Numerical Solution of Quantum Cosmological Model Simulating Boson and Fermion Creation

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A numerical solution of Wheeler-De Witt equation for a quantum cosmological model simulating boson and fermion creation in the early Universe evolution is presented. This solution is based on a Wheeler-De Witt equation obtained by Krechet, Fil'chenkov, and Shikin, in the framework of quantum geometrodynamics for a Bianchi-I metric.

1 Introduction

It is generally aserted that in the early stage of Universe evolution, the quantum phase predominated the era. Therefore there are numerous solutions have been found corresponding to the Wheeler-DeWitt equation which governs this phase [2]. In the present paper we present another numerical solution of Wheeler-De Witt equation for a quantum cosmological model simulating boson and fermion creation in the early Universe evolution for a Bianchi-type I metric [1].

The solution is based on Wheeler-De Witt equation for a Bianchi-I metric obtained by Krechet, Fil'chenkov, and Shikin [1], in the framework of quantum geometrodynamics. Albeit the essence of the solution is quite similar from the solution given in [1] using Bessel function, in the present paper we present numerical result using Maxima. For comparison with other solutions of 1-d hydrogen problem, see [3] and [4].

2 Solution of Wheeler-DeWitt equation for boson and fermion creation

In the evolution of the Universe after inflation, a scalar field describing de Sitter vacuum was supposed to decay and its energy is converted into the energy of fermions and heavy vector-particles (the so-called X and Y bosons) [2].

In the framework of quantum geometrodynamics, and for a Bianchi-I metric, the Wheeler-De Witt equation has been obtained by Krechet, Fil'chenkov, and Shikin, which reduces to become (Eq. 23 in [1]):

$$T'' - \frac{2iC}{3\tau} T' - (E - V)\, T = 0. \tag{1}$$

where T'' and T' represent second and first differentiation of T with respect to r. The resulting equation appears quite similar to radial 1-dimensional Schrödinger equation for a hydrogen-like atom [3], with the potential energy is given by [1]:

$$U(r) = \frac{\beta}{\tau} + \frac{\varepsilon_o}{\tau^{4/3}}, \tag{2}$$

$$E = \frac{8}{3}\kappa\left(\frac{\Lambda}{\kappa} - \frac{M^2}{2\lambda}\right) \tag{3}$$

has here a continuous spectrum.

The solution of equation (1) has been presented in [1] based on modified Bessel function. Its interpretation is that in this quantum cosmological model an initial singularity is absent.

As an alternative to the method presented in [1], the numerical solution can be found using Maxima software package, as follows. All solutions are given in terms of E as constant described by (3).

(a) *Condition where $V = 0$*

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'diff(y,r,2)−E*y−(2*%i*C/3/t)*y=0;   ode2(%o1,y,r);   (4)
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The result is given by:

$$y = K1 \sin(a) + K2 \cos(a), \tag{5}$$

where:

$$a = (r/\sqrt{3})\sqrt{-3E - 2iC/t}\,. \tag{6}$$

(b) *Condition where $V \leqslant 0$*

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'diff(y,r,2)−E*y−(2*%i*C/3/t)*y−(b/t+e/t^(4/3))*y=0;
ode2(%o2,y,r);   (7)
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The result is given by:

$$y = K1 \sin(d) + K2 \cos(d), \tag{8}$$

where:

$$d = (r/(\sqrt{3}\, t^{2/3}))\sqrt{-3Et^{4/3} - 2iCt^{1/3} - 3e - 3bt^{1/3}}\,. \tag{9}$$

As a result, the solution given above looks a bit different compared to the solution obtained in [1] based on the modified Bessel function.

3 A few implications

For the purpose of stimulating further discussions, a few implications of the above solution of Wheeler-DeWitt equation (in the form of 1-d Schrödinger equation) are pointed as follows:

(a) Considering that the Schrödinger equation can be used to solve the Casimir effect (see for instance Silva [5], Alvarez & Mazzitelli [6]), therefore one may expect that there exists some effects of Casimir effect in cosmological scale, in a sense that perhaps quite similar to Unruh radiation which can be derived from the Casimir effective temperature. Interestingly, Anosov [7] has pointed out a plausible deep link between Casimir effect and the fine structure constant by virtue of the entropy of coin-tossing problem. However apparently he did not mention yet another plausible link between the Casimir effective temperature and other phenomena at cosmological scale;

(b) Other implication may be related to the Earth scale effects, considering the fact that Schrödinger equation corresponds to the infinite dimensional Hilbert space. In other words one may expect some effects with respect to Earth eigen oscillation spectrum, which is related to the Earth's inner core interior. This is part of gravitational geophysical effects, as discussed by Grishchuk et al. [8]. Furthermore, this effect may correspond to the so-called Love numbers. Other phenomena related to variation to gravitational field is caused by the Earth inner core oscillation, which yields oscillation period $T \sim 3$–7 hours. Interestingly, a recent report by Cahill [9] based on the Optical fibre gravitational wave detector gave result which suggests oscillation period of around 5hours. Cahill concluded that this observed variation can be attributed to Dynamical 3-space. Nonetheless, the Figure 6c in [9] may be attributed to Earth inner core oscillation instead. Of course, further experiment can be done to verify which interpretation is more consistent.

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