Is There Iso-PT Symmetric Potential in Nature?

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In recent years there are new interests on special symmetry in physical systems, called PT-symmetry with various ramifications. Along with the isodual symmetry popularized by RM Santilli, these ideas form one of cornerstone in hadron physics. In the present article, we argue that it is plausible to generalise both ideas to become iso-PT symmetry which indicate there should be new potential obeying this symmetry. We also discuss some possible interpretation of the imaginary solution of the solution of biquaternionic KGE (BQKGE); which indicate the plausible existence of the propose iso-PT symmetry. Further observation is of course recommended in order to refute or verify this proposition.

Introduction

There were some attempts in literature to generalise the notion of symmetries in Quantum Mechanics, for instance by introducing CPT symmetry, chiral symmetry, etc.

In recent years there are new interests on special symmetry in physical systems, called PT-symmetry with various ramifications. Along with the isodual symmetry popularized by RM Santilli, these ideas form one of cornerstone in hadron physics. In the present article, we argue that it is plausible to generalise both ideas to become iso-PT symmetry which indicate there should be new potential obeying this symmetry. We also discuss some possible interpretation of the imaginary solution of the solution of biquaternionic KGE (BQKGE); which indicate the plausible existence of the propose iso-PT symmetry.

This biquaternion effect may be useful in particular to explore new effects in the context of low-energy reaction (LENR) [14]. Nonetheless, further observation is of course recommended in order to refute or verify this proposition.

Basic ideas: PT-Symmetric Potential and Isoselfdual Symmetry

It has been argued elsewhere that it is plausible to derive a new PT-symmetric Quantum Mechanics (PT-QM) which is characterized by a PT-symmetric potential [1]:

$$V(x) = V(-x)\,. \tag{1}$$

One particular example of such PT-symmetric potential can be found in sinusoidal-form potential:

$$V = \sin\alpha\,. \tag{2}$$

PT-symmetric harmonic oscillator can be written accordingly [2]. Znojil has argued too [1] that condition (1) will yield Hulthen potential:

$$V(\xi) = \frac{A}{\left(1 - e^{2i\xi}\right)^2} + \frac{B}{\left(1 - e^{2i\xi}\right)}\,. \tag{3}$$

Interestingly, the similar Hulthen potential has often been cited with respect to the isodual symmetric proposed by RM. Santilli in a number of published works [3][4]. Therefore it appears quite interesting to find out generalization of these types of symmetries to become (iso-PT symmetry). In other words, we would like to ask in this paper, whether there is isoselfdual-PT symmetric potential in nature, which is the subject of the present paper.

Now we're going to discuss some remarkable result from isoselddual theory popularized by RM. Santilli under the flagship of Hadronic Mechanics (HM theory). With regards to isodual symmetry we note some basic relations in according with [3][4]:

*The imaginary unit is isoselfdual because [3, p.8]:

$$i^d = -\bar{i} = i \tag{4}$$

*The correct left and right multiplicative unit [3, p.6]:

$$A \times^d I^d = I^d \times^{d^d} A = A \tag{5}$$

*The isodual functional analysis also includes [3, p.8]:

$$\sin^d \theta^d = -\sin(-\theta) \tag{6}$$

Therefore, with respect to the aforementioned basic relations of isoselfdual theory, then a new generalization can be suggested, i.e. an isoselfdual-PT symmetry is such that the potential follows this relation:

$$`V^{d}{}_{isoselfdual}(x) = V^{d}(-x)\,. \tag{7}$$

In other words, a possible solution of equation (7), with respect to the isodual functional analysis (6) and (2), can be given by:

$$V^{d} = -\sin(-\alpha)\,. \tag{8}$$

The next section will discuss solution of biquaternion Klein-Gordon equation [5][7] and how it will yield a sinusoidal form potential with appears to be related either to (2) or to (8). See also [8].

Review of solution of biquaternionic Klein-Gordon equation

In our preceding paper [5], we argue that it is possible to write biquaternionic extension of Klein-Gordon equation as follows:

$$\left[\left(\frac{\partial^2}{\partial t^2} - \nabla^2\right) + i\left(\frac{\partial^2}{\partial t^2} - \nabla^2\right)\right]\varphi(x,t) = -m^2\varphi(x,t)\,, \tag{9}$$

Or this equation can be rewritten as:

$$\left(\Diamond\bar{\Diamond} + m^2\right)\varphi(x,t) = 0\,, \tag{10}$$

Provided we use this definition:

$$\begin{aligned}\Diamond = \nabla^q + i\nabla^q &= \left(-i\frac{\partial}{\partial t} + e_1\frac{\partial}{\partial x} + e_2\frac{\partial}{\partial y} + e_3\frac{\partial}{\partial z}\right) \\ &+ i\left(-i\frac{\partial}{\partial T} + e_1\frac{\partial}{\partial X} + e_2\frac{\partial}{\partial Y} + e_3\frac{\partial}{\partial Z}\right)\end{aligned} \tag{11}$$

Where e_1, e_2, e_3 are *quaternion imaginary units* obeying (with ordinary quaternion symbols: e_1=i, e_2=j , e_3 =k):

$$\begin{aligned}&i^2 = j^2 = k^2 = -1\,,\ ij = -ji = k\,,\\ &jk = -kj = i\,, ki = -ik = j\,.\end{aligned} \tag{12}$$

And quaternion *Nabla operator* is defined as [5]:

$$\nabla^q = -i\frac{\partial}{\partial t} + e_1\frac{\partial}{\partial x} + e_2\frac{\partial}{\partial y} + e_3\frac{\partial}{\partial z} \tag{13}$$

Note that equation (11) already included partial time-differentiation.

It is worth nothing here that equation (10) yields solution containing imaginary part, which differs appreciably from known solution of KGE [5]:

$$y(x,t) = \left(\frac{1}{4} - \frac{i}{4}\right) m^2 t^2 + constan\, t \tag{14}$$

Solution of radial biquaternion Klein-Gordon equation and a new sinusoidal form potential

One can expect to use the same method described above to find solution of radial biquaternion KGE [7][8].

First, the standard Klien-Gordon equation reads:

$$\left(\frac{\partial^2}{\partial t^2} - \nabla^2\right)\varphi(x,t) = -m^2\varphi(x,t)\,. \tag{15}$$

At this point we can introduce polar coordinate by using the following transformation:

$$\nabla = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial}{\partial r}\right) - \frac{\ell^2}{r^2} \tag{15a}$$

Therefore by introducing this transformation (15a) into (15) one gets (setting $\ell = 0$):

$$\left(\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial}{\partial r}\right) + m^2\right)\varphi(x,t) = 0\,. \tag{16}$$

Using similar method (15)-(16) applied to equation (10), then one gets radial solution of BQKGE for 1-dimensional condition [7][8]:

$$\left(\frac{1}{r^2}\frac{\partial}{\partial r}\left(\frac{\partial}{\partial r}\right) - \frac{1}{r^2}\frac{\partial}{\partial r}\left(\frac{\partial}{\partial r}\right) + m^2\right)\varphi(x,t) = 0\,, \tag{17}$$

Using Maxima computer package we find solution of (18) as a new potential taking the form of sinusoidal potential:

$$y = k_1 \sin\left(\frac{|m|r}{\sqrt{-i-1}}\right) + k_2 \cos\left(\frac{|m|r}{\sqrt{-i-1}}\right), \qquad (18)$$

Where k1 and k2 are parameters to be determined.

In a recent paper [8], we interpret and compare this result from the viewpoint of EQPEt/TSC model which has been suggested by Prof. Takahashi in order to explain some phenomena related to Condensed matter nuclear Science (CMNS).

Nonetheless what appears to us as more interesting question is whether it is possible to find out proper generalisation of PT-symmetric potential (1) to become isoselfdual-PT symmetric potential (7). Further theoretical and experiments are therefore recommended to verify or refute the proposed new isoselfdual-PT symmetric potential in Nature.

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