A ‘LOGICAL’ APPROACH TO URBAN SUSTAINABILITY INDICATOR DESIGN
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ABSTRACT
The present article proposes a methodology for designing urban sustainability indicators based on fuzzy logic. The aim is to provide a sound basis that increases ‘understandability’ and ‘shareability’ of sustainability indicators.

1_ INTRODUCTION
Our current concerns in relation to the sustainability of our development models have taken to an everyday increasing number of different proposals for measuring urban sustainability, usually in the form of sustainability indexes and assessment tools.

While this constantly increase of measuring models can be understood as something positive [as it increases the number of available options] it conceals a worrying question; the lack of a common approach to sustainability measurement leads every new proposal to ‘start from scratch’ ignoring most of existing knowledge.

This article proposes a methodology for sustainability indicator design that allows us to fully understand what we are measuring and how are we measuring it, increasing ‘shareability’ and ‘understandability’ of sustainability indicators.

The methodology is proposed based on the two approaches to ‘logic’ from Set or ‘Class Theory’1:

- Classic Set Theory or Boolean Logic [Boole 1854] allows us to conceptualize sustainability from ‘a Non-contradiction Principle’ as the complement of ‘unsustainability’
- Fuzzy Sets Theory or Fuzzy Logic [Zadeh 1965] allows us to conceptualize the ‘sustainability degree of a city’ as its ‘Grade of membership’ to the ‘class’ of sustainable cities.

A general definition of a ‘sustainable city’ as a ‘city with the maximum probability to indefinitely endure’ shall suffice for the present proposal.

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1 Although in some contexts there can be a difference between a ‘set’ and ‘class’; for the present work both terms can be considered to be equivalent to ‘class’.

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2. SET THEORY OR BOOLEAN LOGIC: SUSTAINABILITY AND UNSUSTAINABILITY AS COMPLEMENTARY SETS

Classic Set Theory ‘organizes’ objects into different classes assigning to each object ‘x’ a binary membership function to a set or class A; a value ‘0’ means that ‘x’ does not belong to A and a value ‘1’ means that ‘x’ belongs to A.

\[
f_a(x) = 0 \text{ if } x \notin A \\
1 \text{ if } x \in A
\]

*Fig. 1: Binary Membership function 'b' is the value of x the separates null from full membership to class A.*

Membership as proposed by Classic set theory or Boolean logic implies therefore the idea of ‘mutually exclusive classes or concepts’ that we can define as those whose intersection is empty and its union provides the universe of discourse:

\[
X \cup \neg X = \Omega [R] \\
X \cap \neg X = \emptyset
\]  

(1)  

(2)  

This last formula expresses the ‘duality law’ [Boole 1854] as a ‘condition for interpretability of logical functions’, and is in fact a re-formulation of Aristotle’s ‘non-contradiction principle’.

And we can make a first conceptualization of Urban Sustainability if we take the set of all ‘cities’ and divide it into two subsets:

- We designate S or ‘Sustainability’ the set composed by all sustainable cities
- We designate \( \neg S \) or ‘Unsustainability’ the set composed by all non-sustainable cities.

Following the previous criteria the union of S and \( \neg S \) [sustainable and non-sustainable cities] must contain all cities, while their intersection must be empty:

\[
S \cup \neg S = ‘Cities’ = \Omega [R] \\
S \cap \neg S = \emptyset
\]  

(3)  

What we can express graphically as:

*Fig. 2: ‘Sustainability’ and ‘Unsustainability sets are complement in the universe ‘Cities’."

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The drawback of this approach from Boolean logic is that though being theoretically correct [in the long run a binary approach is the only possible; a city does endure or does not] it does not fit the proposed definition; a city may have a continuum range of probabilities to ‘indefinitely endure’.

To solve it, we shall approach ‘sustainability’ from Fuzzy Set Theory or Logic.

