Concept of the Effective Mass Tensor in the GR Gravitomagnetism

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Abstract: *The concept of the effective mass tensor in the General Relativity we discussed in the aspect of the gravitomagnetism. Model predicts the violation in the mass ratio* $\left|\frac{m_I^*}{m_g^*}\right| = 2\frac{v}{c}$ *during the planet year, i.e. measured from the periastron to the periastron. For the planet Earth the predicted violation in the mass ratio is of the order* $6.6 \cdot 10^{-6}$ *during the year, i.e. measured from the perihelion to perihelion.*

Introduction

Gravitomagnetism (GM) refers to a set of formal analogies between the equations for electromagnetism and the General Relativity (GR), specifically: between Maxwell's field equations and the linearized Einstein's field equations in GR [1-3].

In this paper we discussed the concept of *the effective mass tensor* (EMT) in the GR [4] in the aspect of the GM.

GM in EMT world

In the EMT world the Einstein's field equations will becomes *the modified Einstein's field equations* in the form:

$$\tau_{\mu\nu}^* - \frac{1}{2}\frac{m_{\mu\nu}}{m}\tau^* = 8\pi T_{\mu\nu}^* \tag{1}$$

where: $\tau_{\mu\nu}^* = \frac{c^4}{G}R_{\mu\nu}^*$ we will call *the effective energy tensor*, which should include full information regarding the existence of any surrounding fields, the term $\frac{c^4}{G}$ is *the Planck force*, $\tau^* = \frac{c^4}{G}R^*$ we will call *the bare energy*, *c* speed of light, *G* gravitational constant, tensor $T_{\mu\nu}^*$ is *the energy - momentum tensor* in EMT world. The stress–energy tensor $T_{\mu\nu}^*$ is a source of the effective energy.

In the weak gravitational field we can decompose EMT $m_{\mu\nu}$ to a simple form: $m_{\mu\nu} = m_{\mu\nu}^{bare} + m_{\mu\nu}^*$, where: $m_{\mu\nu}^{bare}$ is *the bare mass tensor* and $m_{\mu\nu}^{bare} = m \cdot \eta_{\mu\nu} = \mathrm{diag}(-m,+m,+m,+m)$, $\eta_{\mu\nu}$ is *the Minkowski tensor*, *m* is the bare mass of the body, $m_{\mu\nu}^* = m \cdot \left|h_{\mu\nu}\right| << 1$ is *the small perturbation* [4]. The modified Einstein's field equations in the weak gravitational field has now the form:

$$\left(-\frac{1}{c^2}\frac{\partial^2}{\partial t^2}+\nabla^2\right)\left(\frac{\overline{m}^*_{\mu\nu}}{m}\right)=-\frac{16\pi G}{c^4}T^*_{\mu\nu} \tag{2}$$

where: $\overline{m}^*_{\mu\nu}=m^*_{\mu\nu}-\frac{1}{2}h\cdot\eta_{\mu\nu}$, $h=\mathrm{tr}(m_{\mu\nu})$ and the Lorentz gauge condition $\partial_\nu \overline{m}^{\mu\nu}=0$ has been imposed. The tensor $T^*_{\mu\nu}$ describes the distribution of the matter, which disturbs the gravitational field. Now we will try discuss the equation (2) for the rest and the slowly moving dust.

The rest dust

For the source of the gravitational field like the dust the stress-energy tensor has the form

$$T^*_{\mu\nu}=T_{\mu\nu}=\rho u_\mu u_\nu \tag{3}$$

where ρ is the mass density and u_μ is the four-vector velocity of the dust. When the dust is in the rest the stress-energy tensor has a form

$$T_{00}=\rho c^2 \tag{4}$$

The equation (2) takes the form:

$$\left(-\frac{1}{c^2}\frac{\partial^2}{\partial t^2}+\nabla^2\right)\left(\frac{\overline{m}^*_{00}}{m}\right)=\frac{8\pi G}{c^2}\rho \tag{5}$$

Assume that the expression $\overline{m}^*_{00}$ describes the small perturbations in *the effective gravitational mass* m^*_g and additionally we assume that $m^*_g=\overline{m}^*_{00}$.

The moving dust

For the slowly moving dust the stress-energy tensor has form $T_{\mu\nu}=\rho u_\mu u_\nu$, what gives $T_{00}=\rho c^2$ and $T_{0i}=cj_i$ respectively, $\boldsymbol{j}=\rho\mathbf{v}$ is *the matter current* (the momentum density), $\mathbf{v}$ is a velocity, and $|\mathbf{v}| << c$. The equation (2) takes the form:

$$\left(-\frac{1}{c^2}\frac{\partial^2}{\partial t^2}+\nabla^2\right)\left(\frac{\overline{m}^*_{0i}}{m}\right)=\frac{16\pi G}{c^3}j_i \tag{6}$$

Assume that the expression $\overline{m}^*_{0i}$ describes the small perturbations in *the effective inertial mass* and additionally we assume that $m^*_{Ii}=\overline{m}^*_{0i}$.

