GOLDBACH CONJECTURE RESOLUTION

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Abstract

Goldbach's conjecture is one of the oldest and best-known unsolved problems in number theory and in all of mathematics. It states: Every even integer greater than 2 can be expressed as the sum of two primes. In this study I’ll prove that this conjecture is true.

Keywords: GoldBach conjecture.

Designations:

IC : Impair composite numbers
IN : Impair number
P  : Prime number
PN : Pair number
NS : number of solutions

1. INTRODUCTION

Goldbach’s conjecture is one of the oldest and best-known unsolved problems in number theory and in all of mathematics. It states: Every even integer greater than 2 can be expressed as the sum of two primes [1].
Each pair number higher than 2 can be written as:

\[
\begin{align*}
2 + 2 &= 4 \\
3 + 3 &= 6 \\
3 + 5 &= 8 \\
3 + 7 &= 10 \\
3 + 9 &= 12 \\
3 + 11 &= 14 \\
3 + 13 &= 16 \\
3 + 15 &= 18 \\
3 + 17 &= 20 \\
3 + 19 &= 22 \\
3 + 21 &= 24 \\
\end{align*}
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(1) \quad (2) \quad (3)

In this example we took PN (pair number) max is 24 then we will generalize for higher values, so we see obviously that \((1) \cup (2) = (3)\).

(2) Represents \(3 + P\)

(3) Represents \(3 + IC[2]\)

So in general each PN (pair number) higher than 2 can be written as:

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(1) \quad (2) \quad (3)
So if we that (3) equal to the sum of 2 Prime number then we’ll prove that each pair number higher than 2 is the sum of 2 Prime numbers.

So let’s prove that 3+IC = sum of 2 Prime numbers.

IC ( impair composite number ) , an impair composite number can be written as

\[
IC = \sum_{a-b}^{2P_i} \{ P_i(P_i + 2n) \} \quad n \in \mathbb{N}^+ [2]
\]  

(4)

Let’s put:

\[
\begin{align*}
3 & = 1 + 2 \\
5 & = 3 + 2 \\
7 & = 5 + 2 \\
9 & = 7 + 2 \\
11 & = 9 + 2 \\
13 & = 11 + 2 \\
15 & = 13 + 2 \\
17 & = 15 + 2 \\
19 & = 17 + 2 \\
21 & = 19 + 2 \\
\end{align*}
\]

(5)

And let’s assign (a=1,b=3,c=5,d=7,e=9,f=11,g=13,h=15,i=17,j=19,k=21), changing this in (5) we get:

\[
\begin{align*}
b & = a + 2 \\
c & = b + 2 \\
d & = c + 2 \\
e & = d + 2 \\
f & = e + 2 \\
g & = f + 2 \\
h & = g + 2 \\
i & = h + 2 \\
j & = i + 2 \\
k & = j + 2 \\
\end{align*}
\]

(6)

Our interest is ( e=9=7+2 , h=15=13+2,k=21=19+2) which represents IC ( impair composite numbers that we want to prove that they can be written as the sum of 2 Prime numbers . Here (a=1,b=3,c=5,d=7,f=11,g=13,i=17,j=19) represents the Prime numbers.

In the system (6) we have

\[
\begin{align*}
e + b & = d + 2 + a + 2 \\
e + b & = c + 2 + 2 + b - 2 \\
e + b & = c + 2 + 2 + d - 2 - 2 \\
e + b & = c + d \\
\end{align*}
\]
Changing (e,b,c,d) by their respective values(9,3,5,7) we get

3 + 9 = 5 + 7

In this example we prove that 3+IC= 5+7 (IC=9), if we do the same process for (IC=15 and IC=21) we will find that

3 + 15 = 5 + 13 = 7 + 11 then (3+IC = 5+13=7+11) with (IC=15)

And

3 + 21 = 5 + 19 = 7 + 17 = 11 + 13 then (3+IC = 5+19=7+17=11+13) with (IC=21).

2. GRAPHIC INTERPRETATION

From this example sample example we see clearly that each 3+IC can be written as the sum of 2 prime numbers.

Let’s see this graphically.

| 3 | 5 | 7 | 9 | 11 | 13 | 15 | 17 | 19 | 21 | 23 | 25 | 27 | 29 | 31 | 33 | 35 | 37 | 39 | 41 | 43 | 45 | 47 | 49 | 51 | 53 | 55 |
|---|---|---|---|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|

\[ P_7^2 + 2P_7n \quad P_7 = 7 \]
\[ P_5^2 + 2P_5n \quad P_5 = 5 \]
\[ P_3^2 + 2P_3n \quad P_3 = 3 \]

The numbers framed (9,15,21,25,27,33,35,45,55) are the IC (impair composite numbers and the rest are the prime numbers, from the example above we saw that (3 + 9 = 5 + 7) and (3 + 15 = 5 + 13 = 7 + 11) and (3 + 21 = 5 + 19 = 7 + 17 = 11 + 13), graphically this represents one step on from 3 equal to one step back from the IC, for example (3+9) one step on from 3 is 5, and one step back from 9 is 7 then (3+9=5+7), example for (3+15) one step on from 3 is 5 and one step back from 15 is 13 then (3+15=5+13), two steps on from 3 is 7 and two steps back from 15 is 11 then (3+15=7+11), this means mathematically:

\[ 3 + IC = (3 + 2c) + (IC - 2c) \quad \text{with} \quad c \in \mathbb{N}^+ \quad \text{then} \]

\[ (3 + 2c) + (IC - 2c) = P_i + P_j \quad \text{with} \quad (P_i \text{ and } P_j \text{ are both prime numbers}). \]

Number of solutions:

We saw above that for the examples ((3+15) and (3+21) may have multiple solutions) this means graphically between 3 and 15 we have 4 Prime numbers then the number of solution is 4/2=2. For the example (3+21) graphically we have 6 prime numbers then 6/2=3 solutions, but if we have number of prime number is impair for example (3+25) between 3 and 25 we have 7 prime numbers then 7/2=3.5 but the number of solutions is 3 then the number of solutions is:

\[ NS \leq \frac{\pi(x)}{2} \quad \text{with} \quad \pi(x) \quad \text{is the number of prime numbers in given x which represents the number of impairs numbers}. \]

When \( \pi(x) \) tends to \( +\infty \), \( NS \) tends to \( +\infty \).
In the case where a pair number could be written as the sum of the same prime numbers example (10=5+5, 14=7+7).

Mathematically this means:

$$3 + IC = (3+2c) + (IC-2c) = (3+2c) + (P_n^2 + 2P_n - 2c).$$

When $3+IC = \text{sum of the prime numbers}$, then

$$b - P_i^2$$

$$(3+2c) = (IC-2c) \quad \text{with} \quad IC = \sum_{n=0}^{P_i} \left\{ P_i(P_i + 2n) \right\}$$

we get

$$(3+2c) = (P_n^2 + 2P_n m - 2c) \quad \text{with} \quad m \in \mathbb{N}$$

By solving for $m$ we get

$$m = \frac{3+4c-P_n^2}{2P_n} \quad (7)$$

3. CONCLUSION

The system (6) can be written in general as

$$\begin{align*}
&b = a + 2 \\
&c = b + 2 \\
&d = c + 2 \\
&e = d + 2 \\
&f = e + 2 \\
&g = f + 2 \\
&h = g + 2 \quad \Rightarrow c + n = d + n_{n-1} = e + n_{n-2} = ... \\
&i = h + 2 \\
&j = i + 2 \\
&k = j + 2 \\
&... \\
n_{n-1} = n_{n-2} + 2 \\
n = n_{n-1} + 2
\end{align*}$$

Then each $(3+IC)$ can be written as the sum of two prime numbers and we saw in paragraph 2 the number of solutions we can have and as the number of prime numbers is infinite then we can have infinite solutions that satisfy $3+IC$, here we have proved the Goldbach conjecture and it is true.
5. REFERENCES:
