The Theory of the (E) Question
An alternative scenario for the supra-conduction in graphene v3
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Content
A heuristic scenario explaining the supraconduction ................................................................. 2
Initial considerations .................................................................................................................. 2
The mathematical context ....................................................................................................... 2
The physical context ............................................................................................................... 3
Projecting the Lorentz Einstein Law into the world of differential operators ....................... 3
Remark: trivial solution .......................................................................................................... 4
Perian matrices: definition and examples ................................................................................ 4
Perian matrices: multiplicative properties ............................................................................... 5
Perian matrices: transposed of a Lorentz transformation ...................................................... 5
Perian matrices: inverse element of a Lorentz transformation – trivial case ....................... 5
The proposed scenario .......................................................................................................... 6
Sketching rapidly the actual situation ..................................................................................... 6
The ideal tool we are looking for ............................................................................................ 6
A procedure – first try ............................................................................................................. 7
Justification: an heuristic interpretation .................................................................................. 8
Continuing the procedure ....................................................................................................... 8
An example ............................................................................................................................. 9
Another example .................................................................................................................. 9
Sketching the link with the Ginzburg-Landau theory ............................................................. 9
Domain of validity .................................................................................................................. 10
Limitations relatively to the magnetic field ............................................................................. 10
Limitations related to the relation of coherence .................................................................... 10
Trying to understand the underlying phenomenology .......................................................... 10
Parallel histories .................................................................................................................... 10
A crucial question .................................................................................................................. 11
Mathematics .......................................................................................................................... 11
Basic stones ......................................................................................................................... 11
Definition 01: Perian matrix .................................................................................................. 11
Sub-definition 01.1: positive mono-vector Perian matrix ...................................................... 11
Sub-definition 01.2: negative mono-vector Perian matrix .................................................... 11
Remark 01 ............................................................................................................................. 11
Remark 02 ............................................................................................................................. 11
Definition 02: Dilatation functor ............................................................................................. 11
A heuristic scenario explaining the supraconduction

Initial considerations

The mathematical context
Within a quasi-systematic exploration devoted to the concept of extended, eventually exterior, product, the following mathematical question has been investigated \([F\text{-}01]\): "Is it meaningful to believe that a Lorentz transformation could be in coincidence with some ad hoc trivial divisor; in extenso:

\[
[\omega] = [\Lambda] - \text{Id}_4 = \chi(\Phi(u))?
\]

Readers, even if they were eventually interested by the investigation, were also certainly asking for its utility. My slow progression is actually reaching a point where this previous investigation could be useful in that sense that it suggests a fascinating link between the relation defining a gauge (or, why not, a connection) and one special feature of one of the two fundamental equations of the Ginzburg-Landau theory. Let me explain.
The physical context
When considering supra-conductors, we roughly may imagine objects with two parts: (a) a semi-elastic and stable skeleton (a central structure); (b) free electrons "travelling" around the central structure. For facility, I shall develop that exposé with the image of a graphène lattice in head. One important characteristic for the demonstration I want to expose here is the fact that superconducting electrons in graphène seems to behave as massless particles [F-02]. I shall immediately explain why it is an advantageous property.

Figure n°1: artistic representation of the central structure of graphene.

Hall’s effects, eventually quantized, are occurring in supra-conducting devices. Although the whole piece of matter which is lying for us in a laboratory has seemingly nothing to with an object concerned by the theory of relativity, free electrons in graphène are rapid enough [F-02] to suggest the following interrogation: “Would it not be reasonable to introduce the covariant version of the Lorentz law into a discussion concerning such devices?” Let me answer to the question with “yes”, per hypothesis and, up to now, work with the so-called Lorentz-Einstein law (LEL) [D-02; chapter 20, p. 106, (20.4)]. As a matter of facts, in doing so, I automatically introduce the concept of extended products into my discussion. The prototype of these products is the so-call gravitational term \( \Gamma^\gamma_{\alpha\beta} \).

Projecting the Lorentz Einstein Law into the world of differential operators
That item has also already been investigated in [F-03]. The only result of which I shall make use here is the fact that self-adjoint operators representations of the LEL exist. They spontaneously promote the mixed (up, down) tensor representation of the EM field, \( F \), as an important actor of the play because, in general, it transforms like in a connection. But the main consequence of that previous investigation will be the fact that that connection reduces to a simple gauge relation for massless particles (For the nomenclature, please see the reference [F-03]).

This will allow me to work with it here, considering that, in average, free electrons of the graphene are indeed massless [F-02]! Although the approach developed in [F-03] is not giving precise indication on the matrices \( [\Lambda] \), the previous gauge relation is a relatively clear reminiscence of the usual relation [F-04; p.44, (5.70)]:

\[
F^{\mu\nu}(x) = \Lambda^\nu_\alpha . \Lambda^\mu_\beta . F^{\alpha\beta}(x) \quad \text{which is a direct consequence of the tensor calculus. The latter may be mixed with the other one [F-04; p. 34, (4.64)] insuring the preservation of the length element: } [\Lambda]^1 . [G]. [\Lambda] = [G] \text{ or, equivalently } [G]. [\Lambda]. [\Lambda]^1 = ([\Lambda]^1)^2 . \text{Since the tensor calculus implies } F = F^{\mu\nu} = g^{\mu\nu}. F^{\alpha\nu} \text{ [D-02; chapter 14, p. 77, (14.29)]}, \text{ it follows } g_{\mu\nu}. F^{\mu\nu}(x) = g_{\mu\nu}. \Lambda^\nu_\alpha . \Lambda^\mu_\beta . F^{\alpha\beta}(x) = g_{\mu\nu}. \Lambda^\nu_\alpha . \Lambda^\mu_\beta . g^{\alpha\beta} . F^{\alpha\beta}(x) . \text{ This is nothing but } jF^\mu_\nu = \Lambda^\mu_\nu . jF^\nu_\delta . g_{\mu\nu}. \Lambda^\delta_\beta . g^{\beta\delta} . \text{ We recognize } jF = [\Lambda] . jF . ([G]. [\Lambda]. [G]^1)^1 = [\Lambda] . jF . ([\Lambda]^1)^2 . \text{ This is suggesting a possible identification with the gauge discovered in [F-03]. That gauge is always realized if, simultaneously:}
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An alternative scenario for the supra-conduction in graphene v3

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\[ [\mathcal{P}] = [\Lambda] \text{ and } [\mathcal{P}]^{-1} = ([\Lambda]^\dagger)^{-1}. \]

Mixing the two relations, that special approach imposes:

\[ [\Lambda]^{-1} = ([\Lambda]^\dagger)^{-1}. \]

“The transposed of the inverse should be equal to the inverse of the transposed”.

Does that condition reduce the generality?

Remark: trivial solution
That relation of coherence (04) has at least one trivial solution; namely:

\[ [\Lambda]^t = [\Lambda]^3. \]

Perian matrices: definition and examples
At this stage, we must absolutely make a mathematical remark, the importance of which will appear crucial soon. Let us consider the matrix representation of a generic Lorentz transformation and the one of some EM field (in fact its 2-0 or 0-2 tensor representation). In both cases, the formalism is exactly the same. We face what I have called (with a little bit pretention) the "Perian" matrices. In fact, as soon as D > 1, any (D-D) matrix is such a Perian matrix. The adjective "Perian" only corresponds to a mental decomposition of the (D-D) matrix.

That decomposition acts in splitting the matrix in different parts: (a) the North-West part is a scalar; (b) the row in the North and the column in the West are two, eventually different, (D-1) vectors and (c) the (D-1 D-1) South East matrix.

\[ \begin{bmatrix} \frac{\delta P}{\delta k} & <(D-1)k> \\ \frac{\delta P}{\delta q} & >(D-1)K \end{bmatrix} = [K]^{-1}. \]

Now let me observe attentively a Lorentz transformation matrix closed to the identity (in these cases \([\Lambda] = \text{Id}_4 + [\omega] [F-04; p. 34, (4.65)]\): it “contains” a Perian matrix corresponding to the following mental decomposition (scalar = 0, vector = boost and sub-matrix = rotation). That matrix summarizes what is also called “torsion”.

\[ \begin{bmatrix} 0 & <(3)\lambda.m> \\ \lambda.m & >(3)[\Phi(J = 0, n)] \end{bmatrix} = [\omega] = [\Lambda] - \text{Id}_4 \]

Let me also observe attentively a (2, 0) or a (0, 2) representation of some EM field: it is a perian matrix corresponding, perhaps up to a minus sign somewhere, to the following mental decomposition (scalar = 0, vector = electric field, sub-matrix = trivial divisor involving the magnetic field and corresponding itself to some rotation).

\[ \begin{bmatrix} 0 & <(3)E> \\ -E & >(3)[\epsilon \Phi(H)] \end{bmatrix} = [F(2, 0)] \]
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An alternative scenario for the supra-conduction in graphene v3
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Short said: for a mathematician, both matrices (the \([\omega]\) matrix and the representation of some EM field) could be (understood as being) elements of the same set. But it is important to notice that if (08) is defining a totally anti-symmetric matrix, the relation (07) does in general not; one exception occurs for \(m^{(3)} = 0\).

Perian matrices: multiplicative properties
Let me write (nota bene: although the following relation has been written for covariant components, the formalism holds true for contra-variant components too): (09)

\[
\begin{bmatrix}
  k & <x| \\
  |x> & [M]
\end{bmatrix}
\begin{bmatrix}
  k' & <x'| \\
  |x'> & [M']
\end{bmatrix}
= \begin{bmatrix}
  k & x_1 & x_2 & x_3 \\
  x_1 & m_{11} & m_{12} & m_{13} \\
  x_2 & m_{21} & m_{22} & m_{23} \\
  x_3 & m_{31} & m_{32} & m_{33}
\end{bmatrix}
\begin{bmatrix}
  k' & x'_1 & x'_2 & x'_3 \\
  x'_1 & m'_{11} & m'_{12} & m'_{13} \\
  x'_2 & m'_{21} & m'_{22} & m'_{23} \\
  x'_3 & m'_{31} & m'_{32} & m'_{33}
\end{bmatrix}
= \begin{bmatrix}
  k.k' + \sum_{i} x_i x'_i & \ldots & k.x'_1 + \sum_{i} x_i m'_{1i} \\
  x_1.k' + \sum_{i} m_{1i}x'_i & \ldots & x_1.x'_1 + \sum_{i} m_{1i} m'_{1i} \\
  x_2.k' + \sum_{i} m_{2i}x'_i & \ldots & x_2.x'_1 + \sum_{i} m_{2i} m'_{2i} \\
  x_3.k' + \sum_{i} m_{3i}x'_i & \ldots & x_3.x'_1 + \sum_{i} m_{3i} m'_{3i}
\end{bmatrix}
\]

Perian matrices: transposed of a Lorentz transformation
As explained previously some important “details” must be verified. The transposed matrix of \([\Lambda]\) is very easy to discover:

(10)

\[
[\Lambda]^t = \begin{bmatrix}
  1 & <^{(3)} \lambda . m > \\
  |^{(3)} \lambda . m > & \text{Id}_3 - <^{(3)} \Phi (J = 0 . n)>
\end{bmatrix}
\]

Perian matrices: inverse element of a Lorentz transformation – trivial case
Can the inverse element really be identified with the transposed \([\Lambda]^t = [\Lambda]^{-1}\)? If it would be possible, then we would be obliged to write:

(11)

\[
\begin{bmatrix}
  1 & <^{(3)} \lambda . m > \\
  |^{(3)} \lambda . m > & \text{Id}_3 + <^{(3)} \Phi (J = 0 . n)>
\end{bmatrix}
\begin{bmatrix}
  1 & <^{(3)} \lambda . m > \\
  |^{(3)} \lambda . m > & \text{Id}_3 - <^{(3)} \Phi (J = 0 . n)>
\end{bmatrix}
= \begin{bmatrix}
  1 & <^{(3)} \lambda . m > \\
  |^{(3)} \lambda . m > & \text{Id}_3 + <^{(3)} \Phi (J = 0 . n)>
\end{bmatrix}
\begin{bmatrix}
  1 & <^{(3)} \lambda . m > \\
  |^{(3)} \lambda . m > & \text{Id}_3 - <^{(3)} \Phi (J = 0 . n)>
\end{bmatrix}
\]
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An alternative scenario for the supra-conduction in graphene v3
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\[
\begin{bmatrix}
1 + \lambda^2 \cdot m^2 \\
2 \cdot I_3 + \begin{bmatrix}
(3) \lambda \cdot m \\
(3) \lambda \cdot m \\
(3) \lambda \cdot m 
\end{bmatrix} \\
\end{bmatrix} < \begin{bmatrix}
(3) \lambda \cdot m \\
(3) \lambda \cdot m \\
(3) \lambda \cdot m 
\end{bmatrix} \cdot \begin{bmatrix}
2 \cdot I_3 - (3) [\Phi(J = 0 \cdot n)] \\
\end{bmatrix}
\]

This is absolutely imposing a unique and seemingly meaningless configuration characterized with: \(m^2 = 0\) and, consequently, with \(n^2 = 0\) too. The theory can only contain that eventuality if it is developed inside a context incorporating complex numbers and, more precisely, what E. Cartan calls “spinors” in a 3 dimensional world. (Exploration to be continued...)

The proposed scenario

Sketching rapidly the actual situation
The supra-conduction has been discovered, explored and theorized for a long time now. Despite of the fantastic progresses that have been accomplished in its understanding, some corners remain unclear and the place for very active prospections, both: experimental in [01], [02], and theoretical, e.g.: [03] [F-02], [D-04].

This activity is based on diverse motivations: first, the properties of some recent materials are better known and the first applications appear [e.g.: (a) in radioprotection; (b) in devices making use of Josephson’s junctions [D-05; and so on]; second: the theories (London, Ginzburg-Landau (GL), GLAG, BCS ...) don’t yet have an explanation for all what we experiment or observe [non trivial pairings, high temperature supra-conducting devices...].

The ideal tool we are looking for
This is why it is reasonable to look for an alternative scenario explaining the supra-conduction. That new scenario should ideally contain all previous theories and be compatible with our accepted visions concerning waves and particles at a subatomic scale.

Looking for that “miraculous” scenario it is worth to notice the progressive introduction of a crucial concept; namely: conducting electrons (also called free electrons in the literature) don’t walk without interacting and alone. They interact with the semi-stable and semi-elastic central structure via phonons (the oldest theories) and they are not isolated in vacuums contained inside that structure because they form pairs with others electrons (the Cooper’s electrons).

We may add to that idealized picture new knowledge inherited from the developments of the quantum field theories. For some of them, vacuums never are synonym of empty regions without any activity. On the contrary, vacuums are places for an actually not exactly quantized activity characterized by a permanent instability. Although we don’t know all details concerning this foam, let us just remember one main idea: anything that would be moving inside that foam would also a priori interact with the particles or quasi-particles contained in it. Let us suppose that the conducting electrons interact permanently with photons (real or virtual) and that the today accepted pairing between two such electrons occur via these photons.

Our purpose is to propose a convenient and pertinent tool allowing the correct description of these interactions. That tool must be general enough to allow further applications to any kind of particles.
In fact this should be a generic procedure, easy to promote in any situation. Let me start with a simplified schema:

![Figurative representation of the trajectory of the photon](image)

\[ e' \]

\[ \bullet \]: the presumable place and time of the future interaction

\[ \nabla A \]: local value of the fundamental cube at the presumable place and time of the future interaction.

\[ \nu \]

Figure n°2: the situation before the interaction.

The rose arrows are figuring the speeds: \( u(e') \) and \( u(\nu) \) before the interaction.

Let me try a description of what happens to the electron.

A procedure – first try

With “what happens to the electron” I mean: “How does its EM field change because of an interaction with, e. g.: a photon. At this stage my specific treatment/scenario consists in the following procedure:

1°) keep in mind that both, the electron and the photon have the same equation of motion (EOM) type; precisely: the Lorentz-Einstein law (LEL) or its so-called simplified version.

2°) remember that:

(a) The main extrinsic divisor of that EOM is the mixed (up, down) tensor representation of the EM field; let us write it \( F(\varepsilon(e')) \) just before the chock.

(b) By approximation (we shall precise which one later), that representation may be identified with \( \Phi(\varepsilon(e')) \) where \( [A] \) denotes the reduced version (in fact a matrix) of the “reduced cube” at the place and the instant of the interaction.

(c) Local “Cartan-Killing” geometric conditions are determining the effective possible values of the \( [A] \). These conditions, of course, are unique for a given place at a given instant. So that they are obligatorily the same for the electron and the photon during the interaction.

(d) The representation of the EOM under “self-adjoint” differential operator formalism yields an interesting result: the mixed tensor representation of the EM field transforms like in a connection.

(e) Exceptionally, that connection reduces to a gauge for massless particles.

(f) Superconducting electrons in graphene are massless.

3°) think about the facts that:

(a) A Lorentz transformation matrix is a convenient representation of the \([P]\) matrix intervening in the differential operator formalism of the EOM, especially when the formula signing the existence of a connection is reduced to the one signing a gauge: mass = 0 \( \rightarrow [P] \sim [A] \) \( (03.a) \).

(b) A Lorentz transformation matrix can reasonably be identified with a trivial divisor for some extended exterior product (EEP); let us believe that it is the same EEP than the one in which the electron is involved in:
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(12)
\[ [\Lambda] - \text{Id}_4 \sim [\alpha] \Phi(1, u(|v|)). \]

4°) Suppose that we face the gauge relation \((02)\) when the interaction between the photon and the electron “starts”:

\[ [\Lambda]^t. F(|\alpha|, e). [\Lambda] = F(|\alpha|, e) \]

Justification: an heuristic interpretation

Let me now consider my “heuristic” interpretation relatively to the concept of supra-conduction. Let me take first a comparison. Please consider a set of persons (the central structure) and, perhaps at some distance of it, a lonely and unique man/woman (a free electron). Because the latter is what he/she his and because the group of persons is what it is, there are mutual interferences. And the influences are determining both, the evolution of the group and the evolution of the isolated person. I would describe this interaction in saying that the group and the person are permanently gauging their mutual influences and that that permanent re-estimation of the situation determinates the future for the group or/and for the lonely person.

But how can I measure my influence on the group? This is exactly the place where the previous mathematical remark comes into the play. Even if I don’t really want to influence the group, I do it anyway because I potentially represent a perturbation for it; that is I am a kind of Lorentz transformation matrix: \([\Lambda]\). The group itself, even if it doesn’t really care of me is potentially more or less polarized; it is a kind of EM field with its tensor representation: \(F\).

Now, I am estimating my influence in calculating: \([\Lambda]^t. F. [\Lambda]\) and this calculation is supposed to yield the next (energetic) state for the group after having been interacting with my modest person. At this stage, two eventualities can occur: (a) either the group has had no influence on me and conversely - its energetic state stays unchanged; (b) or we did interact and its energetic state (or mine) is modified. Until now: there is nothing really revolutionary in that exposé.

Continuing the procedure

5°) Confront the diverse recommendations made previously, precisely \((03-a)\) and \((12)\), and get:

\[(13)\]
\[ F \sim [\Lambda] - \text{Id}_4 \sim \alpha \Phi(u). \]

Inject all this into \((02)\); this is implying:

\[(14)\]
\[ (\text{Id}_4 + [\alpha] \Phi(1, u(|v|)))^t. [\alpha] \Phi(1, u(|e|)) \cdot (\text{Id}_4 + [\alpha] \Phi(1, u(|e|))) = [\alpha] \Phi(1, u(|e|)). \]

6°) Suppose that:

\[(14)\]
\[ |u(e)\rangle = 2 |u(e)\rangle + \delta |u(e)\rangle \]

And:

\[(15)\]
\[ |u(e)\rangle - |u(v)\rangle \]
This is yielding:
\[(16)\]
\[\mathcal{A} \mathcal{F} = \mathcal{A}^3 \mathcal{F}. \mathcal{A} \]
\[\downarrow \]
\[(\mathcal{A} \mathcal{F}(\mathcal{u}(v))) + \mathcal{A} \mathcal{F}(\mathcal{u}(v)). (\mathcal{A} \mathcal{F}(\mathcal{u}(v))) = \mathcal{A} \mathcal{F}(\mathcal{u}(v)) \]
\[\downarrow \]
\[\mathcal{A} \mathcal{F}(\mathcal{u}(v)) = \mathcal{A} \mathcal{F}(\mathcal{u}(v))\]
\[\mathcal{A} \mathcal{F}(\mathcal{u}(v)) + \mathcal{A} \mathcal{F}(\mathcal{u}(v)). (\mathcal{A} \mathcal{F}(\mathcal{u}(v))) = \mathcal{A} \mathcal{F}(\mathcal{u}(v)) \]
\[\mathcal{A} \mathcal{F}(\mathcal{u}(v)) = \mathcal{A} \mathcal{F}(\mathcal{u}(v))\]

