Unobservable Potentials to Explain a Quantum Eraser and a Delayed-Choice Experiment

Masahito Morimoto[1 *]

[1]*Telecommunications & Energy Laboratories of Furukawa Electric Co., Ltd.*
6, Yawata-Kaigandori, Ichihara, Chiba, 290-8555, Japan

We present a new explanation for a quantum eraser. The erasure and reappearance of an interference pattern have been explained that a revolvable linear polarizer erases or marks the information of "which-path markers", which indicate the photon path. Mathematical description of the traditional explanation requires quantum-superposition states. However, the phenomenon can be explained without quantum-superposition states by introducing unobservable potentials which can be identified as an indefinite metric vector. In addition, a delayed choice experiment can also be explained without entangled states under the assumption that an definite orientation of the unobservable potentials configured by a setup of the experiment determines the polarization of the photon pairs in advance.

1. Introduction

Quantum theory has paradoxes related to the reduction of the wave packet typified by "Schrödinger's cat" and "Einstein, Podolsky and Rosen (EPR)".[1,2] In order to interpret the quantum theory without paradoxes, de Broglie and Bohm had proposed so called "hidden variables" theory.[3,4] Although, "hidden variables" has been negated,[5] the theory has been extended to consistent with relativity and ontology.[6–10] However the extension has not been completed so far. A.Aspects' experiments[11–13] have demonstrated that Bell's inequalities are always violated confirming the quantum mechanics theory on the non-locality of the photon and demonstrating the absence of "hidden variables" for the local representation. However, as A.Aspect has confirmed himself, hidden variables may quite well exist within a non-local representation, for example a photon representation with a real wave function.

The author has reported the alternative interpretation for quantum theory utilizing quantum field formalism with unobservable potentials similar to Aharonov-Bohm effect[14–16] and rigorous mathematical treatment using tensor form. The interpretation can omit the quantum paradoxes and be applied to elimination of infinite zero-point energy, spontaneous symmetry breaking, mass acquire mechanism, non-Abelian gauge fields and neutrino oscillation, which can lead to the comprehensive theory. For example, as reported in reference,[15] single photon and electron interference can be calculated without quantum-superposition state by introducing the states represent a substantial (localized) photon or electron and the unobservable (scalar) potentials, which are expressed as following Maxwell equations.

$$
\begin{aligned}
\left(\Delta - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)\mathbf{A} - \nabla\left(\nabla\cdot\mathbf{A} + \frac{1}{c^2}\frac{\partial\phi}{\partial t}\right) &= -\mu_0\mathbf{i} \\
\left(\Delta - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)\phi + \frac{\partial}{\partial t}\left(\nabla\cdot\mathbf{A} + \frac{1}{c^2}\frac{\partial\phi}{\partial t}\right) &= -\frac{\rho}{\varepsilon_0} \qquad (1)
\end{aligned}
$$

When the scalar potential of (1) is quantized, the photon annihilation operator $\hat{A}_0'$ expressing the unobservable (scalar) potential can be expressed as follows.

$$
\begin{aligned}
\hat{A}_0' &= \frac{1}{2}\gamma e^{i\theta/2}\hat{A}_1 - \frac{1}{2}\gamma e^{-i\theta/2}\hat{A}_1 \\
\hat{A}_0'^{\dagger} &= \frac{1}{2}\gamma e^{-i\theta/2}\hat{A}_1^{\dagger} - \frac{1}{2}\gamma e^{i\theta/2}\hat{A}_1^{\dagger} \qquad (2)
\end{aligned}
$$

where $\gamma^2 = -1$ (i. e., γ corresponds to the square root of the determinant of Minkowski metric tensor $\sqrt{|g_{\mu\nu}|} \equiv \sqrt{g} \equiv \sqrt{-1} = \gamma$) which stands for requirement of indefinite metric, $\hat{A}_1$ is the photon annihilation operator quantized vector potentials of (1) and θ is a phase difference derived from a geometry. By using tensor form (covariant quantization), we can explicitly identify these operators $\hat{A}_0'$ as the scalar potential, $\hat{A}_1$ as the vector potentials and spontaneously obtain as described later.

The above $\hat{A}_0'$ bears a remarkable resemblance to the expression of $\tilde{\boldsymbol{\Xi}}$ reported by C. Meis to investigate quantum vacuum state as follows.[17]

$$
\tilde{\boldsymbol{\Xi}}_{0_{k\lambda}} = \xi a_{k\lambda}\hat{\epsilon}_{k\lambda}e^{i\varphi} + \xi^* a_{k\lambda}^{\dagger}\hat{\epsilon}_{k\lambda}^{*}e^{-i\varphi} \qquad (3)
$$

where k, λ, ϵ, ξ and φ stand for k mode, λ polarization, a complex unit vector of polarization, a constant and a phase parameter respectively.

If we identify ξ and ξ^* as $\frac{1}{2}\gamma$ and $-\frac{1}{2}\gamma$ and introduce polarization vectors as described later in (7), then (2) corresponds to (3).

