On PT-Symmetric Periodic Potential, Quark Confinement, and Other Impossible Pursuits

Vic Christianto* and Florentin Smarandache†

*Sciprint.org — a Free Scientific Electronic Preprint Server, http://www.sciprint.org
E-mail: admin@sciprint.org

†Department of Mathematics, University of New Mexico, Gallup, NM 87301, USA
E-mail: smarand@unm.edu

As we know, it has been quite common nowadays for particle physicists to think of six impossible things before breakfast, just like what their cosmology fellows used to do. In the present paper, we discuss a number of those impossible things, including PT-symmetric periodic potential, its link with condensed matter nuclear science, and possible neat link with Quark confinement theory. In recent years, the PT-symmetry and its related periodic potential have gained considerable interests among physicists. We begin with a review of some results from a preceding paper discussing derivation of PT-symmetric periodic potential from biquaternion Klein-Gordon equation and proceed further with the remaining issues. Further observation is of course recommended in order to refute or verify this proposition.

1 Introduction

As we know, it has been quite common nowadays for particle physicists to think of six impossible things before breakfast [1], just like what their cosmology fellows used to do. In the present paper, we discuss a number of those impossible things, including PT-symmetric periodic potential, its link with condensed matter nuclear science, and possible neat link with Quark Confinement theory.

In this regards, it is worth to remark here that there were some attempts in literature to generalise the notion of symmetries in Quantum Mechanics, for instance by introducing CPT symmetry, chiral symmetry etc. In recent years, the PT-symmetry and its related periodic potential have gained considerable interests among physicists [2, 3]. It is expected that the discussions presented here would shed some light on these issues.

We begin with a review of results from our preceding papers discussing derivation of PT-symmetric periodic potential from biquaternion Klein-Gordon equation [4–6]. Thereafter we discuss how this can be related with both Gribov's theory of Quark Confinement, and also with EQPET/TSC model for condensed matter nuclear science (aka low-energy reaction or "cold fusion") [7]. We also highlight its plausible implication to the calculation of Gamow integral for the (periodic) non-Coulomb potential.

In other words, we would like to discuss in this paper, whether there is PT symmetric potential which can be observed in Nature, in particular in the context of condensed matter nuclear science (CMNS) and Quark confinement theory.

Nonetheless, further observation is of course recommended in order to refute or verify this proposition.

2 PT-symmetric periodic potential

It has been argued elsewhere that it is plausible to derive a new PT-symmetric Quantum Mechanics (PT-QM; sometimes it is called pseudo-Hermitian Quantum Mechanics [3, 9]) which is characterized by a PT-symmetric potential [2]

$$V(x) = V(-x)\,. \tag{1}$$

One particular example of such PT-symmetric potential can be found in sinusoidal-form potential

$$V = \sin\varphi\,. \tag{2}$$

PT-symmetric harmonic oscillator can be written accordingly [3]. Znojil has argued too [2] that condition (1) will yield Hulthen potential

$$V(\xi) = \frac{A}{(1-e^{2i\xi})^2} + \frac{B}{(1-e^{2i\xi})}\,. \tag{3}$$

Interestingly, a similar periodic potential has been known for quite a long time as Posch-Teller potential [9], although it is not always related to PT-Symmetry considerations. The Posch-Teller system has a unique potential in the form [9]

$$U(x) = -\lambda\cosh^{-2}x\,. \tag{4}$$

It appears worth to note here that Posch-Teller periodic potential can be derived from conformal D'Alembert equations [10, p.27]. It is also known as the second Posch-Teller potential

$$V_\mu(\xi) = \frac{\mu(\mu-1)}{\sinh^2\xi} + \frac{\ell(\ell+1)}{\cosh^2\xi}. \tag{5}$$

The next Section will discuss biquaternion Klein-Gordon equation [4, 5] and how its radial version will yield a sinusoidal form potential which appears to be related to equation (2).