3. FUZZY SET THEORY AND LOGIC: SUSTAINABILITY DEGREE AS ‘GRADE OF MEMBERSHIP’

A fuzzy set is a ‘class’ characterized by a membership function ‘fa(x)’ that associates each element x of a universe X to a number in the range [0,1]; i.e., a class with a continuum of ‘grades of membership’²:

\[ a = \{ [x, f_a(x)] | x \in X \} \]

\[ f_a(x) \rightarrow [0,1] \]  \hspace{1cm} (4)

Fuzzy Logic is a development of Boolean logic to confront intermediate situations that allow ‘grades of membership’ and ‘exclusion’; ‘widening’ the applicability of the non-contradiction principle.

While classical logic can only be referred to ‘mutually exclusive concepts’ [i.e., concepts that must be true or false applied to an ‘object’] fuzzy logic can be referred to any concept or quality that can ‘partly’ be true; any ‘object’ can be characterized by the ‘degree in which it is a quality and the non-quality’; by the degree it belongs –its ‘grade of membership’- to a class and to its opposite or complement.

A fuzzy membership function can take any value in the range [0,1] and this allows us to measure urban sustainability and unsustainability in terms of ‘grade of sustainability/unsustainability’

- the ‘degree of sustainability’ of a city I at a moment ‘T’ will be its ‘grade of membership’ to S and we designate it as ST[I]

\[ S_T[I] = f_S[I] \] \hspace{1cm} (5)

- the ‘degree of unsustainability of a city I at a moment ‘T’ will be its ‘grade of membership’ to ¬S and we designate it as ¬ST[I]

\[ \neg S_T[I] = f_{\neg S}[I] \] \hspace{1cm} (6)

² This definition and the majority that follow are from ZADEH, 1965
Therefore, the degree of sustainability of a city ‘I’ at any moment ‘T’ will have a value between 0 and 1, and we can differentiate three values/ranges of values with a special meaning:

- \( S_1[I] = 1 \) the membership to ‘Sustainability’ class is complete, and therefore the grade of membership to ‘Unsustainability’ class is zero.
- \( 0 < S_1[I] < 1 \) the city has a grade of membership to ‘Sustainability’ class, complementary to its grade of membership to ‘Unsustainability’ class
- \( S_1[I] = 0 \) the ‘grade of membership’ to ‘Sustainability’ class is zero, and therefore the membership to ‘Unsustainability’ class is complete.

3.1. PROPERTIES OF FUZZY SETS
Fuzzy sets have some properties that are quite interesting for the proposed approach:

**COMPLEMENT**
The complement of a set A is denoted as \( \neg A \) and defined as:

\[
f_A[x] = 1 - f_{\neg A}[x]
\]  

(7)

**CONTAINMENT**
If A is contained in B its membership function \( f_A[x] \) is smaller than \( f_B[x] \) for any \( x \):

\[
\forall x \in X: A \subset B \rightarrow f_A[x] \leq f_B[x]
\]  

This property imposes an important condition to the ‘degree of sustainability’ of a city; it must always be equal or smaller than the environment that contains it.

**UNION**
The union of two fuzzy sets A and B with respective membership functions \( f_A[x] \) \& \( f_B[x] \) is a fuzzy set C, which membership function is \( f_C[x] \):

\[
C = A \cup B \rightarrow \forall x \in X: f_C[x] = \max[f_A[x] \land f_B[x]]
\]  

(9)

**INTERSECTION**
The intersection of two fuzzy sets A and B with respective membership functions \( f_A[x] \) \& \( f_B[x] \) is a fuzzy set C which membership function is \( f_C[x] \):

\[
C = A \cap B \rightarrow \forall x \in X: f_C[x] = \min[f_A[x] \land f_B[x]]
\]  

(10)

**SIGNIFICANCE OR CUT VALUES**
Significance or ‘cut values’ indicate ‘thresholds’ at which membership is held to be null or complete; if we consider for example a membership function with two cut values \( \alpha \) and \( \beta \):

- If \( f_A[x] \) is smaller than \( \alpha \) it means that the membership of \( x \) to A is zero.
- If \( f_A[x] \) is higher than \( \beta \) it means that the membership of \( x \) to A is complete.
- If \( f_A[x] \) is between \( \alpha \) and \( \beta \) it means a ‘Grade of membership’ of \( x \) to A.

<table>
<thead>
<tr>
<th>Significance values</th>
<th>( f_A[x] &lt; \alpha \rightarrow f_A[x] = 0 \land f_{\neg A}[x] = 1 )</th>
<th>( f_A[x] &gt; \beta \rightarrow f_A[x] = 1 \land f_{\neg A}[x] = 0 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \alpha &lt; f_A[x] &lt; \beta \rightarrow f_A[x] \in [0,1] \land f_{\neg A}[x] \in [0,1] )</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

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‘Significance values’ are useful for ‘decision making tools’ allowing incorporating into the indicators ‘sufficient values’ and ‘non-negotiable’ minimums for decisions to be positive.

From previous formulas it is easy to state that:

\[ S_T[I] \cup \neg S_T[I] = \Omega = 1 \]  \hspace{1cm} (12)

So...

\[ S_T[I] = 1 - \neg S_T[I] \]  \hspace{1cm} (13)

Therefore, the ‘Unsustainability degree’ is:

\[ \neg S_T[I] = 1 - S_T[I] \]  \hspace{1cm} (14)

This means that the lack of a ‘complete sustainability’ necessarily implies a ‘degree’ of unsustainability, and \( S_I[I]=0.5 \) becomes a ‘limiting’ value; a line that separates the cities that are more ‘sustainable’ than ‘unsustainable’ \( [S_I[I]>0.5] \) from the cities that are more ‘unsustainable’ than sustainable \( [S_I[I]<0.5] \).

\[ S_T[I] < 0.5 \leftrightarrow S_T[I] < \neg S_T[I] \]  \hspace{1cm} (15)

3.2_GRAPHIC REPRESENTATION OF MEMBERSHIP FUNCTIONS

Graphic representation of membership functions allows us to review a couple of important issues:

If we consider a membership function of a fuzzy variable ‘i’ that defines the grade of membership of an element ‘x’ to a class A we can see the existence of two especially ‘meaningful’ values or points:

- A value ‘\( i_1 \)’ so that if \( i \leq i_1 \) ‘x’ membership to class A becomes ‘zero’ -[and therefore, its membership to \( \neg A \) class becomes ‘complete’]

\[ \exists i_1: i \leq i_1 \leftrightarrow f_A[x] = 0 \land f_{\neg A}[x] = 1 \]  \hspace{1cm} (16)

- A value ‘\( i_2 \)’ so that if \( i \geq i_2 \) ‘x’ membership to class A becomes ‘complete’ [and therefore, its membership to \( \neg A \) class becomes ‘zero’]

\[ \exists i_2: i \geq i_2 \leftrightarrow f_A[x] = 1 \land f_{\neg A}[x] = 0 \]  \hspace{1cm} (17)
Both values are fundamental for the design of an urban sustainability indicator in relation to a certain variable information ‘i’ of a city I; ‘i1’ is its unsustainability threshold or limit, and ‘i2’ is its sustainability limit or goal.

The necessary existence of these ‘limits’ allows us to define a ‘variable relevant for urban sustainability’ as a variable for which at least exist one ‘unsustainability limit’ and one ‘sustainability limit’; i.e.: a variable that can produce both sustainability and unsustainability.

And consequently, the sustainability limits of a variable are the delimiting values for the range of the variable ‘i’ that produces fuzzy membership of ‘i’ to S; i.e., the ‘first” values that produce either complete membership of the city to S or to ¬S classes.