When state vector $|\zeta\rangle$, which represents the unobservable (scalar) potentials, is introduced in Schrödinger picture as follows, the vector can be identified as indefinite metric vector.

$$
|\zeta\rangle \equiv \left(\frac{1}{2}\gamma e^{i\theta/2} - \frac{1}{2}\gamma e^{-i\theta/2}\right)|1\rangle \qquad (4)
$$

Where $|1\rangle$ represents a photon state. Therefore when there is no phase difference the expectation value of arbitrary physical quantity $\hat{A}$ and provability amplitude of $|\zeta\rangle$ are zeros ($\langle\zeta|\hat{A}|\zeta\rangle = 0$, $\langle\zeta|\zeta\rangle = 0$), which means the unobservable potentials can not be observed alone in the literature. More detail treatment of these operators and vectors have been discussed in reference.[15]

Aharonov and Bohm have pointed out the unobservable

*E-mail: morimoto@ch.furukawa.co.jp

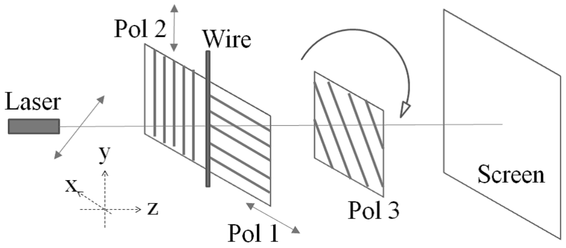

Fig. 1. Typical setup for the Quantum Eraser. Pol1 and Pol2 are fixed linear polarizers with polarizing axes perpendicular (x and y). Pol3 is a revolvable linear polarizer.

potentials can cause electron wave interferences[16] and we should realize all of physical interactions are regulated by gauge fields (gauge principle. the potentials are also gauge fields.), which can not be observed alone.[18–21]

In this letter, we show the existence of the unobservable potentials can explain not only the interferences but also the quantum eraser and delayed choice experiment. In addition, we also shows the interference between photons and the unobservable potentials violates Bell's inequalities in keeping with the locality, which is consistent with relativity. This fact is the most important novel aspect of this paper that the violation of Bell's inequalities can not justify the non-locality of quantum theory and the absence of hidden variables.

2. Traditional explanation for quantum eraser

Figure 1 shows a typical setup for the quantum eraser.[22] Without any polarizers, an interference pattern which is composed of dark and bright fringes can be observed on the screen because light passing on the left of the wire is combining, or "interfering," with light passing on the right-hand side. In other words, we have no information about which path each photon went.

When polarizers 1 and 2, which are called "which-path markers", are positioned right behind the wire as shown in figure 1, the launched light polarized in 45° direction from the Laser is polarized in perpendicular (x-polarized and y-polarized) by these polarizers. Then the interference pattern on the screen is erased because "which-path makers" have made available the information about which path each photon went.

When polarizer 3 is inserted in front of the screen with the polarization angle +45° or -45° in addition to "which-path makers", the interference pattern reappears because polarizer 3 has made the information of "which-path makers" unusable.

We can produce a mathematical description of the erasure and reappearance of the interference pattern as follows. x-polarized and y-polarized photon passing through polarizer 1 and 2 can be expressed by the quantum-superposition state as follows.

$$|x\rangle = \frac{1}{\sqrt{2}}|+\rangle + \frac{1}{\sqrt{2}}|-\rangle \tag{5}$$

and

$$|y\rangle = \frac{1}{\sqrt{2}}|+\rangle - \frac{1}{\sqrt{2}}|-\rangle \tag{6}$$

where "+" and "-" represent polarizations +45° and -45° with respect to x.

The photons pass through polarizer 1 and 2 are polarized at right angles to each other as seen in the left-hand side of (5) and (6), which prevent the interference pattern. In other words, "which-path makers" have made available the information about which path each photon went. Although there are same polarized states in the right-hand side of (5) and (6), the interference patterns consisting of bright and dark fringes made by +45° and -45° polarized states are reverted images and annihilate each other. Therefore sum total of the images has no interference pattern.

When polarizer 3 is inserted with the polarization angle +45° or -45°, only $|+\rangle$ or $|-\rangle$ can pass through polarizer 3. Then the interference pattern made by either $|+\rangle$ or $|-\rangle$ of both (5) and (6) reappears, which means we can not identify which-path the photons had passed through, i.e., polarizer 3 has made the information of "which-path makers" unusable.

3. New explanation for quantum eraser

The mathematical description of the photon states passing through polarizer 1 and 2 for the traditional explanation requires the quantum-superposition states (5) and (6) respectively.

If Maxwell equations are deemed to be classical wave equations whose electro-magnetic fields obey the superposition principle, then the description is valid. However, applying the superposition principle to particle image, e.g., inseparable single photon, leads to quantum paradoxes.

Although tensor form (covariant quantization) is a rigorous treatment as we will describe later, here we conveniently take advantage of the unobservable potentials that can eternally populate the whole of space as waves independent of existence of the substantial photons. Therefore we can replace the photon state $|x\rangle$ with $|x\rangle + |\zeta\rangle$, where $|\zeta\rangle$ is a state represent the unobservable potentials whose probability amplitudes $\langle\zeta|\zeta\rangle = 0$ in initial states as described in (4) (when there are no phase or polarization angle differences as described below.). The unobservable potentials can be polarized by the polarizers because the potentials also the electromagnetic potentials which obey Maxwell equations and populate the whole of space-time.

Note that as we will see later the unobservable potentials, which correspond to the scalar potentials neglected by quantization using Coulomb gauge, and localized vector potentials that represent the substantial photons can exist simultaneously because the both potentials obey the Maxwell equations (1).

