A Simple and General Proof of Beal's Conjecture

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ABSTRACT

Using the same method that we used in [1] to prove *Fermat's Last Theorem* in a simpler and truly marvellous way, we demonstrate that *Beal's Conjecture* yields – in the simplest imaginable manner; to our effort to proving it.

Keywords: Fermat's Last Theorem, Beal's conjecture, Proof.

"*Simplicity is the ultimate sophistication*"

Leonardo *da* Vinci $(1452 - 1519)$.

1. Introduction

Beal's Conjecture is a conjecture in number theory formulated in 1993 while investigating generalizations of *Fermat's Last Theorem* and set forth in 1997 as a Prize Problem by the United States *of* America's Dallas, Texas number theory enthusiast and billionaire banker, Daniel Andrew Beal [2]. As originally stated, the conjecture asserts that:

Beal's Conjecture:

If,

$$A^x + B^y = C^z, \tag{1}$$

were $A, B, C, x, y,$ and z are positive integers with $(x, y, z) > 2$, then $A, B,$ and C have a common prime factor.

For a correct proof or counterexample published in an internationally renowned refereed mathematics journal, Beal initially offered a Prize of US\$5,000.00 in 1997, raising it to \$50,000.00 over ten years by adding US\$5,000.00 each year over the ten year period [2]. Very recently, Andrew Beal upped the stacks and has since raised* it beyond the initial projection of US\$50,000.00 to US\$1,000,000.00.

Herein, we lay down a complete proof of the conjecture not so much for the very "handsome" prize money attached to it, but more for the sheer intellectual challenge that the philanthropist – Andrew Beal, has placed before humanity. We believe that challenges must be tackled heard-on, without fear of failure.

From intuition, we strongly believe or feel that a direct proof of the original statement of Beal conjecture as stated in (1) would be difficult if not impossible to procure. We have to recast this statement into an equivalent form and proceed to a proof by way of contradiction. The equivalent statement to (1) is [2]:

Beal's Conjecture (Recast):

The equation,

$$A^x + B^y = C^z, \tag{2}$$

admits no solutions for any positive integers $A, B, C, x, y,$ and z with $(x, y, z) > 2$ for any piecewise co-prime triple $A, B,$ and C.

In its recast form (2), it becomes clear that Beal's conjecture is a generalization of Fermat's Last Theorem where Fermat's Last Theorem is the special case of Beal's conjecture where $x = y = z = n$. In the parlance of mathematics, Beal's conjecture is a *corollary* to Fermat's Last Theorem.

The proof that we present demonstrates that the triple (A, B, C) can not be co-prime. This is the same method that we used in our "simple, and much more general Proof of Fermat's Last Theorem" [1]. Actually, the present proof is a generalization of the proof of Fermat's Last Theorem presented in [1].

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* G. G. Nyambuya
* The Beal Prize, AMS, `http://www.ams.org/profession/prizes-awards/ams-supported/beal-prize`

2. Lemma

If $(a > 1, b > 1; c > 1; n > 2) \in \mathbb{N}^{+}$ where $(b > c)$, then, the following will hold true always:

$$a^n = a(b+c) \qquad \text{or} \qquad a^n = a(b-c). \tag{3}$$

The above statement is clearly evident and needs no proof. What this statement really means is that the number a^n (for any $n > 2$ and $a > 1$), can always be written as a sum or difference of two numbers p and q where $p \in \mathbb{N}^{+}$ and $q \in \mathbb{N}^{+}$ are not co-prime, *i.e.*:

$$a^n = p + q \ \text{ or } \ a^n = p - q : \gcd(p,q) \neq 1, \tag{4}$$

since one can always find some (p, q) such that a will always be a common factor of (p, q). Equipped with this simple fact, we will demonstrate that as we did with Fermat's Last Theorem, that Beal's Conjecture yields to a proof in the simplest imaginable manner.

3. Proof

The proof that we are going to provide is a proof by contradiction and this proof makes use of Lemma §(2.) whereby we demonstrate that the triple (x, y, z) is such that it will always have a common factor if the equation, $A^x + B^y = C^z$, for any $[(x, y, z) > 2]$; is to hold true. We begin by assuming that the statement:

$$A^x + B^y = C^z, \ \text{ for any } \ [(x, y, z) > 2], \tag{5}$$

to be true for some piecewise co-prime triple $(A, B, C) \in \mathbb{N}^{+}$, the meaning of which is that the greatest common divisor of this triple or any arbitrary pair of the triple is unity (*i.e.*, $\gcd(A, B, C) = 1$).

First, we must realise that if just one of the members of the triple (A, B, C) is equal to unity for any $(x, y, z) > 2$, then, the other two members of this triple can not be integers, hence, from this it follows that if a solution exists, then, all the members of this triple will be greater than unity *i.e.* $(A > 1; B > 1; C) \in \mathbb{N}^{+}$.

Now, for our proof, by way of contradiction, we assert that there exists a set of positive integers $(x, y, z) > 2$ that satisfies the simple relation $A^x + B^y = C^z$ for some piecewise co-prime triple $(A, B, C) > 1$. Having made this assumption, if we can show that $\gcd(A, B, C) > 1$, then, by way of contradiction *Beal's Conjecture* holds true.

If the statement (5) holds true, then – clearly; there must exist some $(p, q) \in \mathbb{N}^{+}$ such that $\gcd(p, q) = 1$, such that A^x, B^y and C^z can be decomposed as follows:

$$\begin{pmatrix} A^x \\ B^y \\ C^z \end{pmatrix} = \begin{pmatrix} p - q \\ 2q \\ p + q \end{pmatrix}. \tag{6}$$

Now, according to the Lemma §(2.), the equation $C^z = p + q$ for any $(z > 2)$ and for any $(C > 1)$, this equation, can always be written such that $p = aC$ and $p = bC$ for some $(a > 1; b > 1) \in \mathbb{N}^{+}$ *i.e.* $C^z = (a+b)C$. Substituting $p = aC$ and $q = bC$ into (6), we will have:

$$\begin{pmatrix} A^x \\ B^y \\ C^z \end{pmatrix} = \begin{pmatrix} (a-b)C \\ 2bC \\ (a+b)C \end{pmatrix}. \tag{7}$$

From (7), it is clear that $\gcd(A^x, B^y, C^z) \neq 1$ since there exists a common divisor $[\mathrm{cd}()]$ of the triple (A^x, B^y, C^z) which is C, that is to say, C is a common divisor of the triple (A^x, B^y, C^z). If $\gcd(A^x, B^y, C^z) \neq 1$, consequently, $\gcd(A, B, C) \neq 1$ and this is in *complete violation of the critical, crucial and sacrosanct assumption that* $\gcd(A, B, C) = 1$.

Alternatively, according to the Lemma §(2.), the equation $A^x = p - q$ for any $(x > 2)$ and for any $(A > 1)$, this equation, can always be written such that $p = aA$ and $q = bA$ for some $(a > 1; b > 1) \in \mathbb{N}^{+}$ *i.e.* $A^n = (a+b)A$. Now, substituting $p = aA$ and $q = bA$ into (6), we will have:

$$\begin{pmatrix} A^x \\ B^y \\ C^z \end{pmatrix} = \begin{pmatrix} (a-b)A \\ 2bA \\ (a+b)A \end{pmatrix}. \tag{8}$$

Again, from (8), it is clear that $\gcd(A^x, B^y, C^z) \neq 1$ since the $\mathrm{cd}(A^x, B^y, C^z) = x$, that is to say, A is a common divisor of triple (A^x, B^y, C^z). From the foregoing, it follows that (A, C) are common divisors of the triple (A^x, B^y, C^z), the meaning of which is that $\gcd(A, B, C) \neq 1$. Therefore, by way of contradiction, *Beal's Conjecture* is true since we arrive at a contradictory result that $\gcd(A, B, C) \neq 1$. What this effectively means is that the equation $A^x + B^y = C^z$ for $(x, y, z) > 2$ may have a solution and this solution is such that the triple (A, B, C) always has a common factor as is the case with all those values of A, B, C that satisfy *Beal's Conjecture*.

4. Discussion and Conclusion

Just as the proof presented in the reading [1], the proof here provided is simple and general. It applies elementary methods of arithmetic that where available even in the days of Fermat. At this point, if anything, we only await the judgement of the world of mathematics as to whether this proof is correct or not. Without any oversight on our confidence in our proof, allow us to say that, until such a time that evidence to the contrary is brought forth, we are at any rate, convinced of the correctness of the proof here presented.

Conclusion

We hereby make the following conclusion that if our proof is correct as we strongly believe, then, *Beal's Conjecture* seizes to be a conjecture but forthwith transforms into a fully-fledged theorem as a – logically and mathematically correct and legitimate; proof has now been supplied.

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