The second interesting issue of graphical representation is that it allows synthesizing the membership to a set and to its complement on the same graphic:

\[ f_{A}(x) = \max \left[ \min \left( \frac{i - i_1}{i_2 - i_1}, 1 \right), 0 \right] \]
\[ f_{\sim A}(x) = 1 - f_{A}(x) \]
\[ f_{-A}(x) = \max \left[ \min \left( 1 - \frac{i - i_1}{i_2 - i_1}, 1 \right), 0 \right] \]

Fig. 4: Membership function of an element x to A and ¬A sets. There is a horizontal symmetry at \( f_{A}[x]=0.5 \), which separates the values of ‘i’ for which x belongs more to A of the values of ‘i’ for which x belongs more to ¬A.

And this property implies that membership to S and ¬S can be represented in the same graphic, but if they are not, the representation of one allows easily obtaining the other.

4. DESIGNING SUSTAINABILITY INDICATORS: SUSTAINABILITY DEGREE OF A CITY IN RELATION TO THE VARIABLES THAT DESCRIBE IT

We have proposed the sustainability degree of a city ‘I’ as its grade of membership to class S, but it is necessary to state that it depends on many different variables and relationships between variables and usually we are not able to calculate it with only one formulation.
Thus, we have to do a ‘progressive’ approach; then analyze the concept ‘Sustainable City’ to detect the concepts or qualities $S_i$ that we expect it to have [the propositions or concepts that we expect to be true when referred to a ‘sustainable city’] and finally review the information that defines the grade of truth of those concepts or ‘propositions’ referred to the city.

For instance; we usually state that a sustainable city must have ‘good employment levels’; ‘accessible public transport service’; ‘adequate provision of green areas’, etc.. And indicators measure the degree of truth of those propositions referred to the city [i.e.: city ‘I’ has good employment levels; accessible transport, etc...]; which can be modeled as ‘membership functions’ to those different classes [to the class of the cities with good employment levels, to the class of the cities with adequate provision of parks, etc...]

Urban sustainability indicators are equivalent to membership functions of the city to the different concepts $S_i$ ‘contained’ in class $S$ for the possible values of different variables ‘i’, and its maximum and minimum values have the following meanings:

- $S[i]=0$ means null membership to $S_i$ [and ‘complete’ membership to $\neg S_i$]; the city does not have at all a quality expected in a sustainable city.
- $S[i]=1$ means ‘complete’ membership to $S_i$ [and null membership to $\neg S_i$]; the city totally fulfills the quality expected in a sustainable city.

The unsustainability/sustainability ‘limits’ of the variables are the values of which null and complete memberships are reached, and they are especially relevant for mathematical formulation of indicators:

### 4.1 SUSTAINABILITY AND UNSUSTAINABILITY LIMITS

A variable ‘$i$’ can only be relevant for the sustainability of a system $I$ if it can produce both its sustainability and unsustainability. The existence of limits is a necessary condition for ‘$i$’ to be relevant for $I$ sustainability, and these limits allow us to formulate its sustainability indicator.

In its more ‘simple’ form, the limits are two parameters that divide in three different zones the impact on the ‘Grade of membership’ of a city ‘$i$’ to any class $S_i$ implied in $S$, for the range of possible values of ‘$i$’:

- The first is the value of ‘$i$’ for which ‘$i$’ reaches null membership to $S_i$ which we have designated as ‘Unsustainability limit or threshold’
- The second is the value of ‘$i$’ for which ‘$i$’ reaches complete membership to $S_i$ which we have designated as ‘sustainability limit or goal’.