Then the following states, which are identified as (4) introducing polarization terms similar to (3), can generate the same interference as the quantum-superposition states (5) and (6).

$$\begin{aligned}
|x\rangle + |\zeta_{\phi,x}\rangle &= |x\rangle + \frac{1}{2}\gamma e^{i\phi}e^{i\theta/2}|x\rangle - \frac{1}{2}\gamma e^{-i\phi}e^{-i\theta/2}|x\rangle \\
|y\rangle + |\zeta_{\phi+\frac{1}{2}\pi,y}\rangle &= |y\rangle + \frac{1}{2}\gamma e^{i(\phi+\frac{1}{2}\pi)}e^{-i\theta/2}|y\rangle \\
&\quad -\frac{1}{2}\gamma e^{-i(\phi+\frac{1}{2}\pi)}e^{i\theta/2}|y\rangle
\end{aligned} \tag{7}$$

where $\gamma^2 = -1$, ϕ and θ are the indefinite metric, the polarization angle of polarizer 3 measured from x-axis and phase difference between left and right paths respectively.

Therefore when we observe only $|x\rangle$ with polarizer 3, i. e., $\theta = 0$, the intensity of the interference $\langle I\rangle$ can be calculated as follows.

$$\begin{aligned}\langle I\rangle &\propto \left(\langle x| + \langle\zeta_{\phi,x}|\right)\left(|x\rangle + |\zeta_{\phi,x}\rangle\right)\\ &= \langle x|x\rangle - \frac{1}{2}\langle x|x\rangle + \frac{1}{2}\langle x|x\rangle\cos(2\phi+\theta)\\ &= \frac{1}{2} + \frac{1}{2}\cos(2\phi+\theta) = \frac{1}{2} + \frac{1}{2}\cos(2\phi) \qquad (8)\end{aligned}$$

Hence the output intensity by rotation angle of polarizer 3 is correctly-reproduced.

When we observe $|x\rangle$ and $|y\rangle$ with polarizer 3, the intensity is obtained as follows.

$$\langle I\rangle \propto \left(\langle x| + \langle\zeta_{\phi,x}| + \langle y| + \langle\zeta_{\phi+\frac{1}{2}\pi,y}|\right)\left(|x\rangle + |\zeta_{\phi,x}\rangle + |y\rangle + |\zeta_{\phi+\frac{1}{2}\pi,y}\rangle\right) \qquad (9)$$

Because $\langle x|y\rangle = \langle y|x\rangle = 0$, then

$$\langle I\rangle \propto \left(\langle x| + \langle\zeta_{\phi,x}|\right)\left(|x\rangle + |\zeta_{\phi,x}\rangle\right) + \left(\langle y| + \langle\zeta_{\phi+\frac{1}{2}\pi,y}|\right)\left(|y\rangle + |\zeta_{\phi+\frac{1}{2}\pi,y}\rangle\right) \qquad (10)$$

By using (8), we can obtain

$$\begin{aligned}\langle I\rangle &\propto \frac{1}{2} + \frac{1}{2}\cos(2\phi+\theta) + \frac{1}{2} + \frac{1}{2}\cos(2\phi+\pi-\theta)\\ &= 1 + \frac{1}{2}\cos(2\phi+\theta) - \frac{1}{2}\cos(2\phi-\theta) \qquad (11)\end{aligned}$$

When $\phi = \pm\pi,\ \pm\frac{1}{2}\pi$ then $\langle I\rangle \propto 1$ and $\phi = \pm\frac{1}{4}\pi$ then $\langle I\rangle \propto 1 \pm \sin\theta$, which reproduces the interference correctly.

In this new explanation, the polarization of substantial photons is fixed and the photons can not pass through the polarizer whose polarization angle is different from that of photons. However, the unobservable potentials create the same interference as the superposition state of $|+\rangle$ and $|-\rangle$ as described above. In case of single photon, the interference can be calculated by (7) replacing $|y\rangle$ with $|0\rangle$. Then $\langle I\rangle \propto 1 + \frac{1}{2}\cos(2\phi+\theta) - \frac{1}{2}\cos(2\phi-\theta)$ is obtained. Note that when we calculate the single photon interference by using photon number operator $\mathbf{n}_1 = \hat{A}_1^\dagger\hat{A}_1$, we can obtain exact expression $\langle I\rangle \propto \frac{1}{2} + \frac{1}{2}\cos(2\phi+\theta)$ because $\langle 0|0\rangle = 1 \neq \langle 0|\mathbf{n}_1|0\rangle = 0$. Where $\hat{A}_1$ is the photon annihilation operator obtained from the vector potentials in (1).[15]

The above calculations are based on Schrödinger picture. We can obtain the same results based on Heisenberg picture. In Heisenberg picture, the photon number operator should be replaced by $\mathbf{n} = (\hat{A}_1^\dagger + \hat{A}_p^\dagger)(\hat{A}_1 + \hat{A}_p)$.[15] Where $\hat{A}_1$ and $\hat{A}_p$ (p : polarization $= x,\ y, \cdots, etc.$) are the photon annihilation operators obtained from the vector and scalar potentials in (1) respectively which represents the substantial photons and modified operator introduce the polarization terms in (2) as follows which represents the polarized unobservable potentials.

$$\begin{aligned}\hat{A}_x &= \frac{1}{2}\gamma e^{i\phi}e^{i\theta/2}\hat{A}_1 - \frac{1}{2}\gamma e^{-i\phi}e^{-i\theta/2}\hat{A}_1\\ \hat{A}_x^\dagger &= \frac{1}{2}\gamma e^{-i\phi}e^{-i\theta/2}\hat{A}_1^\dagger - \frac{1}{2}\gamma e^{i\phi}e^{i\theta/2}\hat{A}_1^\dagger \qquad (12)\end{aligned}$$

We can calculate (8) in Heisenberg picture as follows.