An example
For the special case where the trivial divisor is totally anti-symmetric, terms of degree 2 in \((16)\) vanish and I get with \((15)\):
\[(16a)\]
\[\mathcal{A} \mathcal{F}(\mathcal{u}(v)) = \mathcal{A} \mathcal{F}(\mathcal{u}(v))\]

On the other side, because of \((14)\):
\[(17)\]
\[\mathcal{A} \mathcal{F}(\mathcal{u}(v)) = \mathcal{A} \mathcal{F}(\mathcal{u}(v)) + \mathcal{A} \mathcal{F}(\mathcal{u}(v)) = 2. \mathcal{A} \mathcal{F}(\mathcal{u}(v)) + \mathcal{A} \mathcal{F}(\mathcal{u}(v))\]

The confrontation between \((16a)\) and \((17)\) for that special case yields:
\[(18a)\]
\[\Phi - \Phi^3 = 2. \Phi + \mathcal{A} \mathcal{F}(\mathcal{u}(v)) \Leftrightarrow \Phi^3 + \mathcal{A} \mathcal{F}(\mathcal{u}(v)) = 0\]

Another example
For the special case where the trivial divisor is totally symmetric:
\[(16b)\]
\[\Phi + 2. \Phi^2 + \Phi^3 = \mathcal{A} \mathcal{F}(\mathcal{u}(v))\]

On the other side, because of \((14)\):
\[(17)\]
\[\mathcal{A} \mathcal{F}(\mathcal{u}(v)) = \mathcal{A} \mathcal{F}(\mathcal{u}(v)) + \mathcal{A} \mathcal{F}(\mathcal{u}(v)) = 2. \mathcal{A} \mathcal{F}(\mathcal{u}(v)) + \mathcal{A} \mathcal{F}(\mathcal{u}(v))\]

The confrontation between \((16b)\) and \((17)\) for that special case yields:
\[(18b)\]
\[\Phi^3 + 2. \Phi^2 + \Phi^3 - \mathcal{A} \mathcal{F}(\mathcal{u}(v)) = 0\]

Sketching a probable link with the Ginzburg-Landau theory
The relation \((18a)\) is amazingly mimicking one of the equations appearing in the Ginzburg-Landau (GL) theory; the GL equation I am thinking about is namely:
\[(19)\]
\[\psi^3 - \psi - \frac{\partial^2}{\partial x^2} \psi = 0\]

Indeed, if I introduce here a new variable sketching the wave vector with the help of a trivial kind of linear dilatation function:
\[(20)\]
\[\psi = \pm i. \mathcal{A} \mathcal{F}(\mathcal{u}(v)) \text{ with } i^2 + 1 = 0,\]
it is not difficult to transform (19) into:
(21)
\[-i^3 \Lambda \Phi_1(u) + i \Lambda \Phi(1_u) + i \frac{\partial^2}{\partial^2 x} \Lambda \Phi(1_u) = 0\]
\[\Lambda \Phi_1(u) + \Lambda \Phi(1_u) + \frac{\partial^2}{\partial^2 x} \Lambda \Phi(1_u) = 0.\]

In which I may recognize the relation (18-a), provided the following relation holds true:
(22)
\[\Lambda \Phi(\delta u) \sim \frac{\partial^2}{\partial^2 x} \Lambda \Phi(u).\]

When it is the case, the relation (18-a) can be seen as a representation of (19) which is characterizing a supra-conducting semi-space without magnetic field within the GLAG theory.

**Domain of validity**

**Limitations related to the magnetic field**
The realization of (12), because of the anti-symmetry of the EM field, is limiting the domain of validity to the following matrices:
(23)
\[F \sim \begin{bmatrix} 0 & 0 \\ 0 & \varepsilon(\Phi = 0, n) \end{bmatrix}.\]

The heuristic proposition has consequently also a limited validity; but this is also the case for the relation (19) which is only a peculiar limit case for one of the two GLAG equations.

