, Non-Unitary Octonionic Inflation ended having produced about  $(1/2) 16^{64} = (1/2) (2^4)^{64} = 2^{255} = 6 \times 10^{76}$  Fermion Particles**

**The End of Inflation time was at about  $10^{(-34)}$  sec =  $2^{64}$  Tplanck  
and  
the size of our Universe was then about  $10^{(-24)}$  cm  
which is about the size of a Fermion Schwinger Source Kerr-Newman Cloud.  
( see viXra 1311.0088 )**

**Octonion Inflation produces Gravitational Waves that can now be observed in Polarization Patterns of the Cosmic Microwave Background.**

BICEP2 in arXiv 1403.3985 said:

"... Inflation predicts ... a primordial background of ... gravitational waves ...[that]... would have imprinted a unique signature upon the CMB. **Gravitational waves induce local quadrupole anisotropies** in the radiation field within the last-scattering surface, inducing polarization in the scattered light ... **This polarization pattern will include a “curl” or ... inflationary gravitational wave (IGW) B-mode ... component** at degree angular scales that cannot be generated primordially by density perturbations. The amplitude of this signal depends upon the **tensor-to-scalar ratio ...  $r = 0.20 + 0.07 - 0.05$**  ... which itself is a function of the energy scale of inflation. ...".

**In the Cl(16)-E8 model,**

**Inflation is due to Non-Unitarity of Octonion Quantum Processes**

that occur in 8-dim SpaceTime before freezing out of a preferred Quaternionic Frame ends Inflation and begins Ordinary Evolution in (4+4)-dim M4 x CP2 Kaluza-Klein. The unit sphere in the Euclidean version of 8-dim SpaceTime ( see viXra 1311.0088 for Schwinger's "unitary trick" to allow use of Euclidean SpaceTime ) is the 7-sphere S7.

**Curl-type B-modes** (tensor) are Octonionic Quantum Processes on the surface of SpaceTime S7 which is a **7-dim NonAssociative Moufang Loop Malcev Algebra**.

( image below from Sky and Telescope )

B-modes look like



Spirals on the Surface of S7

**Divergence-type E modes** (scalar and tensor) are Octonionic Quantum Processes from SpaceTime S7

plus a spinor-type S7 representing Dirac Fermions living in SpaceTime plus a 14-dim G2 Octonionic Derivation Algebra connecting the two S7 spheres all of which is a **28-dim D4 Lie Algebra Spin(8)**.

( image below from Sky and Telescope )

E-modes look like Fermion Pair Creation either

off (scalar)



or on (tensor)



the Surface of S7

Therefore: **for E8 Physics Octonionic Inflation the ratio  $r = 7 / 28 = 0.25$**

### End of Inflation and Low Initial Entropy in Our Universe:

Roger Penrose in his book *The Emperor's New Mind* (Oxford 1989, pages 316-317) said: "... in our universe ... Entropy ... increases ... Something forced the entropy to be low in the past. ... **the low-entropy states in the past are a puzzle.** ...".

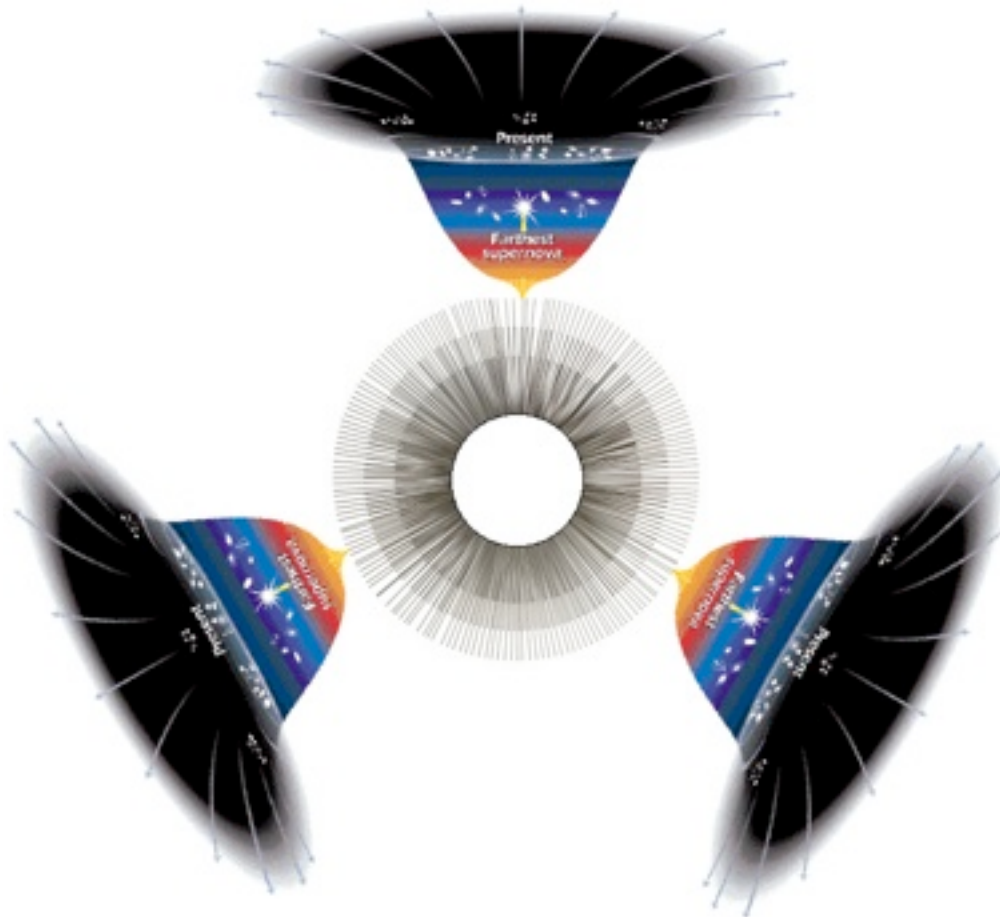
The key to solving Penrose's Puzzle is given by Paola Zizzi in gr-qc/0007006:

"... **The self-reduction of the superposed quantum state is ... reached at the end of inflation ...[at]... the decoherence time ... [  $T_{\text{decoh}} = 10^9 T_{\text{planck}} = 10(-34) \text{ sec}$  ] ... and corresponds to a superposed state of ... [  $10^{19} = 2^{64}$  qubits ]. ...**

... This is also the number of

**superposed tubulins-qubits in our brain ... leading to a conscious event. ...".**

The Zizzi Inflation phase of our universe ends with decoherence "collapse" of the  $2^{64}$  Superposition Inflated Universe into Many Worlds of Quantum Theory,



only one of which Worlds is our World. The central white circle is the Inflation Era in which everything is in Superposition; the boundary of the central circle marks the decoherence/collapse at the End of Inflation; and each line radiating from the central circle corresponds to one decohered/collapsed Universe World (of course, there are many more lines than actually shown), only three of which are explicitly indicated in the image, and only one of which is Our Universe World.

**Since our World is only a tiny fraction of all the Worlds, it carries only a tiny fraction of the entropy of the  $2^{64}$  Superposition Inflated Universe, thus solving Penrose's Puzzle.**

## 6. Quaternionic M4xCP2 Kaluza-Klein SpaceTime

At the end of Non-Unitary Octonionic Inflation Our Universe  
had about  $(1/2) 16^{64} = (1/2) (2^4)^{64} = 2^{255} = 6 \times 10^{76}$  Fermion Particles  
The End of Inflation time was at about  $10^{-34}$  sec =  $2^{64}$  Tplanck  
and  
the size of our Universe was then about  $10^{-24}$  cm  
which is about the size of a Fermion Schwinger Source Kerr-Newman Cloud  
and  
the Real Clifford Algebra of 8-dim SpaceTime was  $Cl(1,7) = Cl(0,8) = M(16,R)$

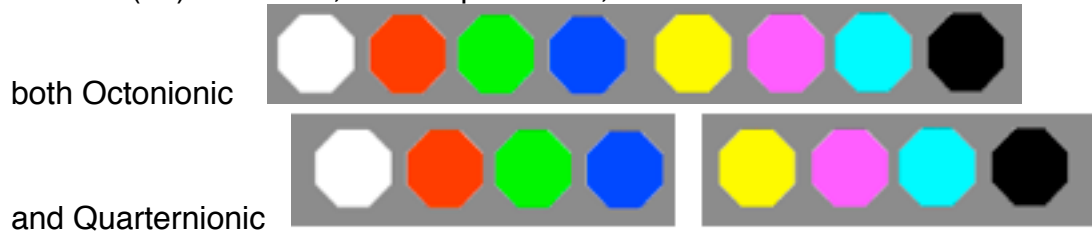
The Event that Ended Inflation was Decoherence of Zizzi Quantum Inflation that also produced decoherence of the D8 brane SpaceTime Planck-Scale Lattice superpositions of the 8 types of E8 Lattice 1E8, iE8, jE8, kE8, EE8, IE8, JE8, KE8 which resulted in a decoherence choice of a particular E8 Lattice. The 240 origin-nearest-neighbor Root Vectors of such a chosen E8 Lattice can be represented as 8 circles of 30 vertices each



with  $4 \times 30 = 120$  vertices (black dots) forming a 600-cell and  
the other  $4 \times 30 = 120$  vertices (white dots) forming another 600-cell at radii expanded  
from that of the black dots by a Golden Ratio factor. Since each 600-cell is 4-dim,  
the Octonionic 8-dim E8 SpaceTime is decomposed into 2 Quaternionic 4-dim parts,

giving the Post-Inflation  $Cl(16)$ -E8 model a (4+4)-dim Kaluza-Klein SpaceTime of the form  $M4 \times CP2$  where  $M4$  is 4-dim Physical Minkowski SpaceTime on which Gravity acts and  $CP2 = SU(3) / U(2)$  is 4-dim Internal Symmetry Space for Standard Model Forces.

In the  $Cl(16)$ -E8 model, 8-dim SpaceTime,



is represented by the 64-dim Adjoint  $D8 / D4 \times D4$  part of E8 which is the  $A7 \times R$  grade-0 part of the Maximal Contraction  $A7 \times h92$  with 5-grading  $28 + 64 + (SL(8,R) + 1) + 64 + 28$

In the  $Cl(16)$ -E8 model Gravity is most often written as in Chapter 18 of this paper in terms of the MacDowell-Mansouri Conformal Group  $Spin(2,4)$  which is the 15-dimensional Conformal BiVector Group of the 64-dim  $Cl(2,4)$  Clifford Algebra but

it can also be written in terms of 64-dim grade-0 Maximal Contraction term  $SL(8,R) + 1$  in which case it is known as Unimodular  $SL(8,R)$  Gravity which effectively describes a generalized checkerboard of 8-dim SpaceTime HyperVolume Elements and, with respect to  $Cl(16) = Cl(8) \times Cl(8)$ , is the tensor product of the two 8v vector spaces of the two  $Cl(8)$  factors of  $Cl(16)$ . If those two  $Cl(8)$  factors are regarded as Fourier Duals, then  $8v \times 8v$  describes Position  $\times$  Momentum in 8-dim SpaceTime.

Conformal  $Spin(2,4) = SU(2,2)$  Gravity and Unimodular  $SL(4,R) = Spin(3,3)$  Gravity seem to be effectively equivalent since, as Bradonjic and Stachel in arXiv 1110.2159 said: "... in ... Unimodular relativity ... the symmetry group of space-time is ... the special linear group  $SL(4,R)$  ... the metric tensor ... break[s up] ... into the conformal structure represented by a conformal metric ... with  $\det = -1$  and a four-volume element ... at each point of space-time ...[that]... may be the remnant, in the ... continuum limit, of a more fundamental discrete quantum structure of space-time itself ...".

Further,

Frampton, Ng, and Van Dam in J. Math. Phys. 33 (1992) 3881-3882 said:

"... Because of the existence of topologically nontrivial solutions, instantons, of the classical field equations associated with quantum chromodynamics (QCD), the quantized theory contains a dimensionless parameter  $\theta$  ( $0 < \theta < 2\pi$ ) not explicit in the classical lagrangian. Since  $\theta$  multiplies an expression odd in CP, QCD predicts violation of that symmetry unless the phase  $\theta$  takes one of the special values ...  $0 \pmod{\pi}$  ... this fine tuning is the strong CP problem ... the quantum dynamics of ... unimodular gravity ... may lead to the relaxation of  $\theta$  to  $\theta = 0 \pmod{\pi}$  without the need ... for a new particle ... such as the axion ...".

## End of Inflation and Quaternionic Structure

In Cl(16)-E8 Physics ( vixra 1405.0030 ) Octonionic symmetry of 8-dim spacetime is broken at the End of Non-Unitary Octonionic Inflation to Quaternionic symmetry of (4+4)-dim Kaluza-Klein M4 x CP2 physical spacetime x internal symmetry space.



Here are some details about that transition:

The basic local entity of Cl(16)-E8 Physics is

$Cl(0,16) = Cl(1,15) = Cl(4,12) = Cl(5,11) = Cl(8,8) = M(R,256) = 256 \times 256$  Real Matrices which contains E8 with 8-dim Octonionic spacetime

and is the tensor product  $Cl(0,8) \times Cl(0,8) = Cl(1,7) \times Cl(1,7)$

where  $Cl(0,8) = Cl(1,7) = M(R,16)$  is the Clifford Algebra of the 8-dim spacetime.

Non-Unitary Octonionic Inflation is based on Octonionic spacetime structure with superposition of E8 integral domain lattices. At the End of Inflation the superposition ends and Octonionic 8-dim structure is replaced by Quaternionic (4+4)-dim structure.

Since  $M(R,16) = M(Q,2) \times M(Q,2)$  and  $M(Q,2) = Cl(1,3) = Cl(0,4)$

$Cl(0,8) = Cl(1,7)$  can be represented as  $Cl(1,3) \times Cl(0,4)$

where

$Cl(1,3)$  is the Clifford Algebra for M4 physical spacetime

and

$Cl(0,4)$  is the Clifford Algebra for  $CP2 = SU(3) / U(2)$  internal symmetry space

thus

making explicit the Quaternionic structure of (4+4)-dim M4 x CP2 Kaluza-Klein.

$Cl(1,3) = Cl(0,4) = M(Q,2)$  has graded structure based on 1 2 1 grading of  $2 \times 2$  matrices and 1 2 1 grading of the Quaternions, so that its total graded structure is

$$\begin{array}{cccc} 1 & 2 & 1 & \\ & 2 & 4 & 2 \\ \hline & & 1 & 2 & 1 \\ 1 & 4 & 6 & 4 & 1 \end{array}$$

and its Spinor structure is  $2 \times 1$  Quaternion matrices

$$\begin{array}{ccc} 1 & 2 & 1 \\ \hline 1 & 2 & 1 \\ 2 & 4 & 2 \end{array} = 1 \ 2 \ 1 + 1 \ 2 \ 1$$

$1 \ 2 \ 1 = 4$ -dim Shilov Boundary for Lie Sphere  $Spin(6) / Spin(4) \times U(1) =$   
 $=$  half-spinors for First Generation Lepton + 3 Quarks

$4s+$  for Electron + 3 Up Quarks  
and



$4s-$  for Neutrino + 3 Down Quarks



One copy of  $Cl(1,3)$  only has room for Particles, no AntiParticles

$Cl(1,3)$  vectors can represent  $M_4$  physical spacetime  
but



the  $CP^2$  part of  $M_4 \times CP^2$  Kaluza-Klein is not directly represented by  $Cl(1,3)$ .



Note that  $Cl(3,1) = Cl(2,2) = M(R,4)$  has the same Clifford Algebra dimension = 16  
as does  $Cl(1,3) = Cl(0,4) = M(Q,2)$   
but

$Cl(3,1) = Cl(2,2) = M(R,4)$  Spinors are  $4 \times 1 = 4$ -dimensional (Real Dirac Gammas)  
so physicists had to Complexify them in order to get realistic results  
while

$Cl(1,3) = Cl(0,4) = M(Q,2)$  Spinors are  $2 \times 4 = 8$ -dimensional and directly give  
the same realistic physical results of Complex Dirac Gammas.

Roughly,

Quaternification of the Clifford Algebra is like Complexification of Spinors.



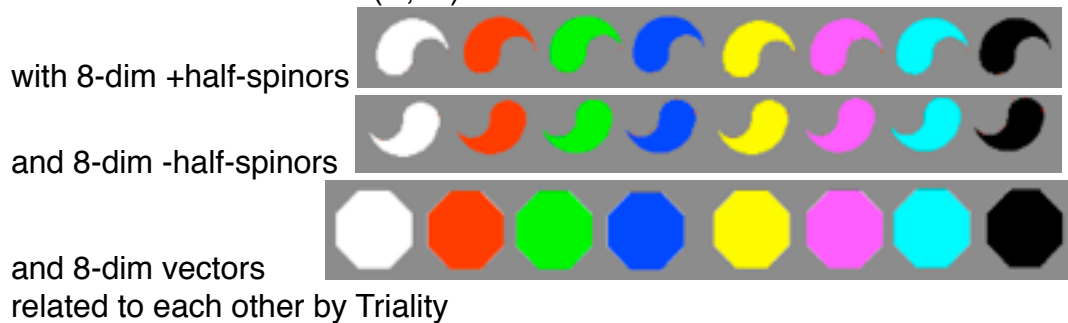
$Cl(0,8) = Cl(1,7) = M(R,16) = M(Q,2) \times M(Q,2)$  has graded structure

1	4	6	4	1					
	4	16	24	16	4				
		6	24	36	24	6			
			4	16	24	16	4		
				1	4	6	4	1	
1	8	28	56	70	56	28	8	1	

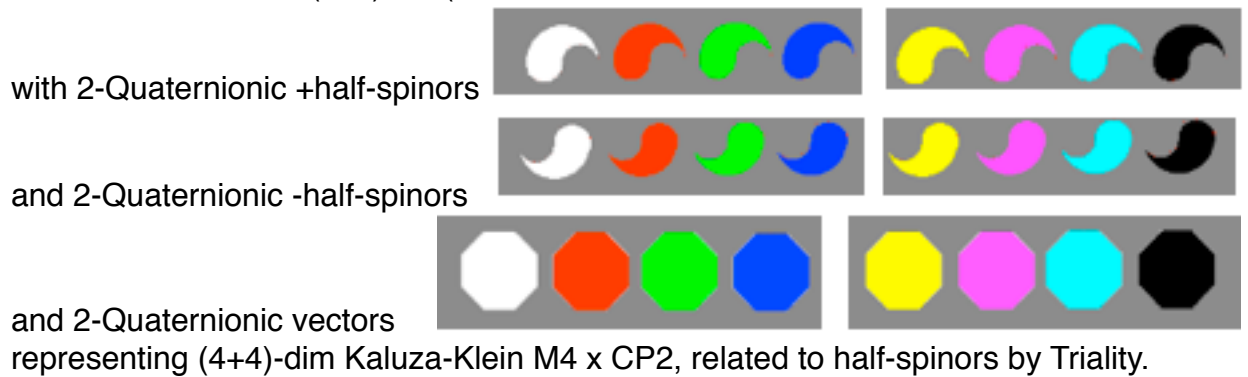
and its Spinor structure based on  $M(Q,2) = 16$ -dim is

Spinors =  $\sqrt{16 \times 16} = 16 = 8+8$

Their Real / Octonionic  $M(R,16)$  structure is:



Their Quaternionic  $M(Q,2) \times M(Q,2)$  structure is



The 8-dim vectors of  $Cl(0,8) = Cl(1,7)$  correspond to  $B_4 / D_4 = OP_1$

Spinors =  $8+8 = F_4 / B_4 = 52-36 = OP_2$

$F_4 = 8 + 28 + (8+8)$

8 = Shilov Boundary for Lie Sphere  $Spin(10) / Spin(8) \times U(1) =$   
 = half-spinors for First Generation Fermion Particles / AntiParticles  
 8s+ for Particles and 8s- for AntiParticles

One copy of  $Cl(8)$  only has room for one Generation, no AntiGeneration  
 The AntiGeneration appears for  $Cl(16) = Cl(8) \times Cl(8)$   
 but is not in  $E_8$  which omits the AntiGeneration half-spinors of  $Cl(16)$

$Cl(0,16) = Cl(1,15) = M(R,256) = M(Q,2) \times M(Q,2) \times M(Q,2) \times M(Q,2)$   
 has graded structure

1	8	28	56	70	56	28	8	1
	8	64	...					
		28	...					
			...					...
1	16	120	...					

and its Spinor structure based on  $M(Q,2) = 16$ -dim is

Spinors =  $\sqrt{16 \times 16 \times 16 \times 16} = 16 \times 16 = 256 = 128 + 128$   
 ( equivalent to  $M(R,256)$  Spinors =  $256 \times 1$  Real =  $256 = 128 + 128$  )

Spinors =  $128 + 128$

$$E8 = 120 + 128$$

$128 = Cl(16)$  half-spinors for One Generation Fermion Particles and AntiParticles

The other 128 is for One AntiGeneration that is not in E8

Quaternionic structure similar to that of  $Cl(1,3) = Cl(0,4) = M(Q,2)$  is seen

in

$Cl(2,4) = M(Q,4) = 4 \times 4$  Quaternion matrices with grading based on  $4 \times 4 = 1 \ 4 \ 6 \ 4 \ 1$

$$\begin{array}{ccccccc}
 1 & 2 & & & & & \\
 & 4 & 8 & & & & \\
 & & 6 & 12 & & & 6 \\
 & & & 4 & 8 & & 4 \\
 & & & & & 1 & 2 & 1 \\
 \hline
 1 & 6 & 15 & 20 & 15 & 6 & 1
 \end{array}$$

Conformal Gravity  $Spin(2,4) = SU(2,2)$  of  $Cl(2,4) = M(Q,4)$   $4 \times 4$  Quaternionic Matrices have  $(4+4) \times 4 = 32$ -dim spinors with



$Cl(2,4)$  vectors are 6-dim but  $Spin(2,4) = SU(2,2)$  so the Twistor Correspondence



produces 1-Quaternionic Twistors that represent the  $M_4$  part of  $M_4 \times CP^2$  Kaluza-Klein



Spinors =  $4 \times 1$  Quaternion  $16 = 4 \ 8 \ 4 = 2 \ 4 \ 2 + 2 \ 4 \ 2$

$2 \ 4 \ 2 = 8 =$  Lie Sphere  $Spin(6) / Spin(4) \times U(1)$  Complex Domain has  $Cl(1,3)$  half-spinor Shilov Boundary  $Cl(2,4)$  is in some sense a  $(1,1)$  Complexification of  $Cl(1,3)$

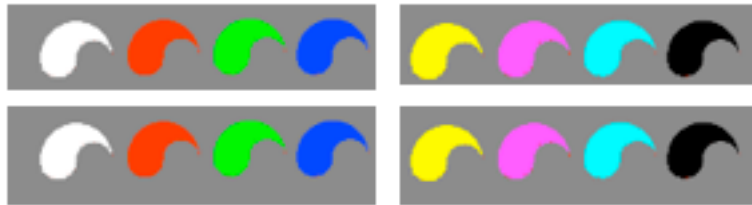
and in

$Cl(2,6) = Cl(3,5) = M(Q,8) = 8 \times 8$  Q-matrix grading based on  $8 \times 8 = 1 \ 6 \ 15 \ 20 \ 15 \ 6 \ 1$

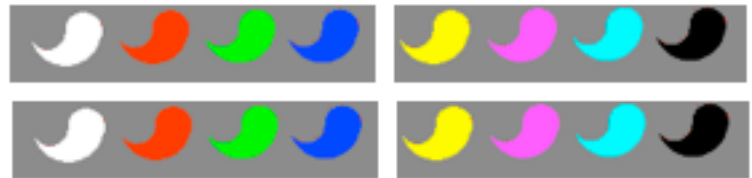
1	2	1						
	6	12	6					
		15	30	15				
			20	40	20			
				15	30	15		
					6	12	6	
						1	2	1
1	8	28	56	70	56	28	8	1

Quaternionic  $M(Q,8)$   $8 \times 8$  Quaternionic Matrices have  $(4+4) \times 4 = 32$ -dim spinors

with 4-Quaternionic +half-spinors



and 4-Quaternionic -half-spinors



and 2-Quaternionic vectors



that represent the two 4-dim spaces of Kaluza-Klein  $M4 \times CP2$

The 8-dim vectors do not correspond to 16-dim  $D5 / D4 \times U(1) = (C \times O)P1$

If you were to expand the vectors to 16-dim you would go to  $Cl(16) = Cl(8) \times Cl(8)$

Spinors =  $8 \times 1$  Quaternion =  $E6 / D5 \times U(1) = 78 - 45 - 1 = 32$

$$E6 = 15 + 30 + 32$$

$$32 = 8 \ 16 \ 8 = 4 \ 8 \ 4 + 4 \ 8 \ 4$$

$$4 \ 8 \ 4 = \text{Lie Sphere Spin}(10) / \text{Spin}(8) \times U(1)$$

The Quaternionic half-spinors in  $Cl(2,6)$  correspond to Lie Sphere Complex Domains whereas

the half-spinors in  $Cl(1,7) = Cl(0,8)$  correspond to Shilov Boundaries

and

the  $E6$  of  $Cl(2,6)$  is in some aspects a Complexification of the  $F4$  of  $Cl(1,7) = Cl(0,8)$ .

## 7. Batakis Standard Model Gauge Groups and Mayer-Trautman Higgs

The Mayer-Trautman Mechanism reduces the Lagrangian integral over the 8-dim SpaceTime whose 8-Position x 8-Momentum is represented by 64-dim D8 / D4xD4 where D8 is the Adjoint part of E8.

$$\int_{8\text{-dim SpaceTime}} \text{Standard Model Gauge Gravity} + \text{Fermion Particle-AntiParticle}$$

to

a Lagrangian integral over the 4-dim M4 Minkowski Physical SpaceTime part of Kaluza-Klein M4 x CP2

$$\int_{4\text{-dim M4}} \text{SM GG} + \text{Fermion Particle-AntiParticle} + \text{Higgs}$$

by integrating out the Lagrangian Density over the CP2 Internal Symmetry Space and so creating a new Higgs term in the Lagrangian Density integrated only over M4.

Since the D4 = U(2,2) of Gauge Gravity acts on the M4, there is no problem with it.

As to the D4 = U(4) of the Standard Model, U(4) contains as a subgroup color SU(3) which is also the global symmetry group of the CP2 = SU(3) / SU(2)xU(1) Internal Symmetry Space of M4 X CP2 Kaluza-Klein SpaceTime.

A. Batakis in Class. Quantum Grav. 3 (1986) L99-L105 said:

“... In a standard Kaluza-Klein framework, M4 x CP2 allows the classical unified description of an SU(3) gauge field with gravity ... [and]

the possibility of an additional SU(2) x U(1) gauge field structure is uncovered. ...”.

Since the CP2 = SU(3) / U(2) has global SU(3) action, the SU(3) can be considered as a local gauge group acting on the M4, so there is no problem with it.

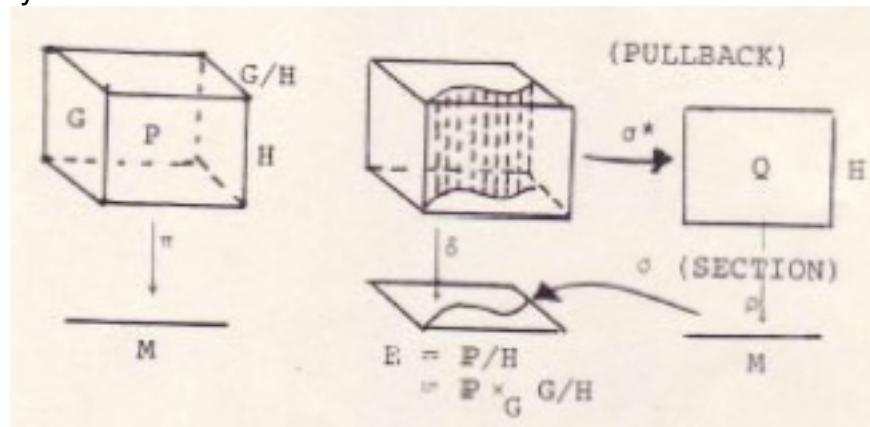
However, the U(2) acts on the CP2 = SU(3) / U(2) as little group, and so has local action on CP2 and then on M4, so **the local action of U(2) on CP2 must be integrated out to get the desired U(2) = SU(2)xU(1) local action directly on M4.**

Since the  $U(1)$  part of  $U(2) = U(1) \times SU(2)$  is Abelian, its local action on  $CP^2$  and then  $M^4$  can be composed to produce a single  $U(1)$  local action on  $M^4$ , so there is no problem with it.

That leaves non-Abelian  $SU(2)$  with local action on  $CP^2$  and then on  $M^4$ , and the necessity to integrate out the local  $CP^2$  action to get something acting locally directly on  $M^4$ .

This is done by a mechanism due to Meinhard Mayer and A. Trautman in "A Brief Introduction to the Geometry of Gauge Fields" and "The Geometry of Symmetry Breaking in Gauge Theories", Acta Physica Austriaca, Suppl. XXIII (1981)

where they say: "...



... We start out from ... four-dimensional  $M$  [ $M^4$ ] ...[and]...  $R$  ...[that is]... obtained from ...  $G/H$  [ $CP^2 = SU(3) / U(2)$ ] ... the physical surviving components of  $A$  and  $F$ , which we will denote by  $A$  and  $F$ , respectively, are a one-form and two form on  $M$  [ $M^4$ ] with values in  $H$  [ $SU(2)$ ] ... the remaining components will be subjected to symmetry and gauge transformations, thus reducing the Yang-Mills action ...[on  $M^4 \times CP^2$ ]... to a Yang-Mills-Ginzburg-Landau action on  $M$  [ $M^4$ ] ... Consider the Yang-Mills action on  $R$  ...

$$S_{YM} = \text{Integral Tr} ( F \wedge *F )$$

... We can ... split the curvature  $F$  into components along  $M$  [ $M^4$ ] (spacetime) and those along directions tangent to  $G/H$  [ $CP^2$ ] .

We denote the former components by  $F_{!!}$  and the latter by  $F_{??}$  , whereas the mixed components (one along  $M$ , the other along  $G/H$ ) will be denoted by  $F_{!?$  ...

Then the integrand ... becomes

$$\text{Tr} ( F_{!!} F^{!!} + 2 F_{! ?} F^{! ?} + F_{??} F^{??} )$$

...

The first term .. becomes the [SU(2)] Yang-Mills action for the reduced [SU(2)] Yang-Mills theory

...

the middle term .. becomes, symbolically,

$$\text{Tr} \sum D_{\mu} \Phi D^{\mu} \Phi$$

where  $\Phi$  is the Lie-algebra-valued 0-form corresponding to the invariance of A with respect to the vector field  $\xi$ , in the G/H [CP2] direction

...

the third term ... involves the contraction  $F_{\mu\nu}$  of F with two vector fields lying along G/H [CP2] ... we make use of the equation [from Mayer-Trautman, Acta Physica Austriaca, Suppl. XXIII (1981) 433-476, equation 6.18]

$$2 F_{\mu\nu} = [\Phi_{,\mu}, \Phi_{,\nu}] - \Phi_{,([\mu, \nu])}$$

... Thus,

the third term ... reduces to what is essentially a Ginzburg-Landau potential in the components of  $\Phi$ :

$$\text{Tr} F_{\mu\nu} F^{\mu\nu} = (1/4) \text{Tr} ([\Phi_{,\mu}, \Phi_{,\nu}] - \Phi_{,([\mu, \nu])})^2$$

...

special cases which were considered show that ...[the equation immediately above]... has indeed the properties required of a Ginzburg-Landau-Higgs potential, and moreover the relative signs of the quartic and quadratic terms are correct, and only one overall normalization constant ... is needed. ...".

See S. Kobayashi and K. Nomizu, Foundations of Differential Geometry, Volume I, Wiley (1963), especially section II.11: "...

*THEOREM 11.7. Assume in Theorem 11.5 that  $\mathfrak{k}$  admits a subspace  $\mathfrak{m}$  such that  $\mathfrak{k} = \mathfrak{j} + \mathfrak{m}$  (direct sum) and  $\text{ad}(J)(\mathfrak{m}) = \mathfrak{m}$ , where  $\text{ad}(J)$  is the adjoint representation of  $J$  in  $\mathfrak{k}$ . Then ...*

*The curvature form  $\Omega$  of the  $K$ -invariant connection defined by  $\Lambda_{\mathfrak{m}}$  satisfies the following condition:*

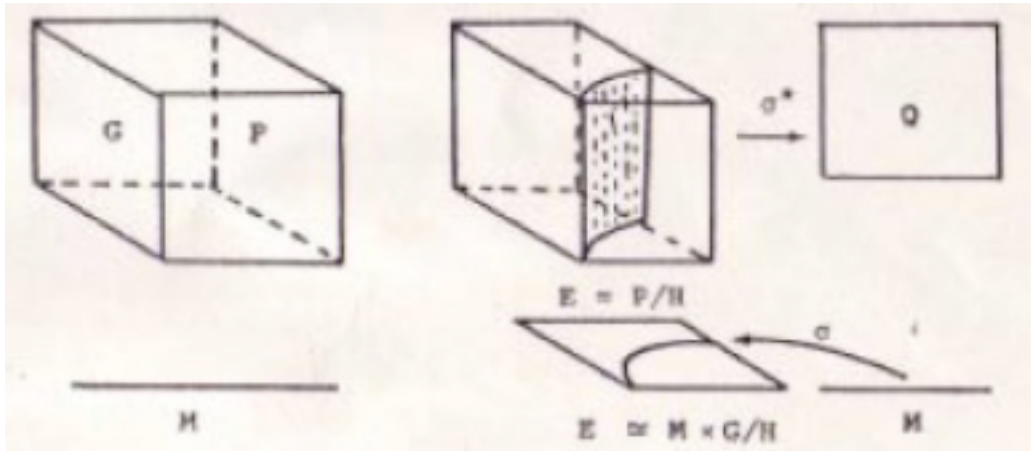
$$2\Omega_{u_0}(\tilde{X}, \tilde{Y}) = [\Lambda_{\mathfrak{m}}(X), \Lambda_{\mathfrak{m}}(Y)] - \Lambda_{\mathfrak{m}}([X, Y]_{\mathfrak{m}}) - \lambda([X, Y]_{\mathfrak{j}})$$

*for  $X, Y \in \mathfrak{m}$ ,*

...".



Along the same lines, Meinhard E. Mayer said (Hadronic Journal 4 (1981) 108-152): “...



... each point of ... the ... fibre bundle ...  $E$  consists of a four-dimensional spacetime point  $x$  [ in  $M_4$  ] to which is attached the homogeneous space  $G/H$  [  $SU(3)/U(2) = CP^2$  ] ... the components of the curvature lying in the homogeneous space  $G/H$  [  $= SU(3)/U(2)$  ] could be reinterpreted as Higgs scalars (with respect to spacetime [  $M_4$  ]) ... the Yang-Mills action reduces to a Yang-Mills action for the  $h$ -components [  $U(2)$  components ] of the curvature over  $M$  [  $M_4$  ] and a quartic functional for the “Higgs scalars”, which not only reproduces the Ginzburg-Landau potential, but also gives the correct relative sign of the constants, required for the BEHK ... Brout-Englert-Higgs-Kibble ... mechanism to work. ...”.

## 8. 2nd and 3rd Generation Fermions

The 8 First Generation Fermion Particles

can each be represented by the 8 basis elements  $\{1,i,j,k,E,I,J,K\}$  of the Octonions  $O$

$1 \Leftrightarrow$  e-neutrino

$i \Leftrightarrow$  red down quark

$j \Leftrightarrow$  green down quark

$k \Leftrightarrow$  blue down quark

$E \Leftrightarrow$  electron

$I \Leftrightarrow$  red up quark

$J \Leftrightarrow$  green up quark

$K \Leftrightarrow$  blue up quark

with AntiParticles being represented similarly.

The Second and Third Generations can be represented by Pairs of Octonions  $OxO$  and Triples of Octonions  $OxOxO$  respectively.

When the non-unitary Octonionic 8-dim spacetime is reduced to the Kaluza-Klein  $M4 \times CP2$  at the End of Inflation, there are 3 possibilities for a fermion propagator from point A to point B:

1 - A and B are both in  $M4$ ,  
so its path can be represented by the single  $O$ ;

2 - Either A or B, but not both, is in  $CP2$ ,  
so its path must be augmented by one projection from  $CP2$  to  $M4$ ,  
which projection can be represented by a second  $O$ ,  
giving a second generation  $OxO$ ;

3 - Both A and B are in  $CP2$ ,  
so its path must be augmented by two projections from  $CP2$  to  $M4$ ,  
which projections can be represented by a second  $O$  and a third  $O$ ,  
giving a third generation  $OxOxO$ .

Combinatorics contributes to Fermion mass ratios. For example:

Blue Down Quark is 1 out of 8 and Blue Up Quark is 1 out of 8  
so the Down Quark : Up Quark mass ratio is 1 : 1

Blue Strange Quark is 3 out of  $8 \times 8 = 64$  and Blue Charm Quark is 17 out of  $8 \times 8 = 64$   
so the Strange Quark : Charm Quark mass ratio is 3 : 17

Blue Beauty Quark is 7 out of  $8 \times 8 \times 8 = 512$  and Blue Truth Quark is 161 out of  $8 \times 8 \times 8 = 512$   
so the Beauty Quark : Truth Quark mass ratio is 7 : 161

**9. Schwinger Sources with inherited Monster Group Symmetry  
have  
Kerr-Newman Black Hole structure size about  $10^{(-24)}$  cm  
and  
Geometry of Bounded Complex Domains and Shilov boundaries**

The  $CI(16)$ -E8 model Lagrangian over 4-dim Minkowski SpaceTime  $M_4$  is

$$\int_{4\text{-dim } M_4} SM GG + \text{Fermion Particle-AntiParticle} + \text{Higgs}$$

Consider the **Fermion Term**.

In the conventional picture, the spinor fermion term is of the form  $m S S^*$  where  $m$  is the fermion mass and  $S$  and  $S^*$  represent the given fermion. The Higgs coupling constants are, in the conventional picture, ad hoc parameters, so that effectively the mass term is, in the conventional picture, an ad hoc inclusion.

The  $CI(16)$ -E8 model does not put in the mass  $m$  in an ad hoc way, but constructs the Lagrangian integral such that the mass  $m$  emerges naturally from the geometry of the spinor fermions by setting the spinor fermion mass term as the volume of the Schwinger Source Fermions.

Effectively the integral over the Schwinger Source spacetime region of its Kerr-Newman cloud of virtual particle/antiparticle pairs plus the valence fermion gives the volume of the Schwinger Source fermion and defines its mass, which, since it is dressed with the particle/antiparticle pair cloud, gives quark mass as constituent mass.

The  $CI(16)$ -E8 model constructs the Lagrangian integral such that the mass  $m$  emerges as the integral over the Schwinger Source spacetime region of its Kerr-Newman cloud of virtual particle/antiparticle pairs plus the valence fermion so that the volume of the Schwinger Source fermion defines its mass, which, being dressed with the particle/antiparticle pair cloud, gives quark mass as constituent mass.

Fermion Schwinger Sources correspond to the Lie Sphere Symmetric space  
 $Spin(10) / Spin(8) \times U(1)$   
 which has local symmetry of the  $Spin(8)$  gauge group  
 from which the first generation spinor fermions are formed as **+half-spinor** and **-half-spinor** spaces  
 and  
 Bounded Complex Domain  $D_8$  of type  $IV_8$  and Shilov Boundary  $Q_8 = RP^1 \times S^7$

Consider the **SM GG** term from Gauge Gravity and Standard Model Gauge Bosons. The process of breaking Octonionic 8-dim SpaceTime down to Quaternionic (4+4)-dim M4 x CP2 Kaluza-Klein creates differences in the way gauge bosons "see" 4-dim Physical SpaceTime

There 4 equivalence classes of 4-dimensional Riemannian Symmetric Spaces with Quaternionic structure consistent with 4-dim Physical SpaceTime:

**S4** = 4-sphere = Spin(5) / Spin(4) where Spin(5) = Schwinger-Euclidean version of the Anti-DeSitter subgroup of the Conformal Group that gives **MacDowell-Mansouiri Gravity**

**CP2** = complex projective 2-space = SU(3) / U(2) with **the SU(3) of the Color Force**

**S2 x S2** = SU(2)/U(1) x SU(2)/U(1) with two copies of **the SU(2) of the Weak Force**

**S1 x S1 x S1 x S1** = U(1) x U(1) x U(1) x U(1) = 4 copies of **the U(1) of the EM Photon** ( 1 copy for each of the 4 covariant components of the Photon )

The Gravity Gauge Bosons (Schwinger-Euclidean versions) live in a Spin(5) subalgebra of the Spin(6) Conformal subalgebra of D4 = Spin(8).



They "see" M4 Physical spacetime as the 4-sphere S4 so that their part of the Physical Lagrangian is

$$\int_{S4} \text{Gravity Gauge Boson Term}$$

an integral over SpaceTime S4.

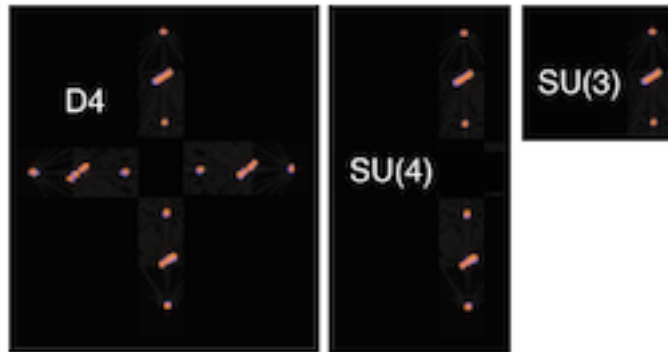
The Schwinger Sources for GRb bosons are the Complex Bounded Domains and Shilov Boundaries for Spin(5) MacDowell-Mansouri Gravity bosons.