$$\begin{aligned}\langle I\rangle &= \langle n|(\hat{A}_1^\dagger + \hat{A}_x^\dagger)(\hat{A}_1 + \hat{A}_x)|n\rangle\\ &= \langle n|\mathbf{n}_1|n\rangle + \langle n|\hat{A}_x^\dagger\hat{A}_x|n\rangle\end{aligned}$$

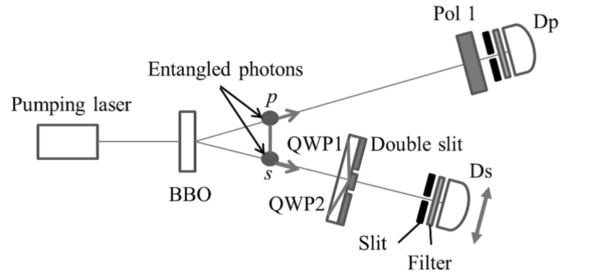

Fig. 2. Typical setup for the Delayed Choice Quantum Eraser. QWP1 and QWP2 are quarter-wave plates aligned in front of the double slit with fast axes perpendicular. Pol1 is a linear polarizer. BBO (β−BaB_2O_4) crystal generates entangled photons by spontaneous parametric down-conversion.[23]

$$\propto 1 - \frac{1}{2} + \frac{1}{2}\cos(2\phi+\theta) = \frac{1}{2} + \frac{1}{2}\cos(2\phi) \qquad (13)$$

Note that x-polarized photon annihilation operator should be represented by $\hat{A}_1 + \hat{A}_x$ instead of $\hat{A}_1$ in Heisenberg picture.[15] Then when there are x- and y-polarized photons, the operator should be represented by $(\hat{A}_1 + \hat{A}_x) + (\hat{A}_2 + \hat{A}_y)$. Where $\hat{A}_2$ is a photon annihilation operator obtained from the quantization of y-polarized vector potential and $\hat{A}_y$ can be obtained by replace ϕ with $\phi + \frac{1}{2}\pi$ and $\hat{A}_1, \hat{A}_1^\dagger$ with $\hat{A}_2, \hat{A}_2^\dagger$ in (12). Then we can calculate (9) in Heisenberg picture as follows.

$$\begin{aligned}\langle I\rangle &= \langle n|(\hat{A}_1^\dagger + \hat{A}_x^\dagger + \hat{A}_2^\dagger + \hat{A}_y^\dagger)(\hat{A}_1 + \hat{A}_x + \hat{A}_2 + \hat{A}_y)|n\rangle\\ &= \langle n|\mathbf{n}_1|n\rangle + \langle n|\hat{A}_x^\dagger\hat{A}_x|n\rangle + \langle n|\mathbf{n}_2|n\rangle + \langle n|\hat{A}_y^\dagger\hat{A}_y|n\rangle\\ &\propto 1 + \frac{1}{2}\cos(2\phi+\theta) - \frac{1}{2}\cos(2\phi-\theta) \qquad (14)\end{aligned}$$

where we identify $\langle n|\mathbf{n}_1|n\rangle \equiv \langle n|\hat{A}_1^\dagger\hat{A}_1|n\rangle = \langle n|\mathbf{n}_2|n\rangle \equiv \langle n|\hat{A}_2^\dagger\hat{A}_2|n\rangle = n$ assuming there are the same number (n) of x- and y-polarized photons. Because under the assumption $|n\rangle \equiv |n\rangle_x + |n\rangle_y$ where $|n\rangle_x, |n\rangle_y$ are the x- and y-polarized n photon states respectively then $\hat{A}_1|n\rangle = \hat{A}_1|n\rangle_x + \hat{A}_1|n\rangle_y = \sqrt{n}|n-1\rangle_x$ and $\hat{A}_2|n\rangle = \hat{A}_2|n\rangle_x + \hat{A}_2|n\rangle_y = \sqrt{n}|n-1\rangle_y$. In addition, $\langle n|\hat{A}_1^\dagger\hat{A}_2|n\rangle = \langle n|\hat{A}_2^\dagger\hat{A}_1|n\rangle = 0$

The new explanation can describe that $\hat{A}_p$ or $|0\rangle + |\zeta\rangle$ which can be identified as vacuum, creates and annihilates the substantial photons through the interference.

Loosely speaking, the unobservable potentials are oriented by the polarizers such as (7) or (12). Then the substantial photons surf on the sea of the oriented potentials which can change into substantial photons through the interference.

Note that (7) are not the superposition states of $|+\rangle$ and $|-\rangle$. Instead, the states are composed of substantial states $|x\rangle$ or $|y\rangle$ and states of unobservable potential $|\zeta\rangle$. These combination of the states create the same interference as the superposition states of $|+\rangle$ and $|-\rangle$. Therefore there is no wave packet reduction and fulfillment of engineering applications utilizing the wave packet reduction such as quantum teleportation or computer will be pessimistic conclusion.

4. New explanation for delayed choice quantum eraser

In this section, we show new explanation for Delayed Choice Quantum Eraser as shown in figure 2 which consists of an entangled photon source and two detectors. The delayed choice has been demonstrated when the distance from BBO to polarizer 1 is longer than that from BBO to the double slit.[23]

Here we should take particular note of the fact that the polarization angle of polarizer 1 has been chosen before the entangled photons are generated. S. P. Walbornet et al.[23] have pointed out that ”the experiment did not allow for the observer to choose the polarization angle in the time period after photon *s* was detected and before detection of *p*”. From the principle of causality, their point will be reasonable.

