Indeterminate masses, elements and models in information fusion

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Abstract: In this paper at the beginning, we make a short history of the logics, from the classical Boolean logic to the most general logic of today neutrosophic logic. We define the general logic space and give the definition of the neutrosophic logic. Then we introduce the indeterminate models in information fusion, which are due either to the existence of some indeterminate elements in the fusion space or to some indeterminate masses. The best approach for dealing with such models is the neutrosophic logic, which is part of neutrosophy. Neutrosophic logic is connected with neutrosophic set and neutrosophic probability and statistics.

Keywords: neutrosophic logic; indeterminacy; indeterminate model; indeterminate element; indeterminate mass; indeterminate fusion rules; DSmT; DST; TBM.

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Biographical notes: Florentin Smarandache is a Professor of Mathematics at the University of New Mexico in the USA. He has published over 240 papers and many books in mathematics, physics, engineering, and computer science. He was invited to lecture at University of Berkeley in 2003, NASA Langley Research Center-USA in 2004, NATO Advance Study Institute-Bulgaria in 2005, and other university and research centres. He received the 2011 Romanian Academy 'Traian Vuia' Award (the highest in the country) for Technical Sciences, the 2012 New Mexico and Arizona Book Award, and the 2011 New Mexico Book Award (together with Dr. W.B. Vasantha Kandasamy) (Albuquerque, USA), the 2010 Gold Medal from the Telesio-Galilei Academy of Science at the University of Pecs – Hungary, and the Outstanding Professional Service and Scholarship from The University of New Mexico (2009, 2005, 2001). Doctor Honoris Causa of Academia DacoRomana from Bucharest – 2011, and of Beijing Jiaotong University from China – 2011.

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1 Introduction

Let Θ be a frame of discernment, defined as:

$$\Theta = \{\phi_1, \phi_2, ..., \phi_n\},\ n \geq 2, \tag{1}$$

and its Super-Power Set (or fusion space):

$$S^{\Theta}(\Theta, \cup, \cap, \complement) \tag{2}$$

which means the set Θ closed under union, intersection, and respectively complement.

As an alternative to the existing logics we have proposed the neutrosophic logic (NL) to represent a mathematical model of uncertainty, vagueness, ambiguity, imprecision, undefined, unknown, incompleteness, inconsistency, redundancy, contradiction. It is a non-classical logic. NL and neutrosophic set are consequences of the neutrosophy.

Neutrosophy is a new branch of philosophy, which studies the origin, nature, and scope of neutralities, as well as their interactions with different ideational spectra.

A logic in which each proposition is estimated to have the percentage of truth in a subset *T*, the percentage of indeterminacy in a subset *I*, and the percentage of falsity in a subset *F*, where *T*, *I*, *F* are defined above, is called *NL*.

(*T*, *I*, *F*) truth-values, where *T*, *I*, *F* are standard or non-standard subsets of the non-standard interval $]^-0, 1^+[$, where $ninf = inf\ T + inf\ I + inf\ F \geq {}^-0$, and $nsup = sup\ T + sup\ I + sup\ F \leq 3^+$. Statically *T*, *I*, *F* are subsets, but dynamically *T*, *I*, *F* are functions/operators depending on many known or unknown parameters.

The truth, indeterminacy and falsity can be approximated: for example, a proposition is between 30% to 40% true and between 60% to 70% false, even worst:

between 30% to 40% or 45% to 50% true (according to various analysers), and 60% or between 66% to 70% false.

The subsets are not necessary intervals, but any sets (discrete, continuous, open or closed or half-open/half-closed interval, intersections or unions of the previous sets, etc.) in accordance with the given proposition.

Statically T, I, F are subsets, but dynamically they are functions/operators depending on many known or unknown parameters.

The classical logic, also called bivalent logic for taking only two values {0, 1}, or Boolean logic from British mathematician George Boole (1815–1964), was named by the philosopher Quine (1981) 'sweet simplicity'.

