Steer by Logic

Einstein's online physics challenge to today's academics — steering clear of false obstacles on the road ahead —

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FQXi 2014 asks, 'How should humanity steer the future?' Recalling false obstacles to medical progress in humanity's recent past — eg, impeding Semmelweis (b.1818), McClintock (1902), Marshall (1951) — we reply, 'Steer by Logic.' Then — with Logic in view and other scientific disciplines in mind — we amplify our answer via an online coaching-clinic/challenge based on Einstein's work. With the future mostly physical, this physics-based challenge shows how we best steer clear of false obstacles — unnecessary barriers that slow humanity's progress. Hoping to motivate others to participate, here's our position: we locate current peer-reviewed claims of 'impossible' — like those from days of old — and we challenge them via refutations and experimental verifications. The case-study identifies an academic tradition replete with 'impossibility-proofs' — with this bonus: many such 'proofs' are challengeable via undergraduate maths and logic. So — at the core of this clinic/challenge; taking maths to be the best logic — we model each situation in agreed mathematical terms, then refute each obstacle in like terms. Of course, upon finding 'impossibilities' that are contradicted by experiments, our next stride is easy: at least one step in such analyses must be false. So — applying old-fashioned commonsense; ie, experimentally verifiable Logic — we find that false step and correct it. With reputable experiments agreeing with our corrections, we thus negate the false obstacles. Graduates of the clinic can therefore more confidently engage in steering our common future: secure in the knowledge that old-fashioned commonsense — genuine Logic — steers well.

[1] Notes to the Reader

> 'In the interest of clearness, it appeared to me inevitable that I should repeat myself frequently, without paying the slightest attention to the elegance of presentation,' Einstein (1916). May this essay bring you many happy hours of fun and critical thinking.

z. History: Submitted to FQXi 's 2014 Essay Contest. Rejected: FQXi's "initial review team has found that your essay fails to meet the minimal standard for publication required by the contest rules, due to a lack of connection to the contest topic." NB: Lightly edited for posting at viXra.org.
a. Pre-requisites: A passion to learn physics! Elementary trig, the nature of functions, will help.
b. Pre-reading: Available online; EPR to the first paragraph on page 778; Bell (1964) to equation (15). Additional hyperlinks are available at Appendix [B]; eg, Goldstein et al. (2011).
c. Notation: In trig functions, $(\mathbf{u}, \mathbf{v})$ = angle between vectors $\mathbf{u}$, $\mathbf{v}$. $\mathbf{u}\cdot\mathbf{v}$ = their inner product.
d. Technical notes: See Appendix [A].
e. Policy: We make weak allowances; take maths to be the best logic; seek experimental validations.
f. Results: Requiring no loopholes, all results here accord with reputable experimental findings.
g. Errors: Please report errors, typos, etc; critical correspondence is especially welcome.
h. Key words: CLR, commonsense local realism; DECs, dynamic equivalence classes; function Q.

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[2] Introduction

In that this essay introduces a plan for an ongoing online coaching-clinic/challenge, readers should freely avail themselves of the accompanying ~~FQXi~~ viXra.org facility for questions/answers, discussion, etc. Past experience shows that — for the benefit of all — simple questions often lead to helpful expansions. Also, to be clear, the coaching-clinic is for all: the challenge-aspect is primarily applicable to academics who publish or hold contrary views.

Further, reader-engagement can often order the priority for expansions. Our own views may also require conversion; eg, to the established languages of maths, physics, social-science. Given our therapeutic-community approach here, subsequent expanded essays will be freely available.

So, hopefully with many minds at ease, we commence with a typical engineering approach: Allowing that natural physical variables and their local interactions alone account for this classical mantra — correlated tests on correlated things produce correlated results, without mystery — we let these natural physical variables (unlike *observables)* be *beables* (after Bell 2004:174): elements of reality, things which exist, their existence independent of *measurement* and *observation.*

As for observables — dynamic physical variables; test outcomes labelled by real numbers — we allow that the beables here may be revealed by observables (by tests/interactions), and confirmed by robust physical experiments and tests.

Coupling Einstein-locality with EPR's *condition of completeness* and their definition of beables (their *elements of physical reality*), we have our fundamental CLR (pronounced *clear*) principles:

> On one supposition we absolutely hold fast; that of local-causality, often called Einstein-locality: "The real *factual* situation of the system S_2 is independent of what is done with the system S_1, which is spatially separated from the former," after Einstein (1949:85).
>
> "Every [relevant] element of the physical reality must have a counterpart in our physical theory," EPR (1935:777).
>
> "If, without any way disturbing a system, we can predict with certainty (ie, with probability equal to unity) the value of a physical quantity, then there exists an element of physical reality [a beable] corresponding to this physical quantity," EPR (1935:777).

Formally, the foregoing specifications together constitute the commonsense local realism (CLR) that, under Einstein's guidance, we bring to any physical situation. For CLR represents the fusion of local-causality (no causal influence propagates superluminally) and physical-realism (some physical properties change interactively).

Under CLR, we now model EPRB: ie, EPR plus B for Bohm (1951:611-623); see also Bell (1964).

$$\downarrow \quad \text{Alice's domain} \quad \downarrow \qquad \downarrow \text{Source} \downarrow \qquad \downarrow \quad \text{Bob's domain} \quad \downarrow$$

$$A^{\pm}=\pm1=(\mathbf{a}\cdot\lambda)\{\pm\mathbf{a}\leftarrow\lambda]\Leftarrow p(\lambda)\cdot\langle\lambda+\lambda'=0\rangle\cdot p'(\lambda')\Rightarrow[\lambda'\rightarrow\pm\mathbf{b}'\}(\lambda'\cdot\mathbf{b}')=\pm1=B^{\pm}. \tag{1}$$

(1) shows the model coded as an equation because it equates perfectly with EPRB — *nothing relevant missing, nothing irrelevant found* — every relevant element of the subject physical reality having its counterpart in the theory.

In (1), with its *gedanken* (mind's eye) *block-time* view, we see a spin-conserving decay $\langle\lambda+\lambda'=0\rangle$ giving birth to twinned spin-half particles which fly apart $\Leftarrow p(\lambda)\cdot\langle\cdot\rangle\cdot p'(\lambda')\Rightarrow$ en-route to their destiny with *gedanken* Stern-Gerlach devices (*SGD*s): function-machines, each built from a squeeze-function, eg $[\lambda\rightarrow\pm\mathbf{a}\}\equiv Q(\pm\mathbf{a})$ and a response-function, eg $(\lambda\cdot\mathbf{a})\equiv R(\mathbf{a})$. Printed outputs $(A^{\pm}=\pm1 : B^{\pm}=\pm1)$ record each Up:Down output; appropriately observed by Alice:Bob.

See [A] for technical notes. Re our use of primes ($'$) see [A].1. Re λ, λ' see [A].2. For R's role as a diagnostic-function: from reporting the output of an *SGD* to analyzing its behavior; see [A].3.

We next show our policy at work: modeling situations in agreed mathematical terms as above, then analyzing each obstacle in like terms. Thus, triggered by Mermin (1988), but in the context of EPRB per (1) above — the experiment in Bell (1964) — we arrive at Bell's theorem.

[3] Bell's theorem

> λ may denote "any number of hypothetical additional complementary variables needed to complete quantum mechanics in the way envisaged by EPR," Bell (2004:242).

Here's a wholly mathematical version of Bell's theorem; Bell (1964:(1)-(3), (12)-(14); 2004:14-21):

$$\textbf{If } A(\lambda, \mathbf{a}) = A^{\pm} = \pm 1;\ B(\lambda', \mathbf{b}') = B^{\pm} = \pm 1 = -A(\lambda, \mathbf{b}'); \int d\lambda\, \rho(\lambda) = 1: \tag{2}$$

$$\textbf{Then } \langle AB \rangle \equiv \int d\lambda\, \rho(\lambda) A(\lambda, \mathbf{a}) B(\lambda', \mathbf{b}') = -\int d\lambda\, \rho(\lambda) A(\lambda, \mathbf{a}) A(\lambda, \mathbf{b}') \neq -\mathbf{a}\cdot\mathbf{b}'; \tag{3}$$

with Bohm and Aharonov (1957) cited as Bell's example. To avoid confusion with later functions, our expectation $\langle AB \rangle$ — the average of AB — replaces Bell's equivalent expression $P(\vec{a}, \vec{b})$.

Introduced with the 'not possible' in the line below his 1964:(3), and based on the paragraph below his 1964:(15), the $\neq$ in (3) is Bell's famous inequality.

However, for us, the identity in LHS (3) is a valid representation of the subject reality (from high-school maths), so the source of Bell's questionable $\neq$ lies elsewhere: to be found. Detective-like, we first turn to those unnumbered equations following Bell 1964:(14). There we refute Bell's analysis: from fundamental first principles — the CLR custom — and thus beyond dispute.

[4] Bell's 1964 analysis refuted

To derive his 1964:(15), Bell goes beyond our (2)-(3) and invokes a third unit-vector $\mathbf{c}$ in unnumbered equations that follow his 1964:(14). If we number them Bell's (14a) to Bell's (14c), Bell — suspiciously, in our view — equates (14b) to (14a).

Since A, B, C are discrete, let's replace Bell's integrals with sums and Bell's 1964:(14a) with discrete variables. For generality, let λ be a random variable in $\mathbb{R}^3$; with a uniform distribution and consequent probability zero that two λs or λ's or two particle-pairs are the same. Then, with index i uniquely numbering each pair, let n be such that, to an adequate accuracy hereafter:

$$\text{Bell's (14a)} = \langle AB \rangle - \langle AC \rangle = -\frac{1}{n}\sum_{i=1}^{n}[A(\mathbf{a}, \lambda_i)A(\mathbf{b}', \lambda_i) - A(\mathbf{a}, \lambda_{n+i})A(\mathbf{c}, \lambda_{n+i})] \tag{4}$$

$$= \frac{1}{n}\sum_{i=1}^{n} A(\mathbf{a}, \lambda_i)A(\mathbf{b}', \lambda_i)[A(\mathbf{a}, \lambda_i)A(\mathbf{b}', \lambda_i)A(\mathbf{a}, \lambda_{n+i})A(\mathbf{c}, \lambda_{n+i}) - 1]. \tag{5}$$

(5) is the correct discrete form of Bell's (14a). And Bell's (14c) is a valid conclusion from his (14b). So, if Bell's (14b) = Bell's (14a), the related components of (5) and Bell's (14c) should be equal. Let $\stackrel{?}{=}$ identify our suspicion of Bell's equality under these conditions. Then,

$$\text{from Bell's (14c): } \langle BC \rangle \equiv -\frac{1}{n}\sum_{i=1}^{n} A(\mathbf{b}', \lambda_i)A(\mathbf{c}, \lambda_i) = -\frac{1}{n}\sum_{i=1}^{n} A(\mathbf{b}', \lambda_{n+i})A(\mathbf{c}, \lambda_{n+i}) \tag{6}$$

$$\stackrel{?}{=} -\frac{1}{n}\sum_{i=1}^{n} A(\mathbf{a}, \lambda_i)A(\mathbf{b}', \lambda_i)A(\mathbf{a}, \lambda_{n+i})A(\mathbf{c}, \lambda_{n+i});\ \text{from (5).} \tag{7}$$

To have Bell's (14a) = Bell's (14b) — and remove our **?** from (7) — we'd require $\lambda_i = \lambda_{n+i}$: the impossible. Impossible because by definition, physical context, and from Bell's own λ-license: $\lambda_i \neq \lambda_{n+i}$. So here's a new — and the first valid — Bell-inequality:

$$\text{Bell 1964 : (14b)} \neq \text{Bell 1964 : (14a)} \tag{8}$$

$$\because \frac{1}{n}\sum_{i=1}^{n} A(\mathbf{a},\lambda_i)A(\mathbf{b}',\lambda_i)A(\mathbf{a},\lambda_{n+i})A(\mathbf{c},\lambda_{n+i}) \neq \frac{1}{n}\sum_{i=1}^{n} A(\mathbf{b}',\lambda_i)A(\mathbf{c},\lambda_i) \tag{9}$$

in general. Of course, a stable cohort of n classical objects — like Bertlmann's socks (Bell 2004:139-158) — would allow a non-destructive test and a follow-up non-destructive retest on that n-member cohort. For such an $n+i$ would denote another run of n tests on the same set as the i-series of tests; and in the same order. Then (7) and (9) would be unfettered equalities. For then

$$A(\mathbf{a},\lambda_i)A(\mathbf{b}',\lambda_i)A(\mathbf{a},\lambda_{n+i})A(\mathbf{c},\lambda_{n+i}) = A(\mathbf{a},\lambda_i)A(\mathbf{b}',\lambda_i)A(\mathbf{a},\lambda_i)A(\mathbf{c},\lambda_i) = A(\mathbf{b}',\lambda_i)A(\mathbf{c},\lambda_i). \tag{10}$$

Thus classical objects like socks (not quantum objects) satisfy Bell's inequality. So we have here the source of those famous (but soon to be shown erroneous) inequalities in (3) and elsewhere. The source too of the error in the CHSH family of inequalities (see also Bell 1980:14), as we show next.

[5] CHSH inequality refuted

Based on Peres' (1995:164) version of the CHSH (1969) inequality, let A_j, B_j, C_j, D_j independently equal ± 1 randomly. Then, in our terms — see (4) and [A].2 — the following conditional truism does not hold in real tests under EPRB:

$$\text{ie, } A_j(B_j - D_j) + C_j(B_j + D_j) \equiv \pm 2 \text{ does not ensure that} \tag{11}$$

$$A_iB_i + B_{n+i}C_{n+i} + C_{2n+i}D_{2n+i} - A_{3n+i}D_{3n+i} = \pm 2 \text{ [sic]}; \tag{12}$$

$$\text{nor that } \mid \langle A_iB_i\rangle + \langle B_{n+i}C_{n+i}\rangle + \langle C_{2n+i}D_{2n+i}\rangle - \langle A_{3n+i}D_{3n+i}\rangle \mid \leq 2 \text{ [sic]}; \tag{13}$$

for (12) is false over particle-pairs indexed by $wn+i$; see [A].2. [With SGD settings denoted by unit-vectors $\mathbf{a}, \mathbf{b}, \mathbf{c}, \mathbf{d}$; (13) is false in the domain $-\pi/3 < \phi < \pi/3$ if $(\mathbf{a}, \mathbf{b}) = (\mathbf{b}, \mathbf{c}) = (\mathbf{c}, \mathbf{d}) = \phi$ and $(\mathbf{a}, \mathbf{d}) = 3\phi$: after (20) below. And false over more that 75% of the range $-\pi < \phi < \pi$.]

Of course, under the same conditions that deliver equality (10), truism (11) will hold. But real experiments are conducted under LHS (12) and LHS (13): refuting such analyses as (11).

So — though naive realism and Bertlmann's socks will wash in (11) — all naively-realistic EPR-based Bell inequalities similarly fall to our CLR particle-by-particle analysis: thanks to that family of unique twins $p(\lambda_{wn+i}), p'(\lambda'_{wn+i})$. And, thanks to them and their first-principle examples, we now move to refute Bell's theorem.

[6] Bell's theorem refuted

Pedagogy moving us to take the opposite tack to Bell in (2) — to show the utility of CLR; it makes no difference to the results — let's here focus on λ'. Allowing λ' to be a random beable uniformly distributed over $\mathbb{R}^3$, λ' will be perturbed by $p'(\lambda')$'s interaction with Bob's $SGD(\mathbf{b}')$: for "each [pristine] particle, considered separately, *is* unpolarized here," Bell (2004:82).

Representing that interaction by $[\lambda' \to \pm\mathbf{b}'\}$, we find $\lambda' \sim \pm\mathbf{b}'$ equiprevalently (ie, with equal prevalence): $\sim$ denoting an equivalence relation on Λ under the function $[\lambda' \to \pm\mathbf{b}'\}$; per [A].3.

So, expanding (2) in our terms — see (14)-(16) next — then using LHS (3) to make (17):

$$A(\lambda, \mathbf{a}) = [\lambda \to \pm\mathbf{a}\}(\lambda{\cdot}\mathbf{a}) = \pm\mathbf{a}{\cdot}\mathbf{a} = \pm 1, \tag{14}$$

$$B(\lambda', \mathbf{b}') = [\lambda' \to \pm\mathbf{b}'\}(\lambda'{\cdot}\mathbf{b}') = (\pm\mathbf{b}'){\cdot}\mathbf{b}' = \pm 1, \tag{15}$$

$$\int d\lambda\, \rho(\lambda) = 1. \tag{16}$$

$$\therefore \langle AB\rangle = \int d\lambda\, \rho(\lambda)[\lambda \to \pm\mathbf{a}\}(\lambda{\cdot}\mathbf{a})[\lambda' \to \pm\mathbf{b}'\}(\lambda'{\cdot}\mathbf{b}'); \tag{17}$$

The function-set of Q- and R-functions under the integral is therefore

$$\{[\lambda \to \pm\mathbf{a}\}\,(\lambda{\cdot}\mathbf{a})\,[\lambda' \to \pm\mathbf{b}'\}\,(\lambda'{\cdot}\mathbf{b}')\} \tag{18}$$

Now, working with functions, any Q may be applied to any element in its domain, in any order, to derive $\langle AB\rangle$. However, since there is just one independent variable in EPRB — from $\lambda + \lambda' = 0$ — one Q is superfluous. So, focussing on λ', the progressively reduced function-sets after (18) are:

$$\{(-\lambda'{\cdot}\mathbf{a})\,[\lambda' \to \pm\mathbf{b}'\}\,(\lambda'{\cdot}\mathbf{b}')\} \Rightarrow \{(-\lambda'{\cdot}\mathbf{a})\,[\lambda' \to \pm\mathbf{b}'\}\,(\pm 1)\} \Rightarrow \{(\mp\mathbf{b}'\cdot\mathbf{a})\,(\pm 1)\} \Rightarrow \{-\mathbf{b}'{\cdot}\mathbf{a}\}: \tag{19}$$

or, equivalently, completing (17):

$$\langle AB\rangle = \int d\lambda\, \rho(\lambda)\,(-\lambda'{\cdot}\mathbf{a})[\lambda' \to \pm\mathbf{b}'\}\lambda'{\cdot}\mathbf{b}' = \int d\lambda\, \rho(\lambda)(\pm 1)(\mp\mathbf{b}'){\cdot}\mathbf{a} = -\mathbf{b}'{\cdot}\mathbf{a} = -\mathbf{a}{\cdot}\mathbf{b}'.\ \text{QED:}\ \blacksquare \tag{20}$$

Bell's theorem — represented in (3) consistent with Bell's formulation — is refuted.

In passing: Since the outputs of (18)-(20) are identical, we see that Q eliminates the need for normalizing integrals in expressions like (20): for Q is a normalizing function when, as here, its arguments are normalized; ie, with Z denoting EPRB: $P(\lambda' \to +\mathbf{b}' \mid Z) = P(\lambda' \to -\mathbf{b}' \mid Z) = {}^1\!/_2$.

(20) is the first in a series of correct CLR *disentanglements*. That series includes GHZ (1989), GHSZ (1990), CRB (1991). But before showing CLR's utility in that department at [9] — via Mermin's (1990; 1990a) 3-particle GHZ-variant — we next extend the refutation in [4] above; this time using continuous variables.

[7] Bell's 1964:(15) refuted

Given (20) — and based on Bell's erroneous (14a) = (14b); see [4]— Bell's 1964:(15) reads:

$$1 + \langle BC\rangle = 1 - \mathbf{b}{\cdot}\mathbf{c} \geq |\mathbf{a}{\cdot}\mathbf{c} - \mathbf{a}{\cdot}\mathbf{b}| = |\,\langle AB\rangle - \langle AC\rangle\,|: \tag{21}$$

But (21) is a false relation (under CLR/EPRB and in general) in the domain $-{}^{\pi}\!/_2 < \phi < {}^{\pi}\!/_2$ if $(\mathbf{a}, \mathbf{b}) = (\mathbf{b}, \mathbf{c}) = \phi$ and $(\mathbf{a}, \mathbf{c}) = 2\phi$. So Bell's 1964:(15) is refuted as a generality.

Instead, Bell 1964:(15) is a typical Bellian relation restricted by Bell's acceptance of naive realism; per d'Espagnat's (1979; 1979a): exemplified by Bell's (1980) use of Bertlmann's socks.

Despite these constraints, Bell (1964:199) concludes:

> "In a theory in which parameters [sic] are added to quantum mechanics to determine the results of individual measurements, without changing the statistical predictions, there must be a mechanism whereby the setting of one measuring device can influence the reading of another instrument, however remote. Moreover, the signal involved must propagate instantaneously, so that such a theory could not be Lorentz-invariant."

To the contrary, we will show that a CLR counter-conclusion prevails:

> In a theory in which hidden properties, revealed by tests, are found to determine the results of individual interactions: there must be a function that accurately tracks the factual inferential consequences of such tests without changing the statistical predictions. Such a theory will be Lorentz-invariant. CLR is such a theory.

Thus, our case against Bell clear, we next refute one of Bell's false opinions.

[8] Bell's 'statistical independence' refuted

> "One general issue raised by the debates over locality is to understand the connection between stochastic independence (probabilities multiply) and genuine physical independence (no mutual influence). It is the latter that is at issue in 'locality,' but it is the former that goes proxy for it in the Bell-like calculations. We need to press harder and deeper in our analysis here," Arthur Fine, in Schlosshauer (2011:45).

In that CLR is devoid of subjective beliefs and non-physical entities, we take 'probable' and its derivatives to be loaded terms here; though we have no problem with technical terms like impossible, probability zero; or certain, probability one. However, to minimize confusion, we allow that P denotes the *normalized prevalence* (aka *objective probability*).

With Z denoting EPRB, let $P(AB = +1|Z)$ denote the *normalized prevalence* of $AB = +1$ given Z. Then, equating (20) to the standard prevalence relation for binary (± 1) outcomes:

$$\langle AB \rangle = -\mathbf{a}{\cdot}\mathbf{b}' = (+1)P(AB = +1|Z) + (-1)[1 - P(AB = +1|Z)]. \tag{22}$$

$$\therefore P(AB = +1|Z) = (1 - \mathbf{a}{\cdot}\mathbf{b}')/2 = \sin^2 \tfrac{1}{2}(\mathbf{a}, \mathbf{b});\ P(AB = -1|Z) = \cos^2 \tfrac{1}{2}(\mathbf{a}, \mathbf{b}). \tag{23}$$

$$\therefore P(A^+B^+|Z) \neq P(A^+|Z)\, P(B^+|Z);\text{ etc.}, \tag{24}$$

when A^+ and B^+ are causally independent; ie, causally independent in the sense that neither exerts any direct causal influence on the other.

That is, just like the apple and pear crop, we expect a dynamic (and hence a mathematico-logical) connection because of the common-cause physical correlation between them. Just as here, with our Q, we expect DECs to be related because of the physical correlations between closely-related (here, twinned) particles.

In this way (from first principles), we refute Bell's opinion (2004:243) and his move there from his (9) to his (10): that *causal independence* should equate to *statistical independence*, seen as a consequence of *local causality*.

Thus, derived from first principles, (24) responds to Fine's urgings and delivers this result: Given EPRB-style physical correlations, *statistical independence* does not equate to *causal independence* under *local causality*: nor with apple and pear crops. Rather, like apple and pear crops, there is a physical correlation and hence a consequential dynamical (and therefore a mathematico-logical) relation between them. Just as, with our Q, we have physical correlations and consequent equivalence relations in our maths/logic.

However, in full accord with reciprocal causal independence and local-causality (ie, no causal influence propagates superluminally), two CLR boundary conditions follow: Causally independent of $SGD(\mathbf{b}'), B^{\pm}, \lambda'$: $A^{\pm}$ may be causally dependent on any property of $SGD(\mathbf{a})$ or λ. Causally independent of $SGD(\mathbf{a}), A^{\pm}, \lambda$: $B^{\pm}$ may be causally dependent on any property of $SGD(\mathbf{b}')$ or λ'.

With (24) another sound result from first principles — and therefore beyond dispute — we finally demonstrate Q's utility in analyzing and disentangling multiparticle experiments.

[9] Understanding Mermin's 3-particle experiment

> Einstein argues that 'EPR correlations can be made intelligible only by completing the quantum mechanical account in a classical way,' after Bell (2004:86). Let's see.

Consider experiment M: Mermin's (1990; 1990a) 3-particle GHZ-variant. Respectively: Three spin-half particles with spin beables λ, μ, ν emerge from a spin-conserving decay such that

$$\lambda + \mu + \nu = \pi. \tag{25}$$

Any pristine beable may thus be represented in terms of its siblings — eg, as (25) is used below in the reduction (29)-(30) or in the transition (31)-(32) — thereby allowing still-relevant Q-functions to supply relevant facts re relevant beable properties.

The particles separate along three straight lines in the y-z plane to interact with SGDs that are orthogonal to the related line of flight. Let a, b, c denote the azimuthal angles of each SGD's principal-axis relative to the positive x-axis; let the test results be A, B, C. Then, extending (14)-(15) appropriately; with $\oplus =$ xor:

$$A(a,\lambda) = A^{\pm} = [\lambda \to a \oplus a + \pi\} \cos(\lambda - a) = \pm 1, \tag{26}$$

$$B(b,\mu) = B^{\pm} = [\mu \to b \oplus b + \pi\} \cos(\mu - b) = \pm 1, \tag{27}$$

$$C(c,\nu) = C^{\pm} = [\nu \to c \oplus c + \pi\} \cos(\nu - c) = \pm 1. \tag{28}$$

The function-set of Q-functions and R-functions — as defined at [A].3 — is therefore

$$\{[\lambda \to a \oplus a + \pi\}; \cos(\lambda - a); [\mu \to b \oplus b + \pi\}; \cos(\mu - b); [\nu \to c \oplus c + \pi\}; \cos(\nu - c)\}. \tag{29}$$

Now, working with functions, any Q may be applied to any element in its domain in any order to derive $\langle ABC \rangle$. However, since there are just two independent variables — see (25) — one Q is superfluous. So, taking just one example: (29) may be reduced to:

$$\{[\lambda \to a \oplus a + \pi\}; \cos(\lambda - a); [\mu \to b \oplus b + \pi\}; \cos(\mu - b); \cos(\pi - \lambda - \mu - c)\}. \tag{30}$$

So, as a physically significant shortcut, (30) will yield $\langle ABC \rangle$ correctly. It being understood that — as with any function — each and every Q-function properly maps its domain to its codomain; and consequently onto the domain of every relevant response-function.

For now, bypassing the shortcut, we employ functions (26)-(28) ordered per (29):

$$\langle ABC \rangle = [\lambda \to a \oplus a + \pi\} \cos(\lambda - a)[\mu \to b \oplus b + \pi\} \cos(\mu - b)[\nu \to c \oplus c + \pi\} \cos(\nu - c) \tag{31}$$

$$= [\lambda \to a \oplus a + \pi\} \cos(\lambda - a)[\mu \to b \oplus b + \pi\} \cos(\mu - b)[\nu \to c \oplus c + \pi\} \cos(\pi - \lambda - \mu - c) \tag{32}$$

$$= [\mu \to b \oplus b + \pi\} \cos(\mu - b)[\nu \to c \oplus c + \pi\} \cos(\pi - a - \mu - c) \oplus -\cos(-a - \mu - c) \tag{33}$$

$$\begin{aligned} = [\nu \to c \oplus c + \pi\} \cos(\pi - a - b - c) \oplus -\cos(-a - b - c) \oplus -\cos(-a - b - c) \\ \oplus \cos(-a - b - c - \pi) \end{aligned} \tag{34}$$

$$= \cos(\pi - a - b - c) \oplus -\cos(-a - b - c) \oplus -\cos(-a - b - c) \oplus \cos(-a - b - c - \pi) \tag{35}$$

$$= -\cos(a + b + c).\ QED. \blacksquare \tag{36}$$

$$\therefore P(ABC = +1 \mid M) = \sin^2 \tfrac{1}{2}(a + b + c); \tag{37}$$

$$P(ABC = -1 \mid M) = \cos^2 \tfrac{1}{2}(a + b + c). \tag{38}$$

(36) is the correct result for experiment M, Mermin's (1990a:733) 'crucial minus' sign properly delivered: from (36), $\langle ABC \rangle = -1$ when $a + b + c = 0$. Thus, consistent with the ordinary rules for functions, we classically deliver intelligible EPR correlations. And (30) does the same.

