This article generalizes certain results on the nedianes (see [1], pp. 97-99). One calls *nedianes* the segments of a line that passes through a vertex of a triangle and partitions the opposite side in \( n \) equal parts. A nediane is called to be of order \( i \) if it partitions the opposite side in the rapport \( i/n \).

For \( 1 \leq i \leq n-1 \) the nedianes of order \( i \) (that is \( AA_i, BB_i \) and \( CC_i \)) have the following properties:

1) With these 3 segments one can construct a triangle.

\[
|AA_i|^2 + |BB_i|^2 + |CC_i|^2 = \frac{i^2 - i \cdot n + n^2}{n^2} (a^2 + b^2 + c^2).
\]

Proofs:

\[
\begin{align*}
\overrightarrow{AA_i} &= \overrightarrow{AB} + \overrightarrow{BA_i} = \overrightarrow{AB} + \frac{i}{n} \overrightarrow{BC} \\
\overrightarrow{BB_i} &= \overrightarrow{BC} + \overrightarrow{CB_i} = \overrightarrow{BC} + \frac{i}{n} \overrightarrow{CA} \\
\overrightarrow{CC_i} &= \overrightarrow{CA} + \overrightarrow{AC_i} = \overrightarrow{CA} + \frac{i}{n} \overrightarrow{AB}
\end{align*}
\]

By adding these 3 relations, we obtain:
therefore the 3 medianes can be the sides of a triangle.

(2) By raising to the square the relations and then adding them we obtain:

\[ (AA_i)^2 + (BB_i)^2 + (CC_i)^2 = \frac{i+n}{n} (AB + BC + CA) = 0 \]

Because \(2AB \cdot BC = -2ca \cdot \cos B = b^2 - c^2 - a^2\) (the theorem of cosines), by substituting this in the relation (4), we obtain the requested relation.

Reference: