Abstract

This paper constructs the zeta functions of the Riemann hypothesis for the sporadic finite simple groups.

Although many researchers have tried the proof of the Riemann hypothesis, they have not resulted in the success. The proof of this Riemann hypothesis has been an important issue of mathematics. In this paper, we construct zeta functions for the sporadic finite simple groups as preparation proving the Riemann hypothesis.

We define the polynomial ring of a group in order to add groups and to add the members of a group.

The argument of a zeta function is a complex number. We can interpret a complex number as the subgroup of a quaternion. Therefore, we can expand the argument of a zeta function to a quaternion. On the other hand, a complex number is one-dimensional complex general linear group. And the sporadic finite simple groups are high dimensional complex general linear groups. Therefore, we can expand the argument of a zeta function to sporadic finite simple groups.

We define the product of groups the direct product. We interpret the cyclic groups of prime order as prime groups. We construct the natural groups of natural order with products of prime groups. Sporadic finite simple groups become subgroups of natural groups because arbitrary finite groups become subgroups of natural groups. We can express arbitrary finite groups by zeta function of sporadic finite simple groups.

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1 Introduction

1.1 Subject

According to the Riemann hypothesis, the nontrivial zeros of the Riemann zeta function all have real part 1/2. We construct zeta functions from sporadic finite simple groups as preparation proving the Riemann hypothesis.

1.2 Importance of subject

The proof of the Riemann hypothesis is one of the most important outstanding problems in mathematics.

For this reason, many mathematicians have tried the proof of the Riemann hypothesis. However, those trials were not successful. One of the methods to solve the problem of mathematics is to express the problem in the other mathematics. Therefore, the mathematician has tried to construct the zeta function from the other mathematics. For this reason, it has been an important issue to construct the zeta function from the other mathematics.

1.3 Past construction method

Leonhard Euler constructed the zeta function of the prime numbers, and made the argument of the zeta function the natural number in 1737. Bernhard Riemann expanded the argument of the zeta function to the complex number in 1859.

Srinivasa Ramanujan constructed the zeta function for automorphic form in 1916. Kornblum constructed the zeta function for prime polynomials on the finite fields in 1919. Atle Selberg constructed the zeta function for prime polynomials on the complex number fields in 1952. Initially Selberg's zeta function was considered as a zeta function for groups; however, it is consisted as a zeta function for prime geodesic lines of the manifolds.

Research trends other than the construction of a zeta function are as follows. David Hilbert and George Polya suggested that the zeros of a zeta function were probably eigenvalues of a certain operator around 1914. This conjecture is called "Hilbert-Polya conjecture." Shigenobu Kurokawa is studying the zeta function by the polynomial ring of prime numbers, fields with one element \( Z=F_1[2, 3, 5, 7, ...] \) around 1996. Christopher Deninger is studying the eigenvalue interpretation of the zeros of a zeta function in 1998. Alain Connes showed the relation between noncommutative...
geometry and the Riemann hypothesis in 1999. Nick Katz and Peter Sarnak are studying the
distribution of zeros in arithmetic quantum chaos theory in 1999.

1.4 New construction method of this paper

We define the polynomial ring of a group in order to add groups and to add the members of a
group.

The argument of a zeta function is a complex number. We can interpret a complex number as the
subgroup of a quaternion. Therefore, we can expand the argument of a zeta function to a quaternion.
On the other hand, a complex number is one-dimensional complex general linear group. And the
sporadic finite simple groups are high dimensional complex general linear groups. Therefore, we
can expand the argument of a zeta function to sporadic finite simple groups.

We define the product of groups the direct product. We interpret the cyclic groups of prime order
as prime groups. We construct the natural groups of natural order with products of prime groups.
Sporadic finite simple groups become subgroups of natural groups because arbitrary finite groups
become subgroups of natural groups. We can express arbitrary finite groups by zeta function of
prime groups. And we can express arbitrary noncommutative finite groups by zeta function of
sporadic finite simple groups.