*Fig. 5: Relation between ‘$i$’ values, thresholds and sustainability/unsustainability degrees. For clarity in some graphics we may change notation $f_{\neg i}[I]$ for $f_i[I]$ meaning both ‘grade of membership’ of $I$ to $\neg S_i$.*
There are some important issues that need to be highlighted regarding the ‘limits’:

- They can be ‘exact values but also ‘ranges of values’ or even ‘dynamic’:
  - The ‘state’ of the system may modify the value of the limits
  - Any change in the system–environment may change the limits, including an evolutionary process.
- Containment property implies that sustainability degree of any city must always be equal or lower than that of its environment; which may impose ‘additional limits’
- For some variables urban sustainability may imply more than two limits

4.2. FORMULATION OF SUSTAINABILITY INDICATORS FOR DIFFERENT TYPES OF VARIABLES

Therefore a sustainability indicator may be understood as a membership function of a city ‘I’ to a class $S_i$ contained in class $S$ in relation to the possible values for some relevant variable information ‘I’; and the sustainability/unsustainability limits of ‘I’ become fundamental for its mathematical formulation.

There are many different possible formulations, but they share a common property; the majority can be deduced from a four limits formulation: two sustainability and two unsustainability limits.

<table>
<thead>
<tr>
<th>TABLE 01_ MEMBERSHIP FUNCTION AND GRAPHIC REPRESENTATION OF A FOUR LIMITS VARIABLE ‘I’</th>
</tr>
</thead>
<tbody>
<tr>
<td>$s[I] = \max \left[ \min \left[ \frac{i - a}{b - a}, 1, \frac{d - i}{d - c} \right] \right]$</td>
</tr>
<tr>
<td>And thus: $S[I] = \max \left[ \min \left[ \frac{i - \text{lim}<em>{1}[I]}{\text{lim}</em>{1}[I] - \text{lim}_{1}[I]} \right] , 1; \right.$</td>
</tr>
<tr>
<td>$\left[ \frac{\text{lim}<em>{2}[I] - i}{\text{lim}</em>{2}[I] - \text{lim}_{2}[I]} \right] ; 0 \right]$</td>
</tr>
</tbody>
</table>

Source: own elaboration with the following comments:
1) Codes are the following:
   a. $S[I]$ _ Value of the sustainability indicator $I$ for a system $I$
   b. $I_i$ _ Value of ‘I’
   c. Lim$_{uis}[I]_1$ _ unsustainability threshold 1 for the system $I$ related to variable $i$.
   d. Lim$_{uis}[I]_2$ _ unsustainability threshold 2 for the system $I$ related to variable $i$.
   e. Lim$_{us}[I]_1$ _ sustainability limit or goal 1 for the system $I$ related to variable $i$.
   f. Lim$_{us}[I]_2$ _ sustainability limit or goal 2 for the system $I$ related to variable $i$.

Due to the short extent of the present article, we review other possible formulations directly through examples. However to focus on the procedure, we will avoid designing new indicators and will rather propose adapting some existing ones.

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2 For an explanation related to the limits of global ecosystem ‘Earth’ refer to RÖCKSTROM ET AL [2009], who suggest that if certain variables of a system get close to their unsustainability thresholds, the sustainable range of values for other variables changes.
4.3 EXAMPLES OF INDICATORS ADAPTED FOLLOWING THE PROPOSED APPROACH

**URBAN SUSTAINABILITY: SOCIAL HOUSING**

Can be understood as a four limit variable that can be formulated as:

![Image](image.png)

**TABLE 02_ URBAN SUSTAINABILITY: SOCIAL HOUSING**

<table>
<thead>
<tr>
<th>Indicator Graphic</th>
<th>Formulation</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image.png" alt="Image" /></td>
<td>$s[SH] = \max \left[ \min \left( \frac{t - \lim_{1}^{I_1}}{\lim_{1}^{I_1} - \lim_{1}^{I_2}}, 1; \frac{\lim_{1}^{I_2} - t}{\lim_{1}^{I_2} - \lim_{1}^{I_1}} \right) \right]$</td>
</tr>
</tbody>
</table>

And the simplified formula for the calculation is:

$$SH[I_{hi}] = \min \left( \frac{VP_{1}}{0.30}, 1; \frac{1.00 - VP_{1}}{0.40} \right) \times 100$$

Source: Own elaboration and Indicator 49. Viviendas con Protección Oficial [Aeub 2010, p. 75] with the following comments:

1) Codes mean the following:
   a. SH[I] _ Social Housing Indicator
   b. SHI _ Percentage of social housing in the area related to total number of houses.