However, due to Stabilization of Condensate SpaceTime by virtual Planck Mass Gravitational Black Holes,

for Gravity, the effective force strength that we see in our experiments is not just composed of the S4 volume and the Spin(5) Schwinger Source volume, but is suppressed by the square of the Planck Mass.

The unsuppressed Gravity force strength is the Geometric Part of the force strength.

The Standard Model SU(3) Color Force bosons live in a SU(3) subalgebra of the SU(4) subalgebra of D4 = Spin(8).



They "see" M4 Physical spacetime as the complex projective plane CP2 so that their part of the Physical Lagrangian is

$$\int_{\text{CP}^2} \text{SU(3) Color Force Gauge Boson Term}$$

an integral over SpaceTime CP2.

The Schwinger Sources for SU(3) bosons are the Complex Bounded Domains and Shilov Boundaries for SU(3) Color Force bosons.

The Color Force Strength is given by the SpaceTime CP2 volume and the SU(3) Schwinger Source volume.

Note that since the Schwinger Source volume is dressed with the particle/antiparticle pair cloud, the calculated force strength is for the characteristic energy level of the Color Force (about 245 MeV).

The Standard Model SU(2) Weak Force bosons live in a SU(2) subalgebra of the U(2) local group of CP2 = SU(3) / U(2)

They "see" M4 Physical spacetime as two 2-spheres S2 x S2 so that their part of the Physical Lagrangian is

$$\int_{\text{S}^2 \times \text{S}^2} \text{SU(2) Weak Force Gauge Boson Term}$$

an integral over SpaceTime S2xS2.

The Schwinger Sources for SU(2) bosons are the Complex Bounded Domains and Shilov Boundaries for SU(2) Weak Force bosons.

However, due to the action of the Higgs mechanism, for the Weak Force, the effective force strength that we see in our experiments is not just composed of the S2xS2 volume and the SU(2) Schwinger Source volume, but is suppressed by the square of the Weak Boson masses.

The unsuppressed Weak Force strength is the Geometric Part of the force strength.

The Standard Model U(1) Electromagnetic Force bosons (photons) live in a U(1) subalgebra of the U(2) local group of CP2 = SU(3) / U(2)  
 They "see" M4 Physical spacetime as four 1-sphere circles S1xS1xS1xS1 = T4 (T4 = 4-torus) so that their part of the Physical Lagrangian is

$$\int_{T4} \text{(U(1) Electromagnetism Gauge Boson Term)}$$

an integral over SpaceTime T4.

The Schwinger Sources for U(1) photons are the Complex Bounded Domains and Shilov Boundaries for U(1) photons. The Electromagnetic Force Strength is given by the SpaceTime T4 volume and the U(1) Schwinger Source volume.

Schwinger Sources as described above are continuous manifold structures of Bounded Complex Domains and their Shilov Boundaries but

**the Cl(16)-E8 model at the Planck Scale has spacetime condensing out of Clifford structures forming a Leech lattice underlying 26-dim String Theory of World-Lines** with 8 + 8 + 8 = 24-dim of fermion particles and antiparticles and of spacetime.

The automorphism group of a single 26-dim String Theory cell modulo the Leech lattice is the Monster Group of order about  $8 \times 10^{53}$ .

When a fermion particle/antiparticle appears in E8 spacetime it does not remain a single Planck-scale entity because Tachyons create a cloud of particles/antiparticles. The cloud is one Planck-scale Fundamental Fermion Valence Particle plus an effectively neutral cloud of particle/antiparticle pairs forming a Kerr-Newman black hole.

That cloud constitutes the Schwinger Source.

Its structure comes from the 24-dim Leech lattice part of the Monster Group which is  $2^{(1+24)}$  times the double cover of Co1, for a total order of about  $10^{26}$ .

(Since a Leech lattice is based on copies of an E8 lattice and since there are 7 distinct E8 integral domain lattices there are 7 (or 8 if you include a non-integral domain E8 lattice) distinct Leech lattices. The physical Leech lattice is a superposition of them, effectively adding a factor of 8 to the order.)

The volume of the Kerr-Newman Cloud is on the order of  $10^{27}$  x Planck scale, so the Kerr-Newman Cloud should contain about  $10^{27}$  particle/antiparticle pairs and its size should be about  $10^{(27/3)} \times 1.6 \times 10^{(-33)}$  cm =  
 = roughly  $10^{(-24)}$  cm.

## 10. Fermion Mass Calculation

In the  $Cl(16)$ -E8 model, the first generation spinor fermions are seen as +half-spinor and -half-spinor spaces of  $Cl(1,7) = Cl(8)$ . Due to Triality, Spin(8) can act on those 8-dimensional half-spinor spaces similarly to the way it acts on 8-dimensional vector spacetime.

Take the the spinor fermion volume to be the Shilov boundary corresponding to the same symmetric space on which Spin(8) acts as a local gauge group that is used to construct 8-dimensional vector spacetime: the symmetric space  $Spin(10) / Spin(8) \times U(1)$  corresponding to a bounded domain of type IV8 whose Shilov boundary is  $RP^1 \times S^7$

Since all first generation fermions see the spacetime over which the integral is taken in the same way ( unlike what happens for the force strength calculation ), the only geometric volume factor relevant for calculating first generation fermion mass ratios is in the spinor fermion volume term.  $Cl(16)$ -E8 model fermions correspond to Schwinger Source Kerr-Newman Black Holes, so the quark mass in the  $Cl(16)$ -E8 model is a constituent mass.

Fermion masses are calculated as a product of four factors:

$$V(Q_{\text{fermion}}) \times N(\text{Graviton}) \times N(\text{octonion}) \times \text{Sym}$$

$V(Q_{\text{fermion}})$  is the volume of the part of the half-spinor fermion particle manifold  $S^7 \times RP^1$  related to the fermion particle by photon, weak boson, or gluon interactions.

$N(\text{Graviton})$  is the number of types of Spin(0,5) graviton related to the fermion. The 10 gravitons correspond to the 10 infinitesimal generators of  $Spin(0,5) = Sp(2)$ . 2 of them are in the Cartan subalgebra. 6 of them carry color charge, and therefore correspond to quarks. The remaining 2 carry no color charge, but may carry electric charge and so may be considered as corresponding to electrons. One graviton takes the electron into itself, and the other can only take the first-generation electron into the massless electron neutrino. Therefore only one graviton should correspond to the mass of the first-generation electron. The graviton number ratio of the down quark to the first-generation electron is therefore  $6/1 = 6$ .

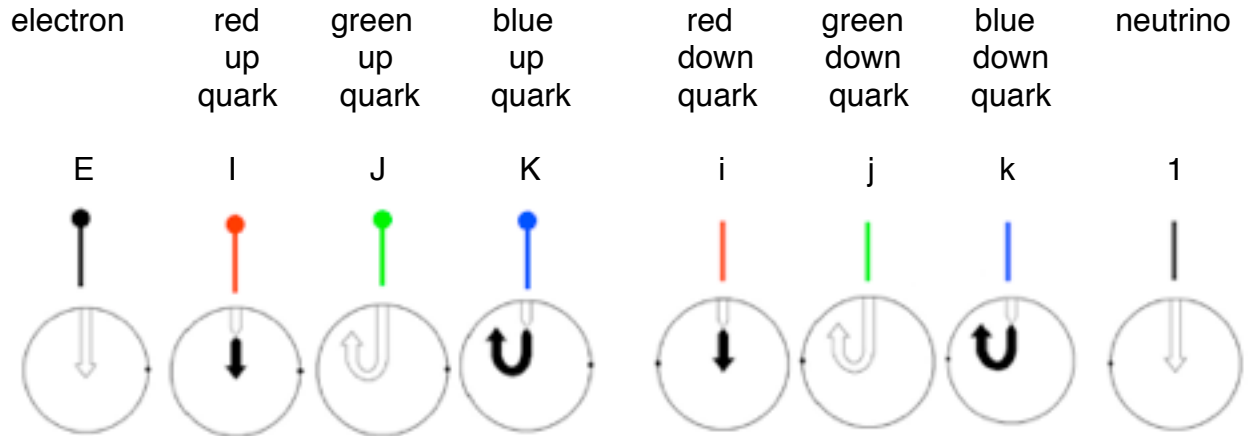
$N(\text{octonion})$  is an octonion number factor relating up-type quark masses to down-type quark masses in each generation.

Sym is an internal symmetry factor, relating 2nd and 3rd generation massive leptons to first generation fermions. It is not used in first-generation calculations.

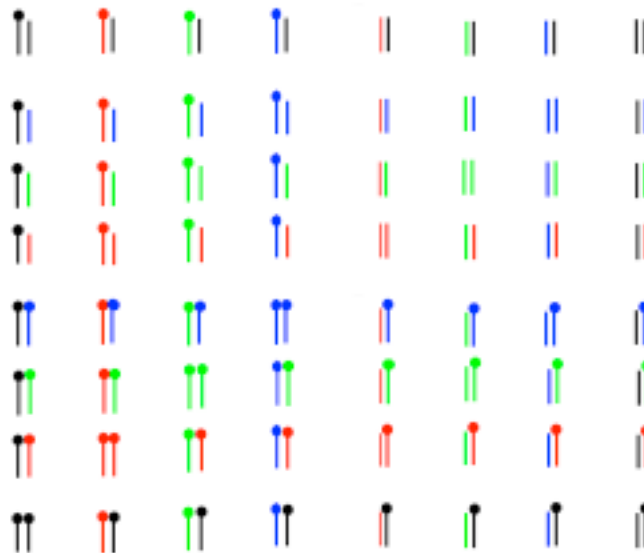
### 3 Generation Fermion Combinatorics

#### First Generation (8)

(geometric representation of Octonions is from arXiv 1010.2979)



#### Second Generation (64)



Mu Neutrino (1)

Rule: a Pair belongs to the Mu Neutrino if:  
 All elements are Colorless (black)  
 and all elements are Associative

(that is, is 1 which is the only Colorless Associative element) .



### Muon (3)

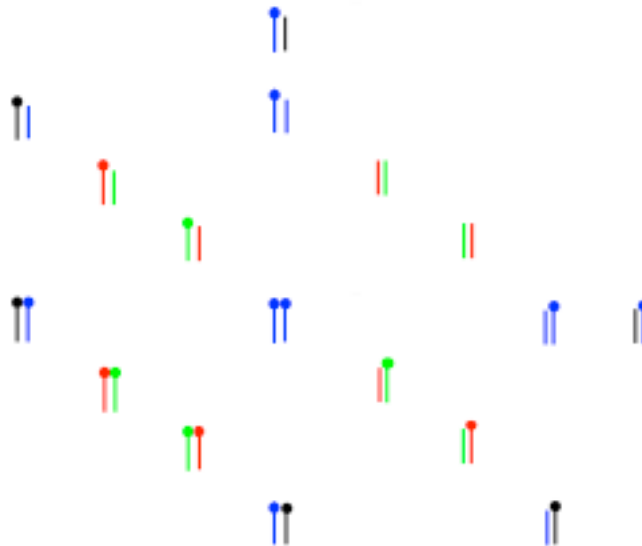
Rule: a Pair belongs to the Muon if:  
All elements are Colorless (black)  
and at least one element is NonAssociative  
(that is, is E which is the only Colorless NonAssociative element).

### Blue Strange Quark (3)

Rule: a Pair belongs to the Blue Strange Quark if:  
There is at least one Blue element and the other element is Blue or Colorless (black)  
and all elements are Associative (that is, is either 1 or i or j or k).


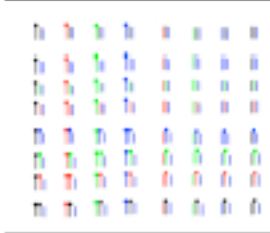

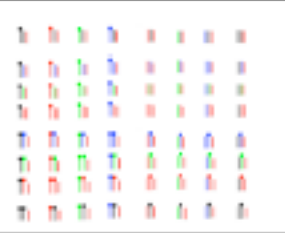
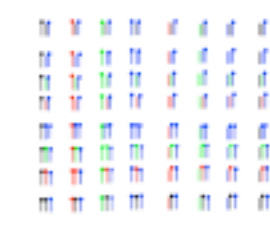
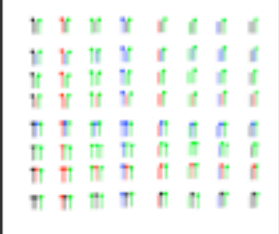
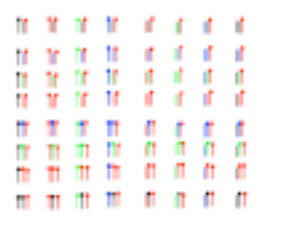
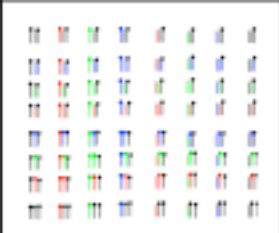
### Blue Charm Quark (17)

- Rules: a Pair belongs to the Blue Charm Quark if:
- 1 - There is at least one Blue element and the other element is Blue or Colorless (black) and at least one element is NonAssociative (that is, is either E or I or J or K)
  - 2 - There is one Red element and one Green element (Red x Green = Blue).



( Red and Green Strange and Charm Quarks follow similar rules )

### Third Generation (512)

#### Tau Neutrino (1)

Rule: a Triple belongs to the Tau Neutrino if:

All elements are Colorless (black)

and all elements are Associative

(that is, is 1 which is the only Colorless Associative element)

#### Tauon (7)

Rule: a Triple belongs to the Tauon if:

All elements are Colorless (black)

and at least one element is NonAssociative (that is, is E which is the only Colorless NonAssociative element)

### Blue Beauty Quark (7)

Rule: a Triple belongs to the Blue Beauty Quark if:

There is at least one Blue element and all other elements are Blue or Colorless (black) and all elements are Associative (that is, is either 1 or i or j or k).

### Blue Truth Quark (161)

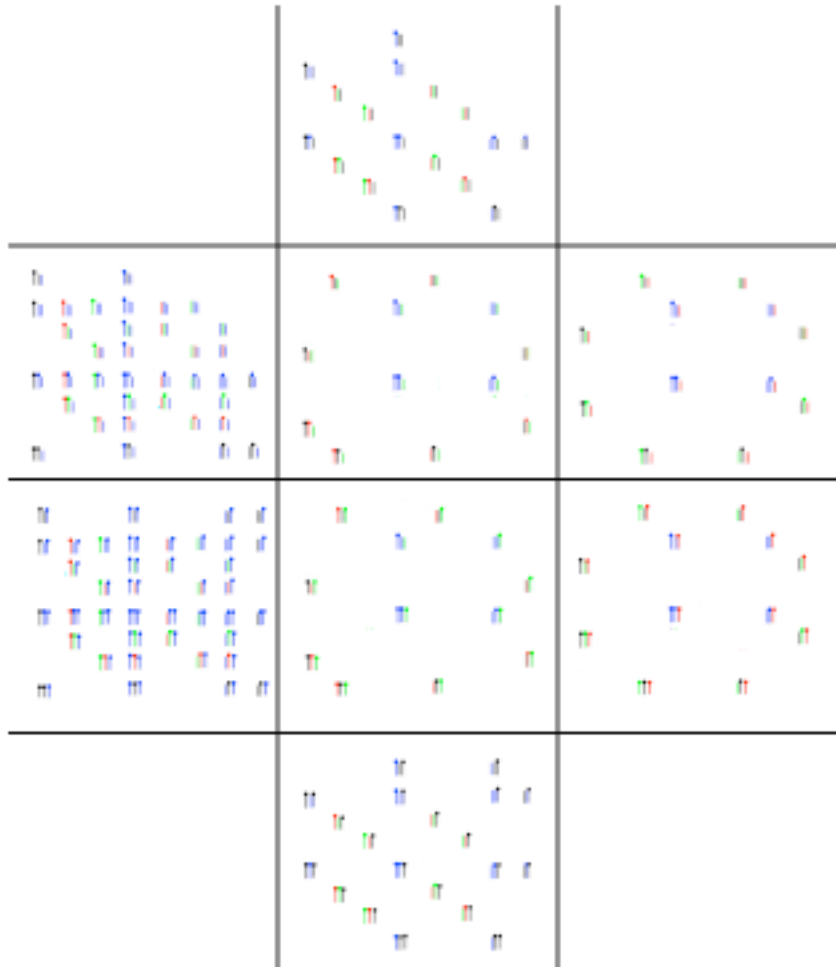
Rules: a Triple belongs to the Blue Truth Quark if:

1 - There is at least one Blue element and all other elements are Blue or Colorless (black)

and at least one element is NonAssociative (that is, is either E or I or J or K)

2 - There is one Red element and one Green element and the other element is Colorless (Red x Green = Blue)

3 - The Triple has one element each that is Red, Green, or Blue, in which case the color of the Third element (for Third Generation) is determinative and must be Blue.



( Red and Green Beauty and Truth Quarks follow similar rules )

**The first generation** down quark constituent mass : electron mass ratio is:

The electron, E, can only be taken into the tree-level-massless neutrino, 1, by photon, weak boson, and gluon interactions.

The electron and neutrino, or their antiparticles, cannot be combined to produce any of the massive up or down quarks.

The neutrino, being massless at tree level, does not add anything to the mass formula for the electron.

Since the electron cannot be related to any other massive Dirac fermion, its volume  $V(Q_{\text{electron}})$  is taken to be 1.

Next consider a red down quark i.

By gluon interactions, i can be taken into j and k, the blue and green down quarks.

By also using weak boson interactions,

it can also be taken into l, J, and K, the red, blue, and green up quarks.

Given the up and down quarks, pions can be formed from quark-antiquark pairs, and the pions can decay to produce electrons and neutrinos.

Therefore the red down quark (similarly, any down quark)

is related to all parts of  $S^7 \times RP^1$ ,

the compact manifold corresponding to  $\{ 1, i, j, k, E, l, J, K \}$

and therefore

a down quark should have

a spinor manifold volume factor  $V(Q_{\text{down quark}})$  of the volume of  $S^7 \times RP^1$ .

The ratio of the down quark spinor manifold volume factor

to the electron spinor manifold volume factor is

$$V(Q_{\text{down quark}}) / V(Q_{\text{electron}}) = V(S^7 \times RP^1) / 1 = \pi^5 / 3.$$

Since the first generation graviton factor is 6,

$$m_d / m_e = 6 V(S^7 \times RP^1) = 2 \pi^5 = 612.03937$$

As the up quarks correspond to l, J, and K, which are the octonion transforms under E of i, j, and k of the down quarks, the up quarks and down quarks have the same constituent mass

$$m_u = m_d.$$

Antiparticles have the same mass as the corresponding particles.

Since the model only gives ratios of masses,

the mass scale is fixed so that the electron mass  $m_e = 0.5110 \text{ MeV}$ .

Then, the constituent mass of the down quark is  $m_d = 312.75 \text{ MeV}$ ,

and the constituent mass for the up quark is  $m_u = 312.75 \text{ MeV}$ .

These results when added up give a total mass of first generation fermion particles:

$$\Sigma_{\text{maf1}} = 1.877 \text{ GeV}$$

As the proton mass is taken to be the sum of the constituent masses of its constituent quarks

$$m_{\text{proton}} = m_u + m_u + m_d = 938.25 \text{ MeV}$$

which is close to the experimental value of 938.27 MeV.

**The third generation** fermion particles correspond to triples of octonions. There are  $8^3 = 512$  such triples.

The triple  $\{ 1, 1, 1 \}$  corresponds to the tau-neutrino.

The other 7 triples involving only 1 and E correspond to the tauon:

$\{ E, E, E \}$   
 $\{ E, E, 1 \}$   
 $\{ E, 1, E \}$   
 $\{ 1, E, E \}$   
 $\{ 1, 1, E \}$   
 $\{ 1, E, 1 \}$   
 $\{ E, 1, 1 \}$

The symmetry of the 7 tauon triples is the same as the symmetry of the first generation tree-level-massive fermions, 3 down, quarks, the 3 up quarks, and the electron, so by the Sym factor the tauon mass should be the same as the sum of the masses of the first generation massive fermion particles.

Therefore the tauon mass is calculated at tree level as 1.877 GeV.

The calculated tauon mass of 1.88 GeV is a sum of first generation fermion masses, all of which are valid at the energy level of about 1 GeV.

However, as the tauon mass is about 2 GeV, the effective tauon mass should be renormalized from the energy level of 1 GeV at which the mass is 1.88 GeV to the energy level of 2 GeV. Such a renormalization should reduce the mass.

If the renormalization reduction were about 5 percent, the effective tauon mass at 2 GeV would be about 1.78 GeV. The 1996 Particle Data Group Review of Particle Physics gives a tauon mass of 1.777 GeV.

All triples corresponding to the tau and the tau-neutrino are colorless.

The beauty quark corresponds to 21 triples.  
They are triples of the same form as the 7 tauon triples involving 1 and E,  
but for 1 and I, 1 and J, and 1 and K,  
which correspond to the red, green, and blue beauty quarks,  
respectively.

The seven red beauty quark triples correspond to the seven tauon triples,  
except that  
the beauty quark interacts with 6 Spin(0,5) gravitons  
while the tauon interacts with only two.

The red beauty quark constituent mass should be the tauon mass times  
the third generation graviton factor  $6/2 = 3$ ,  
so the red beauty quark mass is  $m_b = 5.63111$  GeV.

The blue and green beauty quarks are similarly determined to also be 5.63111 GeV.

The calculated beauty quark mass of 5.63 GeV is a constituent mass,  
that is, it corresponds to the conventional pole mass plus 312.8 MeV.  
Therefore, the calculated beauty quark mass of 5.63 GeV  
corresponds to a conventional pole mass of 5.32 GeV.

The 1996 Particle Data Group Review of Particle Physics gives  
a lattice gauge theory beauty quark pole mass as 5.0 GeV.

The pole mass can be converted to an MSbar mass  
if the color force strength constant  $\alpha_s$  is known.  
The conventional value of  $\alpha_s$  at about 5 GeV is about 0.22.

Using  $\alpha_s(5 \text{ GeV}) = 0.22$ , a pole mass of 5.0 GeV  
gives an MSbar 1-loop beauty quark mass of 4.6 GeV,  
and  
an MSbar 1,2-loop beauty quark mass of 4.3, evaluated at about 5 GeV.

If the MSbar mass is run from 5 GeV up to 90 GeV,  
the MSbar mass decreases by about 1.3 GeV,  
giving an expected MSbar mass of about 3.0 GeV at 90 GeV.

DELPHI at LEP has observed the Beauty Quark  
and found a 90 GeV MSbar beauty quark mass of about 2.67 GeV,  
with error bars  $\pm 0.25$  (stat)  $\pm 0.34$  (frag)  $\pm 0.27$  (theo).

The theoretical model calculated Beauty Quark mass of 5.63 GeV corresponds to a pole mass of 5.32 GeV, which is somewhat higher than the conventional value of 5.0 GeV.

However, the theoretical model calculated value of the color force strength constant  $\alpha_s$  at about 5 GeV is about 0.166, while the conventional value of the color force strength constant  $\alpha_s$  at about 5 GeV is about 0.216, and the theoretical model calculated value of the color force strength constant  $\alpha_s$  at about 90 GeV is about 0.106, while the conventional value of the color force strength constant  $\alpha_s$  at about 90 GeV is about 0.118.

The theoretical model calculations gives a Beauty Quark pole mass (5.3 GeV) that is about 6 percent higher than the conventional Beauty Quark pole mass (5.0 GeV), and a color force strength  $\alpha_s$  at 5 GeV (0.166) such that  $1 + \alpha_s = 1.166$  is about 4 percent lower than the conventional value of  $1 + \alpha_s = 1.216$  at 5 GeV.

Triples of the type  $\{ 1, I, J \}$ ,  $\{ I, J, K \}$ , etc., do not correspond to the beauty quark, but to the truth quark. The truth quark corresponds to those  $512 - 1 - 7 - 21 = 483$  triples, so the constituent mass of the red truth quark is  $161 / 7 = 23$  times the red beauty quark mass, and the red T-quark mass is  $m_t = 129.5155$  GeV

The blue and green truth quarks are similarly determined to also be 129.5155 GeV.

This is the value of the Low Mass State of the Truth calculated in the Cl(16)\_E8 model. The Middle Mass State of the Truth Quark has been observed by Fermilab since 1994. The Low and High Mass States of the Truth Quark have, in my opinion, also been observed by Fermilab (see Chapter 17 of this paper) but the Fermilab and CERN establishments disagree.

All other masses than the electron mass (which is the basis of the assumption of the value of the Higgs scalar field vacuum expectation value  $v = 252.514$  GeV), including the Higgs scalar mass and Truth quark mass, are calculated (not assumed) masses in the Cl(16)-E8 model. These results when added up give a total mass of third generation fermion particles:

$$\text{Sigma}f_3 = 1,629 \text{ GeV}$$

**The second generation** fermion particles correspond to pairs of octonions.  
There are  $8^2 = 64$  such pairs.

The pair  $\{ 1, 1 \}$  corresponds to the mu-neutrino.

The pairs  $\{ 1, E \}$ ,  $\{ E, 1 \}$ , and  $\{ E, E \}$  correspond to the muon.

For the Sym factor, compare the symmetries of the muon pairs to the symmetries of the first generation fermion particles:  
The pair  $\{ E, E \}$  should correspond to the E electron.  
The other two muon pairs have a symmetry group  $S_2$ , which is  $1/3$  the size of the color symmetry group  $S_3$  which gives the up and down quarks their mass of 312.75 MeV.

Therefore the mass of the muon should be the sum of the  $\{ E, E \}$  electron mass and the  $\{ 1, E \}$ ,  $\{ E, 1 \}$  symmetry mass, which is  $1/3$  of the up or down quark mass. Therefore,  $m_{\mu} = 104.76$  MeV .

According to the 1998 Review of Particle Physics of the Particle Data Group, the experimental muon mass is about 105.66 MeV which may be consistent with radiative corrections for the calculated tree-level  $m_{\mu} = 104.76$  MeV as Bailin and Love, in "Introduction to Gauge Field Theory", IOP (rev ed 1993), say: "... considering the order alpha radiative corrections to muon decay ... Numerical details are contained in Sirlin ... 1980 Phys. Rev. D 22 971 ... who concludes that the order alpha corrections have the effect of increasing the decay rate about 7% compared with the tree graph prediction ...". Since the decay rate is proportional to  $m_{\mu}^5$  the corresponding effective increase in muon mass would be about 1.36%, which would bring 104.8 MeV up to about 106.2 MeV.

All pairs corresponding to the muon and the mu-neutrino are colorless.



The red, blue and green strange quark each corresponds to the 3 pairs involving 1 and i, j, or k.

The red strange quark is defined as the three pairs  $\{ 1, i \}$ ,  $\{ i, 1 \}$ ,  $\{ i, i \}$  because i is the red down quark.

Its mass should be the sum of two parts:

the  $\{ i, i \}$  red down quark mass, 312.75 MeV, and

the product of the symmetry part of the muon mass, 104.25 MeV, times the graviton factor.

Unlike the first generation situation, massive second and third generation leptons can be taken, by both of the colorless gravitons that may carry electric charge, into massive particles.

Therefore the graviton factor for the second and third generations is  $6/2 = 3$ .

So the symmetry part of the muon mass times the graviton factor 3 is 312.75 MeV, and the red strange quark constituent mass is  $m_s = 312.75 \text{ MeV} + 312.75 \text{ MeV} = 625.5 \text{ MeV}$

The blue strange quarks correspond to the three pairs involving j, the green strange quarks correspond to the three pairs involving k, and their masses are similarly determined to also be 625.5 MeV.

The charm quark corresponds to the remaining  $64 - 1 - 3 - 9 = 51$  pairs.

Therefore, the mass of the red charm quark should be the sum of two parts:

the  $\{ i, i \}$ , red up quark mass, 312.75 MeV;

and

the product of the symmetry part of the strange quark mass, 312.75 MeV,

and the charm to strange octonion number factor  $51 / 9$ ,

which product is 1,772.25 MeV.

Therefore the red charm quark constituent mass is

$$m_c = 312.75 \text{ MeV} + 1,772.25 \text{ MeV} = 2.085 \text{ GeV}$$

The blue and green charm quarks are similarly determined to also be 2.085 GeV.

The calculated Charm Quark mass of 2.09 GeV is a constituent mass, that is, it corresponds to the conventional pole mass plus 312.8 MeV.

Therefore, the calculated Charm Quark mass of 2.09 GeV corresponds to a conventional pole mass of 1.78 GeV.

The 1996 Particle Data Group Review of Particle Physics gives a range for the Charm Quark pole mass from 1.2 to 1.9 GeV.

The pole mass can be converted to an MSbar mass if the color force strength constant  $\alpha_s$  is known.

The conventional value of  $\alpha_s$  at about 2 GeV is about 0.39, which is somewhat lower than the theoretical model value.

Using  $\alpha_s(2 \text{ GeV}) = 0.39$ , a pole mass of 1.9 GeV gives an MSbar 1-loop mass of 1.6 GeV, evaluated at about 2 GeV.

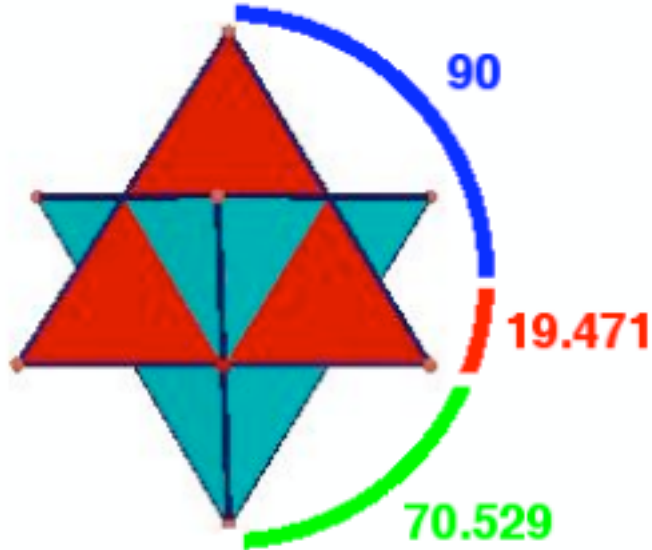
These results when added up give a total mass of second generation fermion particles:

$$\Sigma_{\text{f}2} = 32.9 \text{ GeV}$$

## 11. Kobayashi-Maskawa Parameters

xxx

In E8 Physics the KM Unitarity Triangle angles can be seen on the Stella Octangula



The Kobayashi-Maskawa parameters are determined in terms of the sum of the masses of the 30 first-generation fermion particles and antiparticles, denoted by

$$S_{mf1} = 7.508 \text{ GeV},$$

and the similar sums for second-generation and third-generation fermions, denoted by

$$S_{mf2} = 32.94504 \text{ GeV} \text{ and } S_{mf3} = 1,629.2675 \text{ GeV}.$$

The resulting KM matrix is:

	d	s	b
u	0.975	0.222 0.00249	-0.00388i
c	-0.222 -0.000161i	0.974 -0.0000365i	0.0423
t	0.00698 -0.00378i	-0.0418 -0.00086i	0.999

**Below the energy level of ElectroWeak Symmetry Breaking  
the Higgs mechanism gives mass to particles.**

According to a Review on the Kobayashi-Maskawa mixing matrix by Ceccucci, Ligeti, and Sakai in the 2010 Review of Particle Physics (note that I have changed their terminology of CKM matrix to the KM terminology that I prefer because I feel that it was Kobayashi and Maskawa, not Cabibbo, who saw that 3x3 was the proper matrix structure): "... the charged-current  $W_{\pm}$  interactions couple to the ... quarks with couplings given by ...

$V_{ud}$	$V_{us}$	$V_{ub}$
$V_{cd}$	$V_{cs}$	$V_{cb}$
$V_{td}$	$V_{ts}$	$V_{tb}$

This Kobayashi-Maskawa (KM) matrix is a 3x3 unitary matrix. It can be parameterized by three mixing angles and the CP-violating KM phase ... The most commonly used unitarity triangle arises from  $V_{ud} V_{ub}^* + V_{cd} V_{cb}^* + V_{td} V_{tb}^* = 0$ , by dividing each side by the best-known one,  $V_{cd} V_{cb}^*$

...  $\bar{\rho} + i\bar{\eta} = -(V_{ud} V_{ub}^*)/(V_{cd} V_{cb}^*)$  is phase-convention-independent ...

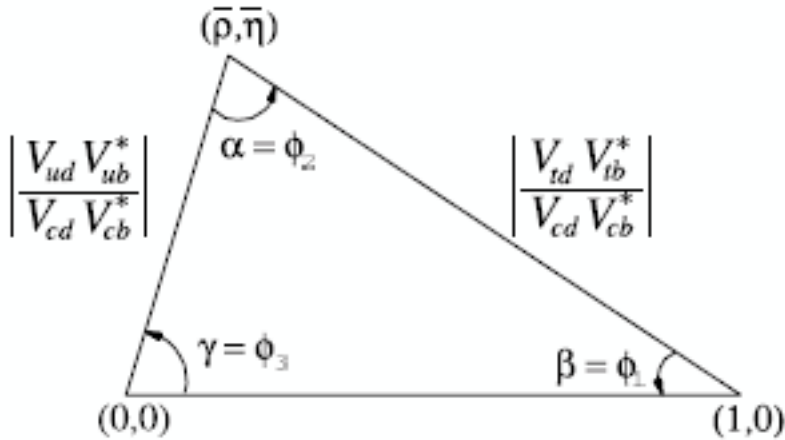


Figure 11.1: Sketch of the unitarity triangle.

...  $\sin 2\beta = 0.673 \pm 0.023$  ...  $\alpha = 89.0 +4.4 -4.2$  degrees ...  $\gamma = 73 +22 -25$  degrees ... The sum of the three angles of the unitarity triangle,  $\alpha + \beta + \gamma = (183 +22 -25)$  degrees, is ... consistent with the SM expectation. ...

The area... of ...[the]... triangle...[is]... half of the Jarlskog invariant,  $J$ , which is a phase-convention-independent measure of CP violation, defined by  $\text{Im } V_{ij} V_{kl} V_{il}^* V_{kj}^* = J \sum_{(m,n)} \epsilon_{ikm} \epsilon_{jln}$

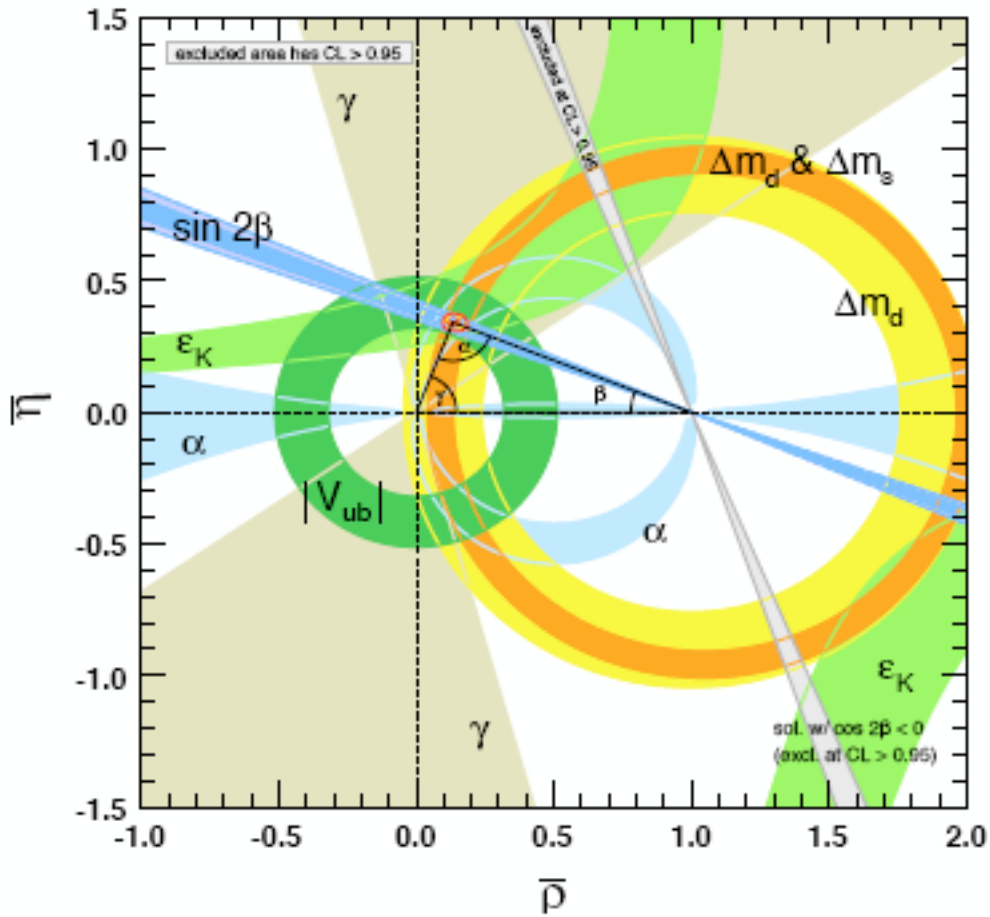


Figure 11.2: Constraints on the  $\bar{\rho}, \bar{\eta}$  plane. The shaded areas have 95% CL.

The fit results for the magnitudes of all nine KM elements are ...

$0.97428 \pm 0.00015$	$0.2253 \pm 0.0007$	$0.00347 +0.00016 -0.00012$
$0.2252 \pm 0.0007$	$0.97345 +0.00015 -0.00016$	$0.0410 +0.0011 -0.0007$
$0.00862 +0.00026 -0.00020$	$0.0403 +0.0011 -0.0007$	$0.999152 +0.000030 -0.000045$

and the Jarlskog invariant is  $J = (2.91 +0.19-0.11) \times 10^{-5}$  ...".

## Above the energy level of ElectroWeak Symmetry Breaking particles are massless.

Kea (Marni Sheppeard) proposed that in the Massless Realm the mixing matrix might be democratic. In Z. Phys. C - Particles and Fields 45, 39-41 (1989) Koide said: "... the mass matrix ... MD ... of the type ...  $1/3 \times m \times$

$$\begin{matrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{matrix}$$

... has name... "democratic" family mixing ... the ... democratic ... mass matrix can be diagonalized by the transformation matrix A ...

$$\begin{matrix} 1/\sqrt{2} & -1/\sqrt{2} & 0 \\ 1/\sqrt{6} & 1/\sqrt{6} & -2/\sqrt{6} \\ 1/\sqrt{3} & 1/\sqrt{3} & 1/\sqrt{3} \end{matrix}$$

as  $A M D A^t =$

$$\begin{matrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & m \end{matrix}$$

...".

Up in the Massless Realm you might just say that there is no mass matrix, just a democratic mixing matrix of the form  $1/3 \times$

$$\begin{matrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{matrix}$$

with no complex stuff and no CP violation in the Massless Realm.

When go down to our Massive Realm by ElectroWeak Symmetry Breaking then you might as a first approximation use  $m = 1$  so that all the mass first goes to the third generation as

$$\begin{matrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{matrix}$$

which is physically like the Higgs being a T-Tbar quark condensate.

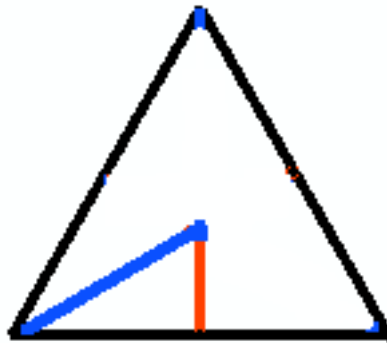
Consider a 3-dim Euclidean space of generations:

The case of mass only going to one generation  
can be represented as a line or 1-dimensional simplex

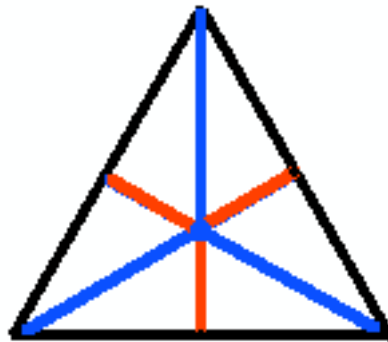


in which the blue mass-line covers the entire black simplex line.

