Neutrosophic Diagram and Classes of Neutrosophic Paradoxes or to the Outer-Limits of Science

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These paradoxes are called “neutrosophic” since they are based on indeterminacy (or neutrality, i.e. neither true nor false), which is the third component in neutrosophic logic. We generalize the Venn diagram to a Neutrosophic Diagram, which deals with vague, inexact, ambiguous, ildefined ideas, statements, notions, entities with unclear borders. We define the neutrosophic truth table and introduce two neutrosophic operators (neuterization and antonymization operators) give many classes of neutrosophic paradoxes.

1 Introduction to the neutrosophics

Let <A> be an idea, or proposition, statement, attribute, theory, event, concept, entity, and <non A> what is not <A>.

Let <anti A> be the opposite of <A>. We have introduced a new notation [1998], <neut A>, which is neither <A> nor <anti A> but in between. <neut A> is related with <A> and <anti A>.

Let’s see an example for vague (not exact) concepts: if <A> is “tall” (an attribute), then <anti A> is “short”, and <neut A> is “medium”, while <non A> is “not tall” (which can be “medium or short”). Similarly for other <A>, <neut A>, <anti A> such as: <good>, <so so>, <bad>, or <perfect>, <average>, <imperfect>, or <high>, <medium>, <small>, or respectively <possible>, <sometimes possible and other times impossible>, <impossible>, etc.

Now, let’s take an exact concept / statement: if <A> is the statement “1 + 1 = 2 in base 10”, then <anti A> is “1 + 1 ≠ 2 in base 10”, while <neut A> is undefined (doesn’t exist) since it is not possible to have a statement in between “1 + 1 = 2 in base 10” and “1 + 1 ≠ 2 in base 10” because in base 10 we have 1+1 is either equal to 2 or 1+1 is different from 2. <non A> coincides with <anti A> in this case, <non A> is “1 + 1 ≠ 2 in base 10”.

Neutrosophy is a theory the author developed since 1995 as a generalization of dialectics. This theory considers every notion or idea <A> together with its opposite or negation <anti A>, and the spectrum of “neutralities” in between them and related to them, noted by <neut A>.

The Neutrosophy is a new branch of philosophy which studies the origin, nature, and scope of neutralities, as well as their interactions with different ideational spectra.

Its Fundamental Thesis:
Any idea <A> is T% true, I% indeterminate (i.e. neither true nor false, but neutral, unknown), and F% false.

Its Fundamental Theory:
Every idea <A> tends to be neutralized, diminished, balanced by <non A> ideas (not only by <anti A> as Hegel asserted) — as a state of equilibrium.

In between <A> and <anti A> there may be a continuous spectrum of particular <neut A> ideas, which can balance <A> and <anti A>.

To neuter an idea one must discover all its three sides: of sense (truth), of nonsense (falsity), and of undecidability (indeterminacy) — then reverse/combine them. Afterwards, the idea will be classified as neutrality.

There exists a Principle of Attraction not only between the opposites <A> and <anti A> (as in dialectics), but also between them and their neutralities <neut A> related to them, since <neut A> contributes to the Completeness of Knowledge.

Hence, neutrosophy is based not only on analysis of oppositional propositions as dialectic does, but on analysis of these contradictions together with the neutralities related to them.

Neutrosophy was extended to Neutrosophic Logic, Neutrosophic Set, Neutrosophic Probability and Neutrosophic Statistics, which are used in technical applications.

In the Neutrosophic Logic (which is a generalization of fuzzy logic, especially of intuitionistic fuzzy logic) every logical variable x is described by an ordered triple x = (T, I, F), where T is the degree of truth, F is the degree of falsehood, and I the degree of indeterminacy (or neutrality, i.e. neither true nor false, but vague, unknown, imprecise), with T, I, F standard or non-standard subsets of the non-standard unit interval [0, 1*]. In addition, these values may vary over time, space, hidden parameters, etc.

Neutrosophic Probability (as a generalization of the classical probability and imprecise probability) studies the chance that a particular event <A> will occur, where that chance is represented by three coordinates (variables): T% chance the event will occur, I% indeterminate (unknown) chance, and F% chance the event will not occur.

Neutrosophic Statistics is the analysis of neutrosophic probabilistic events.

Neutrosophic Set (as a generalization of the fuzzy set, and especially of intuitionistic fuzzy set) is a set such that an element belongs to the set with a neutrosophic probability,
i.e. $T$ degree of appurtenance (membership) to the set, $I$ degree of indeterminacy (unknown if it is appurtenance or non-appurtenance to the set), and $F$ degree of non-appurtenance (non-membership) to the set.

There exist, for each particular idea: PRO parameters, CONTRA parameters, and NEUTER parameters which influence the above values.

Indeterminacy results from any hazard which may occur, from unknown parameters, or from new arising conditions. This resulted from practice.

2 Applications of neutrosophics

Neutrosophic logic/set/probability/statistics are useful in artificial intelligence, neural networks, evolutionary programming, neutrosophic dynamic systems, and quantum mechanics.

3 Examples of neutrosophy used in Arabic philosophy (F. Smarandache and S. Osman)

- While Avicenna promotes the idea that the world is contingent if it is necessitated by its causes, Averroes rejects it, and both of them are right from their point of view. Hence $<A>$ and $<anti A>$ have common parts.
- Islamic dialectical theology (kalam) promoting creationism was connected by Avicenna in an extraordinary way with the opposite Aristotelian-Neoplatonic tradition. Actually a lot of work by Avicenna falls into the frame of neutrosophy.
- Averroes’s religious judges (quidis) can be connected with atheists’ beliefs.
- al-Farabi’s metaphysics and general theory of emanation vs. al-Ghazali’s Sufi writings and mystical treatises [we may think about a coherence of al-Ghazali’s “Incoherence of the Incoherence” book].
- al-Kindi’s combination of Koranic doctrines with Greek philosophy.
- Islamic Neoplatonism + Western Neoplatonism.
- Ibn-Khaldun’s statements in his theory on the cyclic sequence of civilizations, says that: Luxury leads to the raising of civilization (because the people seek for comforts of life) but also Luxury leads to the decay of civilization (because its correlation with the corruption of ethics).
- On the other hand, there’s the method of absent—by—present syllogism in jurisprudence, in which we find the same principles and laws of neutrosophy.
- In fact, we can also function a lot of Arabic aphorisms, maxims, Koranic miracles (Ayat Al- Qur’an) and Sunna of the prophet, to support the theory of neutrosophy.

Take the colloquial proverb that “The continuance of state is impossible” too, or “Everything, if it’s increased over its extreme, it will turn over to its opposite”!

4 The Venn diagram

In a Venn diagram we have with respect to a universal set $U$ the following:

Therefore, there are no common parts amongst $<A>$, $<neut A>$, and $<anti A>$, and all three of them are (completely) contained by the universal set $U$. Also, all borders of these sets $<A>$, $<neut A>$, $<anti A>$, and $U$ are clear, exact. All these four sets are well—defined.

While $<neut A>$ means neutralities related to $<A>$ and $<anti A>$, what is outside of $<A>$ $U$ $<neut A>$ $U$ $<anti A>$ but inside of $U$ are other neutralities, not related to $<A>$ or to $<anti A>$.

Given $<A>$, there are two types of neutralities: those related to $<A>$ (and implicitly related to $<anti A>$), and those not related to $<A>$ (and implicitly not related to $<anti A>$)