2 Explanation of zeta function and sporadic finite simple group

2.1 Riemann hypothesis of zeta function

Bernhard Riemann expanded the argument of the zeta function to the complex number in 1859.

\[ s \in \mathbb{C} \]  \hspace{1cm} (2.1)

\[ \zeta(s) = \prod_{p: \text{prime}} \frac{1}{1 - p^{-s}} \]  \hspace{1cm} (2.2)

The arguments "s" of zeta function \( \zeta(s) \) is the member of the set \( \mathbb{C} \) of the complex numbers. The
numbers "p" are the prime numbers. We can express the zeta function with the members "n" of the
set \( \mathbb{N} \) of natural numbers as follows.

\[ n \in \mathbb{N} \]  \hspace{1cm} (2.3)

\[ \zeta(s) = \sum_{n=1}^{\infty} \frac{1}{n^s} \]  \hspace{1cm} (2.4)

Riemann expected the following thing.

Riemann hypothesis:
The real part \( \text{Re}(\rho) \) is 1/2 for all nontrivial zeros \( \rho \) of zeta function \( \zeta(s) \).

We express the examples of nontrivial zeros \( \rho_1 \) and \( \rho_2 \) in the following figure and formula. The
black circles are zeros and the white circle means a pole.
2.2 Monstrous moonshine of sporadic finite simple group

Evariste Galois suggested the concept of the group in the 1830s. Then, many mathematicians achieved the classification of the finite simple groups in 2004 after 1962. These achievements are called the classification theorem of the finite simple group.

According to the theorem, the finite simple group is of the same type of the following.
(1) The cyclic groups of prime order
(2) The 5th more than alternating groups
(3) Lee type simple groups
(4) 26 sporadic simple groups

Although the number of the 1st to the 3rd groups is infinite, the number of the 4th simple group is 26. One of the simple groups, \( M \) "monster" has the following order.

\[
|M| = 2^{46} \cdot 3^{20} \cdot 5^9 \cdot 7^6 \cdot 11^2 \cdot 13^3 \cdot 17 \cdot 19 \cdot 23 \cdot 29 \cdot 31 \cdot 41 \cdot 47 \cdot 59 \cdot 71 \approx 8.08 \times 10^{53}
\]  

This simple group has 194 irreducible representations. The nontrivial smallest irreducible representation has 196883-dimensional matrixes. The simple group monster was discovered in 1973.

John McKay and John G. Thompson suggested the next conjectures in 1979.
McKay-Thompson conjectures (1979):

(a) Module $V_n$ "with some natural meanings" on which the simple group monster $M$ acts exists. It is on $n \geq 0$ and $V_0 = 0$. Then, we can obtain the following formula.

$$H = \{ z \in \mathbb{C} \mid \text{Im}(z) > 0 \}$$ (2.8)

$$z \in H$$ (2.9)

$$q = \exp(2\pi iz)$$ (2.10)

$$c(n) = \text{dim}(V_n)$$ (2.11)

$$J(z) = j(z) - 744$$ (2.12)

$$J(z) = \sum_{n=0}^{\infty} c(n)q^{n-1}$$ (2.13)

$H$ is the space of a complex number. "$j(z)$" is an automorphic function that is invariant on the transformation by the special linear group $\text{SL}(2, \mathbb{C})$. "dim($V_n$)" is a dimension of module "$V_n$".

(b) Discontinuous subgroup $\Gamma_g$ of the special linear group $\text{SL}(2, \mathbb{R})$ exists to each member "$g$" of the simple group monster "$M$." The coset space "$R_g$" which compactified coset space $\Gamma_g \backslash H$ becomes a Riemann surface with genus of zero. The automorphic function "$J_g(z)$" becomes a generator of the meromorphic function field of coset space "$R_g$".

$$R_g = \Gamma_g \backslash H^{*}$$ (2.14)

$$g \in M$$ (2.15)

$$c_g(n) = \text{Tr}(g|V_n)$$ (2.16)

$$J(z) = j(z) - 744$$ (2.17)

$$J_g(z) = \sum_{n=0}^{\infty} c_g(n)q^{n-1}$$ (2.18)

"$H^*$" is "$H$" with cusp in order to compact coset space $\Gamma_g$. "Tr($g|V_n$)" is trace of the matrix of "$g$" which acts "$V_n$."  