[10] Conclusions

Refuting a number of peer-reviewed 'impossibility' claims in modern physics, we have typified the road-blocks that hinder humanity's steering of the future. In responding to 'How should humanity steer the future?' — reinforcing our reply - 'Steer by Logic' — we have justified the capitalization: associating Logic with CLR-based mathematics; consistent with Einstein's hope.

The way is thus cleared for significant educational and motivational consequences — across many disciplines — to influence and shape the heart of that human future.

We conclude by placing our analysis in its historical context: Employing CLR first-principles and elementary functions, we have refuted Bell's famous theorem and all the Bell-supporting arguments known to us; in our view, beyond dispute. Technically, via (20) and (36), we have also explained 'entanglement' in CLR terms; dismissing the $\neq$ in (3).

For the record, here's Bell's (2004:147) explanation of the background to that $\neq$ in (3):

> "To explain this dénouement without mathematics I cannot do better than follow d'Espagnat (1979; 1979a)." Our paraphrase of d'Espagnat (1979:166) follows:
>
> 'One can infer that in every particle-pair [every pair of twins; $p(\lambda)$, $p'(\lambda')$], one particle has the property A^+ and the other has the property A^-, one has property B^+ and one B^-, ... Such conclusions require a subtle but important extension of the meaning assigned to our notation A^+. Whereas previously A^+ was merely one possible outcome of a measurement made on a particle, it is converted by this argument into an attribute of the particle itself.'

For us, preferring 'outcome of a test' to d'Espagnat's (1979:166) 'outcome of a measurement', and concluding that Bell's theorem is based on a restrictive naive realism, we rejected any such tamper with our task. On the contrary — given *the fact* that such pairs are twins, *physically correlated at birth by their tightly choreographed birth in a spin-conserving decay* — this was our position:

> One can infer that in every particle-pair — every pair of twins, per Bell's abandoned 'genetic' hypothesis (Bernstein 1991:84) — one particle has the property $\lambda \sim +\mathbf{a}$ where $\sim +\mathbf{a}$ is not the outcome A^+ of a test but an equivalence revealed by that outcome:

For we allowed that A^+ reveals a previously-hidden preexisting equivalence relation $\sim$ on Λ.

Thus we arrived at the key to our analysis: we included all the CLR elements of such implications in the dynamics. For, in our micro-physics, we allowed that there may be 'no infinitesimals by the aid of which an observation might be made without appreciable perturbation' (Heisenberg 1930:63). But we also allowed that preexisting pristine properties (ie, beables, properties; such as being a member of a DEC) may be revealed by such perturbations.

So, for us: If a test on a particle reveals an associated DEC, then its pristine twin is a member of a related class: *For such twins are physically correlated at birth by their birth in a spin-conserving decay.* We therefore endorsed EPR's elements of physical reality, defined as follows:

> "If, without any way disturbing a system, we can predict with certainty (ie, with probability equal to unity) the value of a physical quantity, then there exists an element of physical reality [a beable] corresponding to this physical quantity," EPR (1935:777).

For — given the symmetries in (A.4)-(A.5) — let Alice test $A(\mathbf{a}, \lambda)$ and find A^+; ie, $\lambda \sim +\mathbf{a}$. Then, without further ado or disturbance anywhere, Alice can predict with certainty that

$$B(\lambda', \mathbf{a}') = B(-\lambda, \mathbf{a}') = B(-\lambda \sim -\mathbf{a}', \mathbf{a}') = B(\lambda' \sim -\mathbf{a}', \mathbf{a}') = -1 = B^- : \tag{39}$$

$\mathbf{a}'$ distinguishing Bob's $SGD(\mathbf{a}')$ from Alice's $SGD(\mathbf{a})$ when $\mathbf{a}' = \mathbf{b}' = \mathbf{a}$; per [A].1.

Now in (39), the first equality has Bell's backing; see (2) or Bell (1964:(13). And the relation $B(\lambda' \sim -\mathbf{a}', \mathbf{a}') = -1$ is CLR's very definition of equivalence in Bob's domain. For under these conditions, for all $\mathbf{a}'$ and any number of such tests, $B(\lambda' \sim -\mathbf{a}', \mathbf{a}')$ equals minus one with certainty: a central experimental fact.

Thus, via the equivalence class to which $p(\lambda \sim +\mathbf{a})$ in this test belongs, the corresponding EPR beable in Bob's test is $p'(\lambda' \sim -\mathbf{a}')$. In other words: $p'(\lambda' \sim -\mathbf{a}')$ — the EPR beable that here corresponds to the test result B^- — allows us to complement EPR with a CLR comment:

> Unsurprisingly: Without in any way disturbing particle $p'(\lambda' \sim -\mathbf{a}')$, we can predict with certainty the result $B^- = -1$ of that particle's interaction with Bob's $SGD(\mathbf{a}')$:
>
> $$\text{ie, } p'(\lambda' \sim -\mathbf{a}') \Rightarrow [\lambda' \to \pm\mathbf{a}'\}(\lambda' \cdot \mathbf{a}') = -\mathbf{a}' \cdot \mathbf{a}' = -1 = B^-. \tag{40}$$
>
> Moreover, to predict with certainty *any* particular pristine particle's interaction with Bob's $SGD(\pm\mathbf{b}')$: we'd let Alice test that particle's twin with her $SGD(\mp\mathbf{b})$; etc.

We have thus shown that Bell's theorem and related experiments negate naive realism, not commonsense local realism: for that famous inequality at the heart of Bell's analysis is false. Moreover, with every relevant element of each studied physical reality included in our physical theory — with no other elements, subjective or otherwise — we show that our classical mantra holds true: correlated tests on correlated things do produce correlated results without mystery.