3 Solution of radial biquaternion Klein-Gordon equation and a new sinusoidal form potential

In our preceding paper [4], we argue that it is possible to write biquaternionic extension of Klein-Gordon equation as follows

$$\left[\left(\frac{\partial^2}{\partial t^2} - \nabla^2\right) + i\left(\frac{\partial^2}{\partial t^2} - \nabla^2\right)\right]\varphi(x,t) = \\ = -m^2\varphi(x,t)\,, \qquad (6)$$

or this equation can be rewritten as

$$\left(\diamondsuit\bar{\diamondsuit} + m^2\right)\varphi(x,t) = 0 \qquad (7)$$

provided we use this definition

$$\diamondsuit = \nabla^q + i\nabla^q = \left(-i\frac{\partial}{\partial t} + e_1\frac{\partial}{\partial x} + e_2\frac{\partial}{\partial y} + e_3\frac{\partial}{\partial z}\right) + \\ + i\left(-i\frac{\partial}{\partial T} + e_1\frac{\partial}{\partial X} + e_2\frac{\partial}{\partial Y} + e_3\frac{\partial}{\partial Z}\right), \qquad (8)$$

where e_1, e_2, e_3 are *quaternion imaginary units* obeying (with ordinary quaternion symbols $e_1 = i$, $e_2 = j$, $e_3 = k$):

$$i^2 = j^2 = k^2 = -1\,, \quad ij = -ji = k\,, \qquad (9)$$

$$jk = -kj = i\,, \quad ki = -ik = j\,, \qquad (10)$$

and quaternion *Nabla operator* is defined as [4]

$$\nabla^q = -i\frac{\partial}{\partial t} + e_1\frac{\partial}{\partial x} + e_2\frac{\partial}{\partial y} + e_3\frac{\partial}{\partial z}\,. \qquad (11)$$

Note that equation (11) already included partial time-differentiation.

Thereafter one can expect to find solution of *radial* biquaternion Klein-Gordon Equation [5, 6].

First, the standard Klein-Gordon equation reads

$$\left(\frac{\partial^2}{\partial t^2} - \nabla^2\right)\varphi(x,t) = -m^2\varphi(x,t)\,. \qquad (12)$$

At this point we can introduce polar coordinate by using the following transformation

$$\nabla = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial}{\partial r}\right) - \frac{\ell^2}{r^2}\,. \qquad (13)$$

Therefore by introducing this transformation (13) into (12) one gets (setting $\ell = 0$)

$$\left(\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial}{\partial r}\right) + m^2\right)\varphi(x,t) = 0\,. \qquad (14)$$

By using the same method, and then one gets radial expression of BQKGE (6) for 1-dimensional condition as follows [5, 6]

$$\left(\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial}{\partial r}\right) - i\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial}{\partial r}\right) + m^2\right)\varphi(x,t) = 0\,. \qquad (15)$$

Using Maxima computer package we find solution of equation (15) as a new potential taking the form of sinusoidal potential

$$y = k_1\sin\left(\frac{|m|\,r}{\sqrt{-i-1}}\right) + k_2\cos\left(\frac{|m|\,r}{\sqrt{-i-1}}\right), \qquad (16)$$

where k_1 and k_2 are parameters to be determined. It appears very interesting to remark here, when k_2 is set to 0, then equation (16) can be written in the form of equation (2)

$$V = k_1\sin\varphi\,, \qquad (17)$$

by using definition

$$\varphi = \sin\left(\frac{|m|\,r}{\sqrt{-i-1}}\right). \qquad (18)$$

In retrospect, the same procedure which has been traditionally used to derive the Yukawa potential, by using radial biquaternion Klein-Gordon potential, yields a PT-symmetric periodic potential which takes the form of equation (1).

4 Plausible link with Gribov's theory of Quark Confinement

Interestingly, and quite oddly enough, we find the solution (17) may have deep link with Gribov's theory of Quark confinement [8, 11]. In his Third Orsay Lectures he described a periodic potential in the form [8, p.12]

$$\ddot{\psi} - 3\sin\psi = 0\,. \qquad (19)$$

By using Maxima package, the solution of equation (19) is given by

$$\left.\begin{aligned} x_1 &= k_2 - \frac{\int \frac{1}{\sqrt{k_1-\cos(y)}}\,dy}{\sqrt{6}} \\ x_2 &= k_2 + \frac{\int \frac{1}{\sqrt{k_1-\cos(y)}}\,dy}{\sqrt{6}} \end{aligned}\right\}, \qquad (20)$$

while Gribov argues that actually the equation shall be like nonlinear oscillation with damping, the equation (19) indicates close similarity with equation (2).