If mass only goes to one other generation  
that can be represented by a red line extending to a second dimension  
forming a small blue-red-black triangle



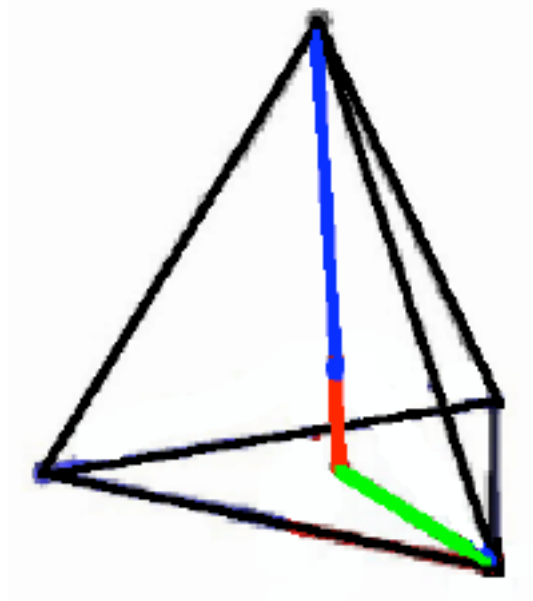
that can be extended by reflection to form six small triangles making up a large triangle



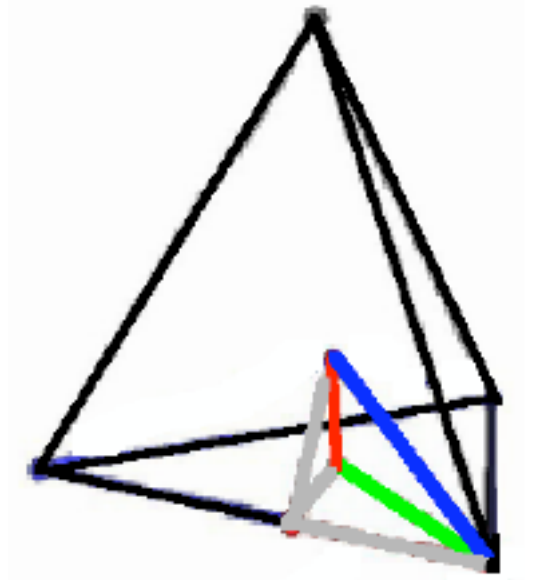
Each of the six component triangles has 30-60-90 angle structure:



If mass goes on further to all three generations  
that can be represented by a green line extending to a third dimension



If you move the blue line from the top vertex to join the green vertex

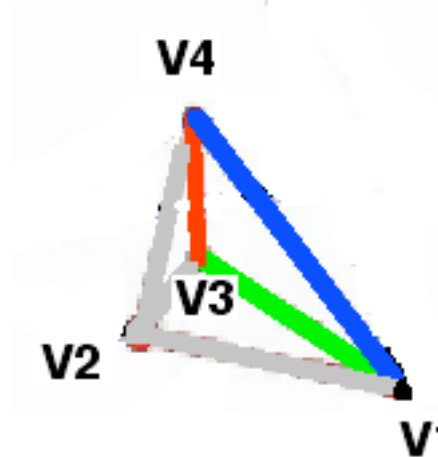


you get a small blue-red-green-gray-gray-gray tetrahedron  
that can be extended by reflection to form 24 small tetrahedra  
making up a large tetrahedron.

Reflection among the 24 small tetrahedra corresponds  
to the  $12+12 = 24$  elements of the Binary Tetrahedral Group.



The basic blue-red-green triangle of the basic small tetrahedron



has the angle structure of the K-M Unitary Triangle.

Using data from R. W. Gray's "Encyclopedia Polyhedra: A Quantum Module" with lengths

$V1.V2 = (1/2) EL \cong$  Half of the regular Tetrahedron's edge length.

$V1.V3 = (1 / \sqrt{3}) EL \cong 0.577\ 350\ 269\ EL$

$V1.V4 = 3 / (2 \sqrt{6}) EL \cong 0.612\ 372\ 436\ EL$

$V2.V3 = 1 / (2 \sqrt{3}) EL \cong 0.288\ 675\ 135\ EL$

$V2.V4 = 1 / (2 \sqrt{2}) EL \cong 0.353\ 553\ 391\ EL$

$V3.V4 = 1 / (2 \sqrt{6}) EL \cong 0.204\ 124\ 145\ EL$

the Unitarity Triangle angles are:

$\beta = V3.V1.V4 = \arccos(2 \sqrt{2} / 3) \cong 19.471\ 220\ 634\ \text{degrees}$  so  $\sin 2\beta = 0.6285$

$\alpha = V1.V3.V4 = 90\ \text{degrees}$

$\gamma = V1.V4.V3 = \arcsin(2 \sqrt{2} / 3) \cong 70.528\ 779\ 366\ \text{degrees}$

which is substantially consistent with the 2010 Review of Particle Properties

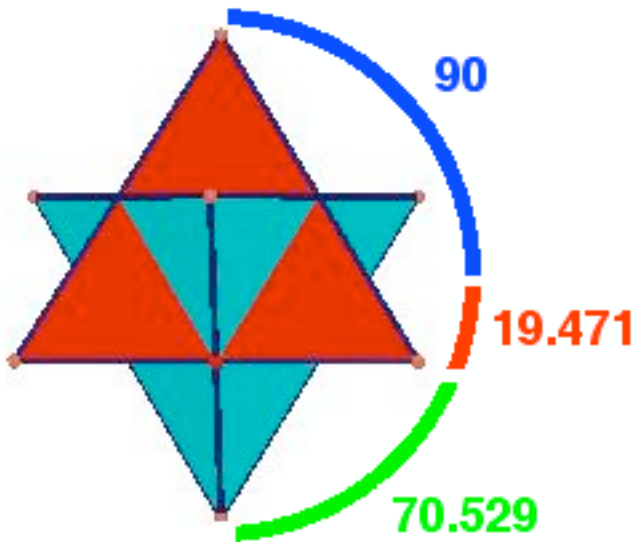
$\sin 2\beta = 0.673 \pm 0.023$  so  $\beta = 21.1495\ \text{degrees}$

$\alpha = 89.0 +4.4 -4.2\ \text{degrees}$

$\gamma = 73 +22 -25\ \text{degrees}$

and so also consistent with the Standard Model expectation.

The constructed Unitarity Triangle angles can be seen on the Stella Octangula configuration of two dual tetrahedra (image from gauss.math.nthu.edu.tw):



In the  $Cl(16)$ -E8 model the Kobayashi-Maskawa parameters are determined in terms of the sum of the masses of the 30 first-generation fermion particles and antiparticles, denoted by  
 $S_{mf1} = 7.508 \text{ GeV}$ ,

and the similar sums for second-generation and third-generation fermions, denoted  
by  $S_{mf2} = 32.94504 \text{ GeV}$  and  $S_{mf3} = 1,629.2675 \text{ GeV}$ .

The reason for using sums of all fermion masses (rather than sums of quark masses only) is that all fermions are in the same spinor representation of  $Spin(8)$ , and the  $Spin(8)$  representations are considered to be fundamental.

The following formulas use the above masses to calculate Kobayashi-Maskawa parameters:

phase angle  $d_{13} = \gamma = 70.529$  degrees

$$\sin(\theta_{12}) = s_{12} = [m_e + 3m_d + 3m_\mu] / \sqrt{[m_e^2 + 3m_d^2 + 3m_\mu^2] + [3m_\nu^2 + 3m_s^2 + 3m_c^2]} = 0.222198$$

$$\sin(\theta_{13}) = s_{13} = [m_e + 3m_d + 3m_\mu] / \sqrt{[m_e^2 + 3m_d^2 + 3m_\mu^2] + [3m_\nu^2 + 3m_b^2 + 3m_t^2]} = 0.004608$$

$$\sin(\theta_{23}) = [3m_\nu + 3m_s + 3m_c] / \sqrt{[3m_\nu^2 + 3m_b^2 + 3m_t^2] + [3m_\nu^2 + 3m_s^2 + 3m_c^2]}$$

$$\sin(\theta_{23}) = s_{23} = \sin(\theta_{23}) \sqrt{(\Sigma_{f2} / \Sigma_{f1})} = 0.04234886$$

The factor  $\sqrt{(\Sigma_{f2} / \Sigma_{f1})}$  appears in  $s_{23}$  because an  $s_{23}$  transition is to the second generation and not all the way to the first generation, so that the end product of an  $s_{23}$  transition has a greater available energy than  $s_{12}$  or  $s_{13}$  transitions by a factor of  $\Sigma_{f2} / \Sigma_{f1}$ .

Since the width of a transition is proportional to the square of the modulus of the relevant KM entry and the width of an  $s_{23}$  transition has greater available energy than the  $s_{12}$  or  $s_{13}$  transitions by a factor of  $\Sigma_{f2} / \Sigma_{f1}$  the effective magnitude of the  $s_{23}$  terms in the KM entries is increased by the factor  $\sqrt{(\Sigma_{f2} / \Sigma_{f1})}$ .

The Chau-Keung parameterization is used, as it allows the K-M matrix to be represented as the product of the following three 3x3 matrices:

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos(\theta_{23}) & \sin(\theta_{23}) \\ 0 & -\sin(\theta_{23}) & \cos(\theta_{23}) \end{pmatrix}$$

$$\begin{pmatrix} \cos(\theta_{13}) & 0 & \sin(\theta_{13})\exp(-i d_{13}) \\ 0 & 1 & 0 \\ -\sin(\theta_{13})\exp(i d_{13}) & 0 & \cos(\theta_{13}) \end{pmatrix}$$

$$\begin{pmatrix} \cos(\theta_{12}) & \sin(\theta_{12}) & 0 \\ -\sin(\theta_{12}) & \cos(\theta_{12}) & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

The resulting Kobayashi-Maskawa parameters  
for  $W^+$  and  $W^-$  charged weak boson processes, are:

	d	s	b
u	0.975	0.222	0.00249 -0.00388i
c	-0.222 -0.000161i	0.974 -0.0000365i	0.0423
t	0.00698 -0.00378i	-0.0418 -0.00086i	0.999

The matrix is labelled by either (u c t) input and (d s b) output,  
or, as above, (d s b) input and (u c t) output.

For  $Z^0$  neutral weak boson processes, which are suppressed by the GIM  
mechanism of cancellation of virtual subprocesses, the matrix is labelled by either  
(u c t) input and (u'c't') output, or, as below, (d s b) input and (d's'b') output:

	d	s	b
d'	0.975	0.222	0.00249 -0.00388i
s'	-0.222 -0.000161i	0.974 -0.0000365i	0.0423
b'	0.00698 -0.00378i	-0.0418 -0.00086i	0.999

Since neutrinos of all three generations are massless at tree level,  
the lepton sector has no tree-level K-M mixing.

In hep-ph/0208080, Yosef Nir says: "... Within the Standard Model,  
the only source of CP violation is the Kobayashi-Maskawa (KM) phase ...  
The study of CP violation is, at last, experiment driven. ...  
The CKM matrix provides a consistent picture  
of all the measured flavor and CP violating processes. ...  
There is no signal of new flavor physics. ...  
Very likely,  
the KM mechanism is the dominant source of CP violation in flavor changing processes.  
... The result is consistent with the SM predictions. ...".

## 12. Proton-Neutron Mass Difference

An up valence quark, constituent mass 313 Mev,  
does not often swap places with a 2.09 Gev charm sea quark,  
but  
a 313 Mev down valence quark  
can more often swap places with a 625 Mev strange sea quark.

Therefore the Quantum color force  
constituent mass of the down valence quark is heavier by about

$$(m_s - m_d) (m_d/m_s)^2 a(w) |V_{ds}| = 312 \times 0.25 \times 0.253 \times 0.22 \text{ Mev} = 4.3 \text{ Mev},$$

(where  $a(w) = 0.253$  is the geometric part of the weak force strength  
and  $|V_{ds}| = 0.22$  is the magnitude  
of the K-M parameter mixing first generation down and second generation strange)

so that the Quantum color force constituent mass  $Q_{md}$  of the down quark is

$$Q_{md} = 312.75 + 4.3 = 317.05 \text{ MeV}.$$

Similarly, the up quark Quantum color force mass increase is about

$$(m_c - m_u) (m_u/m_c)^2 a(w) |V_{uc}| = 1777 \times 0.022 \times 0.253 \times 0.22 \text{ Mev} = 2.2 \text{ Mev},$$

(where  $|V_{uc}| = 0.22$  is the magnitude  
of the K-M parameter mixing first generation up and second generation charm)

so that the Quantum color force constituent mass  $Q_{mu}$  of the up quark is

$$Q_{mu} = 312.75 + 2.2 = 314.95 \text{ MeV}.$$

Therefore, the Quantum color force Neutron-Proton mass difference is

$$m_N - m_P = Q_{md} - Q_{mu} = 317.05 \text{ Mev} - 314.95 \text{ Mev} = 2.1 \text{ Mev}.$$

Since the electromagnetic Neutron-Proton mass difference is roughly

$$m_N - m_P = -1 \text{ MeV}$$

the total theoretical Neutron-Proton mass difference is

$$m_N - m_P = 2.1 \text{ Mev} - 1 \text{ Mev} = 1.1 \text{ Mev},$$

an estimate that is comparable to the experimental value of 1.3 Mev.

### 13. Pion as Sine-Gordon Breather

The quark content of a charged pion is a quark - antiquark pair: either Up plus antiDown or Down plus antiUp. Experimentally, its mass is about 139.57 MeV.

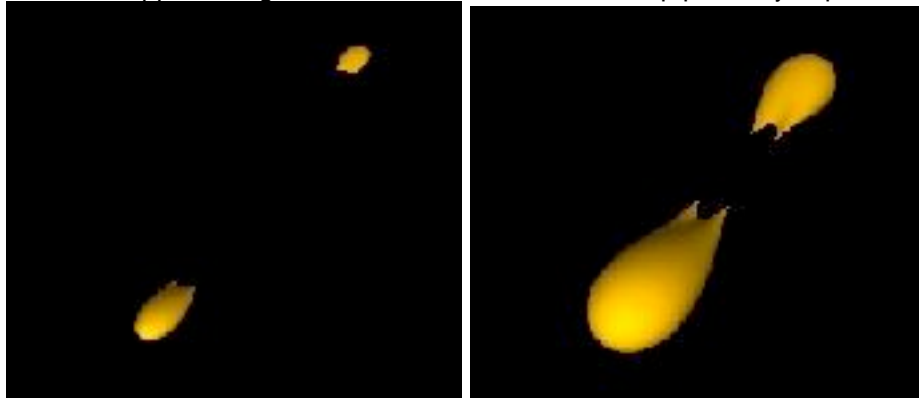
The quark is a Schwinger Source Kerr-Newman Black Hole with constituent mass  $M$  312 MeV.

The antiquark is also a Schwinger Source Kerr-Newman Black Hole, with constituent mass  $M$  312 MeV.

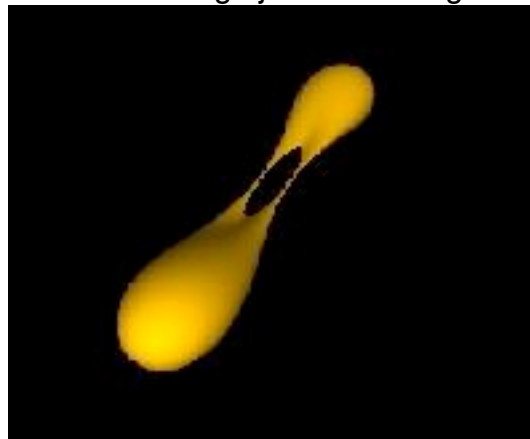
According to section 3.6 of Jeffrey Winicour's 2001 Living Review of the Development of Numerical Evolution Codes for General Relativity (see also a 2005 update):

"... The black hole event horizon associated with ... slightly broken ... degeneracy [ of the axisymmetric configuration ]... reveals new features not seen in the degenerate case of the head-on collision ... If the degeneracy is slightly broken, the individual black holes form with spherical topology but as they approach, tidal distortion produces two sharp pincers on each black hole just prior to merger. ...

Tidal distortion of approaching black holes ... Formation of sharp pincers just prior to merger ..



... toroidal stage just after merger ...



At merger, the two pincers join to form a single ... toroidal black hole.

The inner hole of the torus subsequently [ begins to] close... up (superluminally) ... [ If the closing proceeds to completion, it ]... produce[s] first a peanut shaped black hole and finally a spherical black hole. ...".

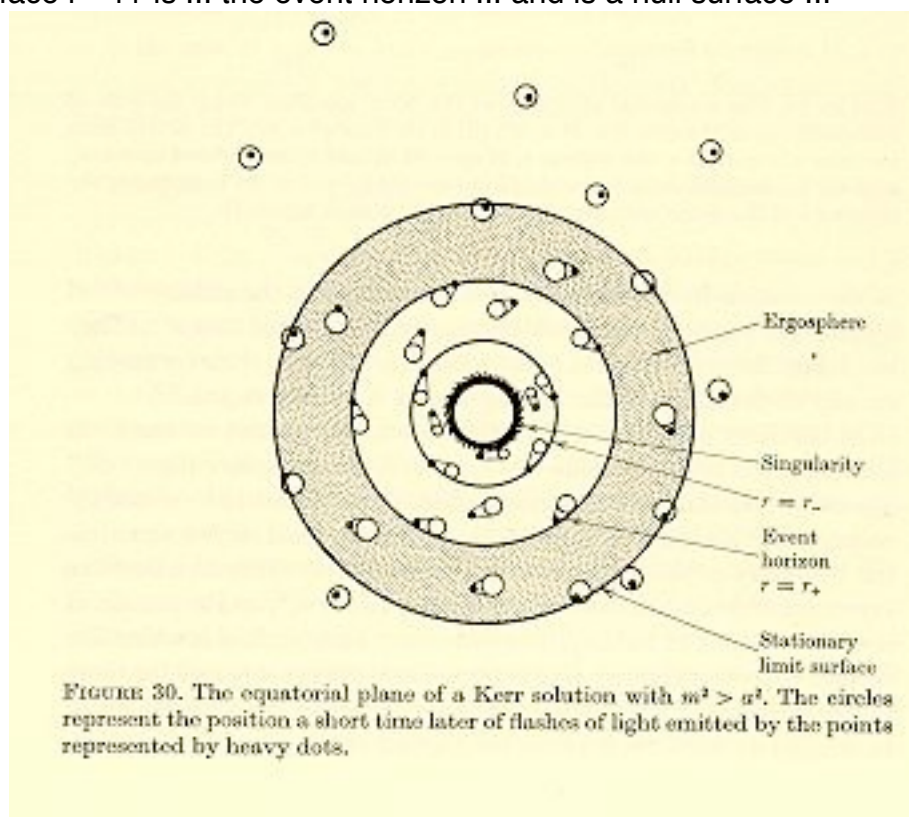
In the physical case of quark and antiquark forming a pion, the toroidal black hole remains a torus.

The torus is an event horizon and therefore is not a 2-spacelike dimensional torus, but is a (1+1)-dimensional torus with a timelike dimension.

The effect is described in detail in Robert Wald's book General Relativity (Chicago 1984). It can be said to be due to extreme frame dragging, or to timelike translations becoming spacelike as though they had been Wick rotated in Complex SpaceTime.

As Hawking and Ellis say in The LargeScale Structure of Space-Time (Cambridge 1973):

"... The surface  $r = r_+$  is ... the event horizon ... and is a null surface ...



... On the surface  $r = r_+$  .... the wavefront corresponding to a point on this surface lies entirely within the surface. ...".

A (1+1)-dimensional torus with a timelike dimension can carry a Sine-Gordon Breather. The soliton and antisoliton of a Sine-Gordon Breather correspond to the quark and antiquark that make up the pion, analogous to the Massive Thirring Model.

Sine-Gordon Breathers are described by Sidney Coleman in his Erica lecture paper Classical Lumps and their Quantum Descendants (1975), reprinted in his book Aspects of Symmetry (Cambridge 1985), where he writes the Lagrangian for the Sine-Gordon equation as ( Coleman's eq. 4.3 ):

$$L = (1 / B^2) ( (1/2) (df)^2 + A ( \cos( f ) - 1 ) )$$

Coleman says: "... We see that, in classical physics, B is an irrelevant parameter: if we can solve the sine-Gordon equation for any non-zero B, we can solve it for any other B.

The only effect of changing B is the trivial one of changing the energy and momentum assigned to a given solution of the equation. This is not true in quantum physics, because the relevant object for quantum physics is not L but [ eq. 4.4 ]

$$L / \hbar = (1 / ( B^2 \hbar )) ( (1/2) (df)^2 + A ( \cos( f ) - 1 ) )$$

An other way of saying the same thing is to say that in quantum physics we have one more dimensional constant of nature, Planck's constant, than in classical physics. ... the classical limit, vanishing  $\hbar$ , is exactly the same as the small-coupling limit, vanishing B ... from now on I will ... set  $\hbar$  equal to one. ...

... the sine-Gordon equation ...[ has ]... an exact periodic solution ...[ eq. 4.59 ]...

$$f( x, t ) = ( 4 / B ) \arctan( ( n \sin( w t ) / \cosh( n w x ) )$$

where [ eq. 4.60 ]  $n = \sqrt{ A - w^2 } / w$  and  $w$  ranges from 0 to A.

This solution has a simple physical interpretation ... a soliton far to the left ...[ and ]... an antisoliton far to the right. As  $\sin( w t )$  increases, the soliton and antisoliton move farther apart from each other. When  $\sin( w t )$  passes through one, they turn around and begin to approach one another. As  $\sin( w t )$  comes down to zero ... the soliton and antisoliton are on top of each other ... when  $\sin( w t )$  becomes negative .. the soliton and antisoliton have passed each other.

... Thus, Eq. (4.59) can be thought of as a soliton and an antisoliton oscillation about their common center-of-mass. For this reason, it is called 'the doublet [ or Breather ] solution'. ... the energy of the doublet ...[ eq. 4.64 ]

$$E = 2 M \sqrt{ 1 - ( w^2 / A ) }$$

where [ eq. 4.65 ]  $M = 8 \sqrt{ A } / B^2$  is the soliton mass.

Note that the mass of the doublet is always less than twice the soliton mass, as we would expect from a soliton-antisoliton pair. ...



Dashen, Hasslacher, and Neveu ... Phys. Rev. D10, 4114; 4130; 4138 (1974).  
 ...[ found that ]... there is only a single series of bound states, labeled by the integer N ...  
 The energies ... are ... [ eq. 4.82 ]

$$E_N = 2 M \sin( B'^2 N / 16 )$$

where  $N = 0, 1, 2 \dots < 8 \pi / B'^2$ , [ eq. 4.83 ]

$B'^2 = B^2 / ( 1 - ( B^2 / 8 \pi ) )$  and M is the soliton mass.

M is not given by Eq. ( 4.65 ), but is the soliton mass corrected by the DHN formula, or, equivalently, by the first-order weak coupling expansion. ...

I have written the equation in this form .. to eliminate A, and thus avoid worries about renormalization conventions.

Note that the DHN formula is identical to the Bohr-Sommerfeld formula, except that B is replaced by B'.

Bohr and Sommerfeld[s] ... quantization formula says that if we have a one-parameter family of periodic motions, labeled by the period, T, then an energy eigenstate occurs whenever [ eq. 4.66 ]

$$[ \text{Integral from 0 to T} ]( dt p \dot{q} = 2 \pi N,$$

where N is an integer. ... Eq.( 4.66 ) is cruder than the WKB formula, but it is much more general;

it is always the leading approximation for any dynamical system ...

Dashen et al speculate that Eq. ( 4.82 ) is exact. ...

the sine-Gordon equation is equivalent ... to the massive Thirring model.

This is surprising,

because the massive Thirring model is a canonical field theory

whose Hamiltonian is expressed in terms of fundamental Fermi fields only.

Even more surprising, when  $B^2 = 4 \pi$ , that sine-Gordon equation is equivalent to a free massive Dirac theory, in one spatial dimension. ...

Furthermore, we can identify the mass term in the Thirring model with the sine-Gordon interaction, [ eq. 5.13 ]

$$M = - ( A / B^2 ) N_m \cos( B f )$$

.. to do this consistently ... we must say [ eq. 5.14 ]

$$B^2 / ( 4 \pi ) = 1 / ( 1 + g / \pi )$$

....[where]... g is a free parameter, the coupling constant [ for the Thirring model ]...

Note that if  $B^2 = 4 \pi$ ,  $g = 0$ ,

and the sine-Gordon equation is the theory of a free massive Dirac field. ...

It is a bit surprising to see a fermion appearing as a coherent state of a Bose field.

Certainly this could not happen in three dimensions,

where it would be forbidden by the spin-statistics theorem.

However, there is no spin-statistics theorem in one dimension,

for the excellent reason that there is no spin. ...

the lowest fermion-antifermion bound state of the massive Thirring model

is an obvious candidate for the fundamental meson of sine-Gordon theory. ...

equation ( 4.82 ) predicts that

all the doublet bound states disappear when  $B^2$  exceeds  $4 \pi$ .

This is precisely the point where the Thirring model interaction switches from attractive to repulsive. ... these two theories ... the massive Thirring model .. and ... the sine-Gordon equation ... define identical physics. ...

I have computed the predictions of ...[various]... approximation methods for the ration of the soliton mass to the meson mass for three values of  $B^2$  :  $4\pi$  (where the qualitative picture of the soliton as a lump totally breaks down),  $2\pi$ , and  $\pi$  . At  $4\pi$  we know the exact answer ... I happen to know the exact answer for  $2\pi$ , so I have included this in the table. ...

Method	$B^2 = \pi$	$B^2 = 2\pi$	$B^2 = 4\pi$
Zeroth-order weak coupling expansion eq2.13b	2.55	1.27	0.64
Coherent-state variation	2.55	1.27	0.64
First-order weak coupling expansion	2.23	0.95	0.32
Bohr-Sommerfeld eq4.64	2.56	1.31	0.71
DHN formula eq4.82	2.25	1.00	0.50
Exact	?	1.00	0.50

...[eq. 2.13b ]

$$E = 8 \sqrt{A} / B^2$$

...[ is the ]... energy of the lump ... of sine-Gordon theory ... frequently called 'soliton...' in the literature ...

[ Zeroth-order is the classical case, or classical limit. ] ...

... Coherent-state variation always gives the same result as the ... Zeroth-order weak coupling expansion ... . The ... First-order weak-coupling expansion ... explicit formula ... is  $( 8 / B^2 ) - ( 1 / \pi )$ . ...".

Using the Cl(16)-E8 model constituent mass of the Up and Down quarks and antiquarks, about 312.75 MeV, as the soliton and antisoliton masses, and setting  $B^2 = \pi$  and using the DHN formula, the mass of the charged pion is calculated to be  $( 312.75 / 2.25 )$  MeV = 139 MeV which is close to the experimental value of about 139.57 MeV.

**Why is the value  $B^2 = \pi$  the special value that gives the pion mass ?**

( or, using Coleman's eq. ( 5.14 ), the Thirring coupling constant  $g = 3\pi$  )

**Because  $B^2 = \pi$  is where the First-order weak coupling expansion substantially coincides with the ( probably exact ) DHN formula.** In other words,

**The physical quark - antiquark pion lives where the first-order weak coupling expansion is exact.**

## 14. Neutrino Masses Beyond Tree Level

Consider the three generations of neutrinos:

$\nu_e$  (electron neutrino);  $\nu_\mu$  (muon neutrino);  $\nu_\tau$

and three neutrino mass states:  $\nu_1$  ;  $\nu_2$  :  $\nu_3$

and

the division of 8-dimensional spacetime into

4-dimensional physical Minkowski spacetime

plus

4-dimensional CP2 internal symmetry space.

The heaviest mass state  $\nu_3$  corresponds to a neutrino

whose propagation begins and ends in CP2 internal symmetry

space, lying entirely therein. According to the Cl(16)-E8 model

the mass of  $\nu_3$  is zero at tree-level

but it picks up a first-order correction

propagating entirely through internal symmetry space by merging

with an electron through the weak and electromagnetic forces,

effectively acting not merely as a point

but

as a point plus an electron loop at beginning and ending points

so

the first-order corrected mass of  $\nu_3$  is given by

$M_{\nu_3} \times (1/\sqrt{2}) = M_e \times GW(m_{\text{proton}}^2) \times \alpha_E$

where the factor  $(1/\sqrt{2})$  comes from the Ut3 component

of the neutrino mixing matrix

so that

$$M_{\nu_3} = \sqrt{2} \times M_e \times GW(m_{\text{proton}}^2) \times \alpha_E =$$

$$= 1.4 \times 5 \times 10^5 \times 1.05 \times 10^{(-5)} \times (1/137) \text{ eV} =$$

$$= 7.35 / 137 = 5.4 \times 10^{(-2)} \text{ eV}.$$

The neutrino-plus-electron loop can be anchored by weak force

action through any of the 6 first-generation quarks

at each of the beginning and ending points, and that the

anchor quark at the beginning point can be different from

the anchor quark at the ending point,

so that there are  $6 \times 6 = 36$  different possible anchorings.

The intermediate mass state  $\nu_2$  corresponds to a neutrino whose propagation begins or ends in CP2 internal symmetry space and ends or begins in M4 physical Minkowski spacetime, thus having only one point (either beginning or ending) lying in CP2 internal symmetry space where it can act not merely as a point but as a point plus an electron loop.

According to the Cl(16)-E8 model the mass of  $\nu_2$  is zero at tree-level but it picks up a first-order correction at only one (but not both) of the beginning or ending points so that so that there are 6 different possible anchorings for  $\nu_2$  first-order corrections, as opposed to the 36 different possible anchorings for  $\nu_3$  first-order corrections, so that the first-order corrected mass of  $\nu_2$  is less than the first-order corrected mass of  $\nu_3$  by a factor of 6, so

the first-order corrected mass of  $\nu_2$  is  
$$M_{\nu_2} = M_{\nu_3} / \text{Vol}(\text{CP}2) = 5.4 \times 10^{(-2)} / 6$$
$$= 9 \times 10^{(-3)} \text{eV}.$$

The low mass state  $\nu_1$  corresponds to a neutrino whose propagation begins and ends in physical Minkowski spacetime. thus having only one anchoring to CP2 interna symmetry space.

According to the Cl(16)-E8 model the mass of  $\nu_1$  is zero at tree-level but it has only 1 possible anchoring to CP2 as opposed to the 36 different possible anchorings for  $\nu_3$  first-order corrections or the 6 different possible anchorings for  $\nu_2$  first-order corrections so that the first-order corrected mass of  $\nu_1$  is less than the first-order corrected mass of  $\nu_2$  by a factor of 6, so

the first-order corrected mass of  $\nu_1$  is  
$$M_{\nu_1} = M_{\nu_2} / \text{Vol}(\text{CP}2) = 9 \times 10^{(-3)} / 6$$
$$= 1.5 \times 10^{(-3)} \text{eV}.$$

Therefore:

$$\begin{aligned} \text{the mass-squared difference } D(M_{23}^2) &= M_{\nu_3}^2 - M_{\nu_2}^2 = \\ &= (2916 - 81) \times 10^{(-6)} \text{ eV}^2 = \\ &= 2.8 \times 10^{(-3)} \text{ eV}^2 \end{aligned}$$

and

$$\begin{aligned} \text{the mass-squared difference } D(M_{12}^2) &= M_{\nu_2}^2 - M_{\nu_1}^2 = \\ &= (81 - 2) \times 10^{(-6)} \text{ eV}^2 = \\ &= 7.9 \times 10^{(-5)} \text{ eV}^2 \end{aligned}$$

The 3x3 unitary neutrino mixing matrix neutrino mixing matrix U

	nu_1	nu_2	nu_3
nu_e	Ue1	Ue2	Ue3
nu_m	Um1	Um2	Um3
nu_t	Ut1	Ut2	Ut3

can be parameterized (based on the 2010 Particle Data Book)  
by 3 angles and 1 Dirac CP violation phase

$$U = \begin{pmatrix} c_{12} c_{13} & s_{12} c_{13} & s_{13} e^{-i\delta} \\ -s_{12} c_{23} - c_{12} s_{23} s_{13} e^{i\delta} & c_{12} c_{23} - s_{12} s_{23} s_{13} e^{i\delta} & s_{23} c_{13} \\ s_{12} s_{23} - c_{12} c_{23} s_{13} e^{i\delta} & -c_{12} s_{23} - s_{12} c_{23} s_{13} e^{i\delta} & c_{23} c_{13} \end{pmatrix}$$

where  $c_{ij} = \cos(\theta_{ij})$  ,  $s_{ij} = \sin(\theta_{ij})$

The angles are

$\theta_{23} = \pi/4 = 45$  degrees

because

$\nu_3$  has equal components of  $\nu_m$  and  $\nu_t$  so  
that  $U_{m3} = U_{t3} = 1/\sqrt{2}$  or, in conventional  
notation, mixing angle  $\theta_{23} = \pi/4$

so that  $\cos(\theta_{23}) = 0.707 = \sqrt{2}/2 = \sin(\theta_{23})$

$\theta_{13} = 9.594$  degrees =  $\arcsin(1/6)$

and  $\cos(\theta_{13}) = 0.986$

because  $\sin(\theta_{13}) = 1/6 = 0.167 = |U_{e3}| =$  fraction of  $\nu_3$  that is  $\nu_e$

$\theta_{12} = \pi/6 = 30$  degrees

because

$\sin(\theta_{12}) = 0.5 = 1/2 = U_{e2} =$  fraction of  $\nu_2$  begin/end points  
that are in the physical spacetime where massless  $\nu_e$  lives

so that  $\cos(\theta_{12}) = 0.866 = \sqrt{3}/2$

$\delta = 70.529$  degrees is the Dirac CP violation phase

$e^{i(70.529)} = \cos(70.529) + i \sin(70.529) = 0.333 + 0.943 i$

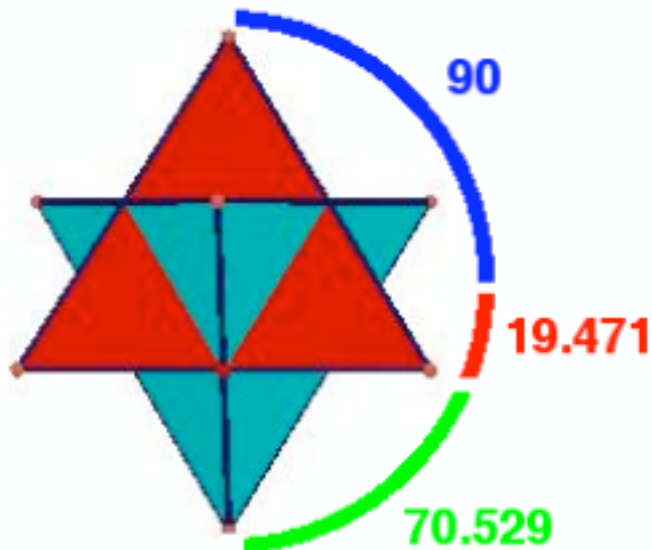
This is because the neutrino mixing matrix has 3-generation structure  
and so has the same phase structure as the KM quark mixing matrix  
in which the Unitarity Triangle angles are:

$\beta = V_3.V_1.V_4 = \arccos( 2 \sqrt{2} / 3 ) \cong 19.471 220 634$  degrees so  $\sin 2\beta = 0.6285$

$\alpha = V_1.V_3.V_4 = 90$  degrees

$\gamma = V_1.V_4.V_3 = \arcsin( 2 \sqrt{2} / 3 ) \cong 70.528 779 366$  degrees

The constructed Unitarity Triangle angles can be seen on the Stella Octangula configuration of two dual tetrahedra (image from [gauss.math.nthu.edu.tw](http://gauss.math.nthu.edu.tw)):



Then we have for the neutrino mixing matrix:

	nu_1	nu_2	nu_3
nu_e	0.866 x 0.986	0.50 x 0.986	0.167 x e-id
nu_m	-0.5 x 0.707 -0.866 x 0.707 x 0.167 x eid	0.866 x 0.707 -0.5 x 0.707 x 0.167 x eid	0.707 x 0.986
nu_t	0.5 x 0.707 -0.866 x 0.707 x 0.167 x eid	-0.866 x 0.707 -0.5 x 0.707 x 0.167 x eid	0.707 x 0.986

	nu_1	nu_2	nu_3
nu_e	0.853	0.493	0.167 e-id
nu_m	-0.354 -0.102 eid	0.612 -0.059 eid	0.697
nu_t	0.354 -0.102 eid	-0.612 -0.059 eid	0.697

Since  $e^{i(70.529)} = \cos(70.529) + i \sin(70.529) = 0.333 + 0.943 i$   
and  $.333e^{-i(70.529)} = \cos(70.529) - i \sin(70.529) = 0.333 - 0.943 i$

	nu_1	nu_2	nu_3
nu_e	0.853	0.493	0.056 - 0.157 i
nu_m	-0.354 -0.034 - 0.096 i	0.612 -0.020 - 0.056 i	0.697
nu_t	0.354 -0.034 - 0.096 i	-0.612 -0.020 - 0.056 i	0.697

for a result of

	nu_1	nu_2	nu_3
nu_e	0.853	0.493	0.056 - 0.157 i
nu_m	-0.388 - 0.096 i	0.592 - 0.056 i	0.697
nu_t	0.320 - 0.096 i	0.632 - 0.056 i	0.697

which is consistent with the approximate experimental values of mixing angles shown in the Michaelmas Term 2010 Particle Physics handout of Prof Mark Thomson if the matrix is modified by taking into account the March 2012 results from Daya Bay observing non-zero  $\theta_{13} = 9.54$  degrees.

## 15. Planck Mass as Superposition Fermion Condensate

At a single spacetime vertex, a Planck-mass black hole is the Many-Worlds quantum sum of all possible virtual first-generation particle-antiparticle fermion pairs allowed by the Pauli exclusion principle to live on that vertex.

Once a Planck-mass black hole is formed, it is stable in the E8 model. Less mass would not be gravitationally bound at the vertex. More mass at the vertex would decay by Hawking radiation.

There are 8 fermion particles and 8 fermion antiparticles for a total of 64 particle-antiparticle pairs. Of the 64 particle-antiparticle pairs, 12 are bosonic pions.

A typical combination should have about 6 pions so it should have a mass of about  $.14 \times 6 \text{ GeV} = 0.84 \text{ GeV}$ .

Just as the pion mass of  $.14 \text{ GeV}$  is less than the sum of the masses of a quark and an antiquark, pairs of oppositely charged pions may form a bound state of less mass than the sum of two pion masses.

If such a bound state of oppositely charged pions has a mass as small as  $.1 \text{ GeV}$ , and if the typical combination has one such pair and 4 other pions, then the typical combination could have a mass in the range of  $0.66 \text{ GeV}$ .

Summing over all  $2^{64}$  combinations, the total mass of a one-vertex universe should give a Planck mass roughly around  $0.66 \times 2^{64} = 1.217 \times 10^{19} \text{ GeV}$ .

The value for the Planck mass given in by the 1998 Particle Data Group is  $1.221 \times 10^{19} \text{ GeV}$ .



## 16. Force Strength and Boson Mass Calculation

Cl(8) bivector Spin(8) is the D4 Lie algebra two copies of which are in the Cl(16)-E8 model Lagrangian (as the D4xD4 subalgebra of the D8 subalgebra of E8)

$$\int_{4\text{-dim } M4} \text{GG SM} + \text{Fermion Particle-AntiParticle} + \text{Higgs}$$

with the Higgs term coming from integrating over the CP2 Internal Symmetry Space of M4 x CP2 Kaluza-Klein by the Mayer-Trautman Mechanism

This shows that the **Force Strength is made up of two parts:**

**the relevant spacetime manifold of gauge group global action**  
and  
**the relevant symmetric space manifold of gauge group local action.**

The 4-dim spacetime Lagrangian **GG SM** gauge boson term is:  
the integral over spacetime as seen by gauge boson acting globally  
of the gauge force term of the gauge boson acting locally  
for the gauge bosons of each of the four forces:

U(1) for electromagnetism

SU(2) for weak force

SU(3) for color force

Spin(5) - compact version of antiDeSitter Spin(2,3) subgroup of Conformal Spin(2,4) for gravity by the MacDowell-Mansouri mechanism.

### **In the conventional picture,**

for each gauge force the gauge boson force term contains the force strength, which in Feynman's picture is the amplitude to emit a gauge boson, and can also be thought of as the probability = square of amplitude, in an explicit ( like  $g |F|^2$  ) or an implicit ( incorporated into the  $|F|^2$  ) form. Either way, the conventional picture is that the force strength  $g$  is an ad hoc inclusion.

**The Cl(16)-E8 model** does not put in force strength  $g$  ad hoc, but constructs the integral such that the force strength emerges naturally from the geometry of each gauge force.

To do that, for each gauge force:

1 - make the spacetime over which the integral is taken be spacetime as it is seen by that gauge boson, that is, in terms of the symmetric space with global symmetry of the gauge boson:

the U(1) photon sees 4-dim spacetime as  $T^4 = S^1 \times S^1 \times S^1 \times S^1$   
the SU(2) weak boson sees 4-dim spacetime as  $S^2 \times S^2$   
the SU(3) weak boson sees 4-dim spacetime as  $CP^2$   
the Spin(5) of gravity sees 4-dim spacetime as  $S^4$

2 - make the gauge boson force term have the volume of the Shilov boundary corresponding to the symmetric space with local symmetry of the gauge boson. The nontrivial Shilov boundaries are:

for SU(2) Shilov =  $RP^1 \times S^2$   
for SU(3) Shilov =  $S^5$   
for Spin(5) Shilov =  $RP^1 \times S^4$

The result is (ignoring technicalities for exposition) the geometric factor for force strengths.

Each gauge group is the global symmetry of a symmetric space

$S^1$  for U(1)  
 $S^2 = SU(2)/U(1) = Spin(3)/Spin(2)$  for SU(2)  
 $CP^2 = SU(3)/SU(2) \times U(1)$  for SU(3)  
 $S^4 = Spin(5)/Spin(4)$  for Spin(5)

Each gauge group is the local symmetry of a symmetric space

U(1) for itself  
SU(2) for Spin(5) /  $SU(2) \times U(1)$   
SU(3) for SU(4) /  $SU(3) \times U(1)$   
Spin(5) for Spin(7) /  $Spin(5) \times U(1)$

The nontrivial local symmetry symmetric spaces correspond to bounded complex domains

SU(2) for Spin(5) /  $SU(2) \times U(1)$  corresponds to IV3  
SU(3) for SU(4) /  $SU(3) \times U(1)$  corresponds to  $B^6$  (ball)  
Spin(5) for Spin(7) /  $Spin(5) \times U(1)$  corresponds to IV5

The nontrivial bounded complex domains have Shilov boundaries

SU(2) for Spin(5) /  $SU(2) \times U(1)$  corresponds to IV3 Shilov =  $RP^1 \times S^2$   
SU(3) for SU(4) /  $SU(3) \times U(1)$  corresponds to  $B^6$  (ball) Shilov =  $S^5$   
Spin(5) for Spin(7) /  $Spin(5) \times U(1)$  corresponds to IV5 Shilov =  $RP^1 \times S^4$

Very roughly, think of the force strength as  
integral over global symmetry space of physical (ie Shilov Boundary) volume =  
= strength of the force.