We have also shown that, for us at least, mathematics is the best logic. For, though associated with hidden-variables, the now discovered dynamic equivalence classes (DECs) are physically real and wholly amenable to mathematical analysis and experimental confirmation. We further note that the antipodean dichotomies associated with the DECs here are powerful discriminators.

Then, making EPR correlations intelligible by completing the quantum mechanical account in a classical way, CLR also corrects the view — eg, Bell (2004:243) and Bell's move there from his (9) to his (10) — that *causal independence* should equate to *statistical independence*, seen as a consequence of *local causality*. For we have shown that a chain of equivalence, based on physical correlations — not causal influences — links the causally independent outcomes in (2) and in (14)-(15) and in (26)-(28) to the appropriate local-realistic expectations $\langle . \rangle$.

And with (23) and (37)-(38) typifying our work on EPRB correlations: we associate the $\frac{1}{2}$ in our trigonometric arguments with the intrinsic spin $s = \frac{1}{2}$ of the spin-half particles. Similar analysis with photons — eg, in Aspect (2002) — yields $s = 1$.

Finally, working from first principles, showing that Bell's work is limited by his naive realism, we also eliminate the source of Bell's discomfort (expressed in Bernstein 1991:84). So, refuting Bell at every step and honoring Einstein similarly, we here rephrase and reverse Bell's lament:

> Perfect quantum correlations demand something like the 'genetic' hypothesis: like the triplets linked by λ, μ, ν in (25). It's so reasonable to assume that the particles carry with them programs, correlated in advance, telling them how to behave. This is so rational that when Einstein saw that, and the others refused to see it, he was the rational man. The others were burying their heads in the sand. So it's great that Einstein's idea of a classical locally-causal reality works. The Logical thing works.

[11] Acknowledgments

For their encouraging engagement with this work, it is a pleasure to acknowledge my indebtedness to Lazar Mayants (Amherst), Olivier Costa de Beauregard (Paris), Fritz Froehner (Karlsruhe); especially of late to Roger Mc Murtrie (Canberra) and Edwin Eugene Klingman (San Gregorio).

[A] Technical notes

[A].1 The use of primes ($'$)

Primes ($'$) helpfully distinguish elements in Bob's domain from similar elements in Alice's domain. In (1), parameter $\mathbf{a}$ represents the principal-axis alignment of Alice's $SGD(\mathbf{a})$, $\mathbf{a}$ freely and independently chosen by Alice. Parameter $\mathbf{b}'$ represents the principal-axis alignment of Bob's $SGD(\mathbf{b}')$; $\mathbf{b}'$ freely and independently chosen by Bob.

So $SGD(\mathbf{a}')$ means that Bob's setting (indicated by the prime) is equal to Alice's setting (indicated by the $\mathbf{a}$). That is: Bob and Alice have identical settings with $\mathbf{a}' = \mathbf{b}' = \mathbf{a}$; agreeing, from *their* common perspective, on Up/Down. Their settings are then antiparallel with $-\mathbf{a}' = \mathbf{b}' = -\mathbf{a}$; agreeing, *from a particle perspective*, on Up/Down (since λ, λ' are themselves antiparallel).

However, in many ways, a fact over-rides such considerations: Bob — alone and independent of anything that Alice might do — can prove $p'(\lambda' \sim -\mathbf{a}')$. To do so, Bob simply tests $p'(\lambda')$ with $SGD(\mathbf{a}')$, revealing $\lambda \sim -\mathbf{a}'$ directly. To thus make $p'(\lambda' \sim -\mathbf{a}')$ his own; as well as ours.

[A].2 λ and λ'

In (1), primes ($'$) show $p'(\lambda')$ and other elements in Bob's domain. λ, λ' are index-suppressed twinned antiparallel — from $\lambda + \lambda' = 0$ — beables from the set of twinned particles

$$\{p(\lambda_{wn+i}), p'(\lambda'_{wn+i}) \mid w = 0, 1, 2, ..; i = 1, 2, .., n\}; w = \text{run-number when required, eg (12).} \quad \text{(A.1)}$$

λ and λ' are thus spin-half related CLR beables; separable hidden-variables: $\lambda, \lambda' \in \Lambda \subset \mathbb{R}^3$.

[A].3 $SGD(\mathbf{a})$, Q-function $Q(\pm\mathbf{a}) \equiv [\lambda \to \pm\mathbf{a}\}$, R-function $R(\mathbf{a})$, DECs

Each SGD is a composite function-machine: squeeze-function Q feeds response-function R. In the context of Alice's device $SGD(\mathbf{a})$: $Q(\pm\mathbf{a}) = [\lambda \to \pm\mathbf{a}\}$; $R(\mathbf{a}) = (\lambda{\cdot}\mathbf{a})$; with related print-out (± 1).

Turning to R in its role as a diagnostic-function: If $R = (\lambda{\cdot}\mathbf{a}) = \pm 1$, then $\lambda = \pm\mathbf{a} \oplus \lambda \sim \pm\mathbf{a}$; $\oplus$ denoting xor. But under our policy of weak allowances, λ is a uniformly-distributed random beable: $\lambda \in \Lambda \subset \mathbb{R}^3$. So $P(\lambda = \pm\mathbf{a} \mid Z) = 0$. However, independent of this supportive fact but in full accord with our CLR policy of weak allowances: $\sim$ is the diagnostic message; $\sim$ being a coarser relation than $=$. It follows that:

$$\mathbf{a}^+ \equiv \{\lambda \in \Lambda \subset \mathbb{R}^3 | \lambda \sim +\mathbf{a} \in V \subset \mathbb{R}^3\},\ \mathbf{a}^- \equiv \{\lambda \in \Lambda \subset \mathbb{R}^3 | \lambda \sim -\mathbf{a} \in V \subset \mathbb{R}^3\}; \quad \text{(A.2)}$$

where $\mathbf{a}^\pm$ denotes a dynamic equivalence class (DEC); termed *dynamic* because subject to such transformations as $Q(\pm\mathbf{b}') : \mathbf{a}^\pm \to \mathbf{b}'^\pm$, or $Q(\pm\mathbf{a}) : \mathbf{b}'^\pm \to \mathbf{a}^\pm$, *with relevant prevalencies.*