Therefore one may think that PT-symmetric periodic potential in the form of (2) and also (17) may have neat link with the Quark Confinement processes, at least in the context of Gribov's theory. Nonetheless, further observation is of course recommended in order to refute or verify this proposition.

5 Implication to condensed matter nuclear science. Comparing to EQPET/TSC model. Gamow integral

In accordance with a recent paper [6], we interpret and compare this result from the viewpoint of EQPET/TSC model which has been suggested by Prof. Takahashi in order to explain some phenomena related to Condensed matter nuclear Science (CMNS).

Takahashi [7] has discussed key experimental results in condensed matter nuclear effects in the light of his EQPET/TSC model. We argue here that his potential model with inverse barrier reversal (STTBA) may be comparable to the periodic potential described above (17).

In [7] Takahashi reported some findings from condensed matter nuclear experiments, including intense production of helium-4, ^{4}He atoms, by electrolysis and laser irradiation experiments. Furthermore he [7] analyzed those experimental results using EQPET (Electronic Quasi-Particle Expansion Theory). Formation of TSC (tetrahedral symmetric condensate) were modeled with numerical estimations by STTBA (Sudden Tall Thin Barrier Approximation). This STTBA model includes strong interaction with negative potential near the center.

One can think that apparently to understand the physics behind Quark Confinement, it requires fusion of different fields in physics, perhaps just like what Langland program wants to fuse different branches in mathematics.

Interestingly, Takahashi also described the Gamow integral of his STTBA model as follows [7]

$$\Gamma_n = 0.218 \left(\mu^{1/2}\right) \int_{r_0}^{b} (V_b - E_d)^{1/2}\, dr\,. \tag{21}$$

Using $b = 5.6$ fm and $r = 5$ fm, he obtained [7]

$$P_{4D} = 0.77, \tag{22}$$

and

$$V_B = 0.257 \text{ MeV}, \tag{23}$$

which gave significant underestimate for 4D fusion rate when rigid constraint of motion in 3D space attained. Nonetheless by introducing different values for λ_{4D} the estimate result can be improved. Therefore we may conclude that Takahashi's STTBA potential offers a good approximation (just what the name implies, STTBA) of the fusion rate in condensed matter nuclear experiments.

It shall be noted, however, that his STTBA lacks sufficient theoretical basis, therefore one can expect that a sinusoidal periodic potential such as equation (17) may offer better result.

All of these seem to suggest that the cluster deuterium may yield a different inverse barrier reversal which cannot be predicted using the D-D process as in standard fusion theory. In other words, the standard procedure to derive Gamow factor should also be revised [12]. Nonetheless, it would need further research to determine the precise Gamow energy and Gamow factor for the cluster deuterium with the periodic potential defined by equation (17); see for instance [13].

In turn, one can expect that Takahashi's EQPET/TSC model along with the proposed PT-symmetric periodic potential (17) may offer new clues to understand both the CMNS processes and also the physics behind Quark confinement.

6 Concluding remarks

In recent years, the PT-symmetry and its related periodic potential have gained considerable interests among physicists.

In the present paper, it has been shown that one can find a new type of PT-symmetric periodic potential from solution of the radial biquaternion Klein-Gordon Equation. We also have discussed its plausible link with Gribov's theory of Quark Confinement and also with Takahashi's EQPET/TSC model for condensed matter nuclear science. All of which seems to suggest that the Gribov's Quark Confinement theory may indicate similarity, or perhaps a hidden link, with the Condensed Matter Nuclear Science (CMNS). It could also be expected that thorough understanding of the processes behind CMNS may also require revision of the Gamow factor to take into consideration the cluster deuterium interactions and also PT-symmetric periodic potential as discussed herein.

Further theoretical and experiments are therefore recommended to verify or refute the proposed new PT symmetric potential in Nature.

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