5 The neutrosophic diagram, as extension of the Venn diagram

Yet, for ambiguous, vague, not-well-known (or even unknown) imprecise ideas / notions / statements / entities with unclear frontiers amongst them the below relationships may occur because between an approximate idea noted by $<A>$ and its opposite $<anti A>$ and their neutralities $<neut A>$ there are not clear delimitations, not clear borders to distinguish amongst what is $<A>$ and what is not $<A>$. There are buffer zones in between $<A>$ and $<anti A>$ and $<neut A>$, and an element $x$ from a buffer zone between $<A>$ and $<anti A>$ may or may not belong to both $<A>$ and $<anti A>$ simultaneously. And similarly for an element $y$ in a buffer zone between $<A>$ and $<neut A>$, and an element $z$ in the buffer zone between $<neut A>$ and $<anti A>$. We may have a buffer zone where the confusion of appurtenance to $<A>$, or to $<neut A>$, or to $<anti A>$ is so high, that we can consider that an element $w$ belongs to all of them simultaneously (or to none of them simultaneously).

We say that all four sets $<A>$, $<neut A>$, $<anti A>$, and the neutrosophic universal set $U$ are ill-defined, inexact, unknown (especially if we deal with predictions; for example...
if $<A>$ is a statement with some degree of chance of occurring, with another degree of change of not occurring, plus an unknown part. In the general case, none of the sets $<A>$, $<\text{neut } A>$, $<\text{anti } A>$, $<\text{non } A>$ are completely included in $U$, and neither $U$ is completely known; for example, if $U$ is the neutrosophic universal set of some specific given events, what about an unexpected event that might belong to $U$? That’s why an approximate $U$ (with vague borders) leaves room for expecting the unexpected.

The Neutrosophic Diagram in the general case is the following (Fig. 2): the borders of $<A>$, $<\text{anti } A>$, and $<\text{neut } A>$ are dotted since they are unclear.

Similarly, the border of the neutrosophic universal set $U$ is dotted, meaning also unclear, so $U$ may not completely contain $<A>$, nor $<\text{neut } A>$ or $<\text{anti } A>$, but $U$ “approximately” contains each of them. Therefore, there are elements in $<A>$ that may not belong to $U$, and the same thing for $<\text{neut } A>$ and $<\text{anti } A>$. Or elements, in the most ambiguous case, there may be elements in $<A>$ and in $<\text{neut } A>$ and in $<\text{anti } A>$ which are not contained in the universal set $U$.

Even the neutrosophic universal set is ambiguous, vague, and with unclear borders.

Of course, the intersections amongst $<A>$, $<\text{neut } A>$, $<\text{anti } A>$, and $U$ may be smaller or bigger or even empty depending on each particular case.

See below an example of a particular neutrosophic diagram (Fig. 3), when some intersections are contained by the neutrosophic universal set:

A neutrosophic diagram is different from a Venn diagram since the borders in a neutrosophic diagram are vague. When all borders are exact and all intersections among $<A>$, $<\text{neut } A>$, and $<\text{anti } A>$ are empty, and all $<A>$, $<\text{neut } A>$, and $<\text{anti } A>$ are included in the neutrosophic universal set $U$, then the neutrosophic diagram becomes a Venn diagram.

The neutrosophic diagram, which complies with the neutrosophic logic and neutrosophic set, is an extension of the Venn diagram.

6 Classes of neutrosophic paradoxes

The below classes of neutrosophic paradoxes are not simply word puzzles. They may look absurd or unreal from the classical logic and classical set theory perspective. If $<A>$ is a precise / exact idea, with well-defined borders that delimit it from others, then of course the below relationships do not occur.

But let $<A>$ be a vague, imprecise, ambiguous, not-well-known, not-clear-boundary entity, $<\text{non } A>$ means what is not $<A>$, and $<\text{anti } A>$ means the opposite of $<A>$. $<\text{neut } A>$ means the neutralities related to $<A>$ and $<\text{anti } A>$, neutralities which are in between them.

When $<A>$, $<\text{neut } A>$, $<\text{anti } A>$, $<\text{non } A>$, $U$ are uncertain, imprecise, they may be selfcontradictory. Also, there are cases when the distinction between a set and its elements is not clear.

Although these neutrosophic paradoxes are based on “pathological sets” (those whose properties are considered atypically counterintuitive), they are not referring to the theory of Meinongian objects (Gegenstandstheorie) such as round squares, unicorns, etc. Neutrosophic paradoxes are not reported to objects, but to vague, imprecise, unclear ideas or predictions or approximate notions or attributes from our everyday life.