Peirce, before 1910, developed a semantics for three-valued logic in an unpublished note, but Emil Post's dissertation (1920s) is cited for originating the three-valued logic. Here '1' is used for truth, '1/2' for indeterminacy, and '0' for falsehood. Also, Reichenbach, leader of the logical empiricism, studied it.

The three-valued logic was employed by Hallden (1949), Korner (1960), and Tye (1994) to solve Sorites Paradoxes. They used truth tables, such as Kleene's, but everything depended on the definition of validity. A three-valued paraconsistent system (LP) has the values: 'true', 'false', and 'both true and false'. The ancient Indian metaphysics considered four possible values of a statement: 'true (only)', 'false (only)', 'both true and false', and 'neither true nor false'; J.M. Dunn (1976) formalised this in a four-valued paraconsistent system as his first degree entailment semantics.

The Buddhist logic added a fifth value to the previous ones, 'none of these' (called *catushkoti*).

The $\{0, a_1, \ldots, a_n, 1\}$ multi-valued, or plurivalent, logic was develop by Lukasiewicz, while post originated the m valued calculus.

The many-valued logic was replaced by Goguen (1969) and Zadeh (1975) with an infinite-valued logic (of continuum power, as in the classical mathematical analysis and classical probability) called fuzzy logic, where the truth-value can be any number in the closed unit interval [0, 1]. The fuzzy set was introduced by Zadeh in 1965.

Applications of neutrosophic logic/set have been used to information fusion (Smarandache and Dezert, 2004–2009), extension logic (Smarandache, 2013; Vladareanu et al., 2013), and to robotics (Smarandache and Vladareanu, 2011; Smarandache, 2011; Okuyama et al., 2013).

With imprecise data has been worked in magnetic bearing systems (Anantachaisilp and Lin, 2013), signal processing (Golpira and Golpira, 2013), water pollution control system (Wang and Wu, 2013), neutrosophic soft set (Broumi and Smarandache, 2013), and especially to robotica and mechatronics systems (Vladareanu et al., 2012a, 2012b).

This paper is organised as follows: we present the NL, the indeterminate masses, elements and models, and give an example of indeterminate intersection.

2 Indeterminate mass

2.1 Neutrosophic logic

NL (Smarandache, 1998, 2002) started in 1995 as a generalisation of the fuzzy logic, especially of the intuitionistic fuzzy logic (IFL). A logical proposition P is characterised by three neutrosophic components:

$$NL(P) = (T, I, F) \tag{3}$$

where T is the degree of truth, F the degree of falsehood, and I the degree of indeterminacy (or *neutral*, where the name 'neutro-sophic' comes from, i.e., neither truth nor falsehood but in between – or included-middle principle), and with:

$$T, I, F \subseteq \left]{}^{-}0, 1^{+}\right[\tag{4}$$

where $]^{-}0, 1^{+}[$ is a non-standard interval.

In this paper, for technical proposal, we can reduce this interval to the standard interval [0, 1].

The main distinction between NL and IFL is that in NL the sum $T + I + F$ of the components, when T, I, and F are crisp numbers, does not need to necessarily be 1 as in IFL, but it can also be less than 1 (for incomplete/missing information), equal to 1 (for complete information), or greater than 1 (for paraconsistent/contradictory information).

The combination of neutrosophic propositions is done using the neutrosophic operators (especially $\wedge$, $\vee$).

2.2 Neutrosophic mass

We recall that a classical mass $m(.)$ is defined as:

$$m : S^{\Theta} \to [0,1] \tag{5}$$

such that

$$\sum_{X \in S^{\Theta}} m(X) = 1 \tag{6}$$

We extend this classical basic belief assignment (mass) $m(.)$ to a neutrosophic basic belief assignment (NBBA) (or neutrosophic mass) $m_n(.)$ in the following way.

$$m_n : S^{\Theta} \to [0,1]^3 \tag{7}$$

with

$$m_n(A) = \left(T(A), I(A), F(A)\right) \tag{8}$$

where $T(A)$ means the (local) chance that hypothesis A occurs, $F(A)$ means the (local) chance that hypothesis A does not occur (non-chance), while $I(A)$ means the (local) indeterminate chance of A (i.e., knowing neither if A occurs nor if A does not occur), such that:

$$\sum_{X \in S^{\Theta}} \left[T(X) + I(X) + F(X)\right] = 1. \tag{9}$$

In a more general way, the summation (9) can be less than 1 (for incomplete neutrosophic information), equal to 1 (for complete neutrosophic information), or greater than 1 (for paraconsistent/conflicting neutrosophic information). But in this paper we only present the case when summation (9) is equal to 1.