That is:

the geometric strength of the force is given by the product of  
the volume of a 4-dim thing with global symmetry of the force and  
the volume of the Shilov Boundary for the local symmetry of the force.

When you calculate the product volumes (using some tricky normalization stuff),  
you see that roughly:

Volume product for gravity is the largest volume  
so since (as Feynman says) force strength = probability to emit a gauge boson means  
that the highest force strength or probability should be 1  
the gravity Volume product is normalized to be 1, and so (approximately):

$$\begin{aligned}\text{Volume product for gravity} &= 1 \\ \text{Volume product for color} &= 2/3 \\ \text{Volume product for weak} &= 1/4 \\ \text{Volume product for electromagnetism} &= 1/137\end{aligned}$$

There are two further main components of a force strength:

- 1 - for massive gauge bosons, a suppression by a factor of  $1 / M^2$
- 2 - renormalization running (important for color force)

Consider Massive Gauge Bosons:

Gravity as curvature deformation of SpaceTime, with SpaceTime as a condensate of  
Planck-Mass Black Holes, must be carried by virtual Planck-mass black holes,  
so that the geometric strength of gravity should be reduced by  $1/M_p^2$

The weak force is carried by weak bosons,  
so that the geometric strength of the weak force should be reduced by  $1/M_W^2$

That gives the result (approximate):

$$\begin{aligned}\text{gravity strength} &= G \text{ (Newton's } G\text{)} \\ \text{color strength} &= 2/3 \\ \text{weak strength} &= G_F \text{ (Fermi's weak force } G\text{)} \\ \text{electromagnetism} &= 1/137\end{aligned}$$

Consider Renormalization Running for the Color Force:: That gives the result:

gravity strength = G (Newton's G)  
color strength = 1/10 at weak boson mass scale  
weak strength =  $G_F$  (Fermi's weak force G)  
electromagnetism = 1/137

the use of compact volumes is itself a calculational device,  
because it would be more nearly correct,  
instead of the integral over the compact global symmetry space of  
the compact physical (ie Shilov Boundary) volume=strength of the force  
to use  
the integral over the hyperbolic spacetime global symmetry space  
of the noncompact invariant measure of the gauge force term.

However, since the strongest (gravitation) geometric force strength is to be normalized  
to 1, the only thing that matters is ratios,  
and the compact volumes (finite and easy to look up in the book by Hua)  
have the same ratios as the noncompact invariant measures.

In fact, I should go on to say that continuous spacetime and gauge force geometric  
objects are themselves also calculational devices,  
and  
that it would be even more nearly correct to do the calculations with respect to a  
discrete generalized hyperdiamond Feynman checkerboard.

**Here are less approximate more detailed force strength calculations:**

The force strength of a given force is

$$\text{alphaforce} = (1 / M\text{force}^2) ( \text{Vol}(\text{MISforce}) ) ( \text{Vol}(\text{Qforce}) / \text{Vol}(\text{Dforce})^{( 1 / m\text{force} )} )$$

where:

alphaforce represents the force strength;

Mforce represents the effective mass;

MISforce represents the relevant part of the target Internal Symmetry Space;

Vol(MISforce) stands for volume of MISforce and is sometimes also denoted by Vol(M);

Qforce represents the link from the origin to the relevant target for the gauge boson;

Vol(Qforce) stands for volume of Qforce;

Dforce represents the complex bounded homogeneous domain of which Qforce is the Shilov boundary;

mforce is the dimensionality of Qforce, which is

4 for Gravity and the Color force,

2 for the Weak force (which therefore is considered to have two copies of QW for SpaceTime),

1 for Electromagnetism (which therefore is considered to have four copies of QE for SpaceTime)

$\text{Vol}(\text{Dforce})^{( 1 / m\text{force} )}$  stands for a dimensional normalization factor (to reconcile the dimensionality of the Internal Symmetry Space of the target vertex with the dimensionality of the link from the origin to the target vertex).

The Qforce, Hermitian symmetric space, and Dforce manifolds for the four forces are:

Spin(5)	Spin(7) / Spin(5)xU(1)	IV5	4	RP^1xS^4
SU(3)	SU(4) / SU(3)xU(1)	B^6(ball)	4	S^5
SU(2)	Spin(5) / SU(2)xU(1)	IV3	2	RP^1xS^2
U(1)	-	-	1	-

The geometric volumes needed for the calculations are mostly taken from the book Harmonic Analysis of Functions of Several Complex Variables in the Classical Domains (AMS 1963, Moskva 1959, Science Press Peking 1958) by L. K. Hua [unit radius scale].

Force	M	Vol(M)
gravity	$S^4$	$8\pi^2/3$ - $S^4$ is 4-dimensional
color	$CP^2$	$8\pi^2/3$ - $CP^2$ is 4-dimensional
weak	$S^2 \times S^2$	$2 \times 4\pi$ - $S^2$ is a 2-dim boundary of 3-dim ball 4-dim $S^2 \times S^2$ = topological boundary of 6-dim 2-polyball Shilov Boundary of 6-dim 2-polyball = $S^2 + S^2$ = = 2-dim surface frame of 4-dim $S^2 \times S^2$
e-mag	$T^4$	$4 \times 2\pi$ - $S^1$ is 1-dim boundary of 2-dim disk 4-dim $T^4 = S^1 \times S^1 \times S^1 \times S^1$ = topological boundary of 8-dim 4-polydisk Shilov Boundary of 8-dim 4-polydisk = $S^1 + S^1 + S^1 + S^1$ = = 1-dim wire frame of 4-dim $T^4$

Note ( thanks to Carlos Castro for noticing this ) also that the volume listed for  $CP^2$  is unconventional, but physically justified by noting that  $S^4$  and  $CP^2$  can be seen as having the same physical volume, with the only difference being structure at infinity.

Note that for U(1) electromagnetism, whose photon carries no charge, the factors Vol(Q) and Vol(D) do not apply and are set equal to 1, and from another point of view, the link manifold to the target vertex is trivial for the abelian neutral U(1) photons of Electromagnetism, so we take QE and DE to be equal to unity.

Force	M	Vol(M)	Q	Vol(Q)	D	Vol(D)
gravity	$S^4$	$8\pi^2/3$	$RP^1 \times S^4$	$8\pi^3/3$	$IV_5$	$\pi^5/2^4 5!$
color	$CP^2$	$8\pi^2/3$	$S^5$	$4\pi^3$	$B^6(\text{ball})$	$\pi^3/6$
Weak	$S^2 \times S^2$	$2 \times 4\pi$	$RP^1 \times S^2$	$4\pi^2$	$IV_3$	$\pi^3/24$
e-mag	$T^4$	$4 \times 2\pi$	-	-	-	-

Note ( thanks to Carlos Castro for noticing this ) that the volume listed for  $S^5$  is for a squashed  $S^5$ , a Shilov boundary of the complex domain corresponding to the symmetric space  $SU(4) / SU(3) \times U(1)$ .

Using the above numbers, the results of the calculations are the relative force strengths at the characteristic energy level of the generalized Bohr radius of each force:

Spin(5)	gravity	approx $10^{19}$ GeV	1	$G G m_{\text{proton}}^2$	approx $5 \times 10^{-39}$
SU(3)	color	approx 245 MeV	0.6286		0.6286
SU(2)	weak	approx 100 GeV	0.2535	$G W m_{\text{proton}}^2$	approx $1.05 \times 10^{-5}$
U(1)	e-mag	approx 4 KeV	1/137.03608		1/137.03608

The force strengths are given at the characteristic energy levels of their forces, because the force strengths run with changing energy levels.

The effect is particularly pronounced with the color force.

The color force strength was calculated using a simple perturbative QCD renormalization group equation at various energies, with the following results:

Energy Level	Color Force Strength
245 MeV	0.6286
5.3 GeV	0.166
34 GeV	0.121
91 GeV	0.106

Taking other effects, such as Nonperturbative QCD, into account, should give a Color Force Strength of about 0.125 at about 91 GeV

## Higgs:

As with forces strengths, the calculations produce ratios of masses, so that only one mass need be chosen to set the mass scale.

In the Cl(16)-E8 model, the value of the fundamental mass scale vacuum expectation value  $v = \langle \text{PHI} \rangle$  of the Higgs scalar field is set to be the sum of the physical masses of the weak bosons,  $W^+$ ,  $W^-$ , and  $Z^0$ , whose tree-level masses will then be shown by ratio calculations to be 80.326 GeV, 80.326 GeV, and 91.862 GeV, respectively, and therefore the electron mass will be 0.5110 MeV.

The relationship between the Higgs mass and  $v$  is given by the Ginzburg-Landau term from the Mayer Mechanism as

$$(1/4) \text{Tr} ( [ \text{PHI} , \text{PHI} ] - \text{PHI} )^2$$

or, i

n the notation of quant-ph/9806009 by Guang-jiong Ni

$$(1/4!) \lambda \text{PHI}^4 - (1/2) \sigma \text{PHI}^2$$

where the Higgs mass  $M_H = \sqrt{2 \sigma}$

Ni says:

"... the invariant meaning of the constant  $\lambda$  in the Lagrangian is not the coupling constant, the latter will change after quantization ... The invariant meaning of  $\lambda$  is nothing but the ratio of two mass scales:

$$\lambda = 3 ( M_H / \text{PHI} )^2$$

which remains unchanged irrespective of the order ...".

Since  $\langle \text{PHI} \rangle^2 = v^2$ , and assuming that  $\lambda = ( \cos( \pi / 6 ) )^2 = 0.866^2$  ( a value consistent with the Higgs-Tquark condensate model of Michio Hashimoto, Masaharu Tanabashi, and Koichi Yamawaki in their paper at hep-ph/0311165 ) we have

$$M_H^2 / v^2 = ( \cos( \pi / 6 ) )^2 / 3$$

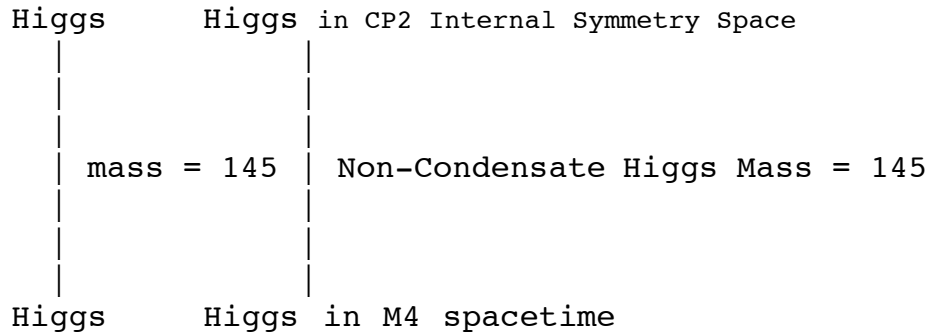
In the Cl(16)-E8 model, the fundamental mass scale vacuum expectation value  $v$  of the Higgs scalar field is the fundamental mass parameter that is to be set to define all other masses by the mass ratio formulas of the model and  $v$  is set to be 252.514 GeV so that

$$M_H = v \cos( \pi / 6 ) / \sqrt{1 / 3} = 126.257 \text{ GeV}$$

This is the value of the Low Mass State of the Higgs observed by the LHC. Middle and High Mass States come from a Higgs-Tquark Condensate System. The Middle and High Mass States may have been observed by the LHC at 20% of the Low Mass State cross section, and that may be confirmed by the LHC 2015-1016 run.

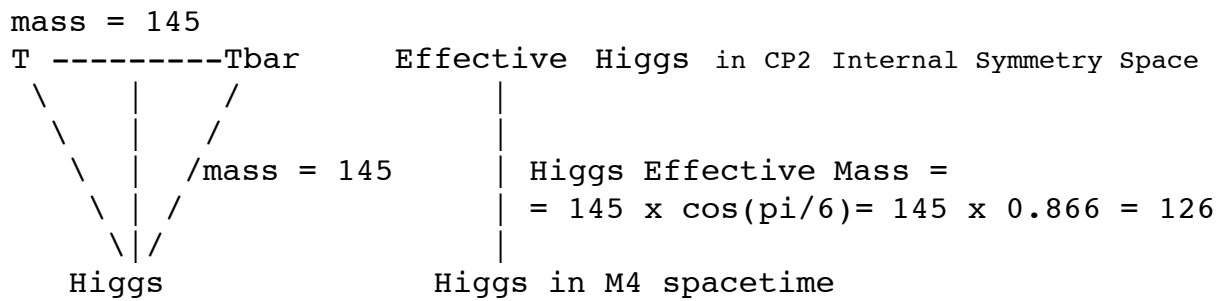


A Non-Condensate Higgs is represented by a Higgs at a point in M4 that is connected to a Higgs representation in CP2 ISS by a line whose length represents the Higgs mass



and the value of lambda is  $1 = 1^2$   
 so that the Higgs mass would be  $M_H = v / \sqrt{3} = 145.789 \text{ GeV}$

However, in the  $Cl(16)$ -E8 model, the Higgs has structure of a Tquark condensate



in which the Higgs at a point in M4 is connected to a T and Tbar in CP2 ISS so that the vertices of the Higgs-T-Tbar system are connected by lines forming an equilateral triangle composed of 2 right triangles (one from the CP2 origin to the T and to the M4 Higgs and another from the CP2 origin to the Tbar and to the M4 Higgs).

In the T-quark condensate picture  
 $\lambda = 1^2 = \lambda(T) + \lambda(H) = (\sin(\pi / 6))^2 + (\cos(\pi / 6))^2$   
 and  
 $\lambda(H) = (\cos(\pi / 6))^2$

Therefore the Effective Higgs mass observed by LHC is:

$$\text{Higgs Mass} = 145.789 \times \cos(\pi/6) = 126.257 \text{ GeV.}$$

To get W-boson masses,  
denote the 3 SU(2) high-energy weak bosons  
(massless at energies higher than the electroweak unification)  
by  $W_+$ ,  $W_-$ , and  $W_0$ ,  
corresponding to the massive physical weak bosons  $W_+$ ,  $W_-$ , and  $Z_0$ .

The triplet  $\{ W_+, W_-, W_0 \}$  couples directly with the T - Tbar quark-antiquark pair,  
so that the total mass of the triplet  $\{ W_+, W_-, W_0 \}$  at the electroweak unification  
is equal to the total mass of a T - Tbar pair, 259.031 GeV.

The triplet  $\{ W_+, W_-, Z_0 \}$  couples directly with the Higgs scalar,  
which carries the Higgs mechanism by which the  $W_0$  becomes the physical  $Z_0$ ,  
so that the total mass of the triplet  $\{ W_+, W_-, Z_0 \}$   
is equal to the vacuum expectation value  $v$  of the Higgs scalar field,  $v = 252.514$  GeV.

What are individual masses of members of the triplet  $\{ W_+, W_-, Z_0 \}$  ?

First, look at the triplet  $\{ W_+, W_-, W_0 \}$  which can be represented by the 3-sphere  $S^3$ .  
The Hopf fibration of  $S^3$  as

$$S^1 \rightarrow S^3 \rightarrow S^2$$

gives a decomposition of the W bosons into the neutral  $W_0$  corresponding to  $S^1$   
and the charged pair  $W_+$  and  $W_-$  corresponding to  $S^2$ .

The mass ratio of the sum of the masses of  $W_+$  and  $W_-$  to the mass of  $W_0$   
should be the volume ratio of the  $S^2$  in  $S^3$  to the  $S^1$  in  $S^3$ .

The unit sphere  $S^3$  in  $R^4$  is normalized by  $1/2$ .

The unit sphere  $S^2$  in  $R^3$  is normalized by  $1/\sqrt{3}$ .

The unit sphere  $S^1$  in  $R^2$  is normalized by  $1/\sqrt{2}$ .

The ratio of the sum of the  $W_+$  and  $W_-$  masses to the  $W_0$  mass should then be  
 $(2/\sqrt{3}) V(S^2) / (2/\sqrt{2}) V(S^1) = 1.632993$

Since the total mass of the triplet  $\{ W_+, W_-, W_0 \}$  is 259.031 GeV,  
the total mass of a T - Tbar pair, and the charged weak bosons have equal mass,  
we have

$$M_{W_+} = M_{W_-} = 80.326 \text{ GeV and } M_{W_0} = 98.379 \text{ GeV.}$$

The charged  $W_{\pm}$  neutrino-electron interchange must be symmetric  
with the electron-neutrino interchange, so that the tree-level absence  
of right-handed neutrino particles requires that

the charged  $W_{\pm}$  SU(2) weak bosons act only on left-handed electrons.

Each gauge boson must act consistently on the entire Dirac fermion particle sector,  
so that the  
charged  $W_{\pm}$  SU(2) weak bosons act only on left-handed fermion particles of all types.

The neutral  $W_0$  weak boson does not interchange Weyl neutrinos with Dirac fermions, and so is not restricted to left-handed fermions, but also has a component that acts on both types of fermions, both left-handed and right-handed, conserving parity.

However, the neutral  $W_0$  weak bosons are related to the charged  $W_{\pm}$  weak bosons by custodial  $SU(2)$  symmetry, so that the left-handed component of the neutral  $W_0$  must be equal to the left-handed (entire) component of the charged  $W_{\pm}$ .

Since the mass of the  $W_0$  is greater than the mass of the  $W_{\pm}$ , there remains for the  $W_0$  a component acting on both types of fermions.

Therefore the full  $W_0$  neutral weak boson interaction is proportional to  $(M_{W_{\pm}}^2 / M_{W_0}^2)$  acting on left-handed fermions and  $(1 - (M_{W_{\pm}}^2 / M_{W_0}^2))$  acting on both types of fermions.

If  $(1 - (M_{W_{\pm}}^2 / M_{W_0}^2))$  is defined to be  $\sin(\theta_w)^2$  and denoted by  $K$ , and if the strength of the  $W_{\pm}$  charged weak force (and of the custodial  $SU(2)$  symmetry) is denoted by  $T$ , then the  $W_0$  neutral weak interaction can be written as  $W_0L = T + K$  and  $W_0LR = K$ .

Since the  $W_0$  acts as  $W_0L$  with respect to the parity violating  $SU(2)$  weak force and as  $W_0LR$  with respect to the parity conserving  $U(1)$  electromagnetic force, the  $W_0$  mass  $m_{W_0}$  has two components: the parity violating  $SU(2)$  part  $m_{W_0L}$  that is equal to  $M_{W_{\pm}}$  and the parity conserving part  $M_{W_0LR}$  that acts like a heavy photon.

As  $M_{W_0} = 98.379 \text{ GeV} = M_{W_0L} + M_{W_0LR}$ , and as  $M_{W_0L} = M_{W_{\pm}} = 80.326 \text{ GeV}$ , we have  $M_{W_0LR} = 18.053 \text{ GeV}$ .

Denote by  $\alpha_E = e^2$  the force strength of the weak parity conserving  $U(1)$  electromagnetic type force that acts through the  $U(1)$  subgroup of  $SU(2)$ .

The electromagnetic force strength  $\alpha_E = e^2 = 1 / 137.03608$  was calculated above using the volume  $V(S^1)$  of an  $S^1$  in  $R^2$ , normalized by  $1 / \sqrt{2}$ .

The  $\alpha_E$  force is part of the  $SU(2)$  weak force whose strength  $\alpha_W = w^2$  was calculated above using the volume  $V(S^2)$  of an  $S^2 \subset R^3$ , normalized by  $1 / \sqrt{3}$ .

Also, the electromagnetic force strength  $\alpha_E = e^2$  was calculated above using a 4-dimensional spacetime with global structure of the 4-torus  $T^4$  made up of four  $S^1$  1-spheres, while the  $SU(2)$  weak force strength  $\alpha_W = w^2$  was calculated above using two 2-spheres  $S^2 \times S^2$ , each of which contains one 1-sphere of the  $\alpha_E$  force.

Therefore

$$\begin{aligned} *alphaE &= alphaE ( \sqrt{ 2 } / \sqrt{ 3 } )(2 / 4) = alphaE / \sqrt{ 6 } , \\ *e &= e / (4th \text{ root of } 6) = e / 1.565 , \end{aligned}$$

and

the mass mW0LR must be reduced to an effective value

$$M\_W0LReff = M\_W0LR / 1.565 = 18.053/1.565 = 11.536 \text{ GeV}$$

for the \*alphaE force to act like an electromagnetic force in the E8 model:

$$*e M\_W0LR = e (1/1.565) M\_W0LR = e M\_Z0,$$

where the physical effective neutral weak boson is denoted by Z0.

Therefore, the correct Cl(16)-E8 model values for weak boson masses and the Weinberg angle theta\_w are:

$$M\_W+ = M\_W- = 80.326 \text{ GeV};$$

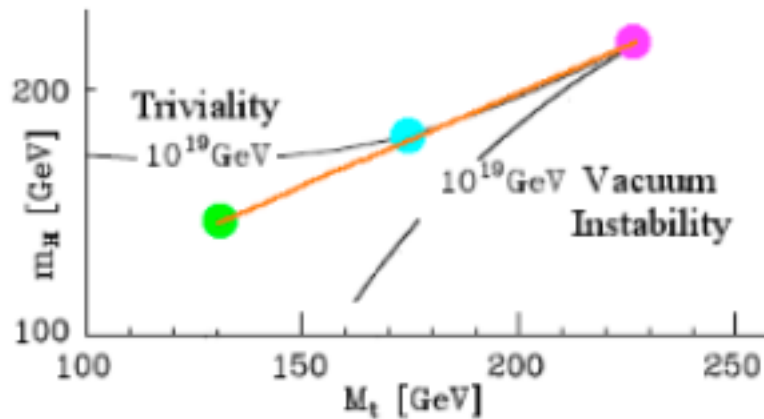
$$M\_Z0 = 80.326 + 11.536 = 91.862 \text{ GeV};$$

$$\text{Sin}(\text{theta}_w)^2 = 1 - (M\_W+/- / M\_Z0)^2 = 1 - ( 6452.2663 / 8438.6270 ) = 0.235.$$


Radiative corrections are not taken into account here, and may change these tree- level values somewhat.

## 17. Higgs - Truth Quark Condensate System with 3 Mass States


The Cl(16)-E8 model identifies the Higgs with Primitive Idempotents of the Cl(8) real Clifford algebra, whereby the Higgs is not seen as a simple-minded single fundamental scalar particle, but rather the Higgs is seen as a quantum process that creates a fermionic condensate and effectively a 3-state Higgs-Tquark System.




( from hep-ph/0307138 )

The Magenta Dot  is the high-mass state of a 220 GeV Truth Quark and a 240 GeV Higgs. It is at the critical point of the Higgs-Tquark System with respect to Vacuum Instability and Triviality. It corresponds to the description in hep-ph/9603293 by Koichi Yamawaki of the Bardeen-Hill-Lindner model. That high-mass Higgs is around 250 GeV in the range of the Higgs Vacuum Instability Boundary which range includes the Higgs VEV.

The Gold Line leading down from the Critical Point roughly along the Triviality Boundary line is based on Renormalization Group calculations with the result that  $M_H / M_T = 1.1$  as described by Koichi Yamawaki in hep-ph/9603293 .

The Cyan Dot  where the Gold Line leaves the Triviality Boundary to go into our Ordinary Phase is the middle-mass state of a 174 GeV Truth Quark and Higgs around 200 GeV. It corresponds to the Higgs mass calculated by Hashimoto, Tanabashi, and Yamawaki in hep-ph/0311165 where they show that for 8-dimensional Kaluza-Klein spacetime with the Higgs as a Truth Quark condensate  $172 < M_T < 175$  GeV and  $178 < M_H < 188$  GeV.

That mid-mass Higgs is around the 200 GeV range of the Higgs Triviality Boundary at which the composite nature of the Higgs as T-Tbar condensate in (4+4)-dim Kaluza-Klein becomes manifest.

The Green Dot  where the Gold Line terminates in our Ordinary Phase is the low-mass state of a 130 GeV Truth Quark and a 126 GeV Higgs.

The conventional Standard Model has structure:

spacetime is a base manifold  
 particles are representations of gauge groups  
 gauge bosons are in the adjoint representation  
 fermions are in other representations (analogous to spinor)  
 Higgs boson is in scalar representation

The Cl(16)-E8 model has structure

(from 248-dim E8 = 120-dim adjoint D8 + 128-dim half-spinor D8):

spacetime is in the adjoint D8 part of E8 (64 of 120 D8 adjoints)  
 gauge bosons are in the adjoint D8 part of E8 (28+28 = 56 of the 120 D8 adjoints)  
 fermions are in the half-spinor D8 part of E8 (64+64 of the 128 D8 half-spinors).

There is no room for a fundamental Higgs directly appearing in the E8, rather, it emerges from the Mayer-Trautman Mechanism with formation of Quaternionic (4+4)-dim M4 x CP2 Kaluza-Klein SpaceTime. To see how that Higgs works in terms of the Cl(16) = Cl(8)xCl(8) Clifford Algebra, embed 248-dim E8 into the 256-dim real Clifford algebra Cl(8):

Cl(8)	256 =	1 + 8 + 28 + 56 + 70 + 56 + 28 + 8 + 1	
Primitive	16 = 1	+ 6	+ 1
Idempotent		+ 8	
E8 Root Vectors	240 =	8 + 28 + 56 + 56 + 56 + 28 + 8	
E8	248 =	8 + 28 + 56 + 64 + 56 + 28 + 8	

The Cl(8) Primitive Idempotent is 16-dimensional and can be decomposed into two 8-dimensional half-spinor parts each of which is related by Triality to 8-dimensional spacetime and has Octonionic structure.

In that decomposition: the 1+6+1 = (1+3)+(3+1) is related to two copies of a 4-dimensional Associative Quaternionic subspace of the Octonionic structure and

the 8 = 4+4 is related to two copies of a 4-dimensional Co-Associative subspace of the Octonionic structure

( see the book "Spinors and Calibrations" by F. Reese Harvey )

The 8 = 4+4 Co-Associative part of the Cl(8) Primitive Idempotent when combined with the 240 E8 Root Vectors forms the full 248-dimensional E8. It represents a Cartan subalgebra of the E8 Lie algebra.

**The (1+3)+(3+1) Associative part of the Cl(8) Primitive Idempotent corresponds to the Higgs of the Cl(16)-E8 model.**

The half-spinors generated by the Higgs part of the  $Cl(8)$  Primitive Idempotent represent  
neutrino; red, green, blue down quarks; red, green, blue up quarks; electron

so the E8 Higgs effectively creates/annihilates the fundamental fermions  
and

**the E8 Higgs is effectively a condensate of fundamental fermions.**

In the  $Cl(16)$ -E8 model the high-energy 8-dimensional Octonionic spacetime reduces,  
by freezing out a preferred 4-dim Associative Quaternionic subspace,  
to a 4+4 -dimensional Batakis Kaluza-Klein of the form  $M4 \times CP2$   
with 4-dim  $M4$  physical spacetime.

The  $(1+3)+(3+1)$  part of the  $Cl(8)$  Primitive Idempotent includes

the 1 of  $Cl(8)$  grade-0 scalar ( that determines  $M4$  transformation properties )  
and  $3+3 = 6$  of the  $Cl(8)$  grade-4  
and the 1 of  $Cl(8)$  grade-8

so the  $Cl(16)$ -E8 model Higgs transforms as a scalar  
with respect to 4-dim  $M4$  Physical SpaceTime  
and is consistent with LHC observations ( see arXiv 1307.1432 ).

Not only does the  $Cl(16)$ -E8 model Higgs fermion condensate transform  
with respect to 4-dim physical spacetime like the Standard Model Higgs  
but  
the geometry of the reduction from 8-dim Octonionic spacetime  
to (4+4)-dimensional Batakis Kaluza-Klein,  
by the Mayer-Trautman Mechanism,  
gives the  $Cl(16)$ -E8 Higgs ElectroWeak Symmetry-Breaking Ginzburg-Landau structure.

Since the second and third fermion generations emerge dynamically  
from the reduction from 8-dim to 4+4 -dim Kaluza-Klein,  
they are also created/annihilated by the Primitive Idempotent  $Cl(16)$ -E8 Higgs  
and are present in the fermion condensate.

**Since the Truth Quark is so much more massive than the other fermions,  
the  $Cl(16)$ -E8 model Higgs is effectively a Truth Quark condensate.**

When Triviality and Vacuum Stability are taken into account,  
the  $Cl(16)$ -E8 model Higgs and Truth Quark system has 3 mass states.

As to composite Higgs and the Triviality boundary, Pierre Ramond says in his book *Journeys Beyond the Standard Model* ( Perseus Books 1999 ) at pages 175-176:

"... The Higgs quartic coupling has a complicated scale dependence. It evolves according to  $d\lambda/dt = (1/16\pi^2)\beta_\lambda$

where the one loop contribution is given by

$$\beta_\lambda = 12\lambda^2 - \dots - 4H^2$$

The value of  $\lambda$  at low energies is related [to] the physical value of the Higgs mass according to the tree level formula

$$m_H = v\sqrt{2\lambda}$$

while the vacuum value is determined by the Fermi constant

...

for a fixed vacuum value  $v$ , let us assume that the Higgs mass and therefore  $\lambda$  is large. In that case,  $\beta_\lambda$  is dominated by the  $\lambda^2$  term, which drives the coupling towards its Landau pole at higher energies.

Hence the higher the Higgs mass, the higher  $\lambda$  is and the closer the Landau pole to experimentally accessible regions.

This means that for a given (large) Higgs mass,

we expect the standard model to enter a strong coupling regime

at relatively low energies, losing in the process our ability to calculate.

This does not necessarily mean that the theory is incomplete,

only that we can no longer handle it ...

it is natural to think that this effect is caused by new strong interactions,

and that the Higgs actually is a composite ...

The resulting bound on  $\lambda$  is sometimes called the triviality bound.

The reason for this unfortunate name (the theory is anything but trivial)

stems from lattice studies where the coupling is assumed to be finite everywhere;

in that case the coupling is driven to zero, yielding in fact a trivial theory.

In the standard model  $\lambda$  is certainly not zero. ...".

Composite Higgs as Tquark condensate studies by Yamawaki et al have produced realistic models that are consistent with the CI(16)-E8 model with a 3-State System:

1 - The basic CI(16)-E8 model state

with **Tquark mass = 130 GeV and Higgs mass = 126 GeV**

2 - Triviality boundary 8-dim Kaluza-Klein state described by Hashimoto,

Tanabashi, and Yamawaki in hep-ph/0311165 where they say:

"... We perform the most attractive channel (MAC) analysis in the top mode standard model with TeV-scale extra dimensions, where the standard model gauge bosons and the third generation of quarks and leptons are put in  $D=(6,8,10,\dots)$  dimensions. In such a model, bulk gauge couplings rapidly grow in the ultraviolet region. In order to make the scenario viable, only the attractive force of the top condensate should exceed the critical coupling, while other channels such as the bottom and tau condensates should not. We then find that the top condensate can be the MAC for  $D=8$  ... We predict masses of the top ( $m_t$ ) and the Higgs ( $m_H$ ) ...



based on the renormalization group for the top Yukawa and Higgs quartic couplings with the compositeness conditions at the scale where the bulk top condenses ... for ... [ Kaluza-Klein type ] ... dimension...  $D=8$  ...  
 **$m_t = 172-175$  GeV and  $m_H=176-188$  GeV ...".**

3 - Critical point BHL state

with **Top quark mass =  $218 \pm 3$  GeV and Higgs mass =  $239 \pm 3$  GeV**

As Yamawaki said in hep-ph/9603293: "... the BHL formulation of the top quark condensate ... is based on the RG equation combined with the compositeness condition ... start[s] with the SM Lagrangian which includes explicit Higgs field at the Lagrangian level ... BHL is crucially based on the perturbative picture ... [which] ... breaks down at high energy near the compositeness scale  $\Lambda$  ... [  $10^{19}$  GeV ] ... there must be a certain matching scale  $\Lambda_{\text{Matching}}$  such that the perturbative picture (BHL) is valid for  $\mu < \Lambda_{\text{Matching}}$ , while only the nonperturbative picture (MTY) becomes consistent for  $\mu > \Lambda_{\text{Matching}}$  ... However, thanks to the presence of a quasi-infrared fixed point, BHL prediction is numerically quite stable against ambiguity at high energy region, namely, rather independent of whether this high energy region is replaced by MTY or something else. ... Then we expect  $m_t = m_t(\text{BHL}) = \dots = 1/(\sqrt{2}) y_{\text{bar}} v$  within 1-2%, where  $y_{\text{bar}}$  is the quasi-infrared fixed point given by  $\text{Beta}(y_{\text{bar}}) = 0$  in ... the one-loop RG equation ...

The composite Higgs loop changes  $y_{\text{bar}}^2$  by roughly the factor  $N_c/(N_c + 3/2) = 2/3$  compared with the MTY value, i.e.,  $250 \text{ GeV} \rightarrow 250 \times \sqrt{2/3} = 204 \text{ GeV}$ , while the electroweak gauge boson loop with opposite sign pulls it back a little bit to a higher value. The BHL value is then given by  $m_t = 218 \pm 3 \text{ GeV}$ , at  $\Lambda = 10^{19} \text{ GeV}$ .

The Higgs boson was predicted as a  $t\text{-}\bar{t}$  bound state

with a mass  $M_H = 2m_t$  based on the pure NJL model calculation.

Its mass was also calculated by BHL through the full RG equation ...

the result being ...  $M_H / m_t = 1.1$  ) at  $\Lambda = 10^{19} \text{ GeV}$  ...

... the top quark condensate proposed by Miransky, Tanabashi and Yamawaki (MTY) and by Nambu independently ... entirely replaces the standard Higgs doublet by a composite one formed by a strongly coupled short range dynamics (four-fermion interaction) which triggers the top quark condensate.

The Higgs boson emerges as a  $t\text{-}\bar{t}$  bound state and hence is deeply connected with the top quark itself. ... MTY introduced explicit four-fermion interactions responsible for the top quark condensate in addition to the standard gauge couplings. Based on the explicit solution of the ladder SD equation, MTY found that even if all the dimensionless four-fermion couplings are of  $O(1)$ , only the coupling larger than the critical coupling yields non-zero (large) mass ... The model was further formulated in an elegant fashion by Bardeen, Hill and Lindner (BHL) in the SM language, based on the RG equation and the compositeness condition. BHL essentially incorporates  $1/N_c$  sub-leading effects such as those of the composite Higgs loops and ... gauge boson loops which were disregarded by the MTY formulation. We can explicitly see that BHL is in fact equivalent to MTY at  $1/N_c$ -leading order. Such effects turned out to reduce the above MTY value  $250 \text{ GeV}$  down to  $220 \text{ GeV}$  ...".

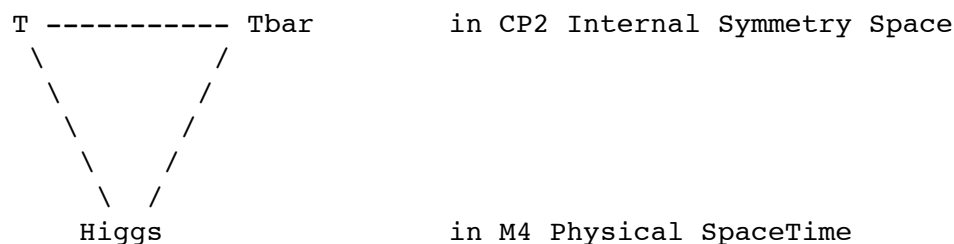
**8-dim Kaluza-Klein spacetime physics as required by Hashimoto, Tanabashi, and Yamawaki for the Middle State of the 3-State System was described by N. A. Batakis in Class. Quantum Grav. 3 (1986) L99-L105 in terms a M4xCP2 structure similar to that of the Cl(16)-E8 model. Although spacetime and Standard Model gauge bosons worked well for Batakis, he became discouraged by difficulties with fermions, perhaps because he did not use Clifford Algebras with natural spinor structures for fermions.**

Calculations of the Low-Mass State of Higgs and Truth Quark have been given in Chapters 10 and 16 of this paper. Here are similar details for Middle and High Mass:

### **Middle Mass State:**

In the Cl(16)-E8 model, the Middle-Mass Higgs has structure that is not restricted to Effective M4 Spacetime as is the case with the Low-Mass Higgs Ground State

but extends to the full  $4+4 = 8$ -dim structure of M4xCP2 Kaluza-Klein.



Therefore the Mid-Mass Higgs looks like a 3-particle system of Higgs + T + Tbar.

The T and Tbar form a Pion-like state.

Since Tquark Mid-Mass State is 174 GeV

the Middle-Mass T-Tbar that lives in the CP2 part of (4+4)-dim Kaluza-Klein has mass  $(174+174) \times (135 / (312+312)) = 75$  GeV.

The Higgs that lives in the M4 part of (4+4)-dim Kaluza-Klein

has, by itself, its Low-Mass Ground State Effective Mass of 125 GeV.

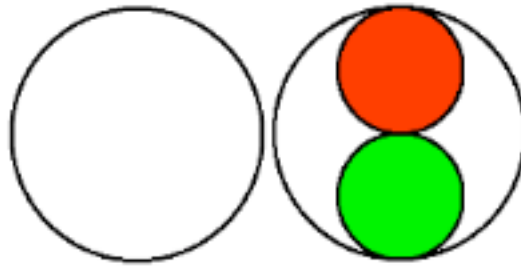
So, the total Mid-Mass Higgs lives in full 8-dim Kaluza-Klein with mass  $75+125 = 200$  GeV.

This is consistent with the Mid-Mass States of the Higgs and Tquark

being on the Triviality Boundary of the Higgs - Tquark System

and with the 8-dim Kaluza-Klein model in hep-ph/0311165 by Hashimoto, Tanabashi, and Yamawaki.

As to the cross-section of the Middle-Mass Higgs



consider that the entire Ground State cross-section lives only in 4-dim M4 spacetime (left white circle)

while the Middle-Mass Higgs cross-section lives in full 4+4 = 8-dim Kaluza-Klein (right circle with red area only in CP2 ISS and white area partly in CP2 ISS with only green area effectively living in 4-dim M4 spacetime)

so that

our 4-dim M4 Physical Spacetime experiments only see for the Middle-Mass Higgs a cross-section that is 25% of the full Ground State cross-section.

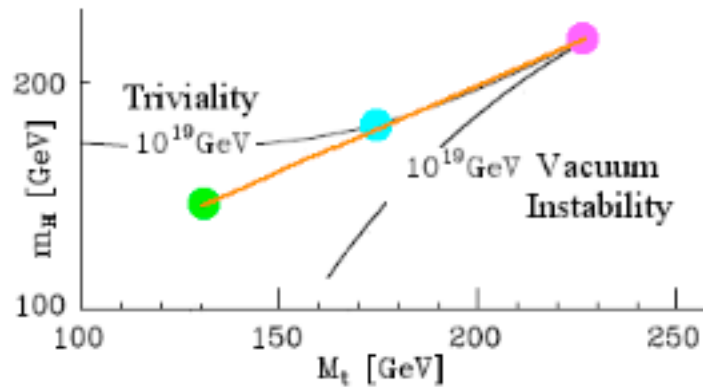
The 25% may also be visualized in terms of 8-dim coordinates  $\{1, i, j, k, E, I, J, K\}$

	1	i	j	k	E	I	J	K
1	11	1i	1j	1k	1E	1I	1J	1K
i	i1	ii	ij	ik	iE	iI	iJ	iK
j	j1	ji	jj	jk	jE	jI	jJ	jK
k	k1	ki	kj	kk	kE	kI	kJ	kK
E	E1	Ei	Ej	Ek	EE	EI	EJ	EK
I	I1	Ii	Ij	Ik	IE	II	IJ	IK
J	J1	Ji	Jj	Jk	JE	JI	JJ	JK
K	K1	Ki	Kj	Kk	KE	KI	KJ	KK

in which  $\{1, i, j, k\}$  represent M4 and  $\{E, I, J, K\}$  represent CP2.

## High Mass State:

In the CI(16)-E8 model, the the High-Mass Higgs State is at the Critical Point of the Higgs-Tquark System



( from hep-ph/0307138 )

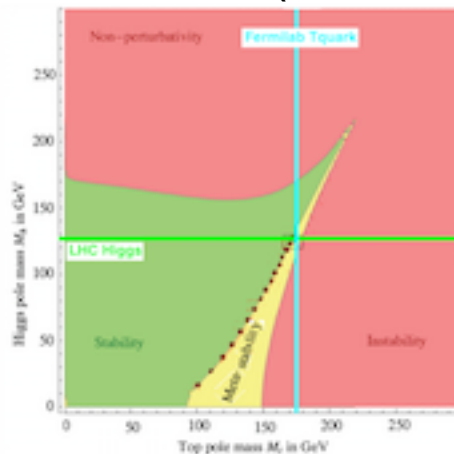
where the Triviality Boundary intersects the Vacuum Instability Boundary which is also at the Higgs Vacuum Expectation Value VEV around 250 GeV.

As with the Middle-Mass Higgs,

the High-Mass Higgs lives in all  $4+4 = 8$  Kaluza-Klein dimensions

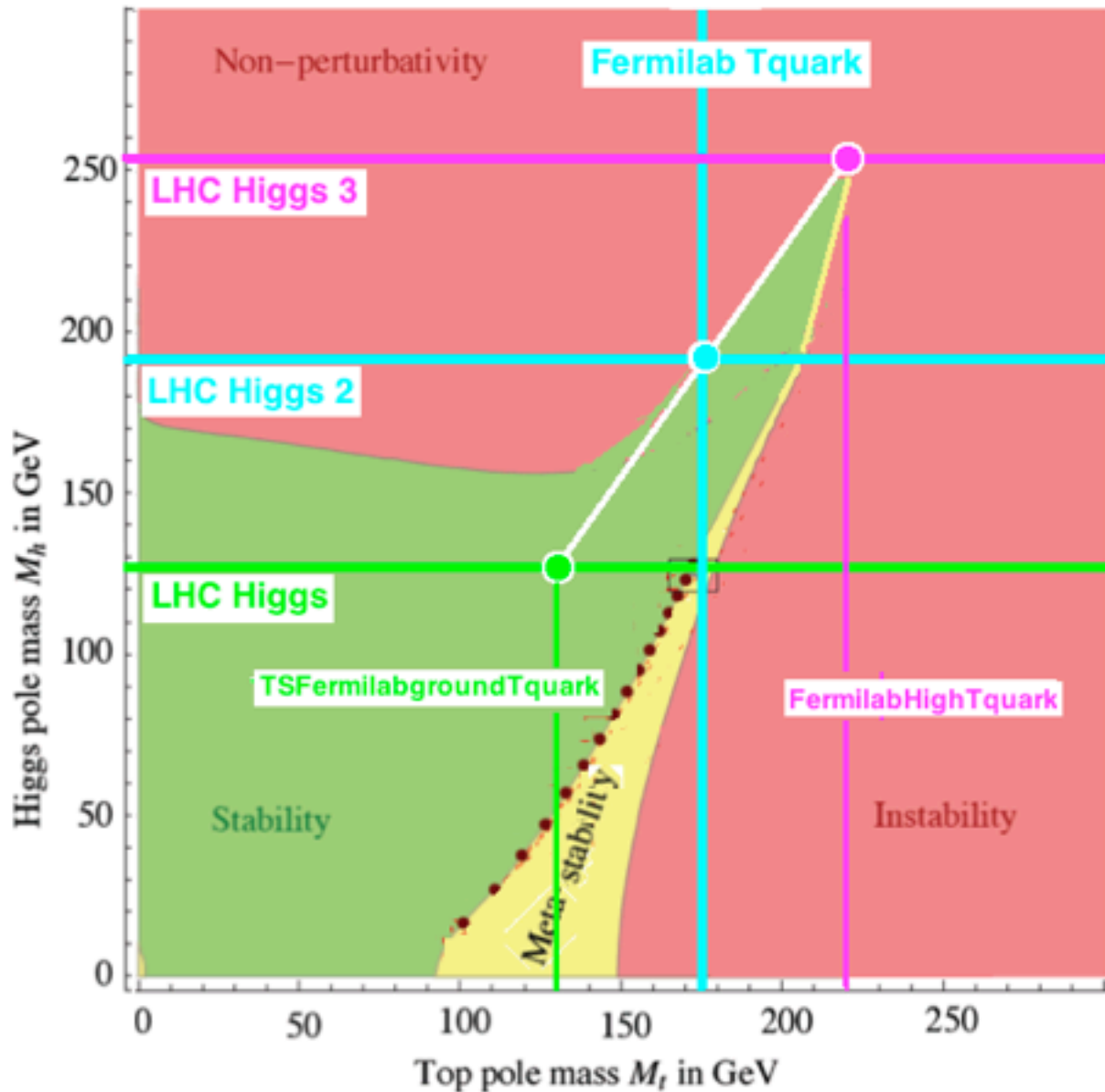
and so has a cross-section that is about 25% of the Higgs Ground State cross-section.

**The CI(16)-E8 model view is 3 Mass States for Higgs and Truth Quark. Opposed to the CI(16)-E8 view is the Fermilab / CERN / Establishment view that there is only one Higgs Mass State ( Low Mass around 126 GeV ) and only one Truth Quark Mass State (Middle Mass around 174 GeV ).**



Their view is represented in the above  $M_H$ - $M_t$  diagram adapted from arXiv 1307.3536 by Buttazzo, Degrandi, Giardino, Giudice, Sala, Salvio, and Strumia who say "... from data ... of the Higgs ... and the ... [Tquark] Yukawa coupling ... we extrapolate ... SM parameters up to large energies ... Then we study the phase diagram of the Standard Model in term of high-energy parameters, finding that the measured Higgs mass roughly corresponds to ... vacuum metastability ... the SM Higgs vacuum is not the true vacuum ... our universe is potentially unstable ...".

**The CI(16)-E8 model has no vacuum metastability problem**  
because it has 3 sets of Higgs-Tquark mass states which modify the phase diagram



so that the Low-Mass Ground State is in the region of Stability and the Middle-Mass State is at the boundary of Non-Perturbativity and the High-Mass State at the Critical Point has Higgs mass = Higgs VEV.

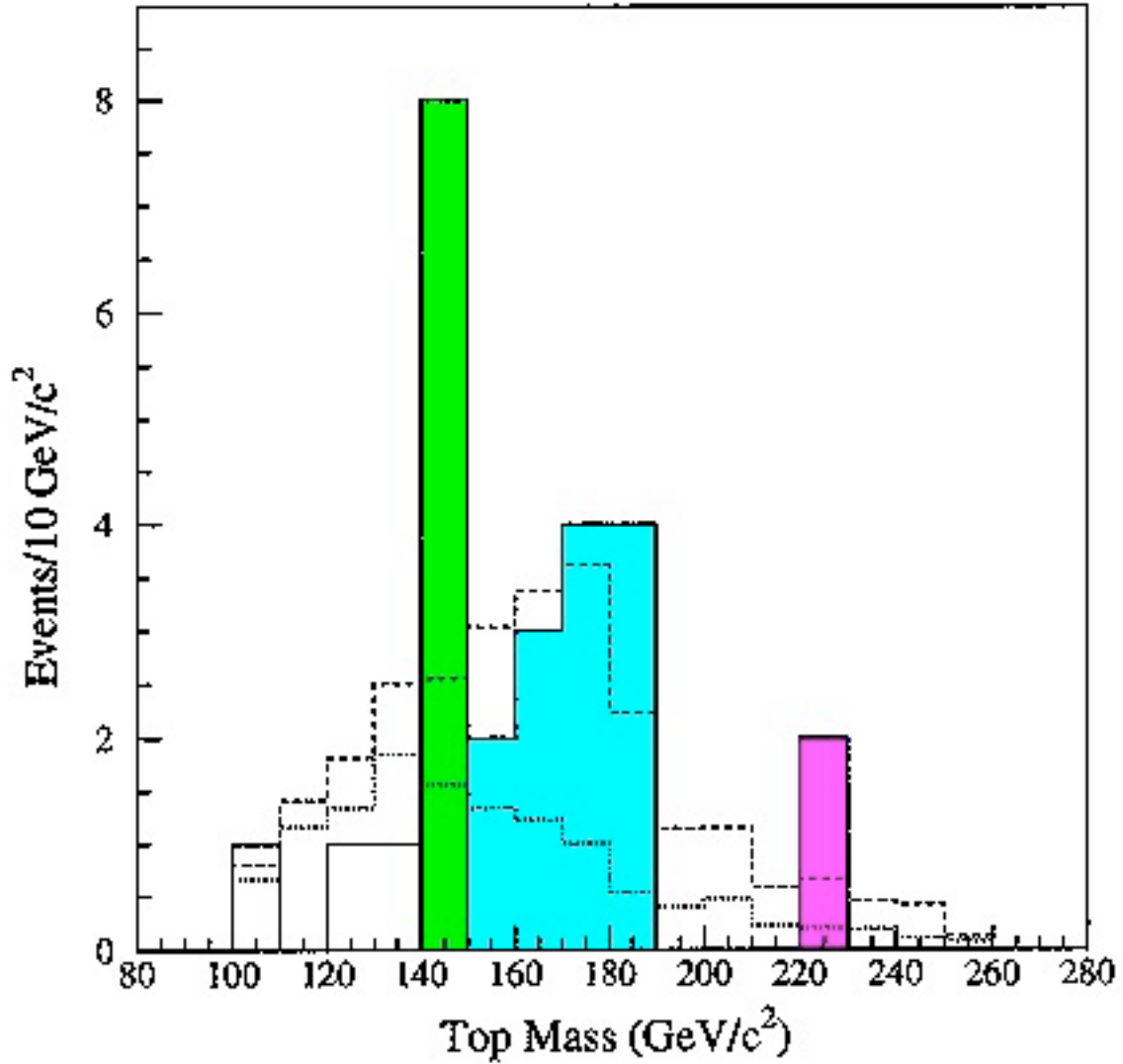
The two additional Tquark mass states and Higgs mass states are not recognized by the Fermilab/CERN/Establishment.

The two Tquark states (TSFermilabgroundTquark and FermilabHighTquark) have been seen at Fermilab and

the LHC has seen indications of the Two Higgs states (LHC Higgs 2 and LHC Higgs 3) whose status should be clarified by the 2015-2016 LHC Run.

Here are details of those additional Fermilab and LHC states:

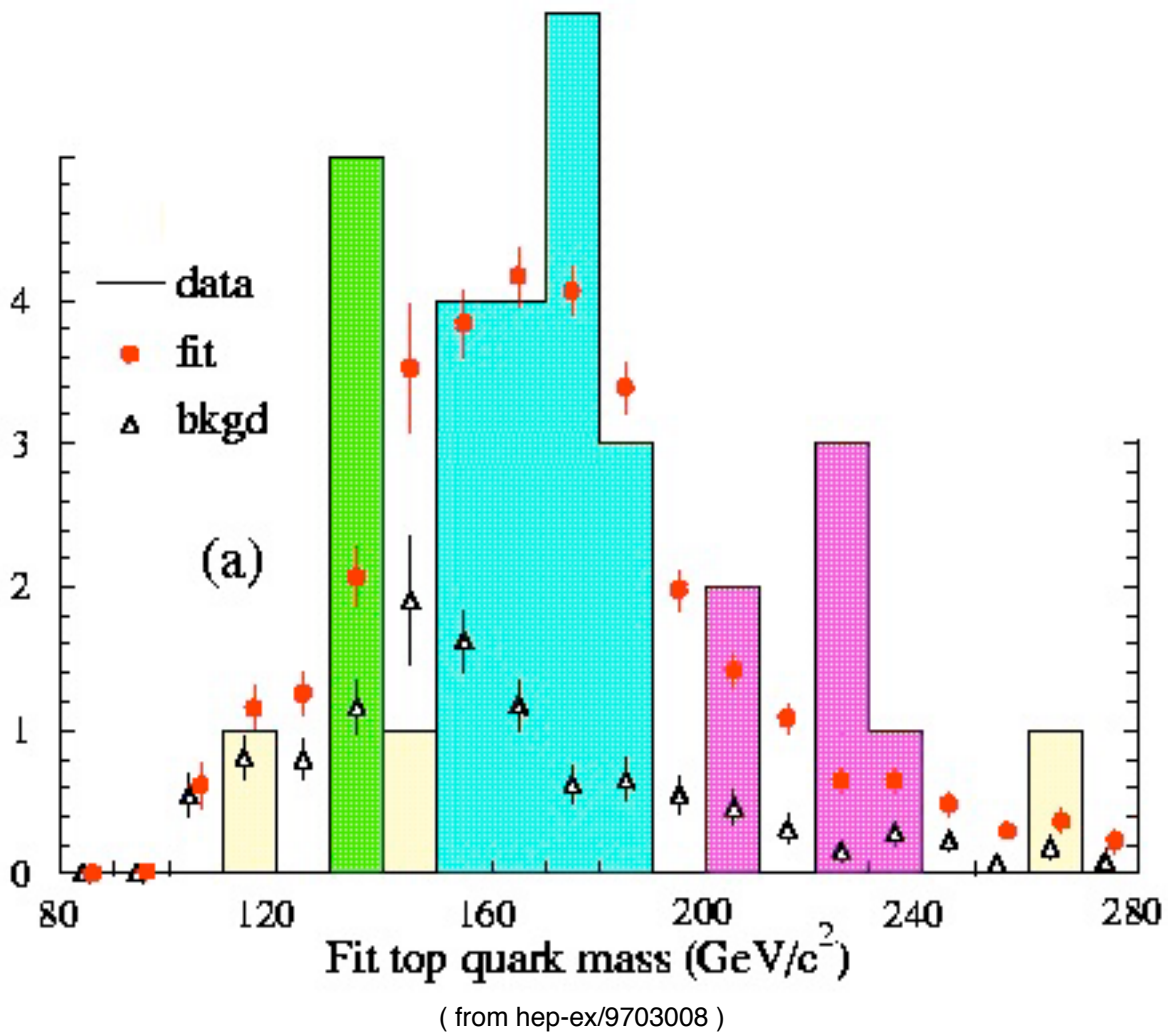
In 1994 a semileptonic histogram from CDF



( from FERMILAB-PUB-94/097-E )

seems to me to show all three states of the T-quark.

In 1997 a semileptonic histogram from D0



also seems to me to show all three states of the T-quark.

The fact that the low (green) state showed up in both independent detectors indicates

a significance of 4 sigma.

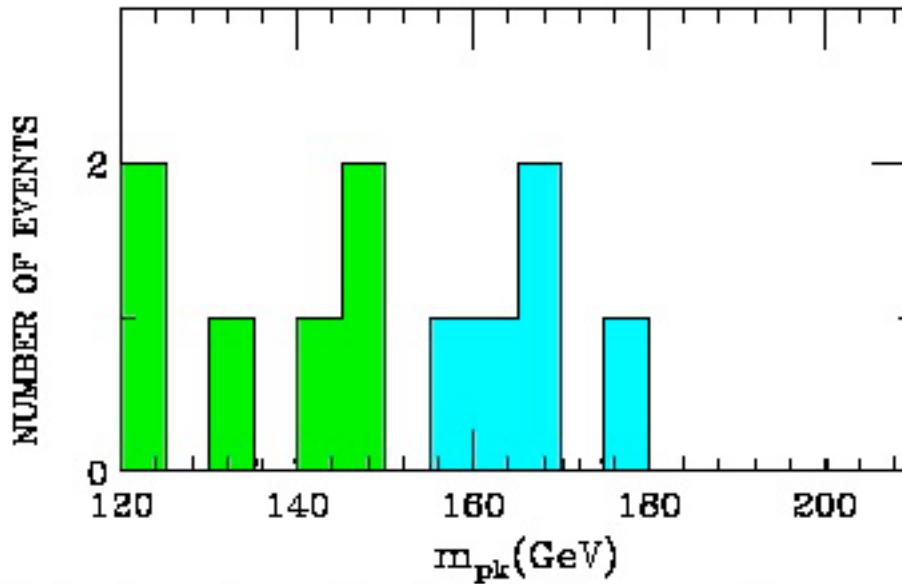
Some object that the low (green) state peak should be as wide as the peak for the middle (cyan) state,

but

my opinion is that the middle (cyan) state should be wide because it is on the Triviality boundary where the composite nature of the Higgs as T-Tbar condensate becomes manifest and

the low (cyan) state should be narrow because it is in the usual non-trivial region where the T-quark acts more nearly as a single individual particle.

In 1998 a dilepton histogram from CDF



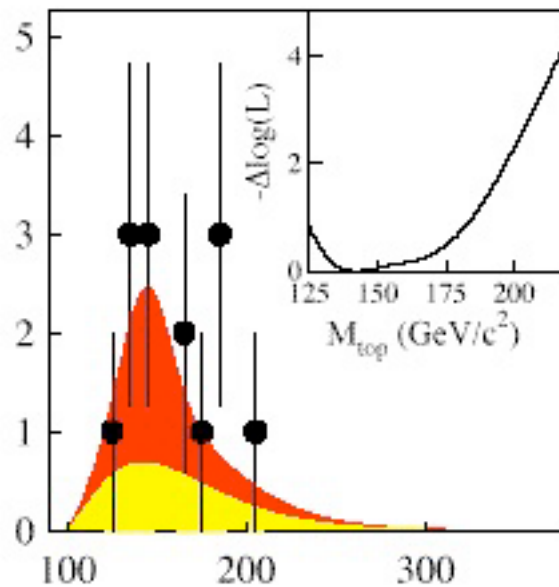
The distribution of  $m_{p\ell}$  values determined from 11 CDF dilepton events available empirically.

( from hep-ex/9802017 )

seems to me to show both the low (green) state and the middle (cyan) T-quark state.

In 1998 an analysis of 14 SLT tagged lepton + 4 jet events by CDF

### SLT Tagged



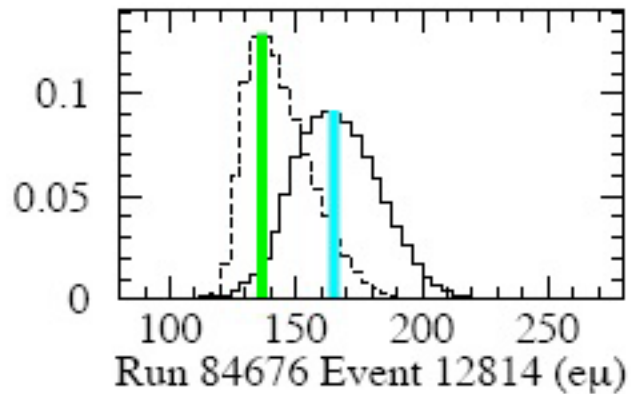
( from hep-ex/9801014 )

showed a T-quark mass of 142 GeV (+33,-14) that seems to me to be consistent with the low (green) state of the T-quark.



In 1997 the Ph.D. thesis of Erich Ward Varnes (Varnes-fermilab-thesis-1997-28) at page 159 said:

"... distributions for the dilepton candidates. For events with more than two jets, the dashed curves show the results of considering only the two highest ET jets in the reconstruction ...



..." (colored bars added by me)

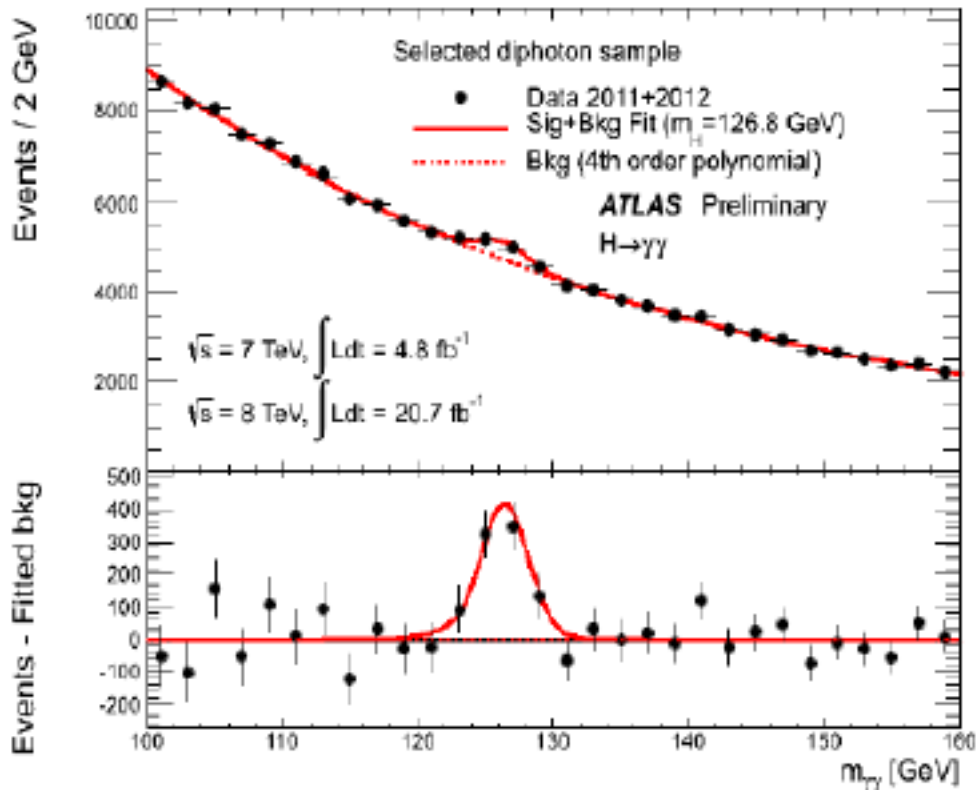
The event for all 3 jets (solid curve) seems to me to correspond to decay of a middle (cyan) T-quark state with one of the 3 jets corresponding to decay from the Triviality boundary down to the low (green) T-quark state, whose immediately subsequent decay is corresponds to the 2-jet (dashed curve) event at the low (green) energy level.

After 1998 Fermilab and CERN have focussed attention on detailed analysis of the middle (cyan) T-quark state, getting much valuable detailed information about it but not producing much information about the low or high Tquark states.

In the 25/fb of data collected through the run ending with the long shutdown at the end of 2012, the LHC has observed a 126 GeV state of the Standard Model Higgs boson.

Here are some details about the LHC observation at 126 GeV and related results shown at Moriond 2013:

The digamma histogram for ATLAS

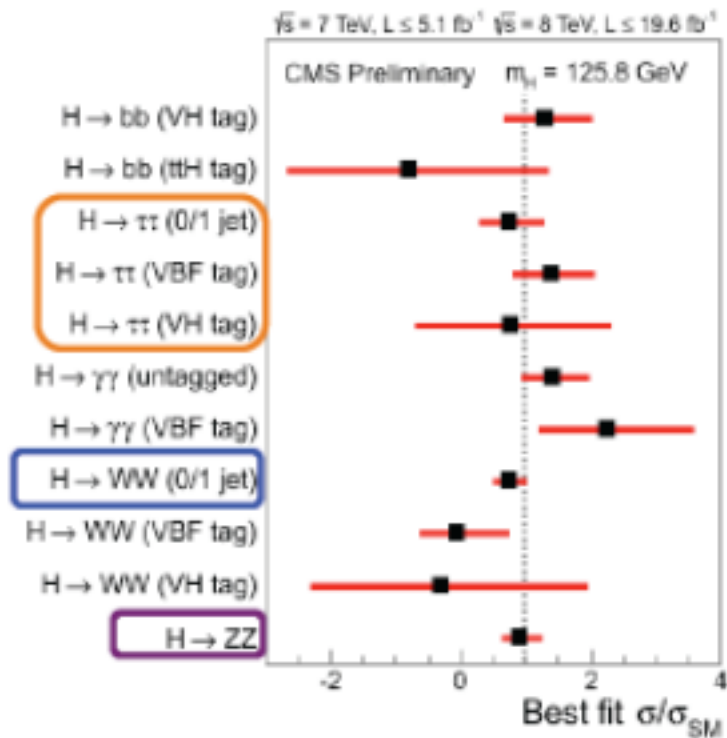


Simple topology: two high- $E_T$  ( $>40,30$  GeV) isolated photons

142681 events in  $100 < m_{\gamma\gamma} [\text{GeV}] < 160$

shows only one peak below 160 GeV and it is around 126 GeV.

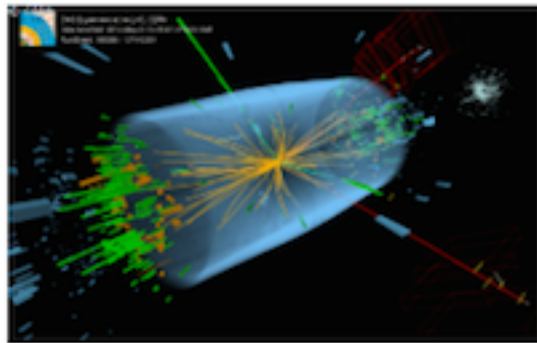
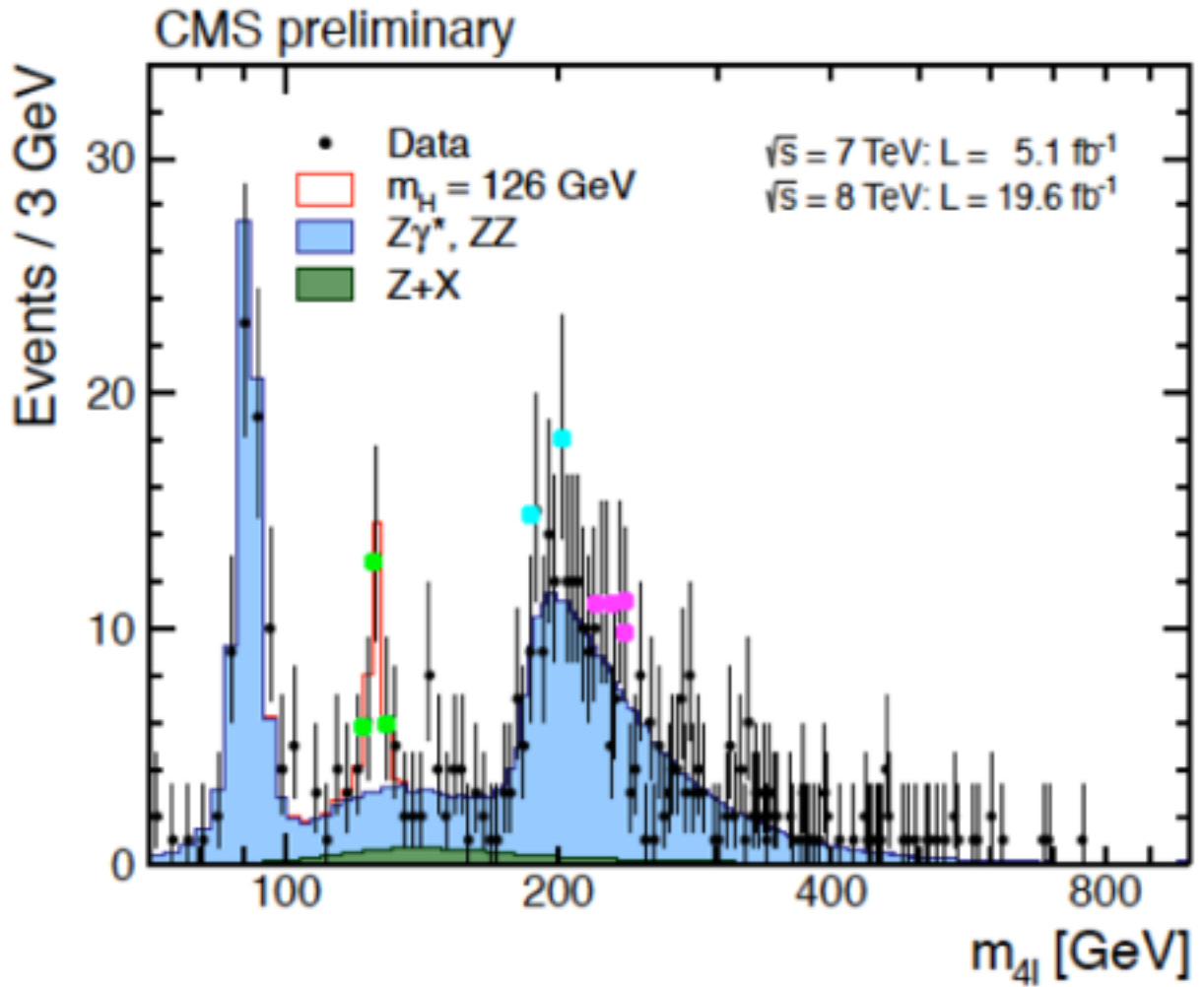
CMS shows the cross sections for Higgs at 125.8 GeV



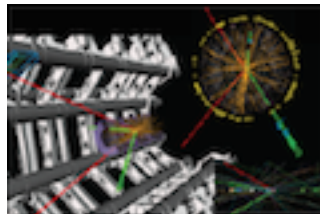
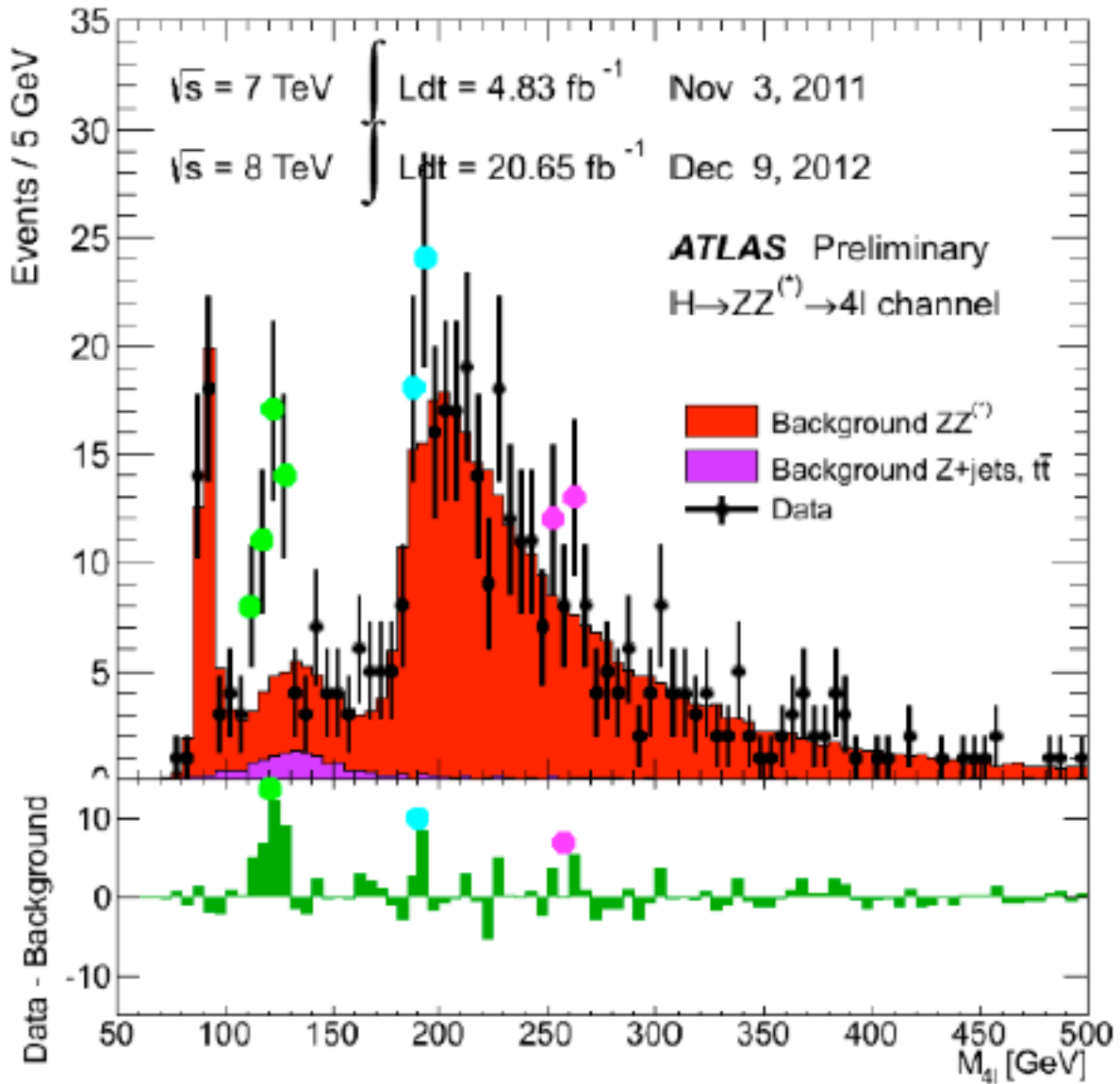
$H \rightarrow ZZ(0/1 \text{ jet}) : 0.84^{+0.32}_{-0.26}$   
 $H \rightarrow ZZ(\text{dijet tag}): 1.22^{+0.84}_{-0.57}$

to be substantially consistent with the Standard Model for the WW and ZZ channels, a bit low for tau-tau and bb channels (but that is likely due to very low statistics there), and a bit high for the digamma channel (but that may be due to phenomena related to the Higgs as a Tquark condensate).

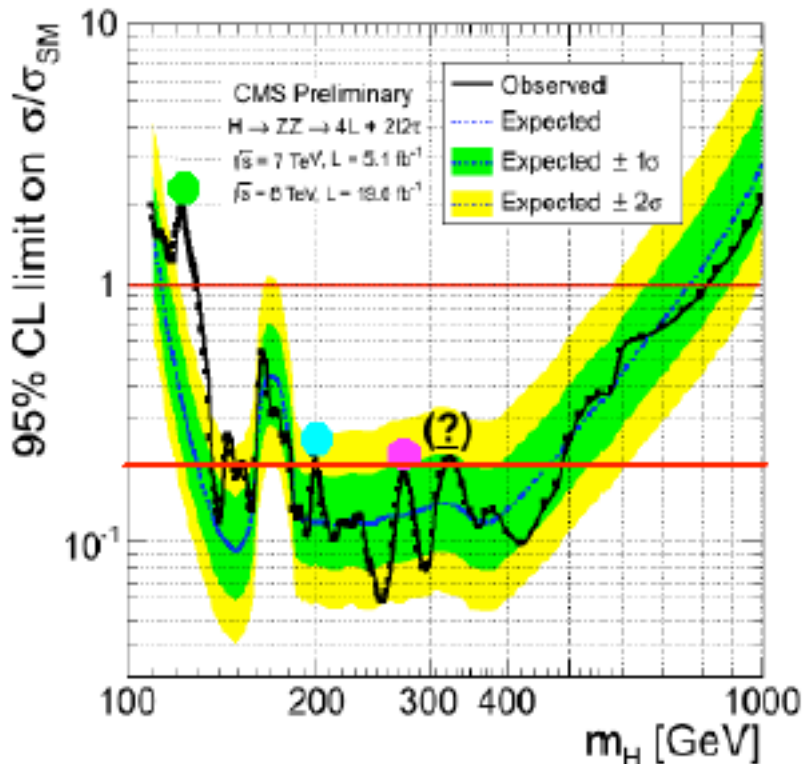
A CMS histogram (some colors added by me) for the Golden Channel Higgs to ZZ to 4l shows the peak around 126 GeV (green dots - lowHiggs mass state). The CMS histogram also indicates other excesses around 200 GeV (cyan dots - midHiggs mass state) and around 250 GeV (magenta dots - highHiggs mass state). An image of one of the events is shown below the histogram.



An ATLAS ZZ to 4l histogram (some colors added by me) show the peak around 126 GeV (green dots - low Higgs mass state). The ATLAS histogram also indicates other excesses around 200 GeV (cyan dots - middle Higgs mass state) and around 250 GeV (magenta dots - high Higgs mass state). An image of one of the events is shown below the histogram.



CMS showed a Brazil Band Plot for the High Mass Higgs to ZZ to 4l/2l2tau channel where the top red line represents the expected cross section of a single Standard Model Higgs and the lower red line represents about 20% of the expected Higgs SM cross section.



The green dot peak is at the 126 GeV Low Mass Higgs state with expected Standard Model cross section.

The cyan dot peak is around the 200 GeV Mid Mass Higgs state expected to have about 25% of the SM cross section.

The magenta dot peak is around the 250 (+/- 20 or so) GeV High Mass Higgs state expected to have about 25% of the SM cross section.

The (?) peak is around 320 GeV where I would not expect a Higgs Mass state and I note that in fact it seems to have gone away in the full ATLAS ZZ to 4l histogram shown above because between 300 and 350 GeV the two sort-of-high excess bins are adjacent to deficient bins .

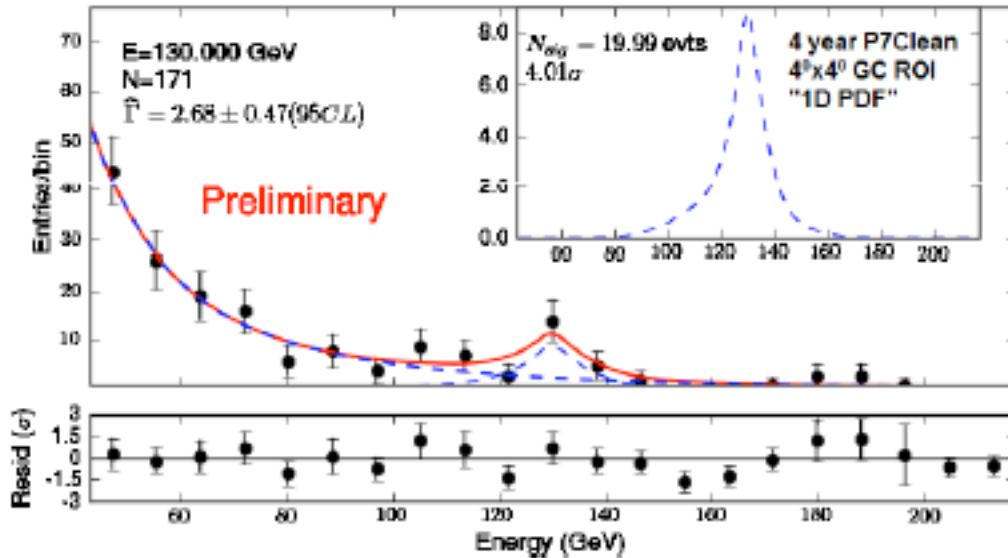
It will probably be no earlier than 2015 (after the long shutdown) that the LHC will produce substantially more data than the 25/fb available at Moriond 2013 and therefore no earlier than 2016 for the green and yellow Brazil Bands to be pushed down (throughout the 170 GeV to 500 GeV region) below 10 per cent (the  $10^{-1}$  line) of the SM cross section as is needed to show whether or not the cyan dot, magenta dot, and/or (?) peaks are real or statistical fluctuations.

My guess (based on the Cl(16)-E8 model) is that the cyan dot and magenta dot peaks will prove to be real and that the (?) peak will go away as a statistical fluctuation.

## Sgr A\* and Higgs = Tquark-Tantiquark Condensate

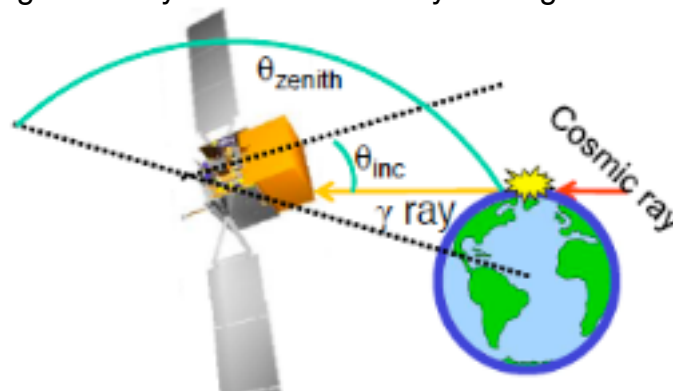
Sagittarius A\* (Sgr A\*) is a very massive black hole in the center of our Galaxy into which large amounts of Hydrogen fall. As the Hydrogen approaches Sgr A\* it increases in energy, ionizing into protons and electrons, and eventually producing a fairly dense cloud of infalling energetic protons whose collisions with ambient protons are at energies similar to the proton-proton collisions at the LHC.

Andrea Albert at The Fermi Symposium 11/2/2012 said: "... gamma rays detectable by the Fermi Large Area Telescope [ FLAT ] ...



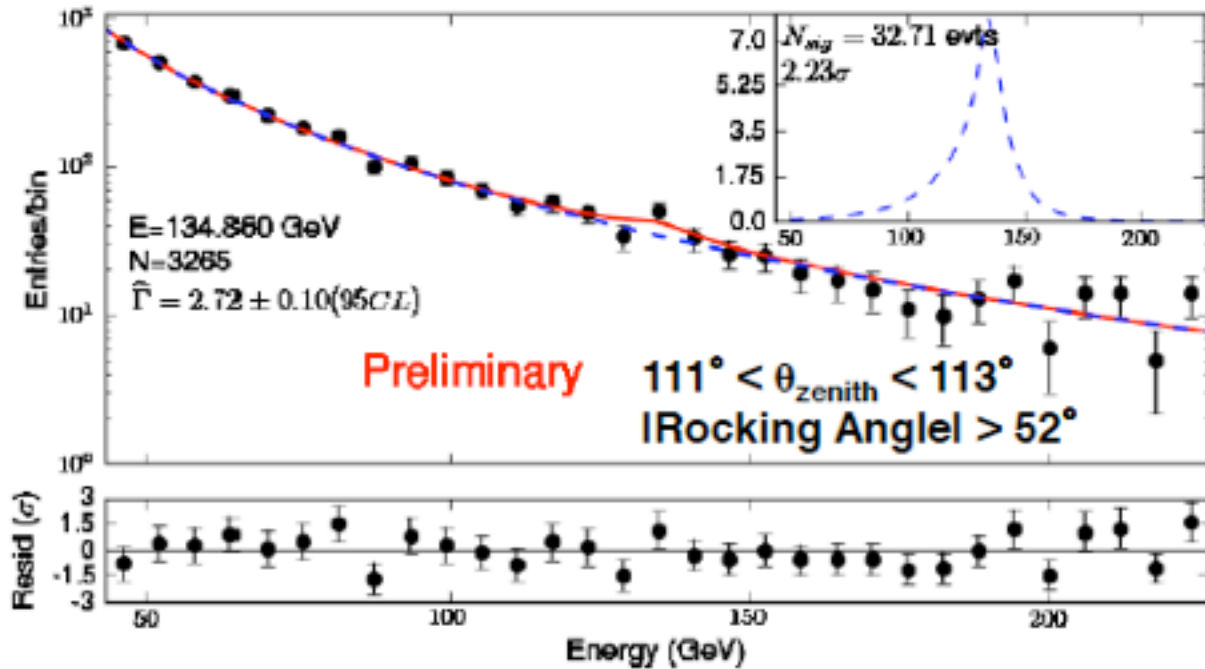
... Line-like Feature near 135 GeV ... localized in the galactic center ...".

In addition to the Galactic Center observations, Fermi LAT looked at gamma rays from Cosmic Rays hitting Earth's atmosphere



by looking at the Earth Limb.

Andrea Albert at The Fermi Symposium 11/2/2012 also said: "... Earth Limb is a bright gamma-ray source ... From cosmic-ray interactions in the atmosphere ...



Fermi LAT Spectral Line Search

11/02/2012

... Line-like feature ... at 135 GeV .. Appears when LAT is pointing at the Limb ...".