7 Neutrosophic operators

Let’s introduce for the first time two new Neutrosophic Operators:

1. An operator that “neuterizes” an idea. To \textit{neuterize} [\textit{neuter}+\textit{ize}, transitive verb; from the Latin word \textit{neuter} = neutral, in neither side], \textit{n}(), means to map an entity to its neutral part. [We use the Segoe Print for “\textit{n}(\,\,\textit{)}”].

   “To neuterize” is different from “to neutralize” [from the French word \textit{neutraliser}] which means to declare a territory neutral in war, or to make ineffective an enemy, or to destroy an enemy.

   \textit{n}(<A>) = <\text{neut } A>. By definition \textit{n}(<\text{neut } A>) = <\text{neut } A>.

   For example, if $<A>$ is “tall”, then \textit{n}(\text{tall}) = \text{medium}, also \textit{n}(\text{short}) = \text{medium}, \textit{n}(\text{medium}) = \text{medium}.

   But if $<A>$ is “1 + 1 = 2 in base 10” then \textit{n}(<1 + 1 = 2 \text{ in base } 10>) is undefined (does not exist), and similarly \textit{n}(<1 + 1 \neq 2 \text{ in base } 10>) is undefined.
2. And an operator that “antonymizes” an idea. To antonymize \[\text{antonymize}(\text{antonym}+\text{ize})\], transitive verb; from the Greek word \[\text{antonymia}\] = instead of, opposite], \(a()\), means to map an entity to its opposite. [We use the Segoe Print for \(n(A)\)] \(a(A)\) = \(<\text{anti } A>\).

For example, if \(<A>\) is “tall”, then \(a(\text{tall}) = \text{short}\), also \(a(\text{short}) = \text{tall}\), and \(a(\text{medium}) = \text{tall or short}\).

But if \(<A>\) is “1 + 1 = 2 in base 10” then \(a(1 + 1 = \text{tall or short}) = 1 + 1 = 2 \text{ in base } 10\) and reciprocally \(a(1 + 1 \neq 2 \text{ in base } 10) = 1 + 1 = 2 \text{ in base } 10\).

The classical operator for negation/complement in logics respectively in set theory, “to negate” (\(\sim\)), which is equivalent in neutrosophy with the operator “to notize” (i.e. to \(\text{not}+\text{ize}\)) or \(\text{non}ization\) (i.e. \(\text{non}+\text{ization}\)), means to map an idea to its neutral or to its opposite (a union of the previous two neutrosophic operators: \(\text{neuterization and antonymization}\): 
\(\neg(A) = \text{non } A = \text{neut } A \cup \text{anti } A = n(A) \cup a(A)\). 

Neutrosophic Paradoxes result from the following neutrosophic logic/set connectives following all apparently impossibilities or semi-impossibilities of neutrosophically connecting \(<A>\), \(<\text{anti } A>\), \(<\text{neut } A>\), \(<\text{non } A>\), and the neutrosophic universal set \(U\).

### 8 Neutrosophic truth tables

For \(<A>\) = “tall”:

<table>
<thead>
<tr>
<th>(&lt;A&gt;)</th>
<th>(a(A))</th>
<th>(n(A))</th>
<th>(&lt;\sim A&gt;)</th>
</tr>
</thead>
<tbody>
<tr>
<td>tall</td>
<td>short</td>
<td>medium</td>
<td>short or tall</td>
</tr>
<tr>
<td>medium</td>
<td>medium</td>
<td>short or medium</td>
<td></td>
</tr>
<tr>
<td>short</td>
<td>tall</td>
<td>medium</td>
<td></td>
</tr>
</tbody>
</table>

To remark that \(n(\text{medium})\) = medium. If \(<A>\) = tall, then \(<\text{neut } A>\) = medium, and \(<\text{anti } A>\) = \(<\text{neut } A>\), or \(n(\text{neut}(A))\) = \(n(\text{A})\).

For \(<A>\) = “1 + 1 = 2 in base 10” we have \(<\text{anti } A>\) = \(<\text{non } A>\) = “1 + 1 = 2 in base 10”, while \(<\text{neut } A>\) is undefined (N/A) — whence the neutrosophic truth table becomes:

<table>
<thead>
<tr>
<th>(&lt;A&gt;)</th>
<th>(a(A))</th>
<th>(n(A))</th>
<th>(&lt;\sim A&gt;)</th>
</tr>
</thead>
<tbody>
<tr>
<td>True</td>
<td>False</td>
<td>N/A</td>
<td>False</td>
</tr>
<tr>
<td>False</td>
<td>True</td>
<td>N/A</td>
<td>True</td>
</tr>
</tbody>
</table>

In the case when a statement is given by its neutrosophic logic components \(<A>\) = \((T, I, F)\), i.e. \(<A>\) is \(T\%\) true, \(I\%\) indeterminate, and \(F\%\) false, then the neutrosophic truth table depends on the defined neutrosophic operators for each application.

### 9 Neutrosophic operators and classes of neutrosophic paradoxes

a) Complement/Negation

\(<\text{anti } A>\neq <\text{non } A>\) and reciprocally \(<\text{anti } A>\neq <A>\).

b) Neutizer

\(n(<A>)\neq <\text{anti } A>\)
\(n(<\text{anti } A>)\neq <\text{anti } A>\)
\(n(<\text{anti } A>)\neq <\text{anti } A>\)
\(n(n(<A>))\neq <A>\)

c) Antonymizer

\(a(<A>)\neq <\text{anti } A>\)
\(a(<\text{anti } A>)\neq <\text{anti } A>\)
\(a(<\text{anti } A>)\neq <\text{anti } A>\)
\(a(a(<A>))\neq <A>\)

d) Intersection/Conjunction

\(<A>\cap <\text{anti } A>\neq \emptyset\) (neutrosophic empty set) [symbolically \((\exists x)(x \in A \land x \in \neg A)\)]

or even more \(<A>\cap <\text{anti } A>\neq \emptyset\) [symbolically \((\exists x)(x \in A \land x \in a(A))\)]

similarly \(<A>\cap <\text{neut } A>\neq \emptyset\) and \(<A>\cap <\text{anti } A>\neq \emptyset\),

up to \(<A>\cap <\text{anti } A>\neq \emptyset\),
The symbolic notations will be in a similar way.

This is Neutrosophic Transdisciplinary, which means to find common features to uncommon entities.

For examples:

There are things which are good and bad in the same time.

There are things which are good and bad and medium in the same time (because from one point of view they may be good, from other point of view they may be bad, and from a third point of view they may be medium).

e) Union/Weak Disjunction

\(<A>\cup <\text{neut } A>\cup <\text{anti } A>\neq U\).
\(<\text{anti } A>\cup <\text{neut } A>\neq <\text{anti } A>\).

Etc.

f) Inclusion/Conditional

\(<A>\subset <\text{anti } A>\)
\((\forall x)(x \in A \rightarrow x \in a(A))\)

All is \(<\text{anti } A>\), the \(<A>\) too.

All good things are also bad.

All is imperfect, the perfect too.

\(<\text{anti } A>\subset <A>\)
\((\forall x)(x \in a(A) \rightarrow x \in A)\)

All is \(<A>\), the \(<\text{anti } A>\) too.

All bad things have something good in them [this is rather a fuzzy paradox].

All is perfect things are imperfect in some degree.

\(<\text{anti } A>\subset <A>\)
We can also take into consideration other logical con-

Combinations of the previous single neutrosophic op-

Equality

which symbolically becomes $\exists x \in \neg A \rightarrow x \notin \neg A$

or even stronger inequality $(\forall x)(x \in \neg A \leftrightarrow x \notin \neg A)$.

Equal Inequalities

$<A> = <\text{anti } A>$

$(\forall x)(x \in A \leftrightarrow x \in a(A))$

All is $<A>$, the $<\text{anti } A>$ too; and reciprocally, all is $<\text{anti } A>$, the $<A>$ too. Or, both combined implications give: All is $<A>$ is equivalent to all is $<\text{anti } A>$.

And so on:

$<A> = <\text{neut } A>$

$<\text{anti } A> = <\text{neut } A>$

$<\text{neut } A> = <\text{neut } A>$

Dilations and Absorptions

$<\text{anti } A> = <\text{neut } A>$,

which means that $<\text{anti } A>$ is dilated to its neutrosophic superset $<\text{neut } A>$, or $<\text{neut } A>$ is absorbed to its neutrosophic subset $<\text{anti } A>$.

Similarly for:

$<\text{neut } A> \cap <\text{anti } A> \neq \emptyset$ [two neutrosophic operators].

$<A> \cup <\text{anti } A> \neq <\text{neut } A>$ and reciprocally $<\text{anti } A> \cup <\text{neut } A> \neq <\text{neut } A>$.

$<A> \cup <\text{neut } A> \neq <\text{anti } A>$ and reciprocally.

Therefore:

$<A> \cup <\text{anti } A> \cup <\text{neut } A> \neq \emptyset$ and reciprocally. Etc.

i) We can also take into consideration other logical connectives, such as strong disjunction (we previously used the weak disjunction), Shaffer’s connector, Peirce’s connector, and extend them to the neutrosophic form.

j) We may substitute $<A>$ by some entities, attributes, statements, ideas and get nice neutrosophic paradoxes, but not all substitutions will work properly.

10 Some particular paradoxes

Quantum Semi-Paradox

Let’s go back to 1931 Schrödinger’s paper. Saul Youssef writes (flipping a quantum coin) in arXiv.org at quant-ph/9509004:

“The situation before the observation could be described by the distribution (1/2,1/2) and after observing
heads our description would be adjusted to (1,0). The problem is, what would you say to a student who then asks: "Yes, but what causes (1/2,1/2) to evolve into (1,0)? How does it happen?"

It is interesting. Actually we can say the same for any probability different from 1: If at the beginning, the probability of a quantum event, \( P(\text{quantum event}) = p, 0 < p < 1 \), and if later the event occurs, we get to \( P(\text{quantum event}) = 1 \); but if the event does not occur, then we get \( P(\text{quantum event}) = 0 \), so still a kind of contradiction.

Torture’s paradox
An innocent person \( P \), who is tortured, would say to the torturer \( T \) whatever the torturer wants to hear, even if \( P \) doesn’t know anything.

So, \( T \) would receive incorrect information that will work against him/her. Thus, the torture returns against the torturer.

Paradoxist psychological behavior
Instead of being afraid of something, say \( \langle A \rangle \), try to be afraid of its opposite \( \langle \text{anti } A \rangle \), and thus – because of your fear – you’ll end up with the \( \langle \text{anti} \langle \text{anti } A \rangle \rangle \), which is \( \langle A \rangle \).

Paradoxically, negative publicity attracts better than positive one (enemies of those who do negative publicity against you will sympathize with you and become your friends).

Paradoxically [word coming etymologically from paradoxism, paradoxist], to be in opposition is more poetical and interesting than being opportunistic.

At a sportive, literary, or scientific competition, or in a war, to be on the side of the weaker is more challenging but on the edge of chaos and, as in Complex Adoptive System, more potential to higher creation.

Law of Self-Equilibrium
(Already cited above at the Neutrosophic Inclusion/Conditional Paradoxes) \( \langle A \rangle \rightarrow \langle B \rangle \) and \( \langle B \rangle \rightarrow \langle \text{anti } A \rangle \), therefore \( \langle A \rangle \rightarrow \langle \text{anti } A \rangle \)

Example: too much work produces sickness; sickness produces less work (absences from work, low efficiency); therefore, too much work implies less work.

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References