However, mathematical description for the phenomenon requires entangled state such as

$$|\psi\rangle = \frac{1}{\sqrt{2}}\left(|x\rangle_s|y\rangle_p + |y\rangle_s|x\rangle_p\right) \tag{15}$$

The entangled state declares that the state of the whole system is a quantum-superposition state consist of $|x\rangle_s|y\rangle_p$ and $|y\rangle_s|x\rangle_p$. Therefore when the state of one photon (*s* or *p*) is observed and determined to be $|x\rangle$, that of the other photon (*p* or *s*) suddenly changes from the quantum-superposition state into $|y\rangle$ even if the photons separate from each other, which postulates the existence of long-range correlation beyond the causality (spooky action at a distance).

Hence we consider physical phenomenon from the moment we choose the polarization angle of polarizer 1 to the moment BBO generates the entangle photon pairs.

The unobservable potentials, which can change from the potentials into substantial photons, eternally populate the whole of space not forgetting the space between BBO and Polarizer 1 independent of substantial photons. Hence the space will be populated by the unobservable potentials which are oriented by polarizer 1 as described above. More precisely, the potentials determine the polarization of substantial photons in the space in advance depending on the polarization angle of polarizer 1.

For example, if we choose the polarization angle of polarizer 1 to ϕ which is measured from the polarization angle ψ of created photons, then the unobservable potential is oriented to $|0\rangle + |\zeta_\phi\rangle = |0\rangle + \frac{1}{2}\gamma e^{i(\phi-\psi)}e^{i\theta/2}|0\rangle - \frac{1}{2}\gamma e^{-i(\phi-\psi)}e^{-i\theta/2}|0\rangle$ at polarizer 1 and propagates to BBO. BBO is forced to generate the photon pair with polarization $p : \phi$ and $s : \phi \pm \frac{1}{2}\pi$ according to the arrival potentials. More precise explanation is as follows. By applying a photon creation operator $\hat{A_\psi}^\dagger$ to the polarized potentials, i. e.,

$$\hat{A_\psi}^\dagger|0\rangle + \hat{A_\psi}^\dagger|\zeta_\phi\rangle = |\psi\rangle + \frac{1}{2}\gamma e^{i(\phi-\psi)}e^{i\theta/2}|\psi\rangle - \frac{1}{2}\gamma e^{-i(\phi-\psi)}e^{-i\theta/2}|\psi\rangle \tag{16}$$

can be calculated as the created photon state at BBO. There is no phase difference $\theta = 0$ because there is no other path in the setup. Then the intensity of the created photon can be calculated as follows.

$$\langle I\rangle \propto \frac{1}{2} + \frac{1}{2}\cos(2\phi - 2\psi) \tag{17}$$

In order to create a photon, i. e., $\langle I\rangle = 1$, $\psi = \phi$ will be required.

Then the polarization of the photon pair is fixed by the unobservable potentials instead of the entangle state (15). Therefore when the polarization angle is set to the fast axis of QWP (Quarter-wave plate) 1 or 2, the interference pattern can be observed.

Because the unobservable potentials can not be observed, we are not aware of the determination of the polarization of the photon pair by the unobservable potentials. This is the reason why the state seems to be ”entangled” and the choice of the polarization angle of polarizer 1 seems to be ”delayed”.

In order to confirm the new explanation, we should make experiments with a shutter between BBO and polarizer 1 as follows. First, close the shutter not to make a definite orientation of the unobservable potentials. After the entangled photon pairs are generated, open the shutter. When the photon *s* is measured by Ds, close the shutter again. After a time period, we excite BBO to generate the next entangled photon pairs. When the next pairs are generated, open the shutter again. By repeating these procedures, we can make a comparison between the traditional results and new result. If the definite orientation of the unobservable potentials as mentioned above is valid, no interference pattern can be observed even if the polarization angle of Polarizer 1 is set to the fast axis of QWP 1 or 2 throughout the experiment.

Note that because the unobservable potentials obeying Maxwell equations propagate at the speed of light, the above time period that prevents the unobservable potentials from being oriented should be longer than the distance between BBO and the shutter divided by the speed of light.

The above new explanation is based on the preselected polarization by the setup. However even if the polarizations of the photon pair are randomly selected, the measurement results seem to have the long-range correlation beyond the causality as follows. From (7), the measurement results of photons *s* and *p* are expressed as follows.

$$\begin{aligned} \langle I_s\rangle &\propto = \frac{1}{2} + \frac{1}{2}\cos(2\phi) \\ \langle I_p\rangle &\propto = \frac{1}{2} - \frac{1}{2}\cos(2\phi) \end{aligned} \tag{18}$$

There is no such a classical correlation and the above results violate Bell's inequalities. Therefore, the confirmation method described the above have to be carefully implemented. When there are no polarizers, the polarization is randomly selected. Hence a detection frequency of photons by D_p which proportional to the intensity of measured photon will be extremely lower than the case when there are polarizers. The difference of the detection frequency will be the only way to distinguish the new explanation from traditional one.

Whatever the results, the interference between the photons and unobservable potentials makes the long-range correlation beyond the causality that does not really exist in nature look exist.

