In this preliminary work, we propose a hypothesis and initiate a step-by-step, systematic upgrade to the cutting-edge iso-electronium model by further equipping it with order parameters of fractional statistics to encode the topological deformations, spontaneous superfluidic gauge symmetry breaking, correlated helices with long range order, and wavepacket wavefunctions for the toroidal polarizations. For this initial case, we consider the singlet planar coupling of two hydrogen atoms that are interlocked with a Santilli-Shillady strong valence bond to form a molecule with iso-electronium. The enhancement results support our hypothesis and are significant because the order parameters arm the iso-electronium model an extra degree of freedom to work with, which may authorize us to further decode and comprehend the underlying physical mechanisms and features associated with the configuration of the toroidal polarizations. Thus, these outcomes should be subjected to additional rigorous scrutiny and improvement via the scientific method.

**Keywords:** Geometry and topology; Topological deformations; Spontaneous gauge symmetry breaking; Superfluids; Iso-electronium; Hadronic mechanics.
1 Introduction

As the cutting-edge discipline of iso-mathematics [1, 2, 3, 4, 5] continues to develop and expand, it relentlessly paves the way for state-of-the-art innovations in science, technology, and engineering. Recently, R.M. Santilli deployed iso-mathematics to establish the new discipline of hadronic mechanics [6, 7]. Interestingly, one facet of the hadronic interlocking “gear” model [6, 7] predicts the existence of a new iso-chemical model of hydrogen creatures that are characterized by a short-range bond between two valence electrons into a singlet quasi-particle state called iso-electronium [8, 9, 10, 11, 12, 13].

In this preparatory paper, we wield the topological deformation order parameters from the Inopin-Schmidt baryon topology [14] to spark a hypothetical upgrade to the restricted “base case” of iso-electronium [8, 9, 10, 11, 12, 13], where two hydrogen atoms interlocked in the singlet planar coupling of the hadronic gear model with the (short-range) Santilli-Shillady strong covalent bond. More precisely, we propose and investigate the following hypothesis: Santilli’s molecular model of iso-electronium may be upgraded with topological deformation order parameters of fractional statistics to encode spontaneous superfluidic gauge symmetry breaking, correlated helices with long range order, and wavepacket wavefunctions for the toroidal polarizations. So, in Section 2, we launch a step-by-step thought experiment to systematically probe, illustrate, quantify, and formalize some fundamental features of this iso-electronium upgrade conjecture. Finally, we conclude with Section 3, where we briefly recapitulate the obtained results, consider some possible implications, and discuss future modes of research.

2 Iso-electronium upgrade procedure and thought experiment

Here, we propose the following step-by-step procedure which aims to investigate, illustrate, quantify, and formalize the fundamental aspects of the said hypothesis:

1. First, let there exist a Euclidean complex/2D topological space \( X \), where a complex number \( x \in X \) identifies a distinct position-point in \( X \), such that \(|x| \in [0, \infty)\) is the complex amplitude-radius (or “modulus”) and \( \langle x \rangle \in [0, 2\pi] \) is the complex azimuthal-phase for the “synchronized, generalized complex-coordinate-vector-number system” [14], which was subsequently upgraded, refined, and clarified in [15, 16]. Following [14, 15, 16], let the position-point \( O \in X \) be the origin-position-point of \( X \) with the zero-amplitude of \(|O| = 0\) for the 2D polar coordinate-vector \((0, 0)\) with the corresponding 2D Cartesian coordinate-vector \((0, 0)\) \( \rho \) with the corresponding 2D Cartesian coordinate-vector \((0, 0)\) \( C \), where \( O = 0 + 0i \) is the origin’s complex number—see Figure 1. Moreover, we briefly note that \( X \) is a “complex slice” of the triplex/3D topological space \( Y \) as in [15], such that \( X \subset Y \), but for the scope of this paper we need only to focus on \( X \).

2. Second, suppose there are two hydrogen atoms located on \( X \) with centers-of-mass that are equidistant from \( O \), where \( X \subset Y \) represents the topological space of an environment filled only with vacuum energy. In this step, we will construct the Inopin Holographic Confinement Ring (IHCR) based topology for the dual atomic nuclei with single protons [14]. For now, we assert that the hydrogen atoms are in
Fig. 1: The Euclidean complex/2D topological space $X$ equipped with the synchronized, generalized complex-coordinate-vector system [14], where $X \subset Y$ is a “complex slice” of the triplex/3D topological space $Y$ as in [15].
fact protium atoms [17] in a non-Riemannian-specific IHCR-based topology [14, 15, 16] because we wish to illustrate the simplest base case of a nuclei with one proton. Thus, let \( O_1 \in X \) and \( O_2 \in X \) be the exact centers-of-mass for the first and second protium atoms (and their nuclei), respectively: the atomic center-of-mass complex-coordinate-vectors \( O_1, O_2 \in X \) share the equidistant amplitude-radii \( |O_1| = |O_2| \sim I \) (where \( I \) corresponds to Santilli’s iso-unit scale of the iso-electronium [8, 9, 10, 11, 12, 13]) and the counterbalancing azimuthal-phases \( \langle O_1 \rangle = \pi = \langle O_2 \rangle \pm \pi \): using the synchronized 2D complex-coordinate-vector notation of [14, 15], the corresponding 2D Cartesian coordinate-vectors are \((-|O_1|, 0)_C\) and \((|O_2|, 0)_C\), while the respective 2D Polar coordinate-vectors are \((|O_1|, \pi)_P\) and \((|O_2|, 0)_P\) for the atomic and nucleic centers-of-mass. Moreover, in accordance to the Inopin-Schmidt baryon topology [14], let there exist the topological 1-spheres \( P_1 \subset X \) and \( P_2 \subset X \) with the proton’s amplitude-radii \( |P_1| = |P_2| = R_{\text{proton}} \) that are respectively centered on \( O_1, O_2 \in X \) to represent the hadronic horizon confinement boundaries and IHCRs [14] of the two pertinent protons of the protium pair. Here, to encode the proton states, we deploy the red-green-blue triangular lattice (RGB-TL) configurations of [14] on \( P_1, P_2 \subset X \), where the first proton’s RGB-TL locations are \( r_1, g_1, b_1 \in P_1 \) and the second proton’s RGB-TL locations are \( r_2, g_2, b_2 \in P_2 \)—see Figure 2. The idea is to use the RGB-TL locations at \( r_1, g_1, b_1 \in P_1 \) and \( r_2, g_2, b_2 \in P_2 \) to encode 3D structures in \( Y \) on the 2D surface of \( X \) along the 1D IHCR hadronic horizons of \( P_1, P_2 \subset X \) in a similar method to [14] that will comply with the iso-electronium [8, 9, 10, 11, 12, 13]—so in this iso-electronium case the RGB-TL locations do not represent the locations of “quarks” but instead represent a more general trilateral-based geometric platform for encoding the hadronic horizons (that are surrounded by the toroidal polarizations of [8, 9, 10, 11, 12, 13] starting in Step 3) for inferring \( Y \)’s 3D states from \( X \)’s 2D states and vice-versa.

3. Third, let there exist the topological 1-sphere IHCR’s \( E_1 \subset X \) and \( E_2 \subset X \) with amplitude-radii \( |E_1| = |E_2| = R_{\text{iso-electron}} \) that are centered on \( O_1, O_2 \in X \), respectively, to encode the orbits of the two pertinent valence iso-electrons located at \( e_1 \in E_1 \) and \( e_2 \in E_2 \) that pair to form the iso-electronium quasi-particle with the Santilli-Shillady strong valence bond [8, 9, 10, 11, 12, 13] between \( P_1, P_2 \subset X \). The amplitude-radii \( |E_1| \) and \( |E_2| \) precisely comply with Santilli’s iso-electronium wavepacket overlapping constraint for the “oo-shaped orbital” [8, 9, 10, 11, 12, 13]—see Figure 3. Hence, the two protium atoms acquire parallel-but-opposite magnetic polarities (with a null value at sufficient distance) to permit the characteristic diamagnetic alignment under an external strong magnetic field [8, 9, 10, 11, 12, 13]. Therefore, the two protium atoms form a singlet planar coupling on \( X \)—see Figure 4.

4. Fourth, upon recalling the work of [14], we equip the RGB-TL locations of the two protium atoms in the singlet planar coupling with topological deformation order parameters of fractional statistics for spontaneous superfluidic gauge symme-
Fig. 2: The pertinent aspects of the Inopin-Schmidt IHCR baryon topology and RGB-TL configuration of [14] are applied to the two protium atoms with IHCR hadronic horizons of $P_1, P_2 \subset X$ with amplitude-radius $|P_1| = |