Since 90% of high-energy Cosmic Rays are Protons and since their collisions with Protons and other nuclei in Earth's atmosphere produce gamma rays, the 135 GeV Earth Limb Line seen by Fermi LAT is also likely to be the Higgs produced by collisions analagous to those at the LHC.

Olivier K. in a comment in Jester's blog on 10 November 2012 said:

"... Could the 135 GeV bump be related ... to current Higgs ... properties ? ...

The coincidence between GeV figures ...[ for LHC ] Higgs mass and this [ Fermi LAT ] bump is thrilling for an amateur like me...".

Jester in his resonances blog on 17 April 2012 said, about Fermi LAT:

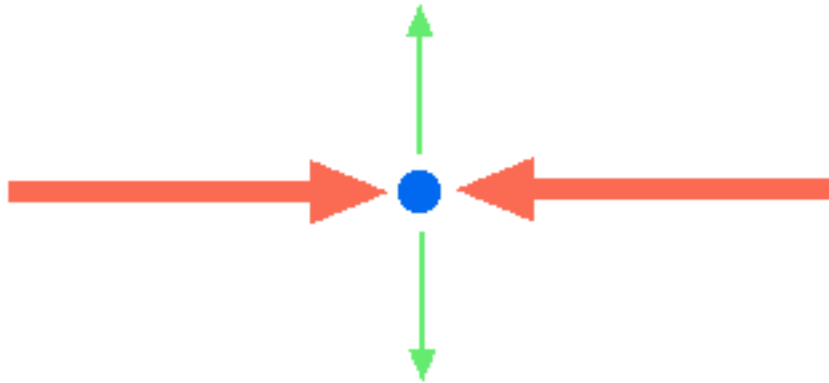
"... the plot shows the energy of \*single\* photons as measured by Fermi, not the invariant mass of photon pairs ...".

Since the LHC 125 GeV peak is for "invariant mass of photon pairs" and the Fermi LAT 135 GeV peak is for ""single" photons"

how could both correspond to a Higgs mass state around 130 GeV ?



The LHC sees collisions of high-energy protons (red arrows) forming Higgs (blue dot)



with the Higgs at rest decaying into a photon pair (green arrows) giving the observed Higgs peak (around 130 GeV) as invariant mass of photon pairs.

Fermi LAT at Galactic Center and Earth Limb sees collisions of one high-energy proton with a low-energy (relatively at rest) proton forming Higgs



with Higgs moving fast from momentum inherited from the high-energy proton decaying into two photons: one with low energy not observed by Fermi LAT and the other being observed by Fermi LAT as a high-energy gamma ray carrying almost all of the Higgs decay energy (around 130 GeV) as a "single" photon.

Therefore, the coincidence noted by Olivier K. is probably a realistic phenomenon.

## 18. Segal-type Conformal gravity with conformal generator structure giving Dark Energy, Dark Matter, and Ordinary Matter ratio

MacDowell-Mansouri Gravity is described by Rabindra Mohapatra in section 14.6 of his book "Unification and Supersymmetry":

### §14.6. Local Conformal Symmetry and Gravity

Before we study supergravity, with the new algebraic approach developed, we would like to discuss how gravitational theory can emerge from the gauging of conformal symmetry. For this purpose we briefly present the general notation for constructing gauge covariant fields. The general procedure is to start with the Lie algebra of generators  $X_A$  of a group

$$[X_A, X_B] = f_{AB}^C X_C, \quad (14.6.1)$$

where  $f_{AB}^C$  are structure constants of the group. We can then introduce a gauge field connection  $h_\mu^A$  as follows:

$$h_\mu = h_\mu^A X_A. \quad (14.6.2)$$

Let us denote the parameter associated with  $X_A$  by  $\epsilon^A$ . The gauge transformations on the fields  $h_\mu^A$  are given as follows:

$$\delta h_\mu^A = \partial_\mu \epsilon^A + h_\mu^B \epsilon^C f_{CB}^A = (D_\mu \epsilon)^A. \quad (14.6.3)$$

We can then define a covariant curvature

$$R_{\mu\nu}^A = \partial_\nu h_\mu^A - \partial_\mu h_\nu^A + h_\nu^B h_\mu^C f_{CB}^A. \quad (14.6.4)$$

Under a gauge transformation

$$\delta_{\text{gauge}} R_{\mu\nu}^A = R_{\mu\nu}^B \epsilon^C f_{CB}^A. \quad (14.6.5)$$

We can then write the general gauge invariant action as follows:

$$I = \int d^4x Q_{AB}^{\mu\nu\rho\sigma} R_{\mu\nu}^A R_{\rho\sigma}^B. \quad (14.6.6)$$

Let us now apply this formalism to conformal gravity. In this case

$$h_\mu = P_\mu^\alpha e_\alpha^n + M_{mn} \omega_\mu^{mn} + K_\mu^n f_\mu^n + D b_\mu. \quad (14.6.7)$$

The various  $R_{\mu\nu}$  are

$$R_{\mu\nu}(P) = \partial_\nu e_\mu^n - \partial_\mu e_\nu^n + \omega_\mu^{mn} e_\nu^n - \omega_\nu^{mn} e_\mu^n - b_\mu e_\nu^n + b_\nu e_\mu^n, \quad (14.6.8)$$

$$R_{\mu\nu}(M) = \partial_\nu \omega_\mu^{mn} - \partial_\mu \omega_\nu^{mn} - \omega_\nu^{rs} \omega_{\mu,p}^n - \omega_\mu^{rs} \omega_{\nu,p}^n - 4(e_\mu^n f_\nu^n - e_\nu^n f_\mu^n), \quad (14.6.9)$$

$$R_{\mu\nu}(K) = \partial_\nu f_\mu^n - \partial_\mu f_\nu^n - b_\mu f_\nu^n + b_\nu f_\mu^n + \omega_\mu^{mn} f_\nu^n - \omega_\nu^{mn} f_\mu^n, \quad (14.6.10)$$

$$R_{\mu\nu}(D) = \partial_\nu b_\mu - \partial_\mu b_\nu + 2e_\mu^n f_\nu^n - 2e_\nu^n f_\mu^n. \quad (14.6.11)$$

The gauge invariant Lagrangian for the gravitational field can now be written down, using eqn. (14.6.6), as

$$S = \int d^4x \epsilon_{\mu\nu\rho\sigma} \epsilon^{\mu\nu\alpha\beta} R_{\mu\nu}^{\alpha\beta}(M) R_{\rho\sigma}^{\alpha\beta}(M). \quad (14.6.12)$$

We also impose the constraint that

$$R_{\mu\nu}(P) = 0, \quad (14.6.13)$$

which expresses  $\omega_\mu^{mn}$  as a function of  $(e, b)$ . The reason for imposing this constraint has to do with the fact that  $P_n$  transformations must be eventually identified with coordinate transformation. To see this point more explicitly let us consider the vierbein  $e_\mu^a$ . Under coordinate transformations

$$\delta_{GC}(\xi^\nu)e_\mu^a = \partial_\nu \xi^\lambda e_\lambda^a + \xi^\lambda \partial_\lambda e_\mu^a. \quad (14.6.14)$$

Using eqn. (14.6.8) we can rewrite

$$\delta_{GC}(\xi^\nu)e_\mu^a = \delta_P(\xi^\nu)e_\mu^a + \delta_M(\xi^\nu \omega^{mn})e_\mu^a + \delta_D(\xi^\nu b) e_\mu^a + \xi^\nu R_{\mu\nu}^a(P),$$

where

$$\delta_P(\xi^\nu)e_\mu^a = \partial_\nu \xi^\mu + \xi^\nu \omega_\mu^{mn} + \xi^{mn} b_\mu. \quad (14.6.15)$$

If  $R^{\mu\nu}(P) = 0$ , the general coordinate transformation becomes related to a set of gauge transformations via eqn. (14.6.15).

At this point we also wish to point out how we can define the covariant derivative. In the case of internal symmetries  $D_\mu = \partial_\mu - iX_A h_\mu^A$ ; now since momentum is treated as an internal symmetry we have to give a rule. This follows from eqn. (14.6.15) by writing a redefined translation generator  $\bar{P}$  such that

$$\delta_{\bar{P}}(\xi) = \delta_{GC}(\xi^\nu) - \sum_A \delta_A(\xi^{mn} h_\mu^A), \quad (14.6.16)$$

where  $A'$  goes over all gauge transformations excluding translation. The rule is

$$\delta_{\bar{P}}(\xi^{mn})\phi = \xi^{mn} D_\mu^C \phi. \quad (14.6.17)$$

We also wish to point out that for fields which carry spin or conformal charge, only the intrinsic parts contribute to  $D_\mu^C$  and the orbital parts do not play any role.

Coming back to the constraints we can then vary the action with respect to  $f_\mu^m$  to get an expression for it, i.e.,

$$e_\nu^m f_{\mu m} = -\frac{1}{4}[e_\mu^a e_\nu^b R_{ab}^{mn} - \frac{1}{2}g_{\mu\nu} R], \quad (14.6.18)$$

where  $f_\mu^m$  has been set to zero in  $R$  written in the right-hand side.

This eliminates (from the theory the degrees of freedom)  $\omega_\mu^{mn}$  and  $f_\mu^m$  and we are left with  $e_\mu^a$  and  $b_\mu$ . Furthermore, these constraints will change the transformation laws for the dependent fields so that the constraints do not change.

Let us now look at the matter coupling to see how the familiar gravity theory emerges from this version. Consider a scalar field  $\phi$ . It has conformal weight  $\lambda = 1$ . So we can write a covariant derivative for it, eqn. (14.6.17)

$$D_\mu^C \phi = \partial_\mu \phi - \phi b_\mu. \quad (14.6.19)$$

We note that the conformal charge of  $\phi$  can be assumed to be zero since  $K_m = x^2 \partial$  and is the dimension of inverse mass. In order to calculate  $\square^C \phi$  we

start with the expression for d'Alembertian in general relativity

$$\frac{1}{e} \partial_\nu (g^{\mu\nu} e D_\mu^c \phi). \quad (14.6.20)$$

The only transformations we have to compensate for are the conformal transformations and the scale transformations. Since

$$\delta b_\mu = -2\xi_k^m e_{m\mu}, \quad \delta(\phi b_\mu) = \phi \delta b_\mu = -2\phi f_\mu^m e_m^\nu = +\frac{1}{2} \phi R, \quad (14.6.21)$$

where, in the last step, we have used the constraint equation (14.6.18). Putting all these together we find

$$\square^c \phi = \frac{1}{e} \partial_\nu (g^{\mu\nu} e D_\mu^c \phi) + b_\mu D_\mu^c \phi + \frac{1}{2} \phi R. \quad (14.6.22)$$

Thus, the Lagrangian for conformal gravity coupled to matter fields can be written as

$$S = \int e d^4x \frac{1}{2} \phi \square^c \phi. \quad (14.6.23)$$

Now we can use conformal transformation to gauge  $b_\mu = 0$  and local scale transformation to set  $\phi = \kappa^{-1}$  leading to the usual Hilbert action for gravity. To summarize, we start with a Lagrangian invariant under full local conformal symmetry and fix conformal and scale gauge to obtain the usual action for gravity. We will adopt the same procedure for supergravity. An important technical point to remember is that,  $\square^c$ , the conformal d'Alembertian contains  $R$ , which for constant  $\phi$ , leads to gravity. We may call  $\phi$  the auxiliary field.

After the scale and conformal gauges have been fixed, the conformal Lagrangian becomes a de Sitter Lagrangian.

Einstein-Hilbert gravity can be derived from the de Sitter Lagrangian, as was first shown by MacDowell and Mansouri (Phys. Rev. Lett. 38 (1977) 739).

( Frank Wilczek, in hep-th/9801184 says that the MacDowell-Mansouri "... approach to casting gravity as a gauge theory was initiated by MacDowell and Mansouri ...

S. MacDowell and F. Mansouri, Phys. Rev. Lett. 38 739 (1977) ... ,

and independently Chamseddine and West ... A. Chamseddine and P. West Nucl. Phys. B 129, 39 (1977); also quite relevant is A. Chamseddine, Ann. Phys. 113, 219 (1978). ...". )

**The minimal group required to produce Gravity,  
and therefore the group that is used in calculating Force Strengths,  
is the [anti] de Sitter group,** as is described by

Freund in chapter 21 of his book Supersymmetry (Cambridge 1986) ( chapter 21 is a Non-Supersymmetry chapter leading up to a Supergravity description in the following chapter 22 ):

"... Einstein gravity as a gauge theory ... we expect a set of gauge fields  $w^{ab}_u$  for the Lorentz group and a further set  $e^a_u$  for the translations, ...

Everybody knows though, that Einstein's theory contains but one spin two field, originally chosen by Einstein as  $g_{uv} = e^a_u e^b_v n_{ab}$  ( $n_{ab}$  = Minkowski metric).

What happened to the  $w^{ab}_u$  ?

The field equations obtained from the Hilbert-Einstein action by varying the  $w^{ab}_u$  are algebraic in the  $w^{ab}_u$  ... permitting us to express the  $w^{ab}_u$  in terms of the  $e^a_u$  ... The  $w$  do not propagate ...

We start from the four-dimensional de-Sitter algebra ...  $so(3,2)$ .

Technically this is the anti-de-Sitter algebra ...

We envision space-time as a four-dimensional manifold  $M$ .

At each point of  $M$  we have a copy of  $SO(3,2)$  (a fibre ...) ...

and we introduce the gauge potentials (the connection)  $h^A_\mu(x)$

$A = 1, \dots, 10$ ,  $\mu = 1, \dots, 4$ . Here  $x$  are local coordinates on  $M$ .

From these potentials  $h^A_\mu$  we calculate the field-strengths

(curvature components) [let  $@$  denote partial derivative]

$R^A_{\mu\nu} = @_\mu h^A_\nu - @_\nu h^A_\mu + f^A_{BC} h^B_\mu h^C_\nu$

...[where]... the structure constants  $f^C_{AB}$  ...[are for]... the anti-de-Sitter algebra ....

We now wish to write down the action  $S$  as an integral over

the four-manifold  $M$  ...  $S(Q) = \text{INTEGRAL}_M R^A \wedge R^B Q_{AB}$

where  $Q_{AB}$  are constants ... to be chosen ... we require

... the invariance of  $S(Q)$  under local Lorentz transformations

... the invariance of  $S(Q)$  under space inversions ...

...[ AFTER A LOT OF ALGEBRA NOT SHOWN IN THIS QUOTE ]...

we shall see ...[that]... the action becomes invariant

under all local [anti]de-Sitter transformations ...[and]... we recognize ... t

he familiar Hilbert-Einstein action with cosmological term in vierbein notation ...

Variation of the vierbein leads to the Einstein equations with cosmological term.

Variation of the spin-connection ... in turn ... yield the torsionless Christoffel

connection ... the torsion components ... now vanish.

So at this level full  $sp(4)$  invariance has been checked.

... Were it not for the assumed space-inversion invariance ...

we could have had a parity violating gravity. ...

Unlike Einstein's theory ...[MacDowell-Mansouri].... does not require Riemannian invertibility of the metric. ... the solution has torsion ... produced by an interference between parity violating and parity conserving amplitudes.

Parity violation and torsion go hand-in-hand.

Independently of any more realistic parity violating solution of the gravity

equations this raises the cosmological question whether

the universe as a whole is in a space-inversion symmetric configuration. ...".

According to gr-qc/9809061 by R. Aldrovandi and J. G. Peireira:

"... If the fundamental spacetime symmetry of the laws of Physics is that given by the de Sitter instead of the Poincare group, the P-symmetry of the weak cosmological-constant limit and the Q-symmetry of the strong cosmological constant limit can be considered as limiting cases of the fundamental symmetry. ...

... N ... [ is the space ]... whose geometry is gravitationally related to an infinite cosmological constant ...[and]... is a 4-dimensional cone-space in which  $ds = 0$ , and whose group of motion is Q. Analogously to the Minkowski case, N is also a homogeneous space, but now under the kinematical group Q, that is,  $N = Q/L$  [ where L is the Lorentz Group of Rotations and Boosts ]. In other words, the point-set of N is the point-set of the special conformal transformations.

Furthermore, the manifold of Q is a principal bundle  $P(Q/L, L)$ , with  $Q/L = N$  as base space and L as the typical fiber. The kinematical group Q, like the Poincare group, has the Lorentz group L as the subgroup accounting for both the isotropy and the equivalence of inertial frames in this space. However, the special conformal transformations introduce a new kind of homogeneity. Instead of ordinary translations, all the points of N are equivalent through special conformal transformations. ...

... Minkowski and the cone-space can be considered as dual to each other, in the sense that their geometries are determined respectively by a vanishing and an infinite cosmological constants. The same can be said of their kinematical group of motions: P is associated to a vanishing cosmological constant and Q to an infinite cosmological constant.

The dual transformation connecting these two geometries is the spacetime inversion  $x^u \rightarrow x^u / \sigma^2$ . Under such a transformation, the Poincare group P is transformed into the group Q, and the Minkowski space M becomes the conespace N. The points at infinity of M are concentrated in the vertex of the conespace N, and those on the light-cone of M becomes the infinity of N. It is concepts of space isotropy and equivalence between inertial frames in the conespace N are those of special relativity. The difference lies in the concept of uniformity as it is the special conformal transformations, and not ordinary translations, which act transitively on N. ..."

**Gravity and the Cosmological Constant come from the MacDowell-Mansouri Mechanism and the 15-dimensional Spin(2,4) = SU(2,2) Conformal Group, which is made up of:**

**3 Rotations  
3 Boosts  
4 Translations  
4 Special Conformal transformations  
1 Dilatation**

The **Cosmological Constant / Dark Energy** comes from the **10 Rotation, Boost, and Special Conformal generators** of the Conformal Group Spin(2,4) = SU(2,2), so the fractional part of our Universe of the Cosmological Constant should be **about 10 / 15 = 67% for tree level.**

Black Holes, including **Dark Matter Primordial Black Holes**, are curvature singularities in our 4-dimensional physical spacetime, and since Einstein-Hilbert curvature comes from the **4 Translations** of the 15-dimensional Conformal Group Spin(2,4) = SU(2,2) through the MacDowell-Mansouri Mechanism (in which the generators corresponding to the 3 Rotations and 3 Boosts do not propagate), the fractional part of our Universe of Dark Matter Primordial Black Holes should be **about 4 / 15 = 27% at tree level.**

Since **Ordinary Matter** gets mass from the Higgs mechanism which is related to the **1 Scale Dilatation** of the 15-dimensional Conformal Group Spin(2,4) = SU(2,2), the fractional part of our universe of Ordinary Matter should be **about 1 / 15 = 6% at tree level.**

However,  
**as Our Universe evolves the Dark Energy, Dark Matter, and Ordinary Matter densities evolve at different rates,**  
so that the differences in evolution must be taken into account from the initial End of Inflation to the Present Time.

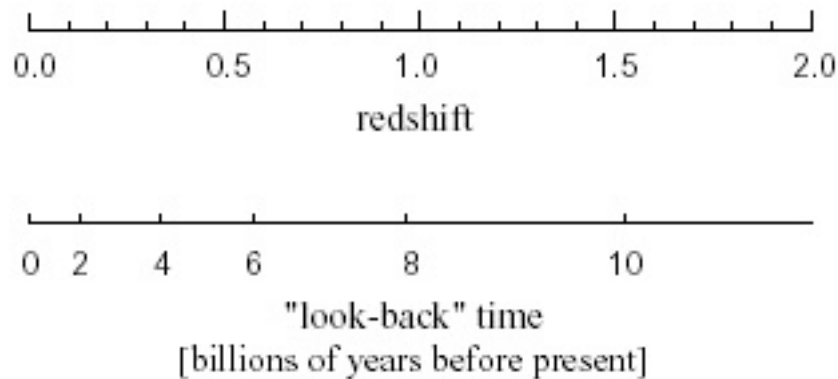
**Without taking into account any evolutionary changes with time,**  
our Flat Expanding Universe should have roughly:

**67% Cosmological Constant  
27% Dark Matter - possibly primordial stable Planck mass black holes  
6% Ordinary Matter**

As Dennis Marks pointed out to me,  
 since density  $\rho$  is proportional to  $(1+z)^3(1+w)$  for red-shift factor  $z$   
 and a constant equation of state  $w$ :  
 $w = -1$  for  $\Lambda$  and the average overall density of  $\Lambda$  Dark Energy remains constant  
 with time and the expansion of our Universe;  
 and  
 $w = 0$  for nonrelativistic matter so that the overall average density of Ordinary  
 Matter declines as  $1 / R^3$  as our Universe expands;  
 and  
 $w = 0$  for primordial black hole dark matter - stable Planck mass black holes - so  
 that Dark Matter also has density that declines as  $1 / R^3$  as our Universe expands;  
 so that the ratio of their overall average densities must vary with time, or scale  
 factor  $R$  of our Universe, as it expands.  
 Therefore,  
 the above calculated ratio  $0.67 : 0.27 : 0.06$  is valid  
 only for a particular time, or scale factor, of our Universe.

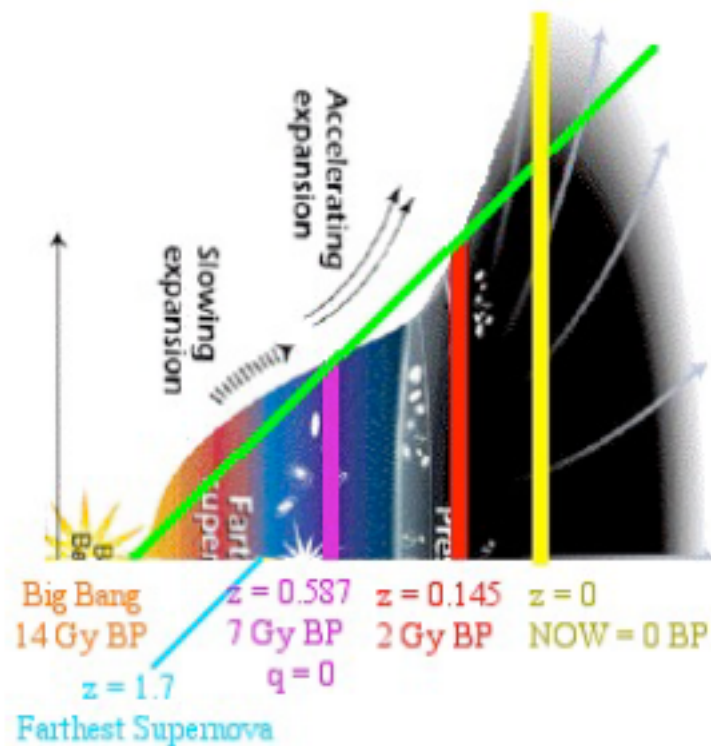
When is that time ? Further, what is the value of the ratio now ?

Since WMAP observes Ordinary Matter at 4% NOW,  
 the time when Ordinary Matter was 6% would be  
 at redshift  $z$  such that  
 $1 / (1+z)^3 = 0.04 / 0.06 = 2/3$ , or  $(1+z)^3 = 1.5$ , or  $1+z = 1.145$ , or  $z = 0.145$ .  
 To translate redshift into time,  
 in billions of years before present, or Gy BP, use this chart



from a [www.supernova.lbl.gov](http://www.supernova.lbl.gov) file SNAPoverview.pdf to see that  
 the time when Ordinary Matter was 6%  
 would have been a bit over 2 billion years ago, or 2 Gy BP.





In the diagram, there are four Special Times in the history of our Universe:  
the Big Bang Beginning of Inflation (about 13.7 Gy BP);

1 - the End of Inflation = Beginning of Decelerating Expansion  
(beginning of green line also about 13.7 Gy BP);

2 - the End of Deceleration ( $q=0$ ) = Inflection Point =  
= Beginning of Accelerating Expansion  
(purple vertical line at about  $z = 0.587$  and about 7 Gy BP).

According to a hubblesite web page credited to Ann Feild, the above diagram "... reveals changes in the rate of expansion since the universe's birth 15 billion years ago. The more shallow the curve, the faster the rate of expansion. The curve changes noticeably about 7.5 billion years ago, when objects in the universe began flying apart as a faster rate. ...".

According to a CERN Courier web page: "... Saul Perlmutter, who is head of the Supernova Cosmology Project ... and his team have studied altogether some 80 high red-shift type Ia supernovae. Their results imply that the universe was decelerating for the first half of its existence, and then began accelerating approximately 7 billion years ago. ...".

According to astro-ph/0106051 by Michael S. Turner and Adam G. Riess: "... current supernova data ... favor deceleration at  $z > 0.5$  ... SN 1997ff at  $z = 1.7$  provides direct evidence for an early phase of slowing expansion if the dark energy is a cosmological constant ...".

3 - the Last Intersection of the Accelerating Expansion of our Universe of Linear Expansion (green line) with the Third Intersection (at red vertical line at  $z = 0.145$  and about 2 Gy BP), which is also around the times of the beginning of the Proterozoic Era and Eukaryotic Life, Fe<sub>2</sub>O<sub>3</sub> Hematite ferric iron Red Bed formations, a Snowball Earth, and the start of the Oklo fission reactor. 2 Gy is also about 10 Galactic Years for our Milky Way Galaxy and is on the order of the time for the process of a collision of galaxies.

4 - Now.

Those four Special Times define four Special Epochs:

The Inflation Epoch, beginning with the Big Bang and ending with the End of Inflation. The Inflation Epoch is described by Zizzi Quantum Inflation ending with Self-Decoherence of our Universe ( see gr-qc/0007006 ).

The Decelerating Expansion Epoch, beginning with the Self-Decoherence of our Universe at the End of Inflation. During the Decelerating Expansion Epoch, the Radiation Era is succeeded by the Matter Era, and the Matter Components (Dark and Ordinary) remain more prominent than they would be under the "standard norm" conditions of Linear Expansion.

The Early Accelerating Expansion Epoch, beginning with the End of Deceleration and ending with the Last Intersection of Accelerating Expansion with Linear Expansion. During Accelerating Expansion, the prominence of Matter Components (Dark and Ordinary) declines, reaching the "standard norm" condition of Linear Expansion at the end of the Early Accelerating Expansion Epoch at the Last Intersection with the Line of Linear Expansion.

The Late Accelerating Expansion Epoch, beginning with the Last Intersection of Accelerating Expansion and continuing forever, with New Universe creation happening many times at Many Times. During the Late Accelerating Expansion Epoch, the Cosmological Constant  $\Lambda$  is more prominent than it would be under the "standard norm" conditions of Linear Expansion.

Now happens to be about 2 billion years into the Late Accelerating Expansion Epoch.

What about Dark Energy : Dark Matter : Ordinary Matter now ?

As to how the Dark Energy  $\Lambda$  and Cold Dark Matter terms have evolved during the past 2 Gy, a rough estimate analysis would be:

$\Lambda$  and CDM would be effectively created during expansion in their natural ratio  $67 : 27 = 2.48 = 5 / 2$ , each having proportionate fraction  $5 / 7$  and  $2 / 7$ , respectively; CDM Black Hole decay would be ignored; and pre-existing CDM Black Hole density would decline by the same  $1 / R^3$  factor as Ordinary Matter, from 0.27 to  $0.27 / 1.5 = 0.18$ .

The Ordinary Matter excess  $0.06 - 0.04 = 0.02$  plus the first-order CDM excess  $0.27 - 0.18 = 0.09$  should be summed to get a total first-order excess of  $0.11$ , which in turn should be distributed to the  $\Lambda$  and CDM factors in their natural ratio  $67 : 27$ , producing, for NOW after 2 Gy of expansion:

CDM Black Hole factor =  $0.18 + 0.11 \times 2/7 = 0.18 + 0.03 = 0.21$   
for a total calculated Dark Energy : Dark Matter : Ordinary Matter ratio for now of

$$0.75 : 0.21 : 0.04$$

so that the present ratio of  $0.73 : 0.23 : 0.04$  observed by WMAP seems to me to be substantially consistent with the cosmology of the E8 model.

2013 Planck Data ( arxiv 1303.5062 ) showed "... anomalies ... previously observed in the WMAP data ... alignment between the quadrupole and octopole moments ... asymmetry of power between two ... hemispheres ... Cold Spot ... are now confirmed at ... 3 sigma ... but a higher level of confidence ...".

Now the Cl(16)-E8 model rough evolution calculation is: DE : DM : OM = 75 : 20 : 05

WMAP: DE : DM : OM = 73 : 23 : 04

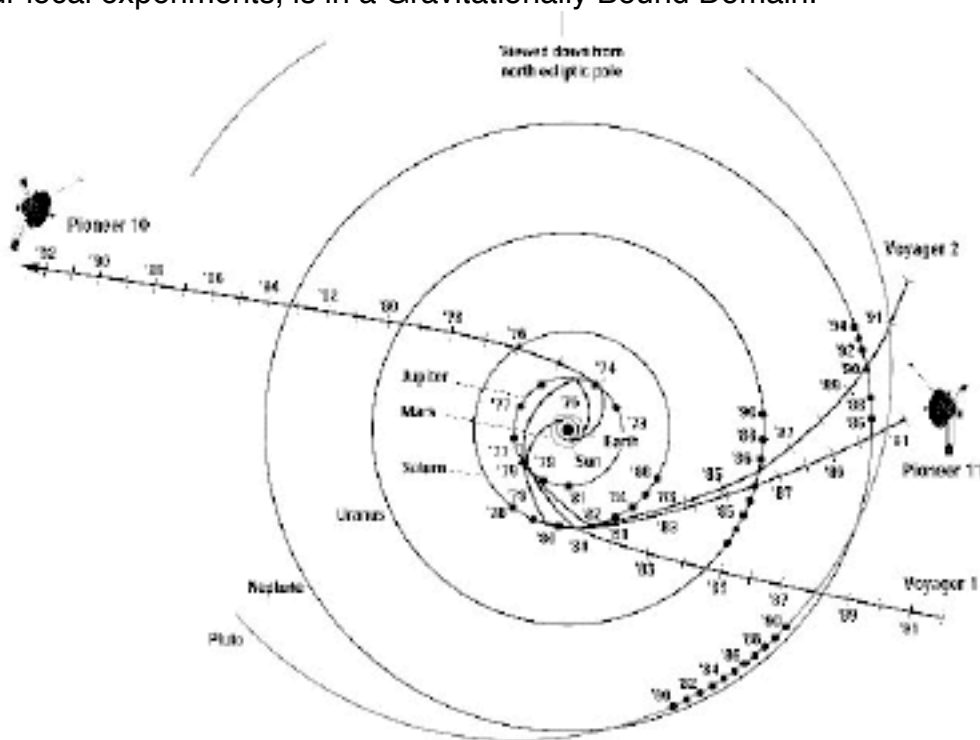
Planck: DE : DM : OM = 69 : 26 : 05

basic E8 Conformal calculation: DE : DM : OM = 67 : 27 : 06

Since uncertainties are substantial, I think that there is reasonable consistency.

## 19. Dark Energy explanations for Pioneer Anomaly and Uranus spin-axis tilt

After the Inflation Era and our Universe began its current phase of expansion, some regions of our Universe become Gravitationally Bound Domains (such as, for example, Galaxies) in which the 4 Conformal GraviPhoton generators are frozen out, forming domains within our Universe like IceBergs in an Ocean of Water. On the scale of our Earth-Sun Solar System, the region of our Earth, where we do our local experiments, is in a Gravitationally Bound Domain.



Pioneer spacecraft are not bound to our Solar System and are experiments beyond the Gravitationally Bound Domain of our Earth-Sun Solar System.

In their Study of the anomalous acceleration of Pioneer 10 and 11 gr-qc/0104064 John D. Anderson, Philip A. Laing, Eunice L. Lau, Anthony S. Liu, Michael Martin Nieto, and Slava G. Turyshev say: "... The latest successful precession maneuver to point ...[Pioneer 10]... to Earth was accomplished on 11 February 2000, when Pioneer 10 was at a distance from the Sun of 75 AU. [The distance from the Earth was [about] 76 AU with a corresponding round-trip light time of about 21 hour.] ... The next attempt at a maneuver, on 8 July 2000, was unsuccessful ... conditions will again be favorable for an attempt around July, 2001. ... At a now nearly constant velocity relative to the Sun of 12.24 km/s, Pioneer 10 will continue its motion into interstellar space, heading generally for the red star Aldebaran ... about 68 light years away ... it should take Pioneer 10 over 2 million years to reach its neighborhood....

[ the above image is ] Ecliptic pole view of Pioneer 10, Pioneer 11, and Voyager

trajectories. Digital artwork by T. Esposito. NASA ARC Image # AC97-0036-3.  
... on 1 October 1990 ... Pioneer 11 ... was [about] 30 AU away from the Sun ...  
The last communication from Pioneer 11 was received in November 1995, when  
the spacecraft was at distance of [about] 40 AU from the Sun. ... Pioneer 11 should  
pass close to the nearest star in the constellation Aquila in about 4 million years ...  
... Calculations of the motion of a spacecraft are made on the basis of the range  
time-delay and/or the Doppler shift in the signals. This type of data was used to  
determine the positions, the velocities, and the magnitudes of the orientation  
maneuvers for the Pioneer, Galileo, and Ulysses spacecraft considered in this  
study. ... The Pioneer spacecraft only have two- and three-way S-band Doppler. ...  
analyses of radio Doppler ... data ... indicated that an apparent anomalous  
acceleration is acting on Pioneer 10 and 11 ... The data implied an anomalous,  
constant acceleration with a magnitude  $a_P = 8 \times 10^{-8}$  cm/cm/s<sup>2</sup>, directed  
towards the Sun ...

... the size of the anomalous acceleration is of the order  $c H$ , where  $H$  is the  
Hubble constant ...

... Without using the apparent acceleration, CHASMP shows a steady frequency  
drift of about  $-6 \times 10^{-9}$  Hz / s, or 1.5 Hz over 8 years (one-way only). ... This  
equates to a clock acceleration,  $-a_t$ , of  $-2.8 \times 10^{-18}$  s / s<sup>2</sup> . The identity with  
the apparent Pioneer acceleration is  $a_P = a_t c$  . ...

... Having noted the relationships

$$a_P = c a_t$$

and that of ...

$$a_H = c H \rightarrow 8 \times 10^{-8} \text{ cm / s}^2$$

if  $H = 82 \text{ km / s / Mpc}$  ...

we were motivated to try to think of any ... "time" distortions that might ... fit the  
CHASMP Pioneer results ... In other words ...

Is there any evidence that some kind of "time acceleration" is being seen?

... In particular we considered ... Quadratic Time Augmentation. This model adds a  
quadratic-in-time augmentation to the TAI-ET ( International Atomic Time -  
Ephemeris Time ) time transformation, as follows

$$ET \rightarrow ET + (1/2) a_{ET} ET^2$$

The model fits Doppler fairly well

...

There was one [other] model of the ...[time acceleration]... type that was  
especially fascinating. This model adds a quadratic in time term to the light time as  
seen by the DSN station:

$$\Delta_{TAI} = TAI_{received} - TAI_{sent} \rightarrow$$

$$\rightarrow \Delta_{TAI} + (1/2) a_{quad} (TAI_{received}^2 - TAI_{sent}^2)$$

It mimics a line of sight acceleration of the spacecraft, and could be thought of as  
an expanding space model.

Note that  $a_{quad}$  affects only the data. This is in contrast to the  $a_t$  ... that affects  
both the data and the trajectory. ... This model fit both Doppler and range very  
well. Pioneers 10 and 11 ... the numerical relationship between the Hubble constant  
and  $a_P$  ... remains an interesting conjecture. ...".

In his book "Mathematical Cosmology and Extragalactic Astronomy" (Academic Press 1976) (pages 61-62 and 72), Irving Ezra Segal says:

"... Temporal evolution in ... Minkowski space ... is

$$H \rightarrow H + s l$$

... unispace temporal evolution ... is ...

$$H \rightarrow (H + 2 \tan(a/2)) / (1 - (1/2) H \tan(a/2)) = H + a l + (1/4) a H^2 + O(s^2)$$

..."

Therefore,

the Pioneer Doppler anomalous acceleration is an experimental observation of a system that is not gravitationally bound in the Earth-Sun Solar System, and its results are consistent with Segal's Conformal Theory.

Rosales and Sanchez-Gomez say, at gr-qc/9810085:

"... the recently reported anomalous acceleration acting on the Pioneers spacecrafts should be a consequence of the existence of some local curvature in light geodesics when using the coordinate speed of light in an expanding spacetime. This suggests that the Pioneer effect is nothing else but the detection of cosmological expansion in the solar system. ... the ... problem of the detected misfit between the calculated and the measured position in the spacecrafts ... this quantity differs from the expected ... just in a systematic "bias" consisting on an effective residual acceleration directed toward the center of coordinates;

its constant value is ...  $H c$  ...

This is the acceleration observed in Pioneer 10/11 spacecrafts. ... a periodic orbit does not experience the systematic bias but only a very small correction ... which is not detectable ... in the old Foucault pendulum experiment ... the motion of the pendulum experiences the effect of the Earth based reference system being not an inertial frame relatively to the "distant stars". ... Pioneer effect is a kind of a new cosmological Foucault experiment, the solar system based coordinates, being not the true inertial frame with respect to the expansion of the universe, mimics the role that the rotating Earth plays in Foucault's experiment ...".

The Rosales and Sanchez-Gomez idea of a 2-phase system in which objects bound to the solar system (in a "periodic orbit") are in one phase (non-expanding pennies-on-a-balloon) while unbound (escape velocity) objects are in another phase (expanding balloon) that "feels" expansion of our universe is very similar to my view of such things as described on this page.

The Rosales and Sanchez-Gomez paper very nicely unites:

the physical 2-phase (bounded and unbounded orbits) view;  
the Foucault pendulum idea; and the cosmological value  $H c$ .

My view, which is consistent with that of Rosales and Sanchez-Gomez, can be summarized as a 2-phase model based on Segal's work which has two phases with different metrics:

a metric for outside the inner solar system, a dark energy phase in which gravity is described in which all 15 generators of the conformal group are effective, some of which are related to the dark energy by which our universe expands;

and

a metric for where we are, in regions dominated by ordinary matter, in which the 4 special conformal and 1 dilation degrees of freedom of the conformal group are suppressed and the remaining 10 generators (antideSitter or Poincare, etc) are effective, thus describing ordinary matter phenomena.

If you look closely at the difference between the metrics in those two regions, you see that the full conformal dark energy region gives an "extra acceleration" that acts as a "quadratic in time term" that has been considered as an explanation of the Pioneer effect by John D. Anderson, Philip A. Laing, Eunice L. Lau, Anthony S. Liu, Michael Martin Nieto, and Slava G. Turyshev in their paper at gr-qc/0104064.

Jack Sarfatti has a 2-phase dark energy / dark matter model that can give a similar anomalous acceleration in regions where  $c^2 \wedge$  dark energy / dark matter is effectively present. If there is a phase transition (around Uranus at 20 AU) whereby ordinary matter dominates inside that distance from the sun and exotic dark energy / dark matter appears at greater distances, then Jack's model could also explain the Pioneer anomaly and it may be that Jack's model with ordinary and exotic phases and my model with deSitter/Poincare and Conformal phases may be two ways of looking at the same thing.

As to what might be the physical mechanism of the phase transition, Jack says "... Rest masses of [ordinary matter] particles ... require the smooth non-random Higgs Ocean ... which soaks up the choppy random troublesome zero point energy ...".

In other words in a region in which ordinary matter is dominant, such as the Sun and our solar system, the mass-giving action of the Higgs mechanism "soaks up" the Dark Energy zero point conformal degrees of freedom that are dominant in low-ordinary mass regions of our universe (which are roughly the intergalactic voids that occupy most of the volume of our universe).

That physical interpretation is consistent with my view.

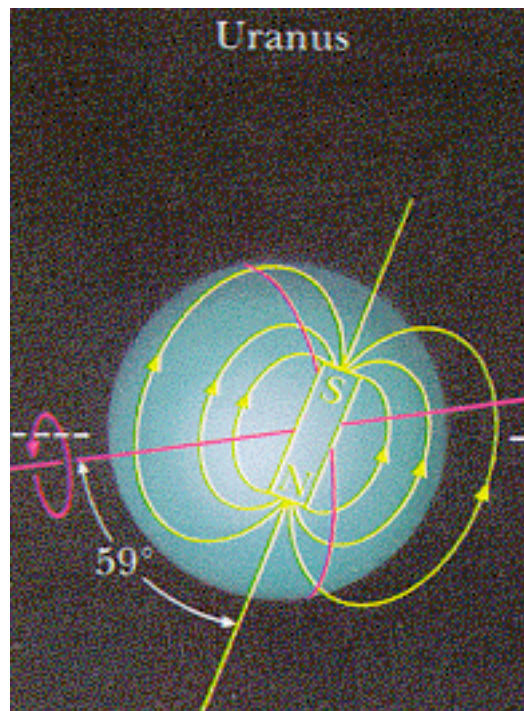
### Transition at Orbit of Uranus:

It may be that the observation of the Pioneer phase transition at Uranus from ordinary to anomalous acceleration is an experimental result that gives us a first look at dark energy / dark matter phenomena that could lead to energy sources that could be even more important than nuclear energy.

In gr-qc/0104064 Anderson et al say:

"... Beginning in 1980 ... at a distance of 20 astronomical units (AU) from the Sun ... we found that the largest systematic error in the acceleration residuals was a constant bias,  $a_P$ , directed toward the Sun. Such anomalous data have been continuously received ever since. ...",

so that the transition from inner solar system Minkowski acceleration to outer Segal Conformal acceleration occurs at about 20 AU, which is about the radius of the orbit of Uranus. That phase transition may account for the unique rotational axis of Uranus,



which lies almost in its orbital plane.

The most stable state of Uranus may be with its rotational axis pointed toward the Sun, so that the Solar hemisphere would be entirely in the inner solar system Minkowski acceleration phase and the anti-Solar hemisphere would be in entirely in the outer Segal Conformal acceleration phase.