2) Four limits need to be established for the indicator:
   c. Sustainability limits are 'SHs' are 30% γ 60%
   d. Unsustainability limits 'SHs' are 0% γ 100%

3) To maintain a balance of housing types through urban continuum, calculation should be referred to units no larger than 25 Ha [aprox. 60 acres]

**URBAN SUSTAINABILITY: POPULATION GROWTH**

Can be understood as a particular case of a four limit variable when $\lim_{1}^{I_1} = \lim_{2}^{I_1}$.

![Image](image.png)

**TABLE 03_ URBAN SUSTAINABILITY: POPULATION GROWTH**

<table>
<thead>
<tr>
<th>Indicator Graphic</th>
<th>Formulation</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image.png" alt="Image" /></td>
<td>$s[PG] = \max \left[ \min \left( \frac{1 - \lim_{1}^{I_1}}{\lim_{1}^{I_1} - \lim_{1}^{I_2}}, 1; \frac{\lim_{1}^{I_2} - t}{\lim_{1}^{I_2} - \lim_{1}^{I_1}} \right) \right]$</td>
</tr>
</tbody>
</table>

And the simplified formula for the calculation is:

$$PG[I_{hi}] = \min \left( 1; 1 - \frac{CP_{1}^{2}}{1.10} \right) \times 100$$

Source: Own elaboration and SHCC. Indicator Population Growth [Graymore et al 2010, p. 463] with the following comments:

1) Though it is in fact a three-limit variable, a symmetry axis appears at PGs1, which allows us to formulate it as a two limits variable.

2) Authors [Graymore Et Al] propose it as a discrete axis which could be modeled as the intersection of three fuzzy membership functions; however the proposed formulation uses only one membership function [equivalent to a two limit variable] which provides similar values through the interval, and maintains the limits.

3) Codes mean the following:
   a. PG[I] _ Population Growth Indicator
   b. PG1 _ Population Growth expressed as a percentage

4) Three limits have been established for the variable:
   c. Sustainability limit is PGs=0%
   d. Unsustainability limits are PGsz ±1.4%
**URBAN SUSTAINABILITY: ADEQUACY OF PROVISION OF PARKS AND OPEN SPACES**

Can be understood as a particular case of a two limit variable when $\lim_{u \to 1} [I] = 0$, that can be formulated as:

$$s[APP] = \max \left[ \min \left[ \frac{i - \lim_{u \to 1} i}{\lim_{u \to 1} i - \lim_{u \to 1} i}; 1 \right]; 0 \right]$$

And the simplified formula for the calculation is:

$$APP[\%] = \min \left[ \frac{APP_i}{APPS}; 1 \right] \times 100$$

Source: Own elaboration and CASBEE for Cities. Q2.1.2. Adequate provision of parks and open spaces [SBC 2011] with the following comments:

1) Codes mean the following:
   - APPs1: Adequacy of Provision of Parks and Open Spaces Indicator
   - APPs2: Provision of Parks in the assessed area [m2/hab]
   - APPs3: Sustainability goal in Adequacy of provision of parks

2) Two limits have been established for the variable:
   a. The ‘sustainability goal’ ‘APPs1’ is to achieve 13m2/hab suggested by JSBC [2011] intermediate value between the 10 and 15 m2 suggested by WHO.
   b. The unsustainability threshold is the existence of no green areas at all ‘APPs1’=0 [from JSBC data a value of 2m2/hab could be understood, but we prefer 0 that is more ‘intuitive’].

3) Many ‘urban sustainability indicators’ can be formulated this way, making this example interesting [especially as it simplifies previous formulations].