Of course,

$$0 \le T(A), I(A), F(A) \le 1 \tag{10}$$

A *basic belief assignment* (or *mass*) is considered *indeterminate* if there exist at least an element $A \in S^{\Theta}$ such that $I(A) > 0$, i.e., there exists some indeterminacy in the chance of at least an element A for occurring or for not occurring. Therefore, a neutrosophic mass which has at least one element A with $I(A) > 0$ is an indeterminate mass.

A classical mass $m(.)$ as defined in equations (5) and (6) can be extended under the form of a neutrosophic mass $m_n'(.)$ in the following way:

$$m_n'(.): S^{\Theta} \to [0,1]^3 \tag{11}$$

with

$$m_n'(A) = \left(m(A), 0, 0\right) \tag{12}$$

but reciprocally it does not work since $I(A)$ has no correspondence in the definition of the classical mass.

We just have $T(A) = m(A)$ and $F(A) = m(C(A))$, where $C(A)$ is the complement of A. The non-null $I(A)$ can, for example, be roughly approximated by the total ignorance mass $m(\Theta)$, or better by the partial ignorance mass $m(\Theta_I)$ where Θ_I is the union of all singletons that have some non-zero indeterminacy, but these mean less accuracy and less refinement in the fusion.

If $I(X) = 0$ for all $X \in S^{\Theta}$, then the neutrosophic mass is simply reduced to a classical mass.

3 Indeterminate element

We have two types of elements in the fusion space S^{Θ}, *determinate elements* (which are well-defined), and *indeterminate elements* (which are not well-defined; for example: a geographical area whose frontiers are vague; or let us say in a murder case there are two suspects, *John* – who is known/determinate element – but he acted together with another man X (since the information source saw *John* together with an unknown/unidentified person) – therefore X is an indeterminate element).

Herein, we gave examples of singletons as indeterminate elements just in the frame of discernment Θ, but indeterminate elements can also result from the combinations (unions, intersections, and/or complements) of determinate elements that form the super-power set S^{Θ}. For example, A and B can be determinate singletons (we call the elements in Θ as singletons), but their intersection $A \cap B$ can be an indeterminate (unknown) element, in the sense that we might not know if $A \cap B = \phi$ or $A \cap B \neq \phi$.

Or A can be a determinate element, but its complement $C(A)$ can be indeterminate element (not well-known), and similarly for determinate elements A and B, but their $A \cup B$ might be indeterminate.

Indeterminate elements in S^{Θ} can, of course, result from the combination of indeterminate singletons too. All depends on the problem that is studied.

A frame of discernment which has at least an indeterminate element is called *indeterminate frame of discernment*. Otherwise, it is called *determinate frame of discernment*. Similarly, we call an *indeterminate fusion space* (S^{Θ}) that fusion space which has at least one indeterminate element. Of course an indeterminate frame of discernment spans an indeterminate fusion space.

An *indeterminate source of information* is a source which provides an indeterminate mass or an indeterminate fusion space. Otherwise it is called a *determinate source of information*.

4 Indeterminate model

An *indeterminate model* is a model whose fusion space is indeterminate, or a mass that characterises it is indeterminate.

Such case has not been studied in the information fusion literature so far. In the next sections, we will present some examples of indeterminate models.

5 Classification of models

In the classical fusion theories, all elements are considered determinate in the closed world, except in Smets' open world where there is some room (i.e., mass assigned to the empty set) for a possible unknown missing singleton in the frame of discernment. So, the open world has a probable indeterminate element, and thus its frame of discernment is indeterminate. While the closed world frame of discernment is determinate.

In the closed world in Dezert-Smarandache theory, there are three models classified upon the types of singleton intersections: Shafer's model (where all intersections are empty), hybrid model (where some intersections are empty, while others are non-empty), and free model (where all intersections are non-empty).

We now introduce a fourth category, called *indeterminate model* (where at least one intersection is indeterminate/unknown, and in general at least one element of the fusion space is indeterminate). We do this because in practical problems we do not always know if an intersection is empty or nonempty. As we still have to solve the problem in the real time, we have to work with what we have, i.e., with indeterminate models.

The *indeterminate intersection* cannot be refined (because not knowing if $A \cap B$ is empty or nonempty, we'd get two different refinements: $\{A, B\}$ when intersection is empty, and $\{A \setminus B, B \setminus A, A \cap B\}$ when intersection is nonempty).

The *percentage of indeterminacy* of a model depends on the number of indeterminate elements and indeterminate masses.

By default: the sources, the masses, the elements, the frames of discernment, the fusion spaces, and the models are supposed determinate.

6 An example of information fusion with an indeterminate model

We present the below example.

Suppose we have two sources, $m_1(.)$ and $m_2(.)$, such that.

Table 1 First part of the fusion with indeterminate model

	A	B	C	$A \cup B \cup C$	$A \cap B$ = *Ind.*	$A \cap C = \phi$	$B \cap C$ = *Ind.*
m_1	0.4	0.2	0.3	0.1			
m_2	0.1	0.3	0.2	0.4			
m_{12}	0.21	0.17	0.20	0.04	0.14	0.11	0.13

Applying the conjunction rule to m_1 and m_2 we get $m_{12}(.)$ as shown in Table 1.

The frame of discernment is $\Theta = \{A, B, C\}$. We know that $A \cap C$ is empty, but we do not know the other two intersections: we note them as $A \cap B$ = *ind.* and $B \cap C$ = *ind,* where *ind.* means indeterminate.

Using the conjunctive rule to fusion m_1 and m_2, we get $m_{12}(.)$:

$$\forall A \in S^{\Theta} \setminus \phi, m_{12}(A) = \sum_{\substack{X,Y \in S^{\Theta} \\ A = X \cap Y}} m_1(X) m_2(Y). \tag{13}$$

Whence $m_{12}(A) = 0.21$, $m_{12}(B) = 0.17$, $m_{12}(C) = 0.20$, $m_{12}(A \cup B \cup C) = 0.04$, and for the intersections:

$$m_{12}(A \cap B) = 0.14, m_{12}(A \cap C) = 0.11, m_{12}(B \cap C) = 0.13.$$

We then use the *PCR5* fusion rule style to redistribute the masses of these three intersections. We recall *PCR5* for two sources:

$$\forall A \in S^{\Theta} \setminus \phi,$$

$$m_{12PCR5}(A) = m_{12}(A) \tag{14}$$

$$+ \sum_{\substack{X \in S^{\Theta} \setminus \{\phi\} \\ X \cap A = \phi}} \left[\frac{m_1(A)^2 m_2(X)}{m_1(A) + m_2(X)} + \frac{m_2(A)^2 m_1(X)}{m_2(A) + m_1(X)} \right]$$

a $m_{12}(A \cap C) = 0.11$ is redistributed back to A and C because $A \cap C = \phi$, according to the PCR5 style.

Let $\alpha 1$ and $\alpha 2$ be the parts of mass 0.11 redistributed back to A, and $\gamma 1$ and $\gamma 2$ be the parts of mass 0.11 redistributed back to C.

We have the following proportionalisations:

$$\frac{\alpha 1}{0.4} = \frac{\gamma 1}{0.2} = \frac{0.4 \cdot 0.2}{0.4 + 0.2} = 0.133333,$$

whence $\alpha 1 = 0.4(0.133333) \approx 0.053333$ and $\gamma 1 = 0.2 (0.13333) \approx 0.026667$.

Similarly:

$$\frac{\alpha 2}{0.1} = \frac{\gamma 2}{0.3} = \frac{0.1 \cdot 0.3}{0.1 + 0.3} = 0.075,$$

whence $\alpha 2 = 0.1(0.075) = 0.0075$ and $\gamma 2 = 0.3(0.075) = 0.0225$.

Therefore, the mass of A, which can also be noted as $T(A)$ in a neutrosophic mass form, receives from 0.11 back:

$$\alpha 1 + \alpha 2 = 0.053333 + 0.0075 = 0.060833,$$

while the mass of C, or $T(C)$ in a neutrosophic form, receives from 0.11 back:

$$\gamma 1 + \gamma 2 = 0.026667 + 0.0225 = 0.049167.$$

We verify our calculations: 0.060833 + 0.049167 = 0.11.

$m_{12}(A \cap B) = 0.14$ is redistributed back to the indeterminate parts of the masses of A and B respectively, namely $I(A)$ and $I(B)$ as noted in the neutrosophic mass form, because $A \cap B$ = *Ind.* We follow the same PCR5 style as done in classical PCR5 for empty intersections (as above).

Let $\alpha 3$ and $\alpha 4$ be the parts of mass 0.14 redistributed back to $I(A)$, and $\beta 1$ and $\beta 2$ be the parts of mass 0.14 redistributed back to $I(B)$.

We have the following proportionalisations:

$$\frac{\alpha 3}{0.4} = \frac{\beta 1}{0.3} = \frac{0.4 \cdot 0.3}{0.4 + 0.3} = 0.171429,$$

whence $\alpha 3 = 0.4(0.171429) \approx 0.068572$ and $\beta 1 = 0.3 (0.171429) \approx 0.051428$.

Similarly:

$$\frac{\alpha 4}{0.1} = \frac{\beta 2}{0.2} = \frac{0.1 \cdot 0.2}{0.1 + 0.2} = 0.066667$$

whence $\alpha 4 = 0.1(0.066667) \approx 0.006667$ and $\beta 2 = 0.2 (0.066667) \approx 0.013333$.

Therefore, the indeterminate mass of A, $I(A)$ receives from 0.14 back:

$$\alpha 3 + \alpha 4 = 0.068572 + 0.006667 = 0.075239$$

and the indeterminate mass of B, $I(B)$, receives from 0.14 back:

$$\beta 1 + \beta 2 = 0.051428 + 0.013333 = 0.064761.$$

Analogously, $m_{12}(B \cap C) = 0.13$ is redistributed back to the indeterminate parts of the masses of B and C respectively, namely $I(B)$ and $I(C)$ as noted in the neutrosophic mass form, because $B \cap C$ = *Ind.* also following the PCR5 style. Whence $I(B)$ gets back 0.065 and $I(C)$ also gets back 0.065.

Finally, we sum all results obtained from firstly using the conjunctive rule (Table 1) and secondly redistributing the intersections masses with PCR5 [sections (a), (b), and (c) from above]:

Table 2 Second part of the fusion with indeterminate model

	T(A)	*T(B)*	*T(C)*	*T(Θ)*	*I(A)*	*I(B)*	*I(C)*
m_{12}	0.21	0.17	0.20	0.04			
Additions	0.0075 0.053333		0.0225 0.026667		0.068572 0.006667	0.051428 0.013333 0.02 0.045	0.04 0.045
$m_{12PCR5I}$	0.270833	0.17	0.249167	0.04	0.075239	0.129761	0.065

where $\Theta = A \cup B \cup C$ is the total ignorance.

7 Believe, disbelieve, and uncertainty

In classical fusion theory, there exist the following functions:

- *Belief in A* with respect to the bba *m*(.) is:

$$Bel(A) = \sum_{\substack{X \in S^{\Theta} \setminus \{\phi\} \\ X \subseteq A}} m(X) \tag{15}$$

- *Disbelief in A* with respect to the bba *m*(.) is:

$$Dis(A) = \sum_{\substack{X \in S^{\Theta} \setminus \{\phi\} \\ X \cap A = \phi}} m(X) \tag{16}$$

- *Uncertainty in A* with respect to the bba *m*(.) is:

$$U(A) = \sum_{\substack{X \in S^{\Theta} \setminus \{\phi\} \\ X \cap A \neq \phi \\ X \cap C(A) \neq \phi}} m(X), \tag{17}$$

  where *C*(*A*) is the complement of *A* with respect to the total ignorance Θ.

- *Plausability of A* with respect to the bba *m*(.) is:

$$Pl(A) = \sum_{\substack{X \in S^{\Theta} \setminus \{\phi\} \\ X \cap A \neq \phi}} m(X) \tag{18}$$

8 Neutrosophic believe, neutrosophic disbelieve, and neutrosophic undecidability

Let us consider a neutrosophic mass $m_n(.)$ as defined in formulas (7) and (8), $m_n(X) = (T(X), I(X), F(X))$ for all $X \in S^{\Theta}$.

We extend formulas (15) to (18) from *m*(.) to $m_n(.)$:

- *Neutrosophic Belief in A* with respect to the nbba $m_n(.)$ is:

$$NeutBel(A) = \sum_{\substack{X \in S^{\Theta} \setminus \{\phi\} \\ X \subseteq A}} T(X) + \sum_{\substack{X \in S^{\Theta} \setminus \{\phi\} \\ X \cap A = \phi}} F(X) \tag{19}$$

- *Neutrosophic Disbelief in A* with respect to the nbba $m_n(.)$ is:

$$NeutDis(A) = \sum_{\substack{X \in S^{\Theta} \setminus \{\phi\} \\ X \cap A = \phi}} T(X) + \sum_{\substack{X \in S^{\Theta} \setminus \{\phi\} \\ X \subseteq A}} F(X) \tag{20}$$

- *Neutrosophic uncertainty in A* with respect to the nbba $m_n(.)$ is

$$\begin{aligned} NeutU(A) &= \sum_{\substack{X \in S^{\Theta} \setminus \{\phi\} \\ X \cap A \neq \phi \\ X \cap C(A) \neq \phi}} T(X) + \sum_{\substack{X \in S^{\Theta} \setminus \{\phi\} \\ X \cap A \neq \phi \\ X \cap C(A) \neq \phi}} F(x) \\ &= \sum_{\substack{X \in S^{\Theta} \setminus \{\phi\} \\ X \cap A \neq \phi \\ X \cap C(A) \neq \phi}} \left[T(X) + F(X) \right] \end{aligned} \tag{21}$$

- We now introduce the *neutrosophic global indeterminacy in A* with respect to the nbba $m_n(.)$ as a sum of local indeterminacies of the elements included in *A*:

$$NeutGlobInd(A) = \sum_{\substack{X \in S^{\Theta} \setminus \{\phi\} \\ X \subseteq A}} I(X) \tag{22}$$

- And afterwards we define another function called *neutrosophic undecidability about A* with respect to the nbba $m_n(.)$:

$$NeutUnd(A) = NeutU(A) + NeutGlobInd(A) \tag{23}$$

  or

$$NeutUnd(A) = \sum_{\substack{X \in S^{\Theta} \setminus \{\phi\} \\ X \cap A \neq \phi \\ X \cap C(A) \neq \phi}} \left[T(X) + F(X) \right] + \sum_{\substack{X \in S^{\Theta} \setminus \{\phi\} \\ X \subseteq A}} I(X) \tag{24}$$

- *Neutrosophic plausability of A* with respect to the nbba $m_n(.)$ is:

$$NeutPl(A) = \sum_{\substack{X \in S^{\Theta} \setminus \{\phi\} \\ X \cap A \neq \phi}} T(X) + \sum_{\substack{Y \in S^{\Theta} \setminus \{\phi\} \\ C(Y) \cap A \neq \phi}} F(Y) \tag{25}$$

In the previous example, let us compute *NeutBel*(.), *NeutDis*(.), and *NeutUnd*(.):

Table 3 Neutrosophic believe, disbelieve and undecidability

	A	*B*	*C*	$A \cup B \cup C$
NeutBel	0.270833	0.17	0.249167	0.73
NeutDis	0.419167	0.52	0.440833	0
NeutGlobInd	0.115239	0.169761	0.105	0
Total	0.805239 ≠ 1	0.859761 ≠ 1	0.795 ≠ 1	0.73 ≠ 1

As we see, for indeterminate model we cannot use the intuitionistic fuzzy set or IFL since the sum *NeutBel*(X) + *NeutDis*(X) + *NeutGlobInd*(X) is less than 1. In this case, we use the neutrosophic set or logic which can deal with incomplete information.

