Abstract
In this work, we calculated the input impedance of dipole antenna coupled with two parasitic resonators. It is shown that the input impedance has negative value in specific configurations. The dipole antenna works as a power generator when its input impedance is negative.

Introduction
Negative input impedance is a special interest in power transmission. If a system has negative input impedance, this system becomes a power generator [1-4]. It is well known of the input impedance reduction in antenna designing. As in Yagi-Uda antenna, the reflected impedance from the parasitic resonators is usually negative so that the input impedance of driven dipole is significant low [5]. It is also observed that, in array antenna, one of the driven dipole may have negative input impedance if there are more than one driven dipoles in the system. However, if there is only one driven dipole, negative input impedance of system has never been reported.

We studied the system of dipole antenna coupled with two parasitic resonators. It is indicated that such a system have negative input impedance in specific configurations.
Methods and Results

For a system composed of one driven dipole and two parasitic resonators as shown in Fig. 1(a), the coupled equations are:

\[ V_1 + Z_{12}I_2 + Z_{13}I_3 = Z_1 I_1 \]  
\[ Z_{21}I_1 + Z_{23}I_3 = Z_2 I_2 \]  
\[ Z_{31}I_1 + Z_{32}I_2 = Z_3 I_3 \]  

Where \( V_1 \) and \( I_1 \) are the voltage and current in the driven dipole respectively; \( I_2 \) and \( I_3 \) are the current in the two parasitic resonators; \( Z_1, Z_2, \) and \( Z_3 \) are the self-impedance of each element; \( Z_{12}, Z_{21}, Z_{13}, Z_{31}, Z_{23}, \) and \( Z_{32} \) are the mutual impedance between each pair. The solution of above equations is

\[ V_1 = Z I_1 \]

\[ Z = Z_1 - \frac{Z_2 Z_{13} Z_{31} + Z_3 Z_{12} Z_{21} + 2 Z_{12} Z_{23} Z_{31}}{Z_2 Z_3 - Z_{23} Z_{32}} \]  

Where \( Z \) is the input impedance of the system. The self-impedance depends on the length of each element. The real and imaginary parts of a dipole’s impedance are shown in Fig. 2 for lengths going from 0.4 \( \lambda \) to 0.6 \( \lambda \) [5]. The mutual impedance depends on the length of each element as well as the distance and orientation between each pair. Mutual impedance between parallel \( \frac{\lambda}{2} \) dipoles not staggered as a function of spacing is shown in Fig. 3 [6]. Due to reciprocity we know that \( Z_{12} = Z_{21}, Z_{13} = Z_{31}, \) and \( Z_{23} = Z_{32}. \) First, we consider the configuration in which the distances between each element pair are all equal, \( d_{12} = d_{23} = d_{13}, \) as shown in Fig. 1(b). All three elements are \( \frac{\lambda}{2} \) length. The calculation of system’s input impedance \( Z \) is shown in Fig. 4. The
real part of input impedance is negative when distance is less than 0.1 λ. Second, we consider the configuration in which all the three elements lay in one plane as shown in Fig. 1(a). For the reason of simplicity, we only calculated couple cases as listed in Table 1.

Table 1: System’s input impedance when three elements lay in one plane. Negative distance indicates opposite location relative to the driven dipole.

<table>
<thead>
<tr>
<th>d_{12}</th>
<th>d_{13}</th>
<th>d_{23}</th>
<th>Re(Z) (Ω)</th>
<th>Im(Z) (Ω)</th>
</tr>
</thead>
<tbody>
<tr>
<td>- 0.1 λ</td>
<td>0.1 λ</td>
<td>0.2 λ</td>
<td>6.6</td>
<td>155.9</td>
</tr>
<tr>
<td>- 0.05 λ</td>
<td>0.05 λ</td>
<td>0.1 λ</td>
<td>-142.0</td>
<td>246.2</td>
</tr>
<tr>
<td>- 0.05 λ</td>
<td>0.1 λ</td>
<td>0.15 λ</td>
<td>-42.1</td>
<td>184.7</td>
</tr>
<tr>
<td>-0.05 λ</td>
<td>0.15 λ</td>
<td>0.2 λ</td>
<td>6.7</td>
<td>146.6</td>
</tr>
<tr>
<td>0.05 λ</td>
<td>0.1 λ</td>
<td>0.05 λ</td>
<td>-167.5</td>
<td>466.3</td>
</tr>
<tr>
<td>0.05 λ</td>
<td>0.15 λ</td>
<td>0.1 λ</td>
<td>-15.5</td>
<td>248.1</td>
</tr>
<tr>
<td>0.1 λ</td>
<td>0.15 λ</td>
<td>0.05 λ</td>
<td>50.0</td>
<td>430.4</td>
</tr>
</tbody>
</table>

Discussion
The input impedance has negative value only in near-field coupling where the distances between elements are less than 0.1 λ. Experimental measurements are desired to verify our calculations. The dipole antenna becomes a power generator when its input impedance is negative and the generated power can be extracted to a load. For the performance of power generation, the imaginary part need be eliminated and the parameters such as length of elements and distances between them need be optimized.
References


Figures

Figure 1: (a) Schematic of one driven dipole antenna coupled with two parasitic resonators. (b) A specific configuration where all the distances between elements are equal.

Figure 2: The real and imaginary parts of a dipole's impedance drawn for lengths going from $0.4\lambda$ to $0.6\lambda$ [5].

Figure 3: Mutual impedance between parallel $\frac{\lambda}{2}$ dipoles not staggered as a function of spacing. Curves $\text{Re}$ and $\text{Im}$ are the resistive and reactive parts of the mutual impedance [6].

Figure 4: System input impedance vs distance between elements for configurations in which the length of driven dipole and parasitic resonators are all $\frac{\lambda}{2}$ and the distance between elements are all equal.
Figure 1
Figure 2
Figure 3
Figure 4