**URBAN SUSTAINABILITY: ACCESS TO PUBLIC TRANSPORT**

Can be understood as a two limit variable that can be formulated as:

$$S[APT] = \max \left[ \min \left[ \frac{\lim_{z \to 1} i - i}{\lim_{z \to 1} i - \lim_{z \to 2} i}; 1 \right]; 0 \right]$$

And the simplified formula for the calculation is:

$$APT[\%] = \min \left[ \frac{\frac{950 - APT_i}{600}}{1}; 1 \right] \times 100$$

Source: Own elaboration and BREEM for Communities 2012. Indicator TM 04. Access to Public Transport [BRE 2012], with the following comments:

1) The indicator provides the accessibility degree of each building, that need to be aggregated for a global accessibility degree, what can be done different ways, for example:

   $$APT[\%] = \frac{1}{n} \sum_{i=1}^{n} APT_i$$

   Optionally, a weighting could be added [number of occupants of each building/ total ‘population’ of the area]

2) Codes mean the following:
   a. APT[\%]: Access To Public Transport Indicator (Global)
   b. APT[\%]: Access To Public Transport Indicator (Building)
   c. APT: distance (in meters) from Access of each building to a public transport station.
   d. ‘n’ total number of buildings in assessed area

3) Two limits have been established for the variable:
   a. ‘Complete Accessibility’ is achieved if distance from each building entrance to a public transport station [train bus, tramway or metro] is ‘APT[s2]’ shorter than 350 m. In ‘rural’ areas distance is increased to 700 m.
   b. ‘null accessibility’ is achieved when distance from building entrance to a public transport stop ‘APT[s2]’ is longer than 950m [own calculation considering 650 m suggested by BRE equivalent to degree of 0.5].

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**ENVIRONMENTAL SUSTAINABILITY: ENERGY CONSUMPTION PER PERSON**

It is a two limit nonlinear variable:

\[ s(E_{C}) = \max \left( \min \left( 1; 1 - \frac{E - \text{lim}_{\text{lower}}}{\text{lim}_{\text{upper}} - \text{lim}_{\text{lower}}} \right)^{1/2}; 0 \right) \]

And the simplified formula for the calculation is:

\[ E_{C}[^{\%}] = \max \left( \min \left( 1; 1 - \frac{E_{C}^{2}}{640^{2}} \right); 0 \right) \times 100 \]

Fuente: Own Elaboration and HWJ; Energy Consumption per person [Prescott Allen et Al 2001, p. 306] with the following comments:

1) Codes mean the following
   a. EC: Environmental Sustainability: Energy Consumption per person
   b. ECI: Annual Energy Consumption per person in Giga Julios

2) Authors [Prescott Allen Et Al] propose it as a discrete variable which could be modeled as the intersection of five fuzzy membership functions; however the proposed formulation uses only one membership function [equivalent to a two limit variable] which provides similar values through the interval, while maintaining the limit-values

3) Two limits have been established for the variable:
   a. ‘Sustainability Goal’ is to achieve zero-consumption 0 GJ.Hab-1Año-1.
   b. ‘Unsustainability Threshold’ is ECus2=640 GJ.Hab-1Año-1

**URBAN SUSTAINABILITY: NETWORK CONNECTIVITY**

Can be understood as two limit variable with the singularity of a ‘significance value’ that can be formulated as:

\[ S[N_{C}] = \max \left( \min \left( 1; 1 - \frac{\text{num}_{\text{inter}}}{\text{val}_{\text{inter}}} \right)^{1/2}; 0 \right) \]

And the simplified formula for the calculation is:

\[ N_{C}[\%] = \min \left( \frac{N_{C}}{155}; 1; \min \left( \frac{N_{C}}{35} \right) \right) \times 100 \]

Source: Own elaboration and LEED [USGBC 2009] with the following comments:

1) This proposal is a partial combination of two different indicators: Indicator SLL Pr 01: ‘Smart Location and Linkage’ and SLL 01: Preferred Locations’ [USGBC, 2009], included in model ‘LEED for Neighbourhood Development’.