The sum is less than 1 because there is missing information (we do not know if some intersections are empty or not).

For example:

$$NeutBel(B) + NeutDis(B) + NeutGlobInd(B) = 0.859761$$
$$= 1 - I(A) - I(C).$$

$$NeutBel(C) + NeutDis(C) + NeutGlobInd(C) = 0.795$$
$$= 1 - I(A) - I(B)$$

and

$$NeutBel(A \cup B \cup C) + NeutDis(A \cup B \cup C)$$
$$+ NeutGlobInd(A \cup B \cup C) = 0.73$$
$$= 1 - I(A) - I(B) - I(C).$$

9 Neutrosophic dynamic fusion

A neutrosophic dynamic fusion is a dynamic fusion where some indeterminacy occurs: with respect to the mass or with respect to some elements.

The solution of the above indeterminate model which has missing information, using the neutrosophic set, is consistent in the classical dynamic fusion in the case we receive part (or total) of the missing information.

In the above example, let us say we find out later in the fusion process that $A \cap B = \phi$. That means that the mass of indeterminacy of A, $I(A)$ = 0.075239, is transferred to A, and the masses of indeterminacy of B (resulted from $A \cap B$ only) – i.e., 0.051428 and 0.13333 – are transferred to B. Thus, we get in Table 4.

The sum *NeutBel*(X) + *NeutDis*(X) + *NeutBlogInd*(X) increases towards 1, as indeterminacy $I(X)$ decreases towards 0, and reciprocally.

When we have complete information we get *NeutBel*(X) + *NeutDis*(X) + *NeutGlobInd*(X) =1 and in this case we have an intuitionistic fuzzy set, which is a particular case of the neutrosophic set.

Let us suppose once more, considering the neutrosophic dynamic fusion, that afterwards we find out that $B \cap C \neq \phi$. Then, from Table 4 the masses of indeterminacies of B, $I(B)$ (0.065 = 0.02 + 0.045, resulted from $B \cap C$ which was considered indeterminate at the beginning of the neutrosophic dynamic fusion), and that of C, $I(C)$ = 0.065, go now to $B \cap C$. Thus, we get in Table 5.

10 More redistribution versions for indeterminate intersections of determinate elements

Besides *PCR5*, it is also possible to employ other fusion rules for the redistribution, such as follows:

a For the masses of the empty intersections we can use *PCR1-PCR4, URR, PURR, Dempster's Rule*, etc. (in general any fusion rule that first uses the conjunctive rule, and then a redistribution of the masses of empty intersections).

b For the masses of the indeterminate intersections we can use *DSm Hybrid* (*DSmH*) rule to transfer the mass $m_{12}(X \cap Y = ind.)$ to $X \cup Y$, since $X \cup Y$ is a kind of uncertainty related to X, Y. In our opinion, a better approach in this case would be to redistributing the empty intersection masses using the *PCR5* and the indeterminate intersection masses using the *DSmH*, so we can combine two fusion rules into one.

Table 4 First neutrosophic dynamic fusion

	A	*B*	*C*	*Θ*	*I(A)*	*I(B)*	*I(C)*	*A∩B*	*A∩C*
m	0.270833	0.17	0.249167	0.04	0	0.065	0.065	0	0
+	0.075239	0.051428 0.013333							
mN	0.346072	0.234761	0.249167	0.04	0	0.065	0.065	0	0

Note: Where Θ = $A \cup B \cup C$ is the total ignorance.