5. Tensor form of the electromagnetic fields

We have introduced the operator by using $\gamma^2 = -1$ such as (12), which expresses the unobservable potentials for convenience in calculation in the above. When we use tensor form of the electromagnetic fields, the operator and results can be spontaneously introduced as following manner. The followings is almost as same as the description for the single photon interference of reference.[15]

The electromagnetic potentials are expressed as following four-vector in Minkowski space.

$$A^\mu = (A^0,\ A^1,\ A^2,\ A^3) = (\phi/c,\ \mathbf{A}) \tag{19}$$

The four-current are also expressed as following four-vector.

$$j^\mu = (j^0,\ j^1,\ j^2,\ j^3) = (c\rho,\ \mathbf{i}) \tag{20}$$

When we set the axises of Minkowski space to $x^0 = ct,\ x^1 = x,\ x^2 = y,\ x^3 = z$, Maxwell equations with Lorentz condition are expressed as follows.

$$\begin{aligned} \Box A^\mu &= \mu_0 j^\mu \\ \partial_\mu A^\mu &= 0 \end{aligned} \tag{21}$$

In addition, the conservation of charge div $\mathbf{i} + \partial\rho/\partial t = 0$ is expressed as $\partial_\mu j^\mu = 0$. Where $\partial_\mu = (1/c\partial t,\ 1/\partial x,\ 1/\partial y,\ 1/\partial z) = (1/\partial x^0,\ 1/\partial x^1,\ 1/\partial x^2,\ 1/\partial x^3)$ and $\Box$ stands for the d'alembertian: $\Box \equiv \partial_\mu \partial^\mu \equiv \partial^2/c^2\partial t^2 - \Delta$.

The transformation between covariance and contravariance vector can be calculated by using the simplest form of Minkowski metric tensor $\mathsf{g}_{\mu\nu}$ as follows.

$$\begin{aligned} \mathsf{g}_{\mu\nu} = \mathsf{g}^{\mu\nu} &= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix} \\ A_\mu &= \mathsf{g}_{\mu\nu} A^\nu \\ A^\mu &= \mathsf{g}^{\mu\nu} A_\nu \end{aligned} \tag{22}$$

The following quadratic form of four-vectors is invariant under a Lorentz transformation.

$$(x^0)^2 - (x^1)^2 - (x^2)^2 - (x^3)^2 \tag{23}$$

The above quadratic form applied a minus sign expresses the wave front equation and can be described by using metric tensor.

$$-\mathsf{g}_{\mu\nu} x^\mu x^\nu = -x^\mu x_\mu = x^2 + y^2 + z^2 - c^2 t^2 = 0 \tag{24}$$

This quadratic form which includes minus sign is also introduced to inner product of arbitrarily vectors and commutation relations in Minkowski space.

The four-vector potential satisfied Maxwell equations with vanishing the four-vector current can be expressed as following Fourier transform in terms of plane wave solutions.[24]

$$A_\mu(x) = \int d\tilde{k} \sum_{\lambda=0}^{3} [a^{(\lambda)}(k)\epsilon_\mu^{(\lambda)}(k) e^{-ik\cdot x} + a^{(\lambda)\dagger}(k)\epsilon_\mu^{(\lambda)*}(k) e^{ik\cdot x}] \tag{25}$$

$$\tilde{k} = \frac{d^3k}{2k_0(2\pi)^3} \quad k_0 = |\mathbf{k}| \tag{26}$$

where the unit vector of time-axis direction n and polarization vectors $\epsilon_\mu^{(\lambda)}(k)$ are introduced as $n^2 = 1,\ n^0 > 0$ and $\epsilon^{(0)} = n$, $\epsilon^{(1)}$ and $\epsilon^{(2)}$ are in the plane orthogonal to k and n

$$\epsilon^{(\lambda)}(k) \cdot \epsilon^{(\lambda')}(k) = -\delta_{\lambda,\lambda'} \quad \lambda\ ,\ \lambda' = 1,\ 2 \tag{27}$$

$\epsilon^{(3)}$ is in the plane $(k,\ n)$ orthogonal to n and normalized

$$\epsilon^{(3)}(k) \cdot n = 0\ ,\ [\epsilon^{(3)}(k)]^2 = -1 \tag{28}$$

Then $\epsilon^{(0)}$ can be recognized as a polarization vector of scalar waves, $\epsilon^{(1)}$ and $\epsilon^{(2)}$ of transversal waves and $\epsilon^{(3)}$ of a longitudinal wave. Then we take these vectors as following the easiest forms.

$$\epsilon^{(0)} = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} \quad \epsilon^{(1)} = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix} \quad \epsilon^{(2)} = \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix} \quad \epsilon^{(3)} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} \tag{29}$$

When the Fourier coefficients of the four-vector potentials are replaced by operators as $\hat{A}_\mu \equiv \sum_{\lambda=0}^{3} \hat{a}^{(\lambda)}(k)\epsilon_\mu^{(\lambda)}(k)$, the commutation relations are obtained as follows.

$$[\hat{A}_\mu(k),\ \hat{A}_\nu^\dagger(k')] = -\mathsf{g}_{\mu\nu}\delta(k-k') \tag{30}$$

The time-axis component (corresponds to $\mu, \nu = 0$ scalar wave, i. e., scalar potential because $\epsilon_\mu^{(0)}(k) = 0\ (\mu \neq 0)$) has the opposite sign of the space axes. Because $\langle 0|\hat{A}_0(k)\hat{A}_0^\dagger(k')|0\rangle = -\delta(k-k')$ then

$$\langle 1|1\rangle = -\langle 0|0\rangle \int d\tilde{k} |f(k)|^2 \tag{31}$$

where $|1\rangle = \int d\tilde{k} f(k)\hat{A}_0^\dagger(k)|0\rangle$. Therefore the time-axis component is the root cause of indefinite metric. Note that the products of the operators replaced from the four-vectors must introduce the same formalism.

$$\hat{A}^\dagger \hat{A} = -\mathsf{g}_{\mu\nu}\hat{A}^{\mu\dagger}\hat{A}^\nu \tag{32}$$

In order to utilize the indefinite metric as followings, Coulomb gauge that removes the scalar potentials should not be used.