Finally we have two field equations for the small perturbation in the gravitational and inertial effective mass in the form:

$$\left(-\frac{1}{c^2}\frac{\partial^2}{\partial t^2}+\nabla^2\right)\left(\frac{m_g^*}{m}\right)=\frac{8\pi G}{c^2}\rho$$

$$\left(-\frac{1}{c^2}\frac{\partial^2}{\partial t^2}+\nabla^2\right)\left(\frac{\boldsymbol{m}_I^*}{m}\right)=\frac{16\pi G}{c^3}\boldsymbol{j} \tag{7}$$

The field equations (7) are consistent with the spirit of Mach's principle, which states that the effective mass of the body (gravitational and inertial) is a result of interactions between the body and all (resting or moving) other bodies (in the Universe).

Is this was what was looking for A. Einstein in their field equations? Please note that the small perturbation in the effective gravitational mass *is the scalar* but the small perturbation in the effective inertial mass *is the vector*.

For the static field and for the planet moving around the star, the field equations (7) takes the form

$$\nabla^2\left(\frac{m_g^*}{m}\right)=\frac{8\pi G}{c^2}\rho$$

$$\nabla^2\left(\frac{\boldsymbol{m}_I^*}{m}\right)=\frac{16\pi G}{c^3}\boldsymbol{j} \tag{8}$$

with the solution

$$\frac{m_g^*}{m}=-\frac{2G}{c^2}\int\frac{\rho dV}{r}$$

$$\frac{\boldsymbol{m}_I^*}{m}=-\frac{4G}{c^3}\int\frac{\boldsymbol{j}\,dV}{r} \tag{9}$$

Let's assume that there exists relations between the effective gravitational mass m_g^* and the scalar gravitational potential V in the form

$$\frac{m_g^*}{m}=\frac{2V}{c^2} \tag{10}$$

where: $V=-\frac{GM}{r}$, r is the distance between planet and star, with mass *M*. Similarly, let's assume that there exists relations between the effective inertial mass $\boldsymbol{m}_I^*$ and the vector gravitational potential $\boldsymbol{A}$ in the form

$$\frac{\boldsymbol{m}_I^*}{m}=-\frac{4\boldsymbol{A}}{c} \tag{11}$$

where: $\boldsymbol{A} = \frac{GM}{c^2 r}\boldsymbol{v}$, v is the linear velocity of the planet. Both relations, (see equations (10) and (11)), show the relationship between the EMT world and GM.

Does the effective inertial mass equal with the effective gravitational mass?

It is a very important question in the our model. From the equations (9) now we calculate the mass ratio

$$\left|\frac{m_I^*}{m_g^*}\right| = \frac{2}{c}\frac{\left|\int \frac{\boldsymbol{j}\,dV}{r}\right|}{\int \frac{\rho dV}{r}} \tag{12}$$

In the particular case the mass ratio has form

$$\left|\frac{m_I^*}{m_g^*}\right| = 2\frac{v}{c} \tag{13}$$

We can see that the mass ratio in equation (13) depends on the planet linear velocity v. If orbit of the planet is a circle then the mass ratio is fixed and does not change during the planet year. But if the orbit of the planet is a ellipse then the mass ratio is change during the planet year: the largest value of changes is in the periastron, the smallest is in apastron.

For the planet Earth the linear velocity in the perihelion is 30.3 km/s and in the aphelion is 29.3 km/s, therefore the expected value changes in the mass ratio is of the order

$$\left|\frac{m_I^*}{m_g^*}\right|_{perihelion} - \left|\frac{m_I^*}{m_g^*}\right|_{aphelion} = 6.6\cdot 10^{-6} \tag{14}$$

during the year, *i.e.* from the perihelion to perihelion.

Conclusion

We solved equation (1) for the weak and the stationary field and we get equations (9). For the planet which is moving in the ellipse orbit around the star, the model **predicts the violation in the mass ratio of the order**

$$\left|\frac{m_I^*}{m_g^*}\right| = 2\frac{v}{c}$$

during the planet year, *i.e.* measured from the periastron to the periastron. For the planet Earth the predicted violation in the mass ratio is of the order

$$\left|\frac{m_I^*}{m_g^*}\right|_{perihelion} - \left|\frac{m_I^*}{m_g^*}\right|_{aphelion} = 6.6\cdot 10^{-6}$$

during the year, *i.e.* measured from the perihelion to perihelion.

This predicted violation in the mass ratio for the planet Earth should be measurable.

References

[1]. B. Mashhoon, *Gravitoelectromagnetism: A Brief Review*, http://arxiv.org/abs/gr-qc/0311030v2 .
[2]. B. Mashhoon, *Gravitoelectromagnetism*, http://arxiv.org/abs/gr-qc/0011014v1 .
[3]. B. Mashhoon, F. Gronwald and H. I. M. Lichtenegger, *Gravitomagnetism and the Clock Effect*, http://arxiv.org/abs/gr-qc/9912027v1 .
[4]. M. J. Kubiak, *Concept of the Effective Mass Tensor in GR. Field Equations*, http://vixra.org/abs/1404.0053 .