2) Codes mean the following:
   a. NCI, Network Connectivity Indicator
   b. NCi, number of intersection locate to a maximum distance of 800 m from limits of assessed area.

3) Two limits and a ‘significance value’ have been established for the variable:
   a. ‘Complete Connectivity’ is achieved when the number of intersections per square kilometer is over NCSi≥155 [LEED ND SLL Cr 01 proposes 400 intersections/square mile as a ‘optimum’ value for which increases no longer improve the connectivity of an area]
   b. ‘null connectivity’ is achieved when the number of intersections per square kilometer is NCus1=0 [own calculation considering that 200 intersections/square suggested by LEED is equivalent to a connectivity degree of 0.5]
   c. A ‘significance value’ is established for NCi=35 intersections per square kilometer [directly taken from SLL Pr 01 which establishes a mandatory minimum of 90 intersections/square mile]

4) All intersections located nearer than 800 m from the limits of the assessed area should be included in calculations.

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4.4 _INDEXES OR INDICATOR DASHBOARDS ASSESSMENT_

The proposed procedure can also be used to check existing indexes or sustainability assessment tools in two ways: consistency and completeness.

- Consistency refers to ‘non-contradiction’; a sustainability index shall only contain sustainability indicators; i.e.: indicators that inform of grade of membership to a class S, contained in class S which complement ¬S, is contained in ¬S class.
- Completeness refers to the fact that indexes should include the effects of all ‘relevant variables’; i.e., all those variables which ranges of ‘possible’ values can modify the ‘grade of membership’ of the city ‘I’ to S or ¬S classes.

These two issues are quite important, yet the revision of many assessment tools show lack of coherence or incompleteness.

However, while any sustainability index or dashboard should always be coherent [i.e: it should not have indicators not referred to sustainability], completeness will allow and even require making some ‘exceptions’

5_ CONCLUSIONS

Fuzzy Set theory provides a framework that allows us to simplify and clarify the design of urban sustainability indicators and indicator dashboards:

- Providing a criterion to determine the relevant variables for the sustainability of an urban ‘model’ or city.
- Allowing conceptualizing sustainability indicators as ‘membership functions’ of the city for each of its relevant variables to any concept S, implied in class S.
- Providing criteria to assess the coherence and completeness of sustainability assessment indexes or indicators dashboards.

Furthermore, the proposed approach to indicator design provides three more applications:

First, it allows us to aggregate the sustainability indicators obtaining a global ‘sustainability degree’ of the city. A formulation and procedure has recently been proposed by the author as a Mathematical Theory of Sustainability [Alvira, 2014].

Second, it allows us to assess other sizes or types of systems; conceptualizing the ‘sustainability degree of a system’ as its ‘grade of membership’ to the set of sustainable systems in its class.

Many different ‘classes’ of Ecological and Socio Ecological systems allow a ‘sufficiently thorough’ description of the qualities that make a system ‘sustainable within its class’; making sustainability indicator design an ‘easy process’.

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4 Otherwise we could have to model the sustainability of the whole universe in order to determine that of a city.
Also the approach to S and ¬S is a useful tool to determine the qualities that a system in any possible class ‘maximizes its possibilities to endure’ [i.e: to belong to S class]. If it is possible to determine the qualities that make a system unsustainable in its class, then it is possible to determine the qualities that make it sustainable, which are the ‘opposite’.

And third point it allows us to measure other concepts different from ‘Sustainability/Unsustainability’; and their aggregation is a measure of their degree of truth referred to different ‘objects’ being an approach to fuzzy propositions logic from Complexity⁵.

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⁵ While a brief overview of this issue is provided in Alvira [2014], the author is currently developing it with more detail as a separate publication expected to appear soon.

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