The above conjectures are also called "monstrous moonshine."

Igor Frenkel, James Lepowsky and Arne Meurman constructed moonshine module "$V$" from Affine Lie algebra on which the monster "$M$" acts in the 1980s.

Richard Borcherds constructed the moonshine vertex operator algebra "$V$" from vertex operator algebra in 1992. The vertex operator algebra has the starting point in the string theory of modern physics.
3 Definition in this paper

3.1 Polynomial ring

The following formulas are called a polynomial.

\[-5x^2yz^2+2xyz−y^2z+z^3−5\]  (3.1)

"x", "y", and "z" are variables. "−5x^2yz^2" is a term. "x^2yz^2" of the term is a monomial. "−5" of the term is a coefficient.

A variable is a character. For example, "x", "y", and "z" are variables.

A monomial is a product of variables. For example, "xyz" is monomial. An exponent is the number of the same variables. The exponent of "x^2" is 2.

\[x^2=xx\]  (3.2)

A degree is the number of the variables of a monomial. For example, the degree of "x^2yz^2" is 5. The same monomial is called a like term.

A coefficient is the number of like terms. The coefficient of "x+1" is 2.

\[2x=x+x\]  (3.3)

The coefficient and the monomial are called a term. For example, "2x" is term. A polynomial is the sum of terms. For example, "5xz+2y" is polynomial.

We can express the monomial "u" with variables \(x_1, x_2, ..., x_n\) as follows.

\[u = \prod_{i=1}^{n} x_i^{a(i)} = x_1^{a(1)}x_2^{a(2)}x_3^{a(3)}...x_n^{a(n)}\]  (3.4)

\[0 \leq a(i) \in Z\]  (3.5)

"a(i)" is an integer function greater than or equal to 0.

A variable, a coefficient, and an exponent are member of the natural number \(N\), the integer \(Z\), the rational number field \(Q\), the real number field \(R\), the complex number field \(C\), the quaternion field \(H\), the general linear matrix \(GL(n,R)\), etc. We express the set of the polynomials with the variables and the coefficients are members of the field \(K\) and the exponents are the natural number \(N\) as follows.

\[K[x] = K[x_1, x_2, ..., x_n]\]  (3.6)

This \(K[x]\) is called \(n\)-variable polynomial ring on the field \(K\) of the variable "x."

3.2 Polynomial ring of groups

We can interpret the variables of polynomials as groups.

We call the following formula "polynomial of groups."

\[-5X^2YZ^2+2XYZ−Y^2Z+Z^3−5\]  (3.7)

"X", "Y", and "Z" are groups. "−5X^2YZ^2" is a term. "X^2YZ^2" of the term is a monomial. "−5" of the term is a coefficient.

The monomial "U" can be expressed as follows by group "G_i."
\[ U = \prod_{i=1}^{n} G_i^{a(i)} = G_1^{a(1)} \times G_2^{a(2)} \times \cdots \times G_n^{a(n)} \]

(3.8)

\[ 0 \leq a(i) \in \mathbb{Z} \]

(3.9)

"a(i)" is an integer function greater than or equal to 0. The product of groups is a direct product.

The polynomial of a group is the sum of the terms.

The terms that have same monomials are called like terms. We express the number of the like terms as the coefficient as follows.

\[ 2G = G + G \]

(3.10)

Coefficients are the rational number field \( \mathbb{Q} \), the real number field \( \mathbb{R} \), the complex number field \( \mathbb{C} \), etc. We express the set of the polynomials of groups with the coefficients of members of the field \( K \) as follows.

\[ K[G] = K[G_1, G_2, G_3, \ldots, G_n] \]

(3.11)

We call \( K[G] \) as the polynomial ring of groups \( G \) on the field \( K \). After this section, we write polynomial ring \( K[G] \) as \( G \) shortly.