Then the rotation of Uranus would not take any material from one phase to the other, and there would be no drag on the rotation due to material going from phase to phase.

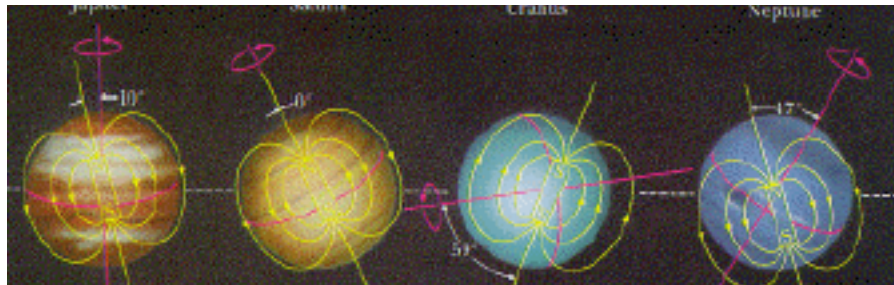


Of course, as Uranus orbits the Sun, it will only be in that most stable configuration twice in each orbit, but an orbit in the ecliptic containing that most stable configuration twice (such as its present orbit) would be in the set of the most stable ground states, although such an effect would be very small now. However, such an effect may have been more significant on the large gas/dust cloud that was condensing into Uranus and therefore it may have caused Uranus to form initially with its rotational axis pointed toward the Sun.

In the pre-Uranus gas/dust cloud, any component of rotation that carried material from one phase to another would be suppressed by the drag of undergoing phase transition, so that, after Uranus condensed out of the gas/dust cloud, the only remaining component of Uranus rotation would be on an axis pointing close to the Sun, which is what we now observe.

In the pre-Uranus gas/dust cloud, any component of rotation that carried material from one phase to another would be suppressed by the drag of undergoing phase transition, so that, after Uranus condensed out of the gas/dust cloud, the only remaining component of Uranus rotation would be on an axis pointing close to the Sun, which is what we now observe.

Much of the perpendicular (to Uranus orbital plane) angular momentum from the original gas/dust cloud may have been transferred (via particles "bouncing" off the phase boundary) to the clouds forming Saturn (inside the phase boundary) or Neptune (outside the phase boundary), thus accounting for the substantial (relative to Jupiter) deviation of their rotation axes from exact perpendicularity (see images above and below from "Universe", 4th ed, by William Kaufmann, Freeman 1994).



According to Utilizing Minor Planets to Assess the Gravitational Field in the Outer Solar System, astro-ph/0504367, by Gary L. Page, David S. Dixon, and John F. Wallin:

"... the great distances of the outer planets from the Sun and the nearly circular orbits of Uranus and Neptune makes it very difficult to use them to detect the Pioneer Effect. ... The ratio of the Pioneer acceleration to that produced by the Sun at a distance equal to the semimajor axis of the planets is 0.005, 0.013, and 0.023 percent for Uranus, Neptune, and Pluto, respectively. ... Uranus' period shortens by 5.8 days and Neptune's by 24.1, while Pluto's period drops by 79.7 days. ... an equivalent change in aphelion distance of  $3.8 \times 10^{10}$ ,  $1.2 \times 10^{11}$ , and  $4.3 \times 10^{11}$  cm for Uranus, Neptune, and Pluto. In the first two cases, this is less than the accepted uncertainty in range of  $2 \times 10^6$  km [ or  $2 \times 10^{11}$  cm ] (Seidelmann 1992). ... Pluto[s] ... orbit is even less well-determined ... than the other outer planets. ... [C]ometes ... suffer ... from outgassing ... [ and their nuclei are hard to locate precisely ] ...".

According to a google cache of an Independent UK 23 September 2002 article by Marcus Chown:

"... The Pioneers are "spin-stabilised", making them a particularly simple platform to understand. Later probes ... such as the Voyagers and the Cassini probe ... were stabilised about three axes by intermittent rocket boosts. The unpredictable accelerations caused by these are at least 10 times bigger than a small effect like the Pioneer acceleration, so they completely cloak it. ...".

## 20. Dark Energy experiment by BSCCO Josephson Junctions and geometry of 600-cell

I. E. Segal proposed a Minkowski-Conformal 2-phase Universe and

Beck and Mackey proposed 2 Photon-GraviPhoton phases:

Minkowski/Photon phase locally Minkowski with ordinary Photons and Gravity weakened by  $1 / (M_{\text{Planck}})^2 = 5 \times 10^{(-39)}$ .

so that we see Dark Energy as only  $3.9 \text{ GeV/m}^3$

Conformal/GraviPhoton phase with GraviPhotons and Conformal symmetry (like the massless phase of energies above Higgs EW symmetry breaking)

With massless Planck the  $1 / M_{\text{Planck}}^2$  Gravity weakening goes away and the Gravity Force Strength becomes the strongest possible = 1

so Conformal Gravity Dark Energy should be enhanced by  $M_{\text{Planck}}^2$  from the Minkowski/Photon phase value of  $3.9 \text{ GeV/m}^3$ .

The Energy Gap of our Universe as superconductor condensate spacetime is from  $3 \times 10^{(-18)} \text{ Hz}$  (radius of universe) to  $3 \times 10^{43} \text{ Hz}$  (Planck length).

Its RMS amplitude is  $10^{13} \text{ Hz} = 10 \text{ THz} = \text{energy of neutrino masses} =$   
 $= \text{critical temperature } T_c \text{ of BSCCO superconducting crystals.}$

Neutrino masses are involved because their mass is zero at tree level

and their masses that we observe come from virtual graviphotons becoming virtual neutrino-antineutrino pairs.

BSCCO superconducting crystals are by their structure natural Josephson Junctions. Dark Energy accumulates (through graviphotons) in the superconducting layers of BSCCO.

Josephson Junction control voltage acts as a valve for access to the BSCCO Dark Energy, an idea due to Jack Sarfatti.

Christian Beck and Michael C. Mackey in astro-ph/0703364 said: "... Electromagnetic dark energy ... is based on a Ginzburg-Landau ... phase transition for the gravitational activity of virtual photons ... in two different phases:

gravitationally active [GraviPhotons] ...

and gravitationally inactive [Photons]

...

Let  $IPI^2$  be the number density of gravitationally active photons ... start from a Ginzburg-Landau free energy density ...

$$F = a IPI^2 + (1/2) b IPI^4$$

... The equilibrium state  $Peq$  is ... a minimum of  $F$  ... for  $T > T_c$  ...

$$Peq = 0 \text{ [and] } Feq = 0$$

... for  $T < T_c$

$$I|Peq|^2 = - a / b \text{ [and] } Fdeq = -(1/2) a^2 / b$$

... temperature  $T$  [of] virtual photons underlying dark energy ... is ..

$$h \nu = \ln 3 k T$$

... dark energy density ...[is]...

$$\rho_{\text{dark}} = (1/2) (\pi h / c^3) (\nu_c)^4$$

... The currently observed dark energy density in the universe of about 3.9 GeV/m<sup>3</sup> implies that the critical frequency  $\nu_c$  is ...

$$\nu_c = 2.01 \text{ THz}$$

... BCS Theory yields ... for Fermi energy ... in copper ... 7.0 eV and the critical temperature of ... YBCO ... around 90 K ...

$$h \nu_c = 8 \times 10^{-3} \text{ eV}$$

... Solar neutrino measurements provide evidence for a neutrino mass of about  $m_\nu c^2 = 9 \times 10^{-3} \text{ eV}$  ...

[ the Cl(16)-E8 model has first-order masses for the 3 generations of neutrinos as

$$1 \times 10^{-3} \text{ and } 9 \times 10^{-3} \text{ and } 5.4 \times 10^{-2} \text{ eV } ]$$

... in solid state physics the critical temperature is essentially determined by the energy gap of the superconductor ... (i.e. the energy obtained when a Cooper pair forms out of two electrons) ...

for [graviphotons] ... at low temperatures (frequencies) Cooper-pair like states [of neutrino-antineutrino pairs] can form in the vacuum ... the ... energy gap would be of the order of typical neutrino mass differences ...".

Clovis Jacinto de Matos and Christian Beck in arXiv 0707.1797 said: "... Tajmar's experiments ... at Austrian Research Centers Gmbh-ARC ... with ... rotating superconducting rings ... demonstrated ...

a clear azimuthal acceleration ... directly proportional to the superconductive ring angular acceleration, and

an angular velocity orthogonal to the ring's equatorial plane ...

In 1989 Cabrera and Tate, through the measurement of the London moment magnetic trapped flux, reported an anomalous Cooper pair mass excess in thin rotating Niobium superconductive rings ...

A non-vanishing cosmological constant (CC)  $\Lambda$  can be interpreted in terms of a non-vanishing vacuum energy density

$$\rho_{\text{vac}} = (c^4 / 8 \pi G) \Lambda$$

which corresponds to dark energy with equation of state  $w = -1$ .

The ... astronomically observed value [is]...  $\Lambda = 1.29 \times 10^{-52} [1/m^2]$  ...

Graviphotons can form weakly bounded states with Cooper pairs, increasing their mass slightly from  $m$  to  $m'$  .

The binding energy is  $E_c = u c^2$  :

$$m' = m + m_y - u$$

... Since the graviphotons are bounded to the Cooper pairs,

their zeropoint energies form a condensate capable of the gravitoelectrodynamic properties of superconductive cavities. ...

Beck and Mackey's Ginzburg-Landau-like theory leads to a finite dark energy density dependent on the frequency cutoff  $\nu_c$  of vacuum fluctuations:

$$\rho^* = (1/2) (\pi h / c^3) (\nu_c)^4$$

in vacuum one may put  $\rho^* = \rho_{\text{vac}}$  from which the cosmological cutoff frequency  $\nu_{\text{cc}}$  is estimated as

$$\nu_{\text{cc}} = 2.01 \text{ THz}$$

The corresponding "cosmological" quantum of energy is:

$$E_{\text{cc}} = h \nu_{\text{cc}} = 8.32 \text{ MeV}$$

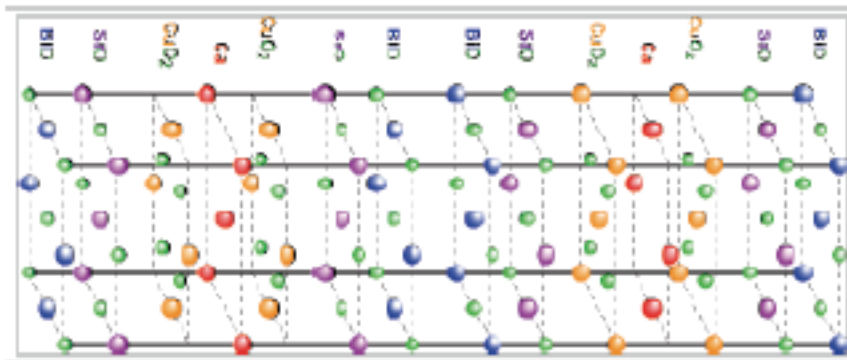
... In the interior of superconductors ... the effective cutoff frequency can be different ...  $h \nu = \ln 3 k T$  ... we find the cosmological critical temperature  $T_{\text{cc}}$

$$T_{\text{cc}} = 87.49 \text{ K}$$

This temperature is characteristic of the BSCCO High-Tc superconductor.

...".

Xiao Hu and Shi-Zeng Lin in arXiv 0911.5371 said: "... The Josephson effect is a phenomenon of current flow across two weakly linked superconductors separated by a thin barrier, i.e. Josephson junction, associated with coherent quantum tunneling of Cooper pairs. ... The Josephson effect also provides a unique way to generate high-frequency electromagnetic (EM) radiation by dc bias voltage ... The discovery of cuprate high-Tc superconductors accelerated the effort to develop novel source of EM waves based on a stack of atomically dense-packed intrinsic Josephson junctions (IJJs), since the large superconductivity gap covers the whole terahertz (THz) frequency band. Very recently, strong and coherent THz radiations have been successfully generated from a mesa structure of  $\text{Bi}_2\text{Sr}_2\text{CaCu}_2\text{O}_{8+d}$  single crystal ... [ BSCCO image from Wikipedia



]

which works both as the source of energy gain and as the cavity for resonance. This experimental breakthrough posed a challenge to theoretical study on the phase dynamics of stacked IJJs, since the phenomenon cannot be explained by the known solutions of the sine-Gordon equation so far. It is then found theoretically that, due to huge inductive coupling of IJJs produced by the nanometer junction separation and the large London penetration depth ... of the material, a novel dynamic state is stabilized in the coupled sine-Gordon system, in which  $\pm \pi$  kinks in phase differences are developed responding to the standing wave of Josephson plasma and are

stacked alternately in the c-axis. This novel solution of the inductively coupled sine-Gordon equations captures the important features of experimental observations.

The theory predicts an optimal radiation power larger than the one observed in recent experiments by orders of magnitude ...".

### **What are some interesting BSCCO JJ Array configurations ?**

Christian Beck and Michael C. Mackey in astro-ph/0605418 describe "... the AC Josephson effect ...

a Josephson junction consists of two superconductors with an insulator sandwiched in between. In the Ginzburg-Landau theory each superconductor is described by a complex wave function whose absolute value squared yields the density of superconducting electrons. Denote the phase difference between the two wave functions ... by  $\phi(t)$ .

...

at zero external voltage a superconductive current given by  $I_s = I_c \sin(\phi)$  flows between the two superconducting electrodes ...  $I_c$  is the maximum superconducting current the junction can support.

...

if a voltage difference  $V$  is maintained across the junction, then the phase difference  $\phi$  evolves according to

$$d\phi / dt = 2 e V / \hbar$$

i.e. the current ... becomes an oscillating current with amplitude  $I_c$  and frequency  $\nu = 2 e V / h$

This frequency is the ... Josephson frequency ... The quantum energy  $h\nu$  ... can be interpreted as the energy change of a Cooper pair that is transferred across the junction ...".

Xiao Hu and Shi-Zeng Lin in arXiv 1206.516 said:

"... to enhance the radiation power in terahertz band based on the intrinsic Josephson Junctions of  $\text{Bi}_2\text{Sr}_2\text{CaCu}_2\text{O}_{8+d}$  single crystal ...

we focus on the case that the Josephson plasma is uniform along a long crystal as established by the cavity formed by the dielectric material. ...

A ...  $\pi$  kink state ... is characterized by static  $\pm\pi$  phase kinks in the lateral directions of the mesa, which align themselves alternately along the c-axis. The  $\pi$  phase kinks provide a strong coupling between the uniform dc current and the cavity modes, which permits large supercurrent flow into the system at the cavity resonances, thus enhances the plasma oscillation and radiates strong EM wave ...

The maximal radiation power ... is achieved when the length of BSCCO single crystal at c-axis equals the EM wave length. ...".

Each long BSCCO single crystal looks geometrically like a line so configure the JJ Array using BSCCO crystals as edges.

The simplest polytope, the **Tetrahedron**, is made of 6 edges:

Feigelman, Ioffe, Geshkenbein, Dayal, and Blatter in cond-mat/0407663 said: "... Superconducting tetrahedral quantum bits ..."

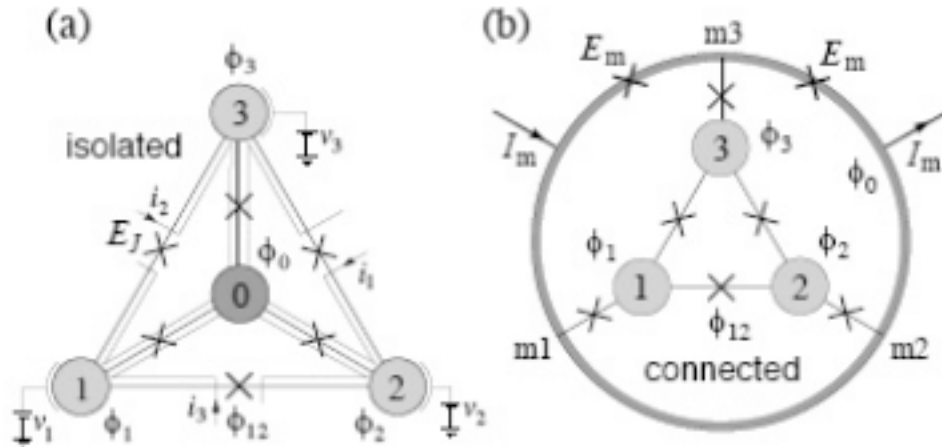
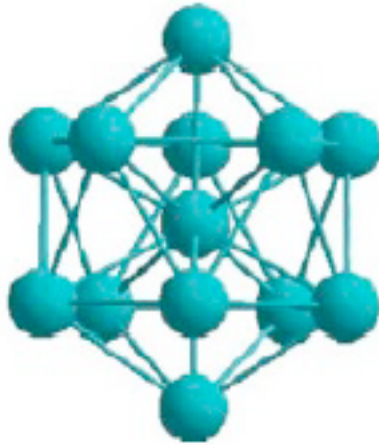


FIG. 1: (a) Tetrahedral superconducting qubit involving four islands and six junctions (with Josephson coupling  $E_J$  and charging energy  $E_C$ ); all islands and junctions are assumed to be equal and arranged in a symmetric way. The islands are attributed phases  $\phi_i, i = 0, \dots, 3$ . The qubit is manipulated via bias voltages  $v_i$  and bias currents  $i_i$ . In order to measure the qubit's state it is convenient to invert the tetrahedron as shown in (b) — we refer to this version as the 'connected' tetrahedron with the inner dark-grey island in (a) transformed into the outer ring in (b). The measurement involves additional measurement junctions with couplings  $E_m \gg E_J$  on the outer ring which are driven by external currents  $I_m$  (schematic, see Fig. 6 for details); the large coupling  $E_m$  effectively binds the ring segments into one island.

... tetrahedral qubit design ... emulates a spin-1/2 system in a vanishing magnetic field, the ideal starting point for the construction of a qubit. Manipulation of the tetrahedral qubit through external bias signals translates into application of magnetic fields on the spin; the application of the bias to different elements of the tetrahedral qubit corresponds to rotated operations in spin space. ...".

**42 edges make an Icosahedron plus its center**

(image from Physical Review B 72 (2005) 115421 by Rogan et al)



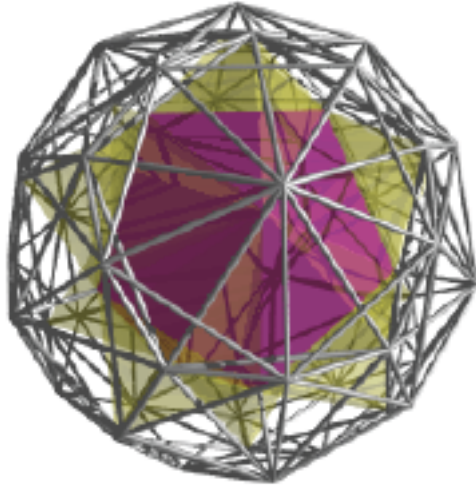
with 30 exterior edges and 12 edges from center to vertices.  
It has 20 cells which are approximate Tetrahedra in flat 3-space  
but become exact regular Tetrahedra in curved 3-space.

Could an approximate-20Tetrahedra-Icosahedron configuration  
of 42 BSCCO JJ tap into Dark Energy so that the Dark Energy  
might regularize the configuration to exact Tetrahedra and so  
curve/warp spacetime from flat 3-space to curved 3-space ?



## 720 edges make a 4-dimensional 600-cell

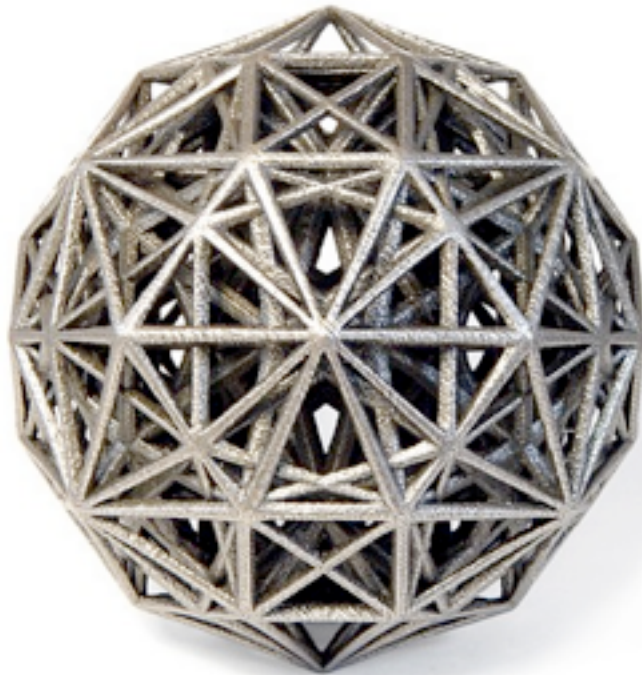
(image from Wikipedia)



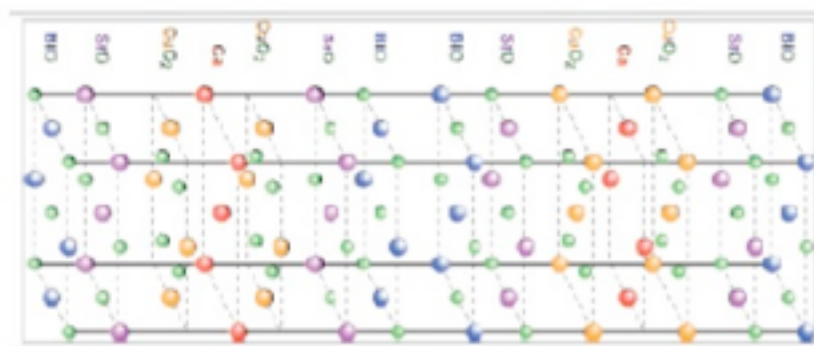
At each vertex 20 Tetrahedral faces meet forming an Icosahedron which is exact because the 600-cell lives on a curved 3-sphere in 4-space. It has 600 Tetrahedral 3-dim faces and 120 vertices

Could a 600 approximate-Tetrahedra configuration of 720 BSCCO JJ approximating projection of a 600-cell into 3-space tap into Dark Energy so that the Dark Energy might regularize the configuration to exact Tetrahedra and an exact 600-cell and so curve/warp spacetime from flat 3-space to curved 3-space ?

The basic idea of Dark Energy from BSCCO Josephson Junctions is based on the 600-cell as follows: Consider 3-dim models of 600-cell such as metal sculpture from Bathsbeba Grossman who says:  
"... for it I used an orthogonal projection rather than the Schlegel diagrams of the other polytopes I build.  
... In this projection all cells are identical, as there is no perspective distortion. ...".



For the Dark Energy experiment each of the 720 lines would be made of a single BSCCO crystal



whose layers act naturally to make the BSCCO crystal an intrinsic Josephson Junction. ( see Wikipedia and arXiv 0911.5371 )

Each of the 600 tetrahedral cells of the 600-cell has 6 BSCCO crystal JJ edges.

Since the 600-cell is in flat 3D space the tetrahedra are distorted.

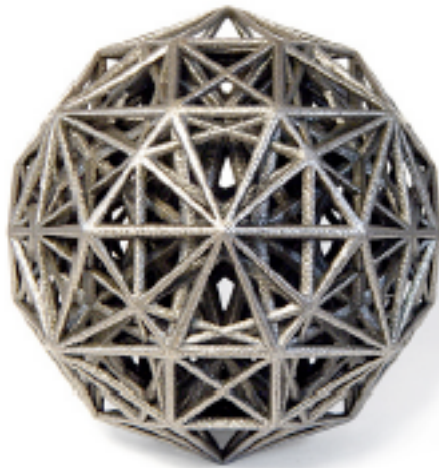
According to the ideas of Beck and Mackey ( astro--ph/0703364 ) and of Clovis Jacinto de Matos ( arXiv 0707.1797 ) the superconducting Josephson Junction layers of the 720 BSCCO crystals will bond with Dark Energy GraviPhotons that are pushing our Universe to expand.

My idea is that the Dark Energy GraviPhotons will not like being configured as edges of tetrahedra that are distorted in our flat 3D space and they will use their Dark Energy to make all 600 tetrahedra to be exact and regular by curving our flat space (and space-time).

My view is that the Dark Energy GraviPhotons will have enough strength to do that because their strength will NOT be weakened by the  $(1 / M_{\text{Planck}})^2$  factor that makes ordinary gravity so weak.

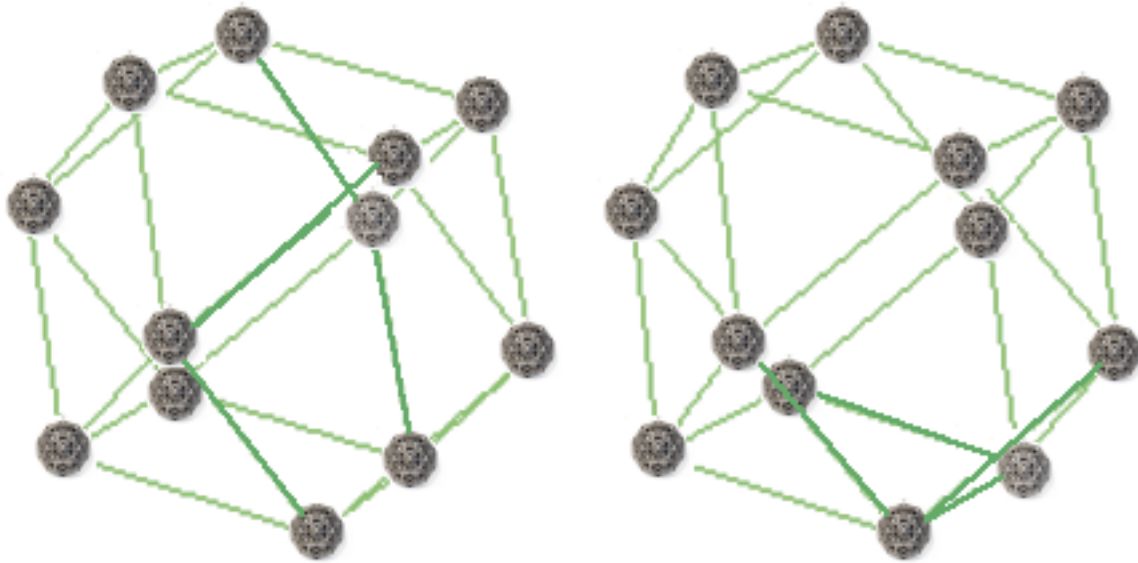
It seems to me to be a clearly designed experiment that will either  
1- not work and show my ideas to be wrong or  
2 - work and open the door for humans to work with Dark Energy.

Consider BSCCO JJ 600-cells

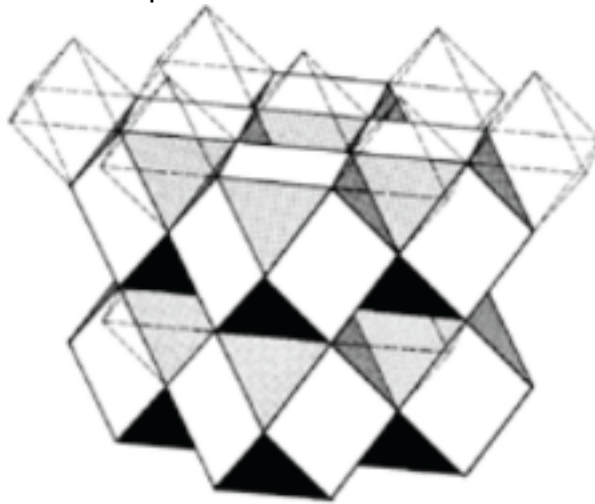


in this configuration:

First put 12 of the BSCCO JJ 600-cells at the vertices of a cuboctahedron shown here as a 3D stereo pair:

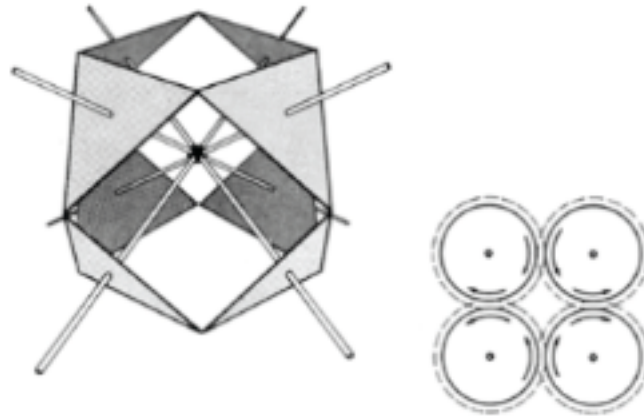


Cuboctahedra do not tile 3D flat space without interstitial octahedra

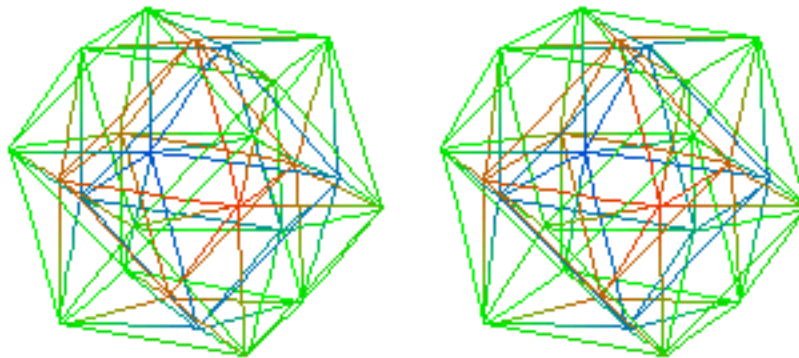


but BSCCO JJ 600-cell cuboctahedra can be put together square-face-to-square-face in flat 3D configurations including flat sheets.

As Buckminster Fuller described, the 8 triangle faces of a cuboctahedron



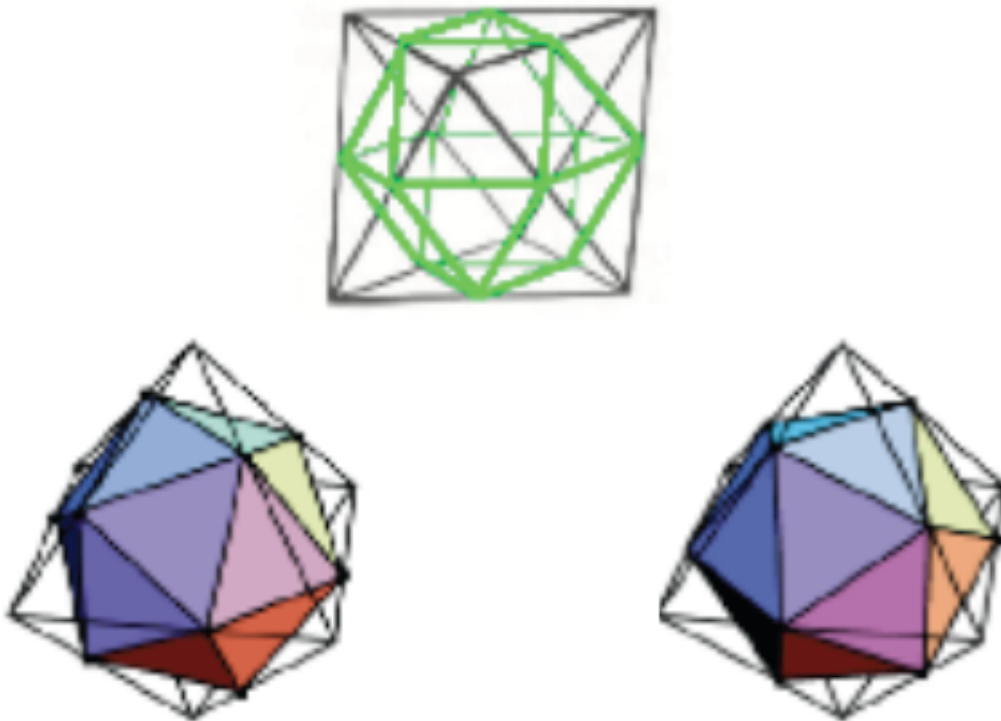
give it an inherently 4D structure consistent with the green cuboctahedron



central figure of a 24-cell (3D stereo 4thD blue-green-red color)  
that tiles flat Euclidean 4D space.

So, cuboctahedral BSCCO JJ 600-cell structure likes flat 3D and 4D space  
but  
if BSCCO JJ Dark Energy act to transform flat space into curved space  
like a 720-edge 600-cell with 600 regular tetrahedra  
then  
Dark Energy should transform cuboctahedral BSCCO JJ 600-cell structure  
into  
a 720-edge BSCCO JJ 600-cell structure that likes curved space.

There is a direct Jitterbug transformation of the 12-vertex cuboctahedron to the 12-vertex icosahedron

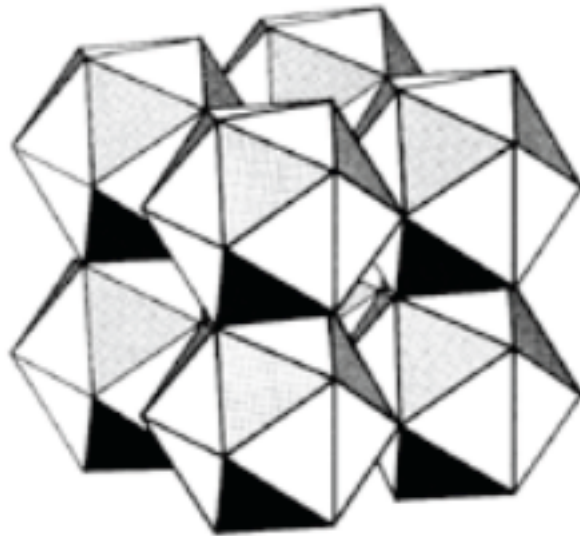


whereby the 12 cuboctahedron vertices as midpoints of octahedral edges are mapped to 12 icosahedron vertices as Golden Ratio points of octahedral edges. There are two ways to map a midpoint to a Golden Ratio point. For the Dark Energy experiment the same choice of mapping should be made consistently throughout the BSCCO JJ 600-cell structure.

The result of the Jitterbug mapping is that each cuboctahedron in the BSCCO JJ 600-cell structure with its 12 little BSCCO JJ 600-cells at its 12 vertices is mapped to an icosahedron with 12 little BSCCO JJ 600-cells at its 12 vertices

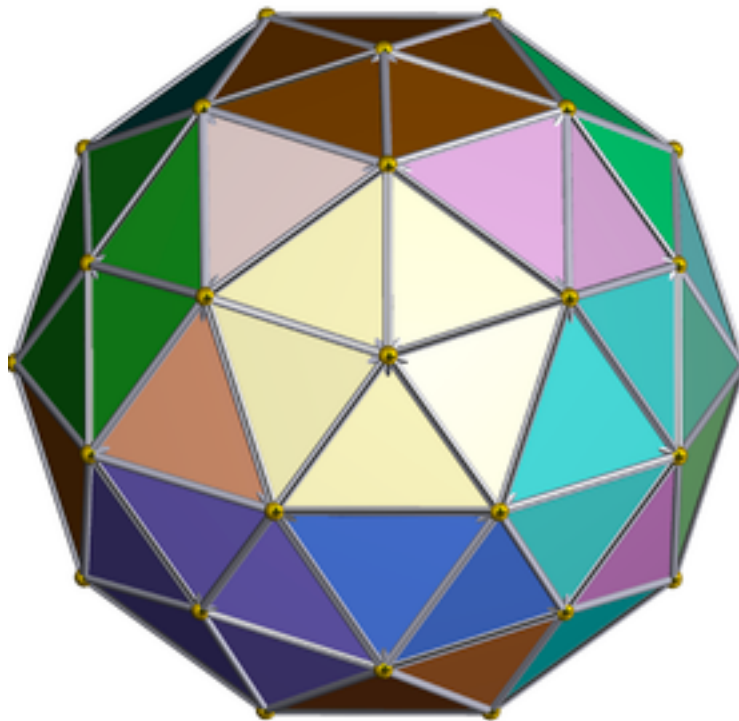


and the overall cuboctahedral BSCCO JJ 600-cell structure  
is transformed into  
an overall icosahedral BSCCO JJ 600-cell structure



does not fit in flat 3D space in a naturally characteristic way  
( This is why icosahedral QuasiCrystal structures do not extend as simply  
throughout flat 3D space as do cuboctahedral structures ).

However, the BSCCO JJ 600-cell structure Jitterbug icosahedra  
do live happily in 3-sphere curved space within the icosahedral 120-cell



which has the same 720-edge arrangement as the 600-cell ( see Wikipedia ).  
 The icosahedral 120-cell is constructed by 5 icosahedra around each edge.  
 It has:

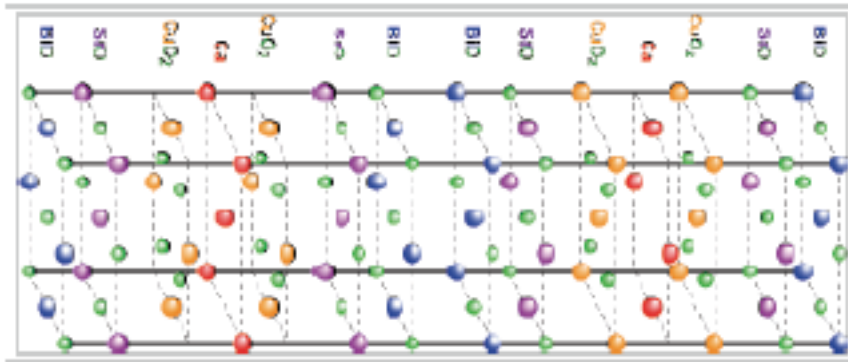
cells - 120 {3,5}  
 faces - 1200 {3}  
 edges - 720  
 vertices - 120  
 vertex figure - {5,5/2}  
 symmetry group  $H_4, [3,3,5]$   
 dual - small stellated 120-cell

In summary,

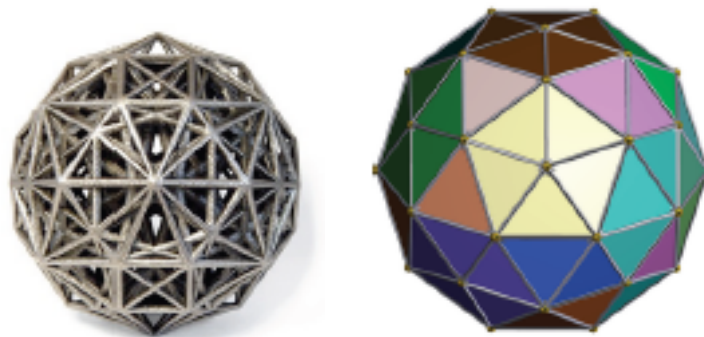
**Jitterbug transformations and BSCCO Josephson Junctions  
 may be the Geometric Key to controlling Dark Energy**

( as were Chain Reactions for Nuclear Fission and Ellipsoidal Focussing for H-Bombs )

The Energy Gap of our Universe as superconductor condensate spacetime  
 is from  $3 \times 10^{-18}$  Hz (radius of universe) to  $3 \times 10^{43}$  Hz (Planck length).  
 Its RMS amplitude is  $10^{13}$  Hz = 10 THz = energy of neutrino masses =  
 = critical temperature  $T_c$  of BSCCO superconducting crystals.



BSCCO superconducting crystals are natural Josephson Junctions.  
 Dark Energy accumulates in the superconducting layers of BSCCO.  
 The basic idea of Dark Energy from BSCCO Josephson Junctions is  
 based on the 600-cell each of whose 720 edge-lines would be made of a single BSCCO  
 crystal. It may be useful to use a Jitterbug-type transformation between a 600-cell  
 configuration and a configuration based on icosahedral 120-cells which also have 720  
 edge-lines:





## 21. 600-cell Geometry of Cl(16)-E8 Physics

Start by building a 600-cell from a 24-cell.