Table 5 Second neutrosophic dynamic fusion

	A	B	C	Θ	I(A)	I(B)	I(C)	$A \cap B$	$A \cap C$	$B \cap C$
m_N	0.346 072	0.234 761	0.249 167	0.04	0	0.065	0.065	0	0	0
–/+						–0.065	–0.065			+0.065 +0.065
m_{NN}	0.346 072	0.234 761	0.249 167	0.04	0	0	0	0	0	0.13

Let $m_1(.)$ and $m_2(.)$ be two masses. Then:

$$m_{12PCR5/DSmH}(A) = \sum_{\substack{X,Y \in S^\Theta \setminus \{\phi\} \\ X \cap Y = A}} m_1(X) m_2(Y)$$

$$+ \sum_{\substack{X \in S^\Theta \setminus \{\phi\} \\ X \cap A = \phi}} \left[\frac{m(A)^2 m_2(X)}{m_1(A) + m_2(X)} + \frac{m_2(A)^2 m_1(X)}{m_2(A) + m_1(X)} \right]$$

$$+ \sum_{\substack{X,Y \in S^\Theta \setminus \{\phi\} \\ X \cap Y = ind. \\ Z \cup Y = A}} m_1(X) m_2(Y) \quad (26)$$

$$\sum_{\substack{X,Y \in S^\Theta \setminus \{\phi\} \\ \{X \cap Y = A\} \vee \{(X \cap Y = ind.) \wedge (X \cup Y = A)\}}} m_1(X) m_2(Y)$$

$$+ \sum_{\substack{X \in S^\Theta \setminus \{\phi\} \\ X \cap A = \phi}} \left[\frac{m_1(A)^2 m_2(X)}{m_1(A) + m_2(X)} + \frac{m_2(A)^2 m_1(X)}{m_2(A) + m_1(X)} \right]$$

Yet, the best approach, for an indeterminate intersection resulted from the combination of two classical masses $m_1(.)$ and $m_2(.)$ defined on a determinate frame of discernment, is the first one:

- Use the *PCR5* to combine the two sources: formula (14).
- Use the *PCR5-ind* [adjusted from classical *PCR5* formula (14)] in order to compute the indeterminacies of each element involved in indeterminate intersections:

  $$\forall A \in S^\Theta \setminus \phi,$$

  $$m_{12PCR5Ind}\left(I(A)\right) = \quad (27)$$

  $$\sum_{\substack{X \in S^\Theta \setminus \{\phi\} \\ X \cap A = ind.}} \left[\frac{m_1(A)^2 m_2(X)}{m_1(A) + m_2(X)} + \frac{m_2(A)^2 m_1(X)}{m_2(A) + m_1(X)} \right]$$

- Compute *NeutBel*(.), *NeutDis*(.), *NeutGlobInd*(.) of each element.

11 Conclusions

In order for the paper to be easier understanding, a short history of logics was made in the introduction. Connection between neutrosophy and NL were established.

In this paper, we introduced for the first time the notions of indeterminate mass (BBA), indeterminate element, indeterminate intersection, and so on. We gave an example of neutrosophic dynamic fusion using two classical masses, defined on a determinate frame of discernment, but having indeterminate intersections in the super-power set S^Θ (the fusion space). We adjusted several classical fusion rules (*PCR5* and *DSmH*) to work for indeterminate intersections instead of empty intersections.

Then we extended the classical *Bel*(.), *Dis*(.) {also called *Dou*(.), i.e., Dough} and the uncertainty *U*(.) functions to their respectively neutrosophic correspondent functions that use the neutrosophic masses, i.e., to the *NeutBel*(.), *NeutDis*(.), *NeutU*(.) and to the undecidability function *NeutUnd*(.). We have also introduced the neutrosophic global indeterminacy function, *NeutGlobInd*(.), which together with *NeutU*(.) form the Neut*Und*(.) function.

In our first example, the mass of $A \cap B$ is determined (it is equal to 0.14), but the element $A \cap B$ is indeterminate (we do not know if it empty or not).

But there are cases when the element is determinate (let us say a suspect John), but its mass could be indeterminate as given by a source of information {for example, $m_n(John)$ = (0.4, 0.1, 0.2), i.e., there is some mass indeterminacy: $I(John) = 0.2 > 0$}.

These are the distinctions between the indeterminacy of an element, and the indeterminacy of a mass.

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