### 3.3 Definition of product of group

We define the product of groups as the direct product of a group. We define the product of groups \( AB \) as follows.

\[ AB = A \times B \]

(3.12)

We define the multiplier of the group \( A \) as follows.

\[ A^2 = A \times A \]

(3.13)

We define the multiplier of the unitary group \( U(1) \) as follows.

\[ U^2(1) = U(1) \times U(1) \]

(3.14)

### 3.4 Definition of sum of group

We define the sum of the groups as the direct sum of a group. We define the sum of groups \( A+B \) as follows.

\[ A+B = A(+)B \]

(3.15)

We define the coefficient of the group as follows.

\[ 2A = A+A \]

(3.16)

We define the coefficient of the unitary group \( U(1) \) as follows.

\[ 2U(1) = U(1)+U(1) \]

(3.17)

### 3.5 Definition of set of all irreducible representation of group

We define the set of the all irreducible representations of the group \( G \) as \( \text{Irr}(G) \). We define an irreducible representation of the set of all irreducible representation \( \text{Irr}(G) \) as \( R(G) \).
\[ R(G) \subseteq \text{Irr}(G) \] (3.18)

We suppose that the maximum dimension of the set of all irreducible representation \text{Irr}(G) is \( n \). We express the irreducible representation \( R(G) \) as a subgroup of \( n \)-dimensional complex general linear group \( GL(n, C) \).

\[ R(G) \subseteq GL(n, C) \] (3.19)

We express the representation matrix of the member "\( g \)" of the group \( G \) as \( r(g) \).

\[ g \in G \] (3.20)
\[ r(g) \in R(G) \] (3.21)

We express the \( m \)-th member of the group \( G \) as "\( g_m \)." We call the "\( m \)" of "\( g_m \)" as a group member number.

We suppose that the member of the set of representations \( R(G) \) is matrix "\( r(g_m) \)." We express the element of the \( i \)-th row and the \( j \)-th column of the matrix as "\( r_{ij}(g_m) \)."

There are many matrix representations of a group. \text{Irr}(g) \) is a set of all representations. The product of \text{Irr}(G) \) is the product of all representations of \text{Irr}(G). The sum of \text{Irr}(G) \) is the sum of the all representation of \text{Irr}(G).

After this section, we write the set of the all irreducible representations \text{Irr}(G) as \( G \) shortly. We write the member \( r(g) \) as \( g \).

### 3.6 Set of all irreducible representations of finite simple groups

The \( i \)-th finite simple group is written as \( F_i \).

A set of the all irreducible representation of a finite simple group is set to \text{Irr}(F_i).

We write \text{Irr}(F_i) as \( F_i \) after this section.

### 3.7 Definition of product of members of a group

We define the product of the members of the group as the normal product of members. We define the product "\( ab \)" of members of the group \( G \) as follows.

\[ a, b, c \in G \] (3.22)
\[ c = ab \] (3.23)

We define the multiplier of member "\( a \)" of a group as follows.

\[ a^2 = aa \] (3.24)

### 3.8 Definition of sum of the members of a group

We define the sum of members of a group as follows.

We define the sum of the members "\( a \)" and "\( b \)" of the group \( G \) as follows.
\[
\begin{align*}
  a, b, c & \in G \\
  r(a), r(b), r(c) & \in R(G) \\
  r_{ij}(c) &= r_{ij}(a) + r_{ij}(b)
\end{align*}
\]

We sum each element of matrices of the members of a group.

4 Expansion of the argument of the zeta function to sporadic finite simple group

4.1 Expansion of the argument of the zeta function to member of quaternion field

The argument "s" of zeta function \(\zeta(s)\) is a member of the complex number field \(C\). We can interpret the complex number field \(C\) as the subfield of the quaternion field \(H\). Therefore, the argument of a zeta function is expandable to the member of a quaternion field.

\[
s \in C \subseteq H
\]

\[
q \in H
\]

\[
\zeta(q) = \prod_{p: \text{prime}} \frac{1}{1 - p^{-q}}
\]