Here we can recognize the potentials before passing through the polarizers 1 and 2 as

$$A_\mu = (A_0,\ A_1,\ A_2,\ 0) \tag{33}$$

where, we neglect the longitudinal wave which is considered to be unphysical presence, i. e., $A_3 = 0$ for simplicity. When there are an x-polarized photon and scalar potential and pass through the each polarizers, then the potentials passing through the polarizers can be expressed as

$$\begin{aligned} A_{(\text{x pol 1})\ \mu} &= \left(\frac{1}{2}e^{i\theta_x/2}A_{(x)0},\ A_{(x)1},\ 0,\ 0\right) \\ A_{(\text{x pol 2})\ \mu} &= \left(\frac{1}{2}e^{-i\theta_x/2}A_{(x)0},\ 0,\ 0,\ 0\right) \end{aligned} \tag{34}$$

When these scalar potentials undergo a $|\phi|$ phase shift, i. e., the angle of the polarizer 3, by passing through the polarizer 3, the phase terms will be shifted to $\pm i\,(|\phi| + \theta_x/2)$. Here we identify the number operators as $\langle 1|A_0^\dagger A_0|1\rangle = \langle 1|A_1^\dagger A_1|1\rangle = \langle 1|A_2^\dagger A_2|1\rangle = 1$ because of the Lorentz invariance. Hence the single photon interference (8) or (18) is obtained as followings.

$$\begin{aligned} A_{(\text{x pol 1, 2}\to 3)\ \mu} &\equiv A_{(\text{x pol 1}\to 3)\ \mu} + A_{(\text{x pol 2}\to 3)\ \mu} \\ &= \left(\cos(|\phi| + \frac{\theta_x}{2})A_{(x)0},\ A_{(x)1},\ 0,\ 0\right) \end{aligned} \tag{35}$$

$$\begin{aligned} \langle I_s\rangle &\propto \langle 1|A^\dagger_{(\text{x pol 1, 2}\to 3)}A_{(\text{x pol 1, 2}\to 3)}|1\rangle \\ &= \frac{1}{2} - \frac{1}{2}\cos(2|\phi| + \theta_x) \end{aligned} \tag{36}$$

Similarly, in case of a y-polarized photon

$$\begin{aligned} A_{(\text{y pol 1})\ \mu} &= \left(\frac{1}{2}e^{i\theta_y/2}A_{(y)0},\ 0,\ 0,\ 0\right) \\ A_{(\text{y pol 2})\ \mu} &= \left(\frac{1}{2}e^{-i\theta_y/2}A_{(y)0},\ 0,\ A_{(y)2},\ 0\right) \end{aligned} \tag{37}$$

$$A_{(\text{y pol 1, 2}\to 3)\ \mu} \equiv A_{(\text{y pol 1}\to 3)\ \mu} + A_{(\text{y pol 2}\to 3)\ \mu}$$

$$= \left(\cos(|\phi| + \frac{\theta_y}{2})A_{(y)0},\ 0,\ A_{(y)2},\ 0\right) \tag{38}$$

Then

$$\begin{aligned} \langle I_p \rangle &\propto \langle 1|A^\dagger_{(\text{y pol 1, 2}\to 3)}A_{(\text{y pol 1, 2}\to 3)}|1\rangle \\ &= \frac{1}{2} - \frac{1}{2}\cos(2|\phi| + \theta_y) \end{aligned} \tag{39}$$

By choosing $\theta \equiv \theta_x = -(\theta_y + \pi)$, i. e., the potentials undergo π phase shift and the relatively-same phase shift at polarizer 1 and 2 when divided,

$$\begin{aligned} \langle I_s \rangle &\propto \frac{1}{2} - \frac{1}{2}\cos(2|\phi| + \theta) \\ \langle I_p \rangle &\propto \frac{1}{2} + \frac{1}{2}\cos(2|\phi| - \theta) \end{aligned} \tag{40}$$

Hence we should choose $\theta = \theta + \pi$ to correct the reversed signs, which is attributed to the difference between using $\gamma^2 = -1$ and tensor form.

In case of both polarization photon exist, the potentials just before the polarizer 3 will be expressed by summation of (34) and (37). Then the potentials undergo a $|\phi|$ phase shift by the polarizer 3 can be expressed as follows.

$$A_{(\text{x, y pol 1, 2}\to 3)\ \mu} = \left(A_{(x)0}\cos(|\phi| + \frac{\theta_x}{2}) + A_{(y)0}\cos(|\phi| + \frac{\theta_y}{2}),\ A_{(x)1},\ A_{(y)2},\ 0\right) \tag{41}$$

Therefore the photon number operator of the output of the polarizer 3 can be calculated as follows.

$$\begin{aligned} & A^\dagger_{(\text{x, y pol 1, 2}\to 3)}A_{(\text{x, y pol 1, 2}\to 3)} \\ = & -A^\dagger_{(x)0}A_{(x)0}\cos^2(|\phi| + \frac{\theta_x}{2}) - A^\dagger_{(y)0}A_{(y)0}\cos^2(|\phi| + \frac{\theta_y}{2})) \\ & -(A^\dagger_{(x)0}A_{(y)0} + A^\dagger_{(y)0}A_{(x)0})\cos(|\phi| + \frac{\theta_x}{2})\cos(|\phi| + \frac{\theta_y}{2}) \\ & +A^\dagger_{(x)1}A_{(x)1} + A^\dagger_{(y)2}A_{(y)2} \end{aligned} \tag{42}$$