**Limitations related to the relation of coherence**
The condition (22) is the second important actor in that scenario. A correct interpretation can only be found within a discussion taking precisely care of the phenomenology implicitly contained in (07). This being said, it is of course crucial to verify if the proposed scenario is coherent with that implicit phenomenology.

**Trying to understand the underlying phenomenology**

**Parallel histories**
That heuristic scenario is in need of a deeper analysis. The underlying approach may be seen as the building of a kind of lattice. Why? Which one? Let us consider the following set of histories.

1°) the history of a given particle with a speed which is changing along its proper time:
(H-1)
\[0_u \rightarrow 1_u \rightarrow \ldots \rightarrow i_u \rightarrow \ldots \rightarrow f_u \rightarrow \ldots \rightarrow \infty_u\]

2°) if that particle is supposed to respect the Lorentz Einstein law (LEL), we can seriously envisage the trivial “dilatation” history of the speed of that particle which, in that case, only is (approximately) the history of the local representation of the EM field in its (1, 1) version:
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An alternative scenario for the supra-conduction in graphene v3

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\[ (H-2) \]
\[ _0F \sim \phi(0u) \rightarrow _2F \sim \phi(0u) \rightarrow \ldots _iF \sim \phi(0u) \rightarrow \ldots \rightarrow \infty F \sim \phi(\infty u) \]

\(3^*\) the allowed Lorentz transformations history which is the set of the transformations connecting the different speeds and, simultaneously, the different representation of the EM field:

\[ (H-3) \]
\[ [0 \rightarrow 1 \Lambda] \rightarrow [1 \rightarrow 2 \Lambda] \rightarrow \ldots \rightarrow [i \rightarrow i+1 \Lambda] \rightarrow \ldots \rightarrow [j \rightarrow j+1 \Lambda] \rightarrow \ldots \]

A crucial question

All this is a very acceptable presentation of the reality. But I have to justify the strange hypothesis (13) which is directly comparing an element of the Lie algebra of the Lorentz group and an EM field. Why should it be true? This way of thinking which is choking at our human scale is in fact acceptable at a subatomic scale where nothing more is clearly separating the particle emitting or receiving a signal from the signal itself. Let us figure it with something like that:

\[ \text{Quark}_0u \rightarrow \text{Gluon}_0 \rightarrow \text{Quark}_1u \]

Figure n°3: artistic representation of the evolution of a quark.

Mathematics

Basic stones

Definition 01: Perian matrix
A perian matrix is, per convention, any matrix in any integer D-dimensional space (\(D > 1\)) interpreted in the way symbolically contained in (06).

Sub-definition 01.1: positive mono-vector Perian matrix
A mono-vector Perian matrix is a matrix interpreted in the way symbolically contained in (06) when \(k = p\).

Sub-definition 01.2: negative mono-vector Perian matrix
A mono-vector Perian matrix is a matrix interpreted in the way symbolically contained in (06) when \(k = -p\).

Remark 01
Any Lorentz transformation matrix represented with (03) is a negative mono-vector Perian matrix.

Remark 02
Any (2-0) or (0-2) tensor representation (04) of the EM field is a positive mono-vector Perian matrix.

Definition 02: Dilatation functor
In general, a so-called “dilatation functor” is a mathematical tool giving a representation of some mathematical object represented by a D-dimensional writing into a set where the same object can/must be represented with the help of a (D + 1)-dimensional writing.

Example 01: dual representation of a vector
Any D-dimensional vector \(^{(D)}u\) which is an element of a vector space \(E_0(K)\) can be represented in the dual space \(E^*_0(K) \cong K^D\) just in writing it, e.g.: \(^{(D)}u^* = [u^0, u^1, \ldots, u^{D-1}]\), which is the row of the components of \((D)u\). In my head, a dilatation functor of order one is any function connecting \((D)u\), a
“physically” zero-dimensional mathematical, with one of its possible “physically” one dimensional representation, for example: $|^{(D)}u\rangle$. The row representation and the column representation of a given vector in the dual space are two concrete examples of that idea.

**Definition 03: Pythagorean table**

Let us consider two vector spaces $E_p(K)$ and $E_q(K)$ with $p$ and $q$ integer numbers strictly greater than one ($p > 1; q > 1$). Let us consider the set of all applications connecting an element of $E_p(K)$ with an element of $E_q(K)$, say $\mathcal{F}(E_p(K), E_q(K))$, and $f$ one element of that set. And let us consider any element $^{(p)}u$ in $E_p(K)$ and any element $^{(q)}w$ in $E_q(K)$. Per definition, a Pythagorean table acting on the ordered pair $^{(p)}u, ^{(q)}w$ is any $(p-q)$ matrix:

$$T_2(f)(^{(p)}u, ^{(q)}w) = [...] f(u^a, w^b) [...] \in M(K; p \times q)$$

**Example 02: Pythagorean table acting on a given vector**

Any Pythagorean table acting on a given vector is de facto defining a dilatation functor of order two “enlarging” a “physically” zero dimensional object, namely a vector, in a “physically” two dimensional object, namely the matrix $T_2(f)(^{(0)}u, ^{(0)}u) = [...] f(u^a, u^a) [...] \in M(K; D \times D) = M_D(K)$.

**Example 03: Trivial divisor**

The application $\Phi_\Box: (^{(D)}u) \rightarrow \Phi_\Box(^{(D)}u)$ where $\Box A$ is any cube of ordered $D^3$ elements arbitrarily chosen in $K$ can be understood as representing a kind of dilatation functor of order two too. It is defining a correspondence between a (physically) zero dimensional object, the vector $^{(D)}u$, and a (physically) two dimensional object, the matrix $\Phi_\Box(^{(D)}u)$. The latter is nothing but the trivial divisor of the generic extended product $\triangle_{\Box A}(^{(D)}u, ...)$. 

**Definition 04: Anti-symmetric cube**

If the components of a cube $\Box A$ are such that $A_{\alpha\beta\chi} + A_{\beta\alpha\chi} = 0$, the cube is said to be “anti-symmetric”. An extended product built on an anti-symmetric cube is called an extended exterior product.

**Definition 05: special cube**

If the components of a cube $\Box A$ are such that $A_{\alpha\beta\chi} + A_{\alpha\chi\beta} = 0$, the cube is said to be “special”.

**Remark 03**

Any extended product built on a special cube is characterized by an anti-symmetric trivial divisor. Demonstration: $A_{\alpha\beta\chi} + A_{\alpha\chi\beta} = 0, \forall (^{(0)}u) \Rightarrow (A_{\alpha\beta\chi} + A_{\alpha\chi\beta}) \cdot u^\alpha = 0 \Rightarrow A_{\alpha\beta\chi} \cdot u^\alpha + A_{\alpha\chi\beta} \cdot u^\beta = 0 \Rightarrow \Phi_\Box(^{(D)}u) + \Phi_\Box(^{(D)}u) = [0]_\Box$.

**Definition 06: reduced cube**

To be an anti-symmetric cube or to be a special cube are two different and disjoint properties. But they can be mixed. In that case, the cube is said to be “reduced”. The adjective meaning in fact that such cube has only a reduced number of independent components. An extended product built with the help of a special cube is not necessarily an extended exterior product. An extended product built with the help of an anti-symmetric cube is per definition an extended exterior product. An extended product built with the help of a reduced cube is consequently an extended exterior product too.

**Remark 04**

Any negative mono-vector Perian matrix can be seen as the representation of some trivial divisor for an extended product built with the help of a reduced cube.
Remark 05
In any 3-dimensional space, a reduced cube has 3 components and could be compared to a 3-dimensional vector; in any 4-dimensional space, a reduced cube has only 6 components and could, because of that, be compared with a bi-vector or with a negative mono-vector Perian matrix.

Theorem (surjection of the trivial dilatation function)
Any negative mono-vector Perian matrix being considered as a trivial divisor (remark 04) has a source with a precise formalism in the vector space of the discussion.

Corollary
Any negative mono-vector Perian matrix induces a Cartan-Killing like metric based on a reduced cube.

These are only a few mathematical evidences building the basic set of the tools that we shall need to go further.

Provisory conclusion
The document presented here is an attempt to promote a new understanding of what the supra-conduction is. It only sketches a first and original path between the traditional construction of the GLAG equations (involving the free energy of the EM field in vacuum) and my vision involving exterior extended products.

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