4.2 Expansion of the argument of zeta function to high dimensional complex general linear group

The argument "s" of zeta function \(\zeta(s)\) is a member of the complex number field \(C\). The complex number field \(C\) is one-dimensional complex general linear group \(GL(1, C)\). One-dimensional complex general linear group \(GL(1, C)\) is a subgroup of complex general linear group \(GL(n, C)\) of \(n\)-dimensional. Therefore, the argument "s" of a zeta function is expandable to member \(g\) of complex general linear group \(GL(n, C)\) of \(n\)-dimensional.

\[
s \in GL(1, C) \subseteq GL(n, C)
\]

\[
g \in GL(n, C)
\]

\[
\zeta(g) = \prod_{p: \text{prime}} \frac{1}{1 - p^{-g}}
\]

4.3 Construction of noncommutative finite group by finite simple group

The arbitrary noncommutative finite group \(G\) is subgroups of the direct product of two or more finite simple group \(F_i\).

\[
G \subseteq \prod_{i=1}^{n} F_i^{a(i)}
\]

Since there are 26 finite simple groups, \(n\) is 26. "\(a(i)\)" is an integer function used greater than or equal to 0.
4.4 Expansion to sporadic finite simple group of argument of zeta function

If the maximum number of dimensions of the irreducible representation of a sporadic finite simple group is \( n \), the irreducible representation of the sporadic finite simple group \( F \) becomes a subgroup of complex general linear group \( GL(n, C) \) of \( n \)-dimensional. Therefore, the argument of a zeta function is expandable to member \( g \) of the sporadic finite simple group \( F \).

\[
g \in F \subseteq GL(n, C)
\]

\[
\zeta(g) = \prod_{p: \text{prime}} \frac{1}{1 - p^{-g}}
\]

\( p \) is a prime number.

5 Conclusion

In this paper we got the following results.

(1) Expansion of argument of zeta function
(1-1) Expansion of the argument of the zeta function to the quaternion
(1-2) Expansion of the argument of the zeta function to the complex general linear group
(1-3) Expansion of the argument of the zeta function to the sporadic finite simple group

6 Supplement: Construction of zeta function of sporadic finite simple group

In supplement, we try to get the following results.

(1) Expansion of prime number of zeta functions
(1-1) Expansion of the prime number of zeta function to the prime group
(1-2) Expansion of the prime number of zeta function to the sporadic finite simple group

6.1 Construction of prime group

We interpret the cyclic groups of prime order as prime groups.

We express a cyclic group \( Z_p \) of prime order \( p \) as prime group \( P_p \).

We express a set of the all irreducible representations of a group as \( \text{Irr}(P_p) \).

We write \( \text{Irr}(P_p) \) as \( P_p \) after this section.

6.2 Construction of finite group by prime group

We can construct groups that have order of natural number, natural group from direct products of prime groups \( P_p \).

\[
N_n = \prod_{p: \text{prime}} P_p^{a(p)}
\]

"\( p \)" is a prime number. "\( a(p) \)" is an integer function greater than or equal to "0".
Can't we construct arbitrary finite groups by subgroups of natural groups?
In order to consider this, we consider the case of infinite groups.

We consider the case of the infinite group of a small number of dimensions.
The two-dimensional unitary group \( U(2) \) becomes a subgroup of \( U^4(1) \).
\[
U(2) \subseteq U^4(1) \quad (6.2)
\]
The three-dimensional orthogonal group \( O(3) \) becomes a subgroup of \( U(3) \).
Three-dimensional unitary group \( U(3) \) becomes a subgroup of \( U^6(1) \).
\[
O(3) \subseteq U(3) \subseteq U^6(1) \quad (6.3)
\]
Next, we consider the case of the infinite group of \( n \)-dimensional.
The \( n \)-dimensional unitary group \( U(n) \) becomes a subgroup of \( U^{2n}(1) \).
\[
U(n) \subseteq U^{2n}(1) \quad (6.4)
\]
The \( n \)-dimensional orthogonal group \( O(n) \) becomes a subgroup of \( U(n) \).
The \( n \)-dimensional unitary group \( U(n) \) becomes a subgroup of \( U^{2n}(1) \).
\[
O(n) \subseteq U(n) \subseteq U^{2n}(1) \quad (6.5)
\]
We can interpret \( U(n) \) and \( O(n) \) as finite groups with large orders. We can interpret \( U(1) \) as a cyclic group with large orders. Therefore, we can guess the following thing from the above mentioned result.