Then by choosing $\theta \equiv \theta_x = -(\theta_y + \pi)$,

$$\begin{aligned} & \langle 1|A^\dagger_{(\text{x, y pol 1, 2}\to 3)}A_{(\text{x, y pol 1, 2}\to 3)}|1\rangle \\ = & \ 1 - \frac{1}{2}\cos(2|\phi| + \theta) + \frac{1}{2}\cos(2|\phi| - \theta) \\ & -\langle 1|(A^\dagger_{(x)0}A_{(y)0} + A^\dagger_{(y)0}A_{(x)0})|1\rangle\cos(|\phi| + \frac{\theta}{2})\sin(|\phi| - \frac{\theta}{2}) \end{aligned} \tag{43}$$

Here we should recognize $|1\rangle = (|1\rangle_x + |1\rangle_y)$ as mentioned above and $A_{(x)0}$ and $A_{(y)0}$ annihilate x and y-polarized photon respectively, i. e., $A_{(x)0}|1\rangle = |0\rangle_x$ and $A_{(y)0}|1\rangle = |0\rangle_y$. Because ${}_x\langle 0|0\rangle_y = 0$, then

$$-\langle 1|(A^\dagger_{(x)0}A_{(y)0} + A^\dagger_{(y)0}A_{(x)0})|1\rangle = 0 \tag{44}$$

Hence (43) corresponds to (11) and (14) except the π phase shift of θ.

6. Conclusions

We have presented the quantum eraser can be explained without quantum-superposition states by introducing the states represent the unobservable (scalar) potentials whose probability amplitudes are zero. The explanation presents a image of vacuum that can create and annihilate the substantial photons.

We have also investigated the delayed choice experiment under the assumption that the polarization of the photon pairs is determined by the unobservable (scalar) potentials which are oriented by the setup of the experiment in advance. In addition to these discussions based on a method for convenience in calculation, we have shown rigorous mathematical treatment using tensor form (covariant quantization).

The new explanations obtained in the present paper are more general and appear to be physically more consistent than traditional explanations which require paradoxical quantum-superposition states and entangled states.

The other experiments and considerations have been reported, which seem like paradoxes.[11–13,25–27] We believe the paradoxes can be avoided by the new explanation and conclude that engineering application utilizing wave packet reduction or entangled states will fail because there are no concepts of quantum-superposition and entangle states in nature.

Acknowledgements

The author would like to thank K. Sato and Dr. S. Takasaka for their helpful discussions.

1) J. D. Trimmer: Proceedings of the American Philosophical Society **124** (1980) 323.
2) A. Einstein, B. Podolsky, and N. Rosen: Phys. Rev **47** (1935) 777.
3) D. Bohm: Phys. Rev. **85** (1952) 166.
4) D. Bohm: Phys. Rev. **85** (1952) 180.
5) J. S. BELL: Rev. Mod. Phys. **38** (1966) 447.
6) X. Oriols and J. Mompart: ArXiv e-prints (2012).
7) P. Pylkkänen, B. J. Hiley, and I. Pättiniemi: ArXiv e-prints (2014).
8) D. Durr, S. Goldstein, T. Norsen, W. Struyve, and N. Zanghi: Royal Society of London Proceedings Series A **470** (2013) 30699.
9) V. Allori, S. Goldstein, R. Tumulka, and N. Zanghi: ArXiv e-prints (2012).
10) G. Dennis, M. A. de Gosson, and B. J. Hiley: Physics Letters A **378** (2014) 2363 .
11) A. Aspect, P. Grangier, and G. Roger: Physical review letters **47** (1981) 460.
12) A. Aspect, P. Grangier, and G. Roger: Physical review letters **49** (1982) 91.
13) A. Aspect, J. Dalibard, and G. Roger: Physical review letters **49** (1982) 1804.
14) W. Ehrenberg and R. E. Siday: Proceedings of the Physical Society. Section B **62** (1949) 8.
15) M. Morimoto: http://vixra.org/abs/1312.0097 (2013).
16) Y. Aharonov and D. Bohm: Phys. Rev. **115** (1959) 485.
17) C. Meis: Physics Research International **2014** (2014) 5.
18) C. N. Yang and R. L. Mills: Phys. Rev. **96** (1954) 191.
19) T. T. Wu and C. N. Yang: Phys. Rev. D **12** (1975) 3845.
20) S. Weinberg: Phys. Rev. Lett. **19** (1967) 1264.
21) R. Utiyama: Phys. Rev. **101** (1956) 1597.
22) P. K. Rachel Hillmer: Scientific American (2007) 9095.
23) S. P. Walborn, M. O. Terra Cunha, S. Pádua, and C. H. Monken: Phys. Rev. A **65** (2002) 033818.
24) C. Itzykson and J. B. Zuber: *Quantum Field Theory* (McGraw-Hill, 1985).
25) D. Salart, A. Baas, J. A. W. van Houwelingen, N. Gisin, and H. Zbinden: Phys. Rev. Lett. **100** (2008) 220404.

26) Y.-H. Kim, R. Yu, S. P. Kulik, Y. Shih, and M. O. Scully: Phys. Rev. Lett. **84** (2000) 1.
27) J. F. Clauser and M. A. Horne: Physical review D **10** (1974) 526.