(A.2) shows that Λ is partitioned dyadically under the mapping $[\lambda \to \pm\mathbf{a}\}$. So $\sim$ on the elements of Q's domain denotes: "has the same output/image under Q." With $[+\mathbf{a} \to +\mathbf{a}\} = [\lambda \to +\mathbf{a}\}$, allowing that $\mathbf{a}$ could be an element of Λ: $[. \to +\mathbf{a}\}$ is well-defined under $\sim$ on Λ.

Representing maximal antipodean discrimination, the quotient set $\Lambda/\sim$ is a set of two diametrically-opposed extremes: $\Lambda/\sim = \{\mathbf{a}^+, \mathbf{a}^-\}$.

[A].4 The fundamental experiment of CLR

$$p(\lambda_i) \Rightarrow [\lambda_i \to \pm\mathbf{v}_k\}(\lambda_i{\cdot}\mathbf{v}_k) = \pm 1 = x : y = \pm 1 = (-\mathbf{v}_k^{'}{\cdot}\lambda_i^{'})\{\mp\mathbf{v}_k^{'} \leftarrow \lambda_i^{'}] \Leftarrow p'(\lambda_i^{'}) :$$
$$xy = +1 : \text{ for all } i = 1, 2, .., n; \text{ for all } k = 1, 2, .., \aleph_0, \quad \text{(A.3)}$$

for all unit-vectors $\mathbf{v}_k \in V \subset \mathbb{R}^3$ and any number of tests: an important proof of exactness.

That is: Under (A.3), the Q-functions are proven to be proper functions: it is impossible for one beable to be mapped to two different images.

In other words: Under the equivalence relation $\sim$ on Λ, two spin-half beables are equivalent because a given Q maps them to the same output/image; ie, to their vector equivalents; it being impossible for one beable to be mapped to two different images.

[A].5 The fundamental findings of CLR

Under (A.3): (a) Q-functions are proven to be such: it is impossible to map one spin-beable to two different outputs/images. (b) The equivalence relation $\sim$ on Λ holds: spin-related beables are equivalent if a given Q maps them to the same output/image.

So $p(\lambda) = p(\lambda \sim +\mathbf{a}) = p(\mathbf{a}^+)$ reveals the previously-hidden (but related) DEC of its unperturbed and *still-pristine* correlate: ie, in general;

$$p(\lambda) = p(\lambda \sim \pm\mathbf{a}) = p(\lambda \in \mathbf{a}^\pm) = p(\mathbf{a}^\pm) \text{ implies} \tag{A.4}$$

$$p'(\lambda') = p'(\lambda' = -\lambda) = p'(\lambda' \sim \mp\mathbf{a}') = p'(\lambda' \in \mathbf{a}'^\mp) = p'(\mathbf{a}'^\mp); \text{ and vice-versa, etc :} \tag{A.5}$$

a range of properties (physical facts) suited to many analytic situations.

More formally: $Q : \Lambda \to V \subset \mathbb{R}^3$ by assigning every object $\lambda \in \Lambda$ to exactly one element $Q(\lambda) \in V$ where V is the space of 3-vectors. Experimental proof of the exactness here is provided by this example from (A.3): The product of the paired outputs (± 1) from $SGD(\pm\mathbf{a})$ on $p(\lambda)$ and $SGD(\mp\mathbf{a}')$ on $p(\lambda')$ — for all $\mathbf{a}$ and any number of tests — equals one.

Allowing λ, λ' to be antiparallel random beables, it follows that the mutually-exclusive collectively-exhaustive equiprevalent outputs in (1) are $\sim \pm\mathbf{a}$ and $\sim \pm\mathbf{b}'$; to thus highlight the symmetries in EPRB.

[A].6 CLR dynamics

CLR dynamics deliver the results of local SGD/particle interactions *as well as their* factual *implications*; updating facts re pristine correlates with a mathematical **If** ...**: Then** ...: Converting the source of our inferences (*physical facts*) to relevant physical properties (*other physical facts*) via the mathematical transmission of such facts; independent of vague words and conjectures.

> 'Surely the big — $SGD(\mathbf{a})$ — and the small — $p(\lambda)$ — should merge smoothly with one another? And surely in fundamental physical theory this merging should be described not just by vague words but by precise mathematics?' after Bell (2004:190).
>
> "The concept of 'measurement' becomes so fuzzy on reflection that it is quite surprising to have it appearing in physical theory *at the most fundamental level*. ...does not any *analysis* of measurement require concepts more *fundamental* than measurement? And should not the fundamental theory be about these more fundamental concepts? One line of development towards greater physical precision would be to have the [quantum] 'jumps' [or mergings] in the equations and not just in the talk — so it would come about as a dynamical process in dynamically defined conditions," Bell (2004:117-118).

In the context of EPRB, we take *transformation* to be a concept 'more fundamental than measurement'. Requiring such transformations/mergings in our equations — and not just in the talk — we allow that local interaction between $SGD(\mathbf{a})$ and $p(\lambda)$ transforms both the particle and the device: transforming hidden beables and revealing DECs; eg, $\lambda \in \mathbf{a}^+$. Importantly, a pristine correlate will have a related DEC: ie, $\lambda' \in \mathbf{a}'^-$ in this example; $\lambda' \in \mathbf{a}'^-$ being confirmed with certainty by Bob's direct pre-, 'simultaneous' or post-testing of that correlate under $SGD(\mathbf{a}')$.

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