The arbitrary finite group \( G \) becomes a subgroup of a certain natural group \( N_n \) with sufficiently big "\( n \)."
\[
G \subseteq N_n \quad (6.6)
\]

6.3 Construction of finite simple group by prime groups

Finite simple groups are finite groups. Therefore, the arbitrary finite simple group \( F \) becomes a subgroup of a certain natural group \( N_n \) with sufficiently big "\( n \)."
\[
F \subseteq N_n \quad (6.7)
\]

6.4 Construction of zeta function for prime groups

We construct zeta function \( \zeta(s) \) of prime groups as follows.
\[
s \in C \quad (6.8)
\]
\[
\zeta(s) = \prod_{p: \text{prime}} \frac{1}{1 - p^{-s}} \quad (6.9)
\]
Prime group \( P_p \) is a cyclic group of the prime order \( Z_p \). "\( s \)" is a member of the complex number field \( C \). \( \zeta(s) \) becomes a member of a polynomial ring of prime groups.

If we suppose the order \(|P_p|\) of the prime group \( P_p \) is a prime number "\( p \)," we can construct a zeta function of the orders of prime groups.
6.5 Construction of zeta function of natural groups

We construct zeta function $\zeta(s)$ of natural groups as follows.

$$s \in C$$  \hspace{1cm} (6.10)

$$\zeta(s) = \prod_{p: \text{prime}} \frac{1}{1 - p^{-s}}$$  \hspace{1cm} (6.11)

Natural group is a group which has order of natural number and constructed from direct products of prime groups $P_p$. "$s" is a member of the complex number field $C$. $\zeta(s)$ becomes a member of a polynomial ring of natural groups.

If we suppose the order $|N_n|$ of the natural group $N_n$ is a natural number "$n,"$ we can construct a zeta function of the orders of natural groups.

$$s \in C$$  \hspace{1cm} (6.12)

$$\zeta(s) = \sum_{n=1}^{\infty} \frac{1}{N_n^s}$$  \hspace{1cm} (6.13)

6.6 Construction of zeta function of sporadic finite simple group

We construct zeta function $\zeta(s)$ of a sporadic finite simple group as follows.

$$s \in C$$  \hspace{1cm} (6.14)

$$\zeta(s) = \sum_{n=1}^{\infty} \frac{1}{n^s}$$  \hspace{1cm} (6.15)

Simple group $F_i$ is the $i$-th simple group. Since there are 26 simple groups, "$n" is 26. "$s" is the member of the complex number field $C$. $\zeta(s)$ becomes a member of a polynomial ring of finite simple groups.

If we suppose the order $|F_i|$ of the finite simple group $F_i$ is a finite simple number "$f_i,"$ we can construct a zeta function of the orders of finite simple groups.

$$s \in C$$  \hspace{1cm} (6.16)

$$\zeta(s) = \prod_{i=1}^{n} \frac{1}{1 - F_i^{-s}}$$  \hspace{1cm} (6.17)

$$\zeta(s) = \prod_{i=1}^{n} \frac{1}{1 - f_i^{-s}}$$  \hspace{1cm} (6.18)

12/13
7 Term arrangement

Table 7.1: Prime group etc.

<table>
<thead>
<tr>
<th>Classification</th>
<th>Prime</th>
<th>Natural</th>
<th>Finite simple</th>
</tr>
</thead>
<tbody>
<tr>
<td>Group</td>
<td>Prime group $P$</td>
<td>Natural group $N$</td>
<td>Finite simple group $F$</td>
</tr>
<tr>
<td>Member number of group</td>
<td>Member number of prime group $m_P$</td>
<td>Member number of natural group $m_N$</td>
<td>Member number of finite simple group $m_F$</td>
</tr>
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<td>Order of group</td>
<td>Order of prime group $</td>
<td>P</td>
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<tr>
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</tbody>
</table>

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