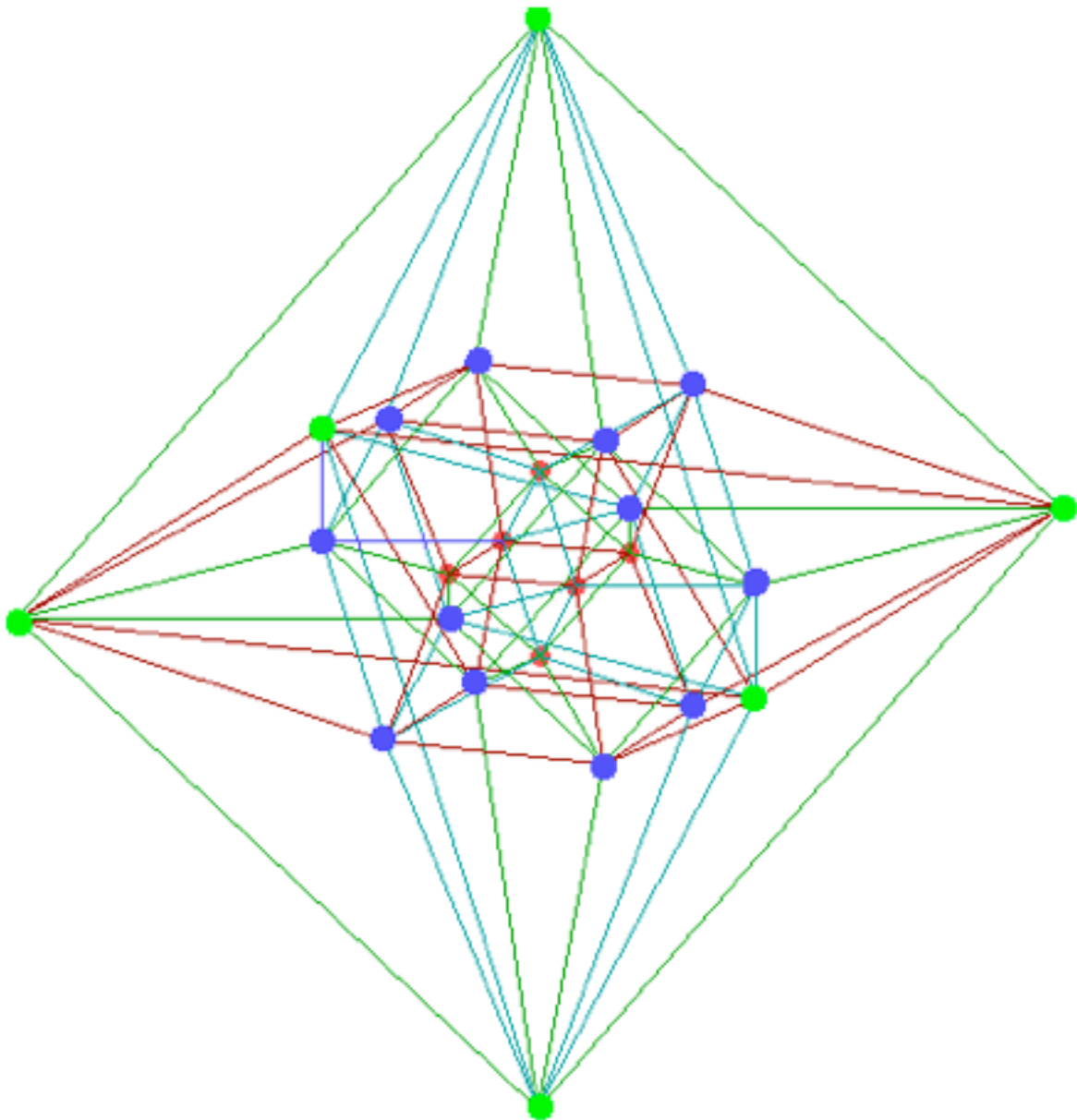
24-cell diagrams here are adapted from those of Frans Marcelis at

<http://members.home.nl/fg.marcelis/24-cell.htm#stereographic%20projection>

The 24-cell is made up of an Outer Octahedron (green),

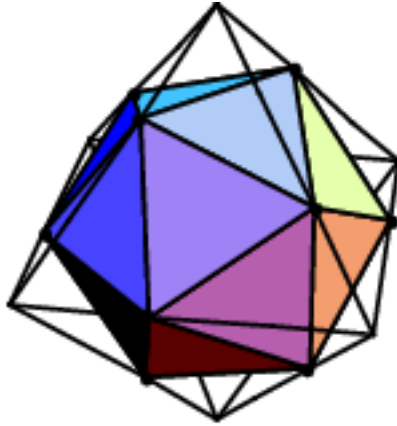
a Central Cuboctahedron (blue), and an Inner Octahedron (red).

Physically, it corresponds to the 24 Root Vectors of a D4 Gauge Group that can represent either Gravity + Dark Energy or the Standard Model.



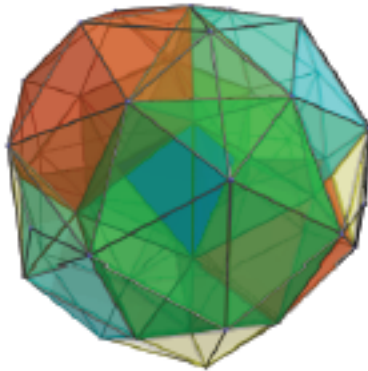
To build a 600-cell, first surround each of the 24 vertices with 5 Tetrahedra which gives you 120 of its 600 Tetrahedra.

Next, look at the 24 Octahedra that fill up the volume of the 24-cell.  
Each Octahedron contains an Icosahedron



(image from wolfram mathworld)

plus some extra volume in each Octahedron.  
The extra volume can be divided into 24 vertex Tetrahedra + 96 edge Tetrahedra  
so the 24-cell becomes a Snub 24-cell with 24 Icosahedra and 120 Tetrahedral cells

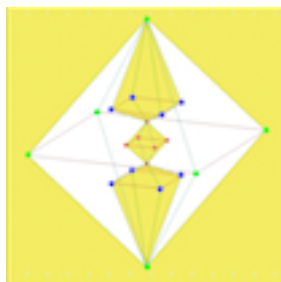
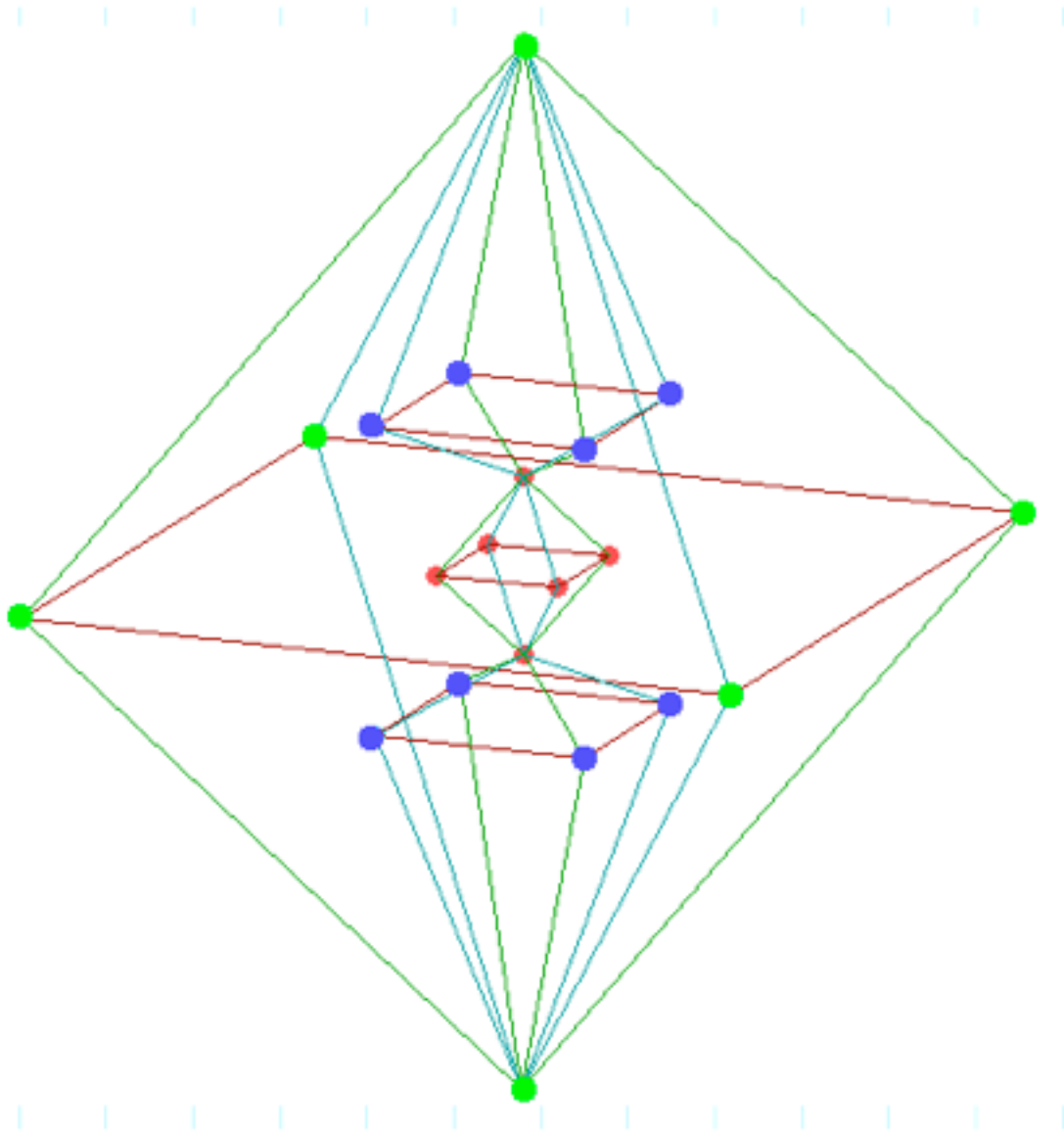


( image from eusebia.dyndns.org )

Each of the 24 Icosahedra contains 20 Tetrahedra for a total of 480 Tetrahedra  
which when added to the  $24+96 = 120$  Tetrahedra outside the Icosahedra

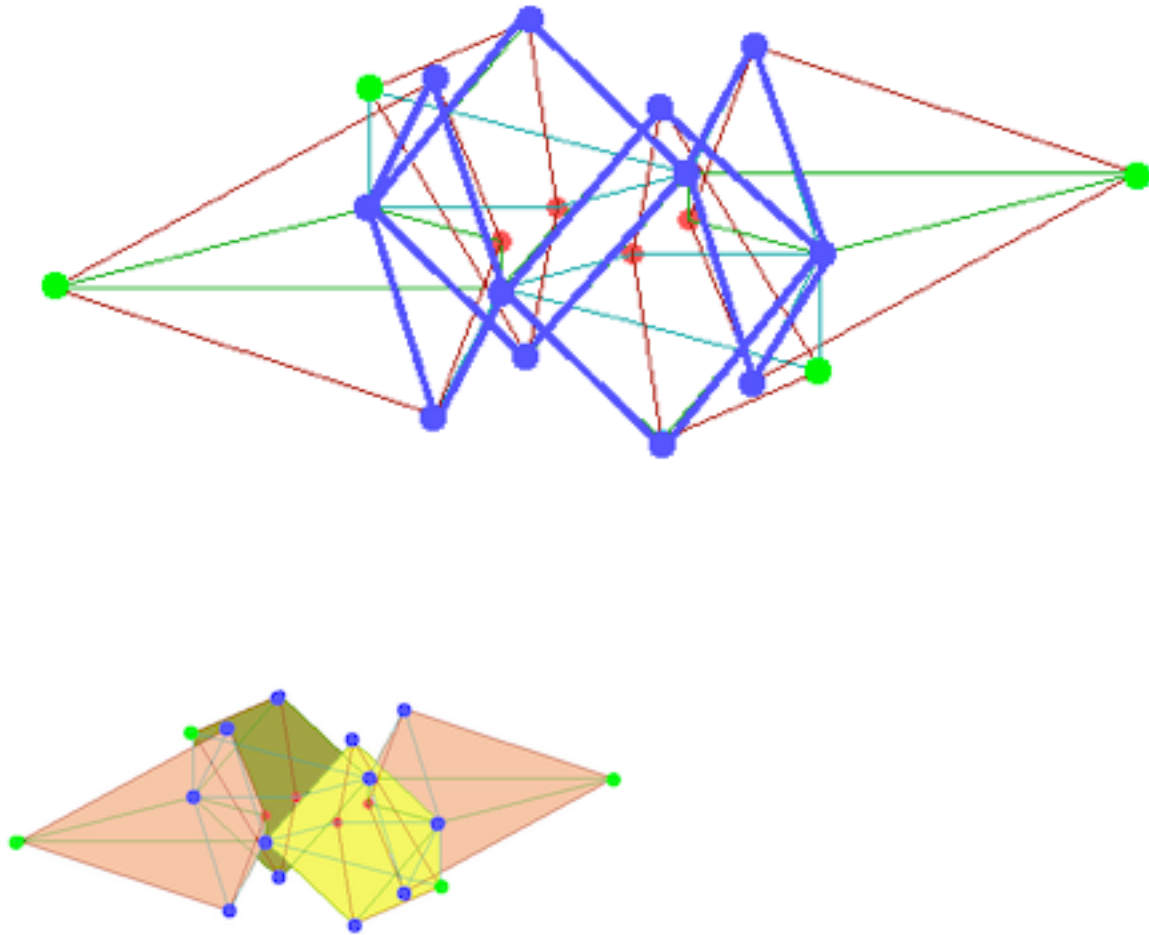
( Tetrahedra are only approximately regular in 3D space but become regular in 4D)  
give you the  $480+120 = 600$  Tetrahedra of the 600-cell.

These are 4 of the 24-cell Octahedra corresponding to 4-dim M4 physical spacetime within (4+4)-dim M4 x CP2 Kaluza-Klein of B4 / D4  
They account for  $4 \times 20 = 80$  of the 600-cell Tetrahedra.



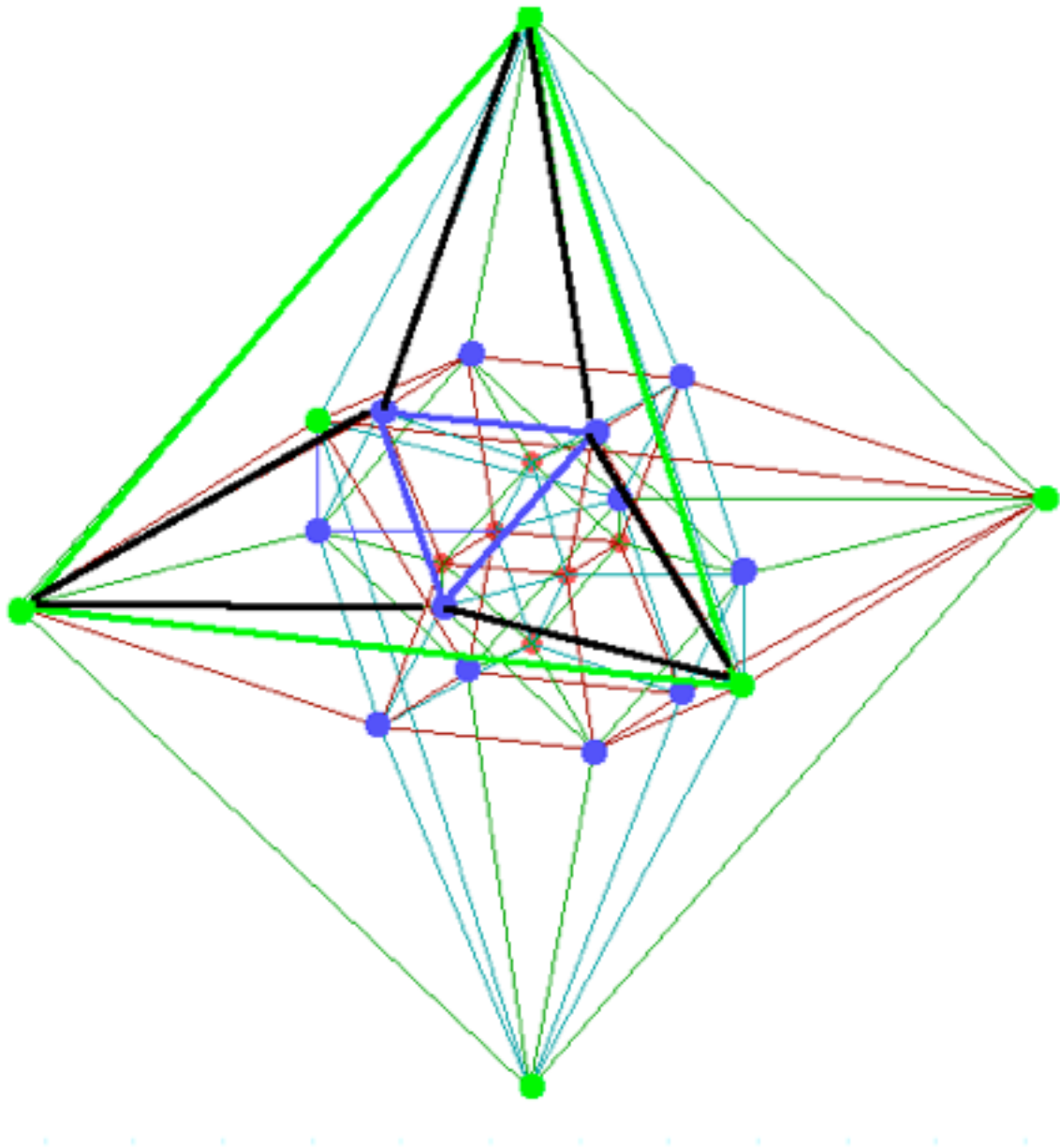
These are 4 of the 24-cell Octahedra corresponding to 4-dim CP2 internal symmetry space within the (4+4)-dim M4 x CP2 Kaluza-Klein of B4 / D4

They account for  $4 \times 20 = 80$  of the 600-cell Tetrahedra.



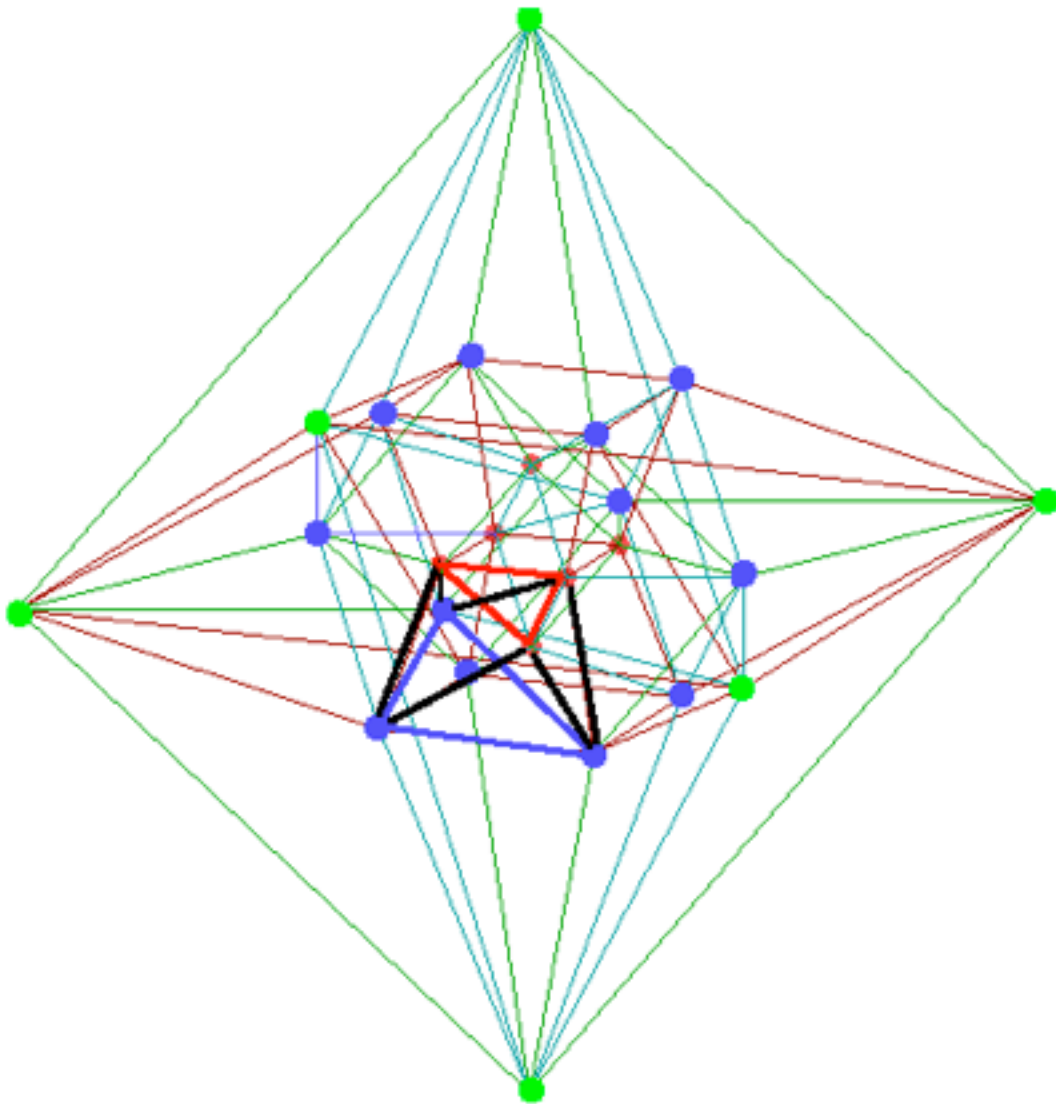
This is one of the 8 Octahedra corresponding to 8 fundamental Fermion Particles within the (8+8)-dim spinor-type Octonionic Projective Plane of F4 / B4. The other 7 are similarly configured on each of the other 7 faces of the outer (green) Octahedron.

These 8 account for  $8 \times 20 = 160$  of the 600-cell Tetrahedra.



This is one of the 8 Octahedra corresponding to 8 fundamental Fermion AntiParticles within the (8+8)-dim spinor-type Octonionic Projective Plane of F4 / B4. The other 7 are similarly configured on each of the other 7 faces of the inner (red) Octahedron.

These 8 account for  $8 \times 20 = 160$  of the 600-cell Tetrahedra.



That is all  $600 = 120 + 80 + 80 + 160 + 160 = 280 + 320$  Tetrahedra of the 600-cell corresponding to Cl(16)-E8 Physics through the structure of F4 as follows:

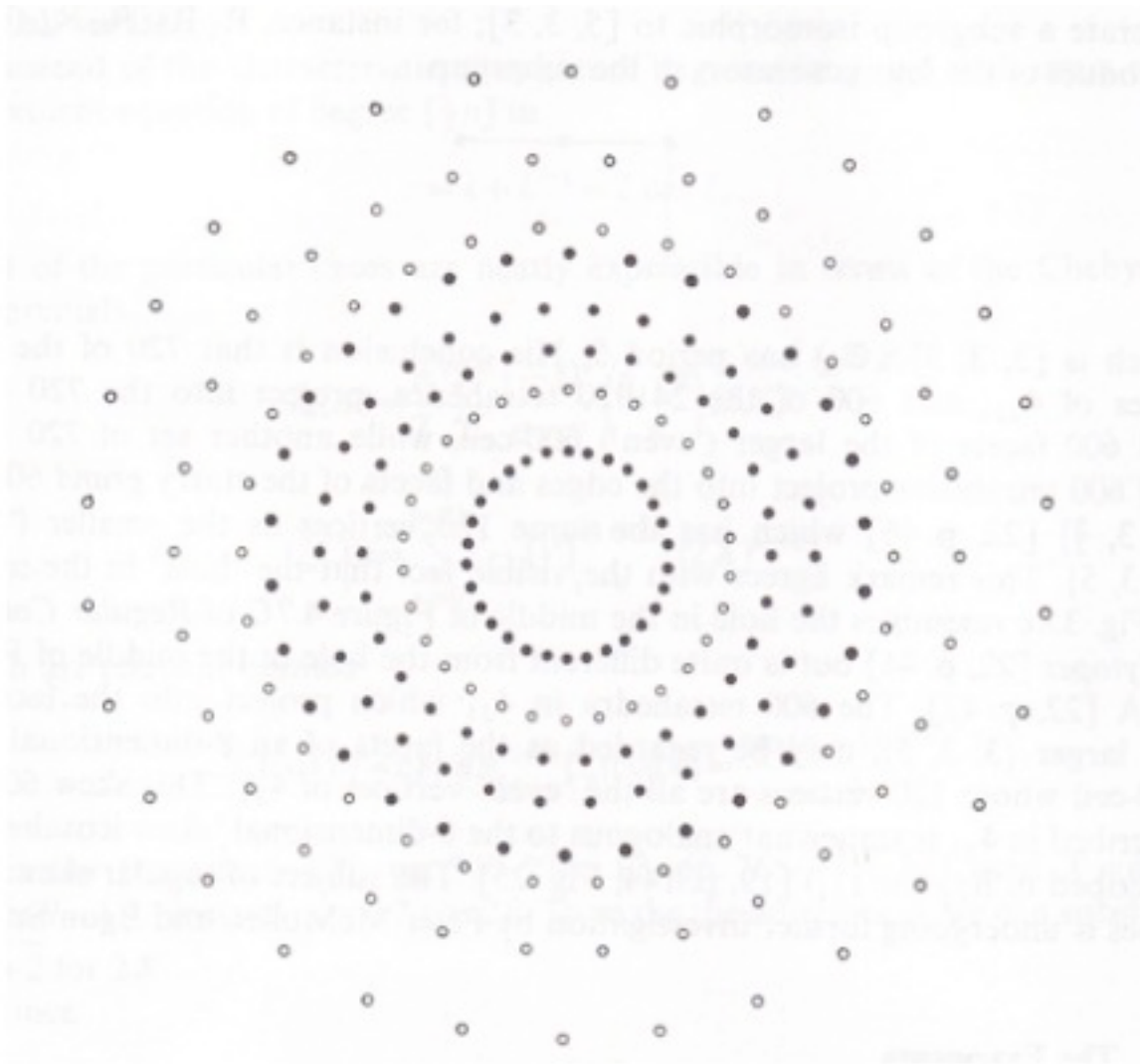
120 to 28-dim D4 for SU(2,2) Gauge Gravity or 28-dim D4 for Standard Model SU(3)  
 80 + 80 to B4 / D4 for (4+4)-dim Kaluza-Klein M4 x CP2  
 160 +160 to F4 / B4 for (8+8)-dim OP2 spinor fermions

Here is another way to look at 600-cells with respect to  $Cl(16)$ -E8 Physics:

The 240 Root Vectors of E8 can be represented by two 120-vertex 600-cells.  
One 600 cell corresponds to 4-dim Minkowski SpaceTime and to Gravity + Dark Energy  
and to 4 of the 8 coordinates of each Fermion Particle and AntiParticle  
while  
the other 600 cell corresponds to  $CP^2$  Internal Symmetry Space and to Standard Model  
Gauge Bosons  
and to the other 4 of the 8 coordinates of each Fermion Particle and AntiParticle.

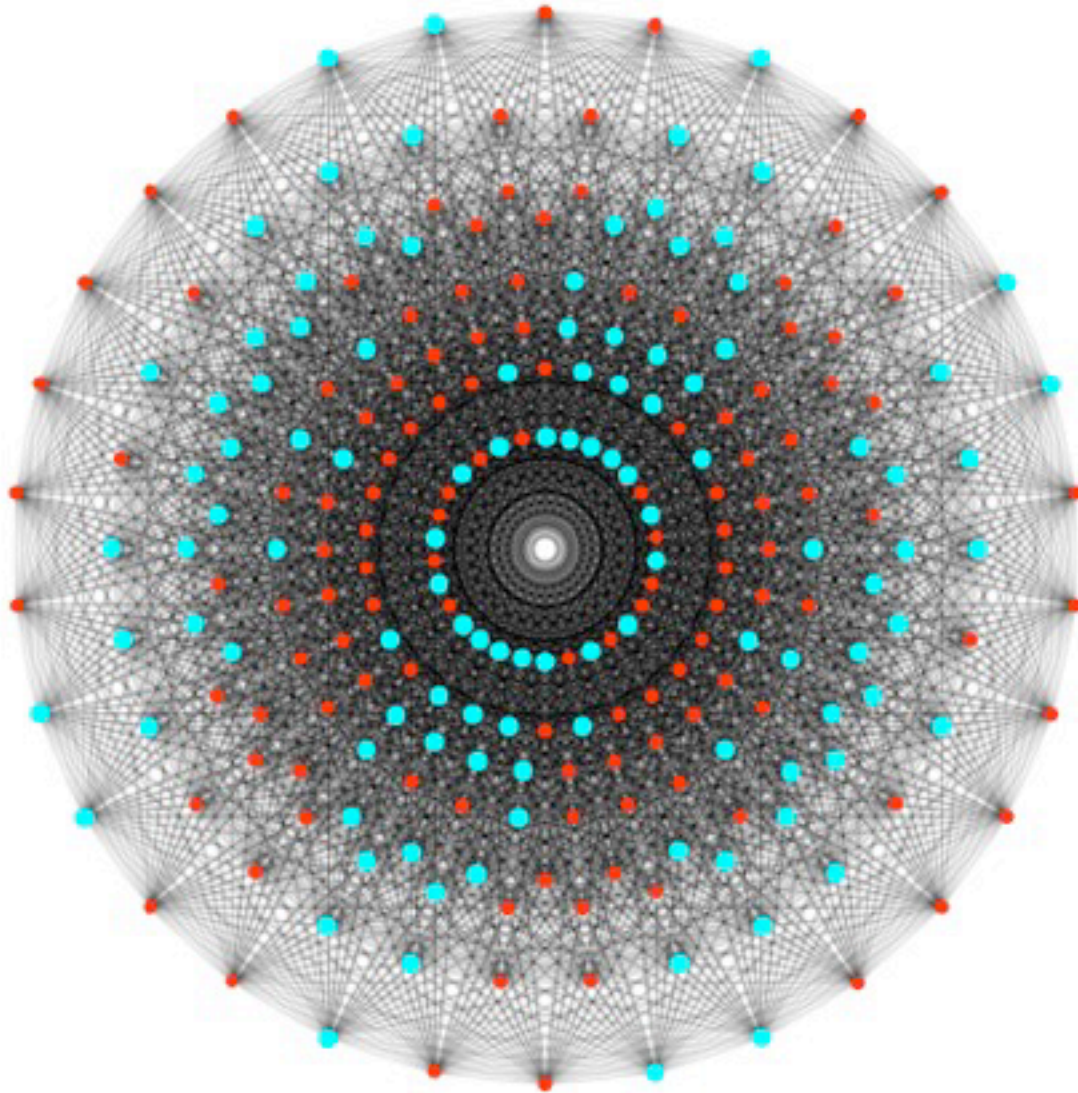
Here is a more detailed discussion:

The 240 Root Vectors of E8 can be projected onto 2D as 8 circles of 30 vertices each  
as shown in this diagram from Regular and Semi-Regular Polytopes III by Coxeter



in which there are 4 circles of white dots and 4 circles of black dots  
with the 120 white-dots being like the 120 black dots expanded by the Golden Ratio.

The 120+120 is not the division into Spacetime+GaugeBosons and Fermion Particles. That division is shown by the 240 Root Vectors of E8 being projected onto 2D as 8 circles of 30 vertices each as shown in this diagram from <http://www.madore.org/~david/math/e8w.html> in which there are 112 large dots (colored cyan) and 128 small dots (colored red).





The 112 large cyan dots correspond to the D8 subalgebra of E8 which represents SpaceTime and Gauge Bosons

The 128 small red dots correspond to Fermion Particles and AntiParticles

The circles break down like this:

inner - black dots	- 18 SpaceTime Gauge Boson and 12 Fermion
second from center - white dots	- 10 SpaceTime Gauge Boson and 20 Fermion
third from center - black dots	- 10 SpaceTime Gauge Boson and 20 Fermion
fourth from center - black dots	- 10 SpaceTime Gauge Boson and 20 Fermion
fifth from center - black dots	- 18 SpaceTime Gauge Boson and 12 Fermion
sixth from center - white dots	- 18 SpaceTime Gauge Boson and 12 Fermion
seventh from center - white dots	- 18 SpaceTime Gauge Boson and 12 Fermion
eighth from center (outer) - white dots	- 10 SpaceTime Gauge Boson and 20 Fermion

There are  $4 \times 12 + 4 \times 20 = 128$  Fermion small red dots

and

$4 \times 18 + 4 \times 10 = 112$  SpaceTime Gauge Boson large cyan dots

The black-dot 4 circles of the small 600-cell contain

56 SpaceTime Gauge Boson cyan dots and 64 Fermion red dots.

You can take the small 600-cell to correspond to M4 4D physical spacetime

so that

24 of the 56 give Gauge Bosons for Gravity + Dark Energy

and

32 of the 56 give 4 M4 spacetime components of 8-dim Momentum

and

32 of the 64 give 4 M4 spacetime components of 8 fundamental Fermion Particles

and

32 of the 64 give 4 M4 spacetime components of 8 fundamental Fermion AntiParticles

The white-dot 4 circles of the large (by Golden ratio) 600-cell also contain

56 SpaceTime Gauge Boson cyan dots and 64 Fermion red dots.

You can take the small 600-cell to correspond to CP2 4D Internal Symmetry Space

so that

24 of the 56 give Gauge Bosons for Standard Model SU(3)

and

32 of the 56 give 4 CP2 internal symmetry space components of 8-dim Momentum

and

32 of the 64 give 4 CP2 internal symmetry components of 8 Fermion Particles

and

32 of the 64 give 4 CP2 internal symmetry components of Fermion AntiParticles

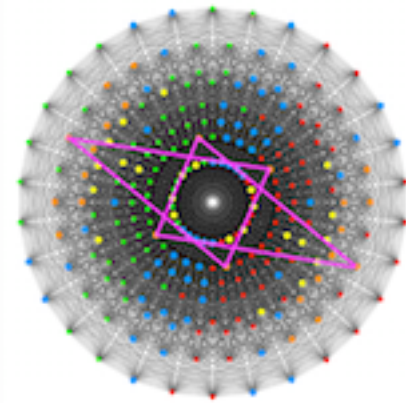
In more detail:

There are 24 yellow dots ( + 4 Cartan Elements) for Gravity + Dark Energy:

- 8 in black circle 1 (inner) =  $M4 \times CP2$  (4+4)-dim Kaluza-Klein
- 2 in black circle 3
- 2 in black circle 4
- 12 in black circle 5 = Conformal  $SU(2,2) = Spin(2,4)$  Root Vectors

There are 24 orange dots ( + 4 Cartan Elements) for the Standard Model:

- 4 in white circle 2
- 10 in white circle 6
- 2+8 = 10 in white circle 7
- 0 in white circle 8 (outer)



4+2 = 6 Standard Model  $SU(3)$  Root Vectors

There are 64 green dots for Fermion Particles:

- 6 in black circle 1 (inner)
- 10 in black circle 3
- 10 in black circle 4
- 6 in black circle 5
- 10 in white circle 2
- 6 in white circle 6
- 6 in white circle 7
- 10 in white circle 8 (outer)

There are 64 red dots for Fermion AntiParticles:

- 6 in black circle 1 (inner)
- 10 in black circle 3
- 10 in black circle 4
- 6 in black circle 5
- 10 in white circle 2
- 6 in white circle 6
- 6 in white circle 7
- 10 in white circle 8 (outer)

There are 64 blue dots for SpaceTime:

10 in black circle 1 (inner)

8 in black circle 3

8 in black circle 4

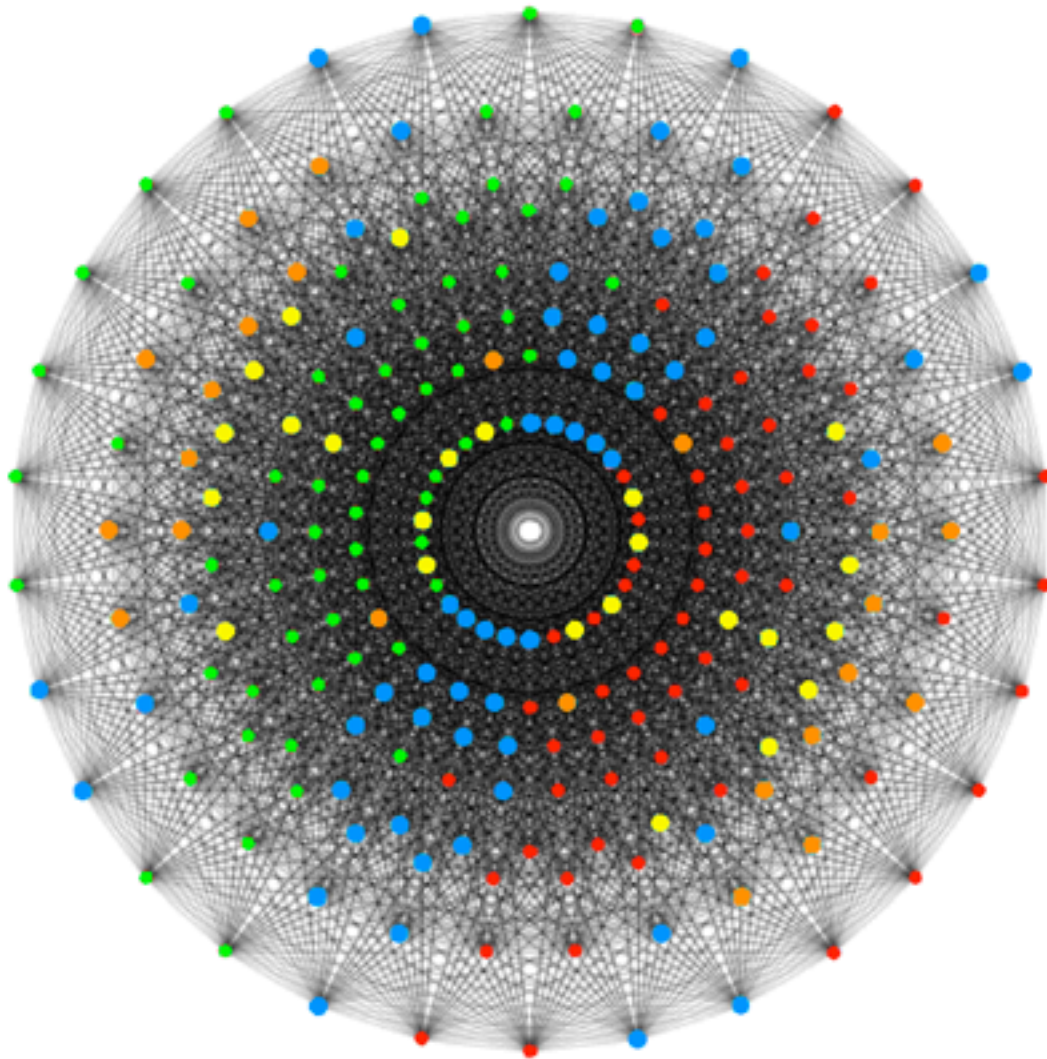
6 in black circle 5

6 in white circle 2

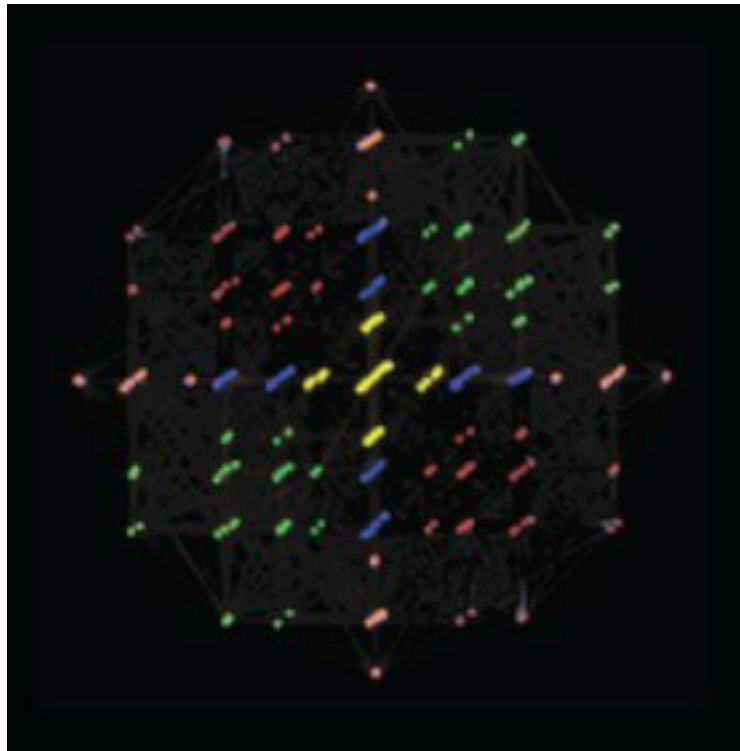
8 in white circle 6

8 in white circle 7

10 in white circle 8 (outer)

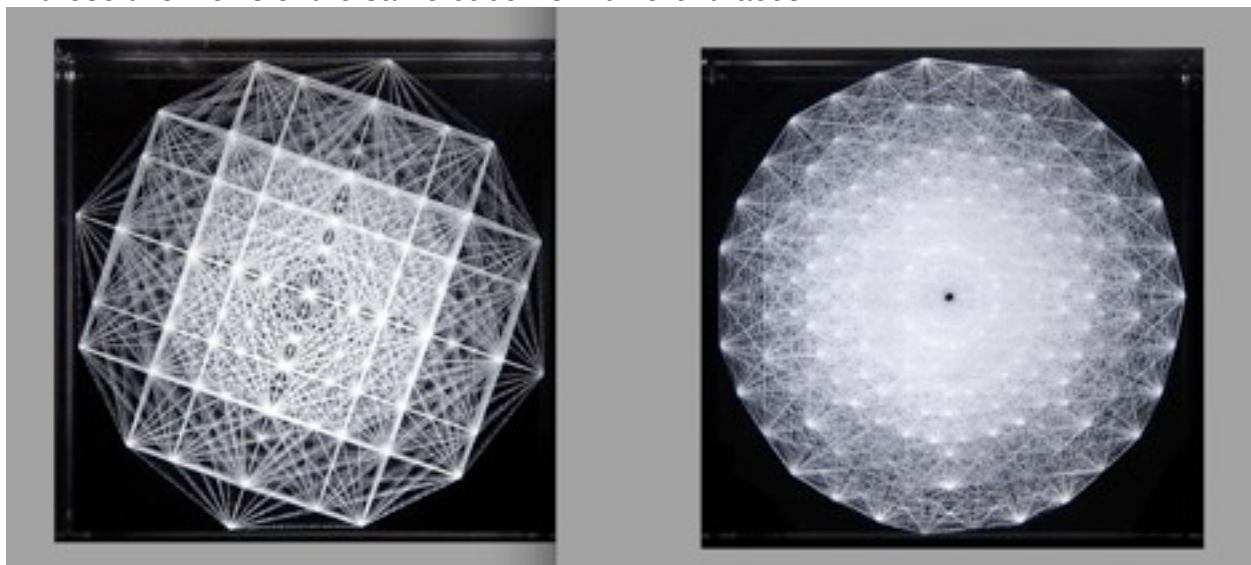


All this works out nicely but the 8-circles-of-30 picture is not as intuitively clear in terms of physical interpretation which is why in this paper I use a more square-type projection



to explain physical interpretation.

As the Bathsheba Grossman E8 crystal cube makes clear in these two views of the same cube from different faces



the square and circle projections are of the same E8 and so give the same physics.