Schrödinger-Langevin Equation with PT-Symmetric Periodic Potential and its Application to Deuteron Cluster

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In this article, we find out some analytical and numerical solutions to the problem of barrier tunneling for cluster deuterium, in particular using Langevin method to solve the time-independent Schrödinger equation.

1 Introduction

One of the most reported problem related to the CMNS (condensed matter nuclear science, or LENR), is the low probability of Coulomb barrier tunneling. It is supposed by standard physics that tunneling is only possible at high enough energy (by solving Gamow function).

However, a recent study by Takahashi (2008, 2009) and experiment by Arata etc. (2008) seem to suggest that it is not impossible to achieve a working experiment to create the CMNS process.

In accordance with Takahashi's EQPET/TSC model [1–3], the proposed study will find out some analytical and numerical solutions to the problem of barrier tunneling for cluster deuterium, in particular using Langevin method to solve the time-independent Schrödinger equation. It is hoped that the result can answer some of these mysteries.

One of the results of recent experiments is the lack of signature of D-D reaction as in standard fusion process; this is part of the reason to suggest that D-D fusion doesn't take place [1]. However, Takahashi suggests new possible reaction in the context of cluster deuterium, called 4D fusion [1–3], this mechanism seems to enable reaction at low temperature (CMNS). His result (2009) can be summarized as follows:

> "The ultimate condensation is possible only when the double Platonic symmetry of 4D/TSC is kept in its dynamic motion. The sufficient increase (super screening) of barrier factor is also only possible as far as the Platonic symmetric 4D/TSC system is kept. Therefore, there should be always 4 deuterons in barrier penetration and fusion process, so that 4d simultaneous fusion should take place predominantly. The portion of 2D (usual) fusion rate is considered to be negligible".

In this respect it can be noted that there are recent reports suggesting that hydrogen cluster can get reaction at very low temperature, forming the condition of superfluidity [4]. This seems to happen too in the context of Takahashi TSC condensate dynamics. Other study worth mentioning here is one that discussed molecular chessboard dynamics of deuterium [5].

The difference between this proposed study and recent work of Takahashi based on Langevin equation for cluster deuterium is that we focus on solution of Schrödinger-Langevin equation [6, 7] with PT-Symmetric periodic potential as we discussed in the preceding paper and its Gamow integral. The particular implications of this study to deuteron cluster will be discussed later.

Another differing part from the previous study is that in this study we will also seek clues on possibility to consider this low probability problem as an example of self-organized criticality phenomena. In other words, the time required before CMNS process can be observed is actually the time required to trigger the critical phenomena. To our present knowledge, this kind of approach has never been studied before, although self-organized criticality related to Schrödinger equation approximation to Burger's turbulence has been discussed in Boldyrev [8]. Nonetheless there is recent study suggesting link between diffusion process and the self-organized criticality phenomena.

The result of this study will be useful to better understanding of anomalous phenomena behind Condensed matter nuclear science.

2 Schrödinger-Langevin equation

The Langevin equation is considered as equivalent and therefore has often been used to solve the time-independent Schrödinger, in particular to study molecular dynamics.

Here we only cite the known Langevin equation [3, p. 29]

$$dX_t = p_t\, dt\,, \tag{1}$$

$$dp = -\partial_x \lambda_0(X_t)dt + K p_t dt + dW_t \sqrt{2TK}\,. \tag{2}$$

Takahashi and Yabuuchi also used quite similar form of the stochastic non-linear Langevin equation [7] in order to study the dynamics of TSC condensate motion.

3 Schrödinger equation with PT-symmetric periodic potential

Consider a PT-Symmetric potential of the form [9, 10]

$$V = K_1 \sin(b\, r)\,, \tag{3}$$

where

$$b = \frac{|m|}{\sqrt{-i-1}}\,. \tag{4}$$

Hence, the respective Schrödinger equation with this potential can be written as follows

$$\Psi''(r) = -k^2(r)\,\Psi(r)\,, \tag{5}$$

where

$$k(r) = \frac{2m}{\hbar^2}[E - V(r)] = \frac{2m}{\hbar^2}\Big[E - k_1 \sin(br)\Big]. \tag{6}$$

For the purpose of finding Gamow function, in area near x=a we can choose linear approximation for Coulomb potential, such that

$$V(x) - E = -\,\alpha(x - a)\,. \tag{7}$$

Substitution to Schrödinger equation yields

$$\Psi'' + \frac{2m\alpha}{\hbar^2}\,(x - a)\,\Psi = 0\,, \tag{8}$$

which can be solved by virtue of Airy function.

4 Gamow integral

In principle, the Gamow function can be derived as follows [11]

$$\frac{d^2y}{dx^2} + P(x)y = 0\,. \tag{9}$$

Separating the variables and integrating, yields

$$\int \frac{d^2y}{y} = \int -P(x)\,dx \tag{10}$$

or

$$y\,dy = \exp\left(-\int P(x)\,dx + C\right). \tag{11}$$

To find solution of Gamow function, therefore the integral below must be evaluated:

$$\gamma = \sqrt{\frac{2m}{\hbar^2}\,[V(x) - E]}\,. \tag{12}$$

For the purpose of analysis we use the same data from Takahashi's EQPET model [3,12], i.e. $b = 5.6$ fm, and $r_0 = 5$ fm. Here we assume that $E = V_b = 0.257$ MeV. Therefore the integral becomes

$$\Gamma = 0.218\,\sqrt{m}\int_{r_0}^{b}\sqrt{k_1 \sin(br) - 0.257}\;dr\,. \tag{13}$$

By setting boundary condition (either one or more of these conditions)

(a) at $r = 0$ then $V_0 = -V_b - 0.257$MeV;

(b) at $r = 5.6\,fm$ then $V_1 = k_1 \sin(br) - 0.257 = 0.257$MeV, therefore, one can find estimate of m;

(c) Using this procedure solution of the equation (11) can be found.

The interpretation of this Gamow function is the tunneling rate of the fusion reaction of cluster of deuterium (for the given data) corresponding to Takahashi data [12], with the difference that here we consider a PT-symmetric periodic potential.

The numerical study will be performed with standard package like Maxima etc. Some plausible implications in cosmology modeling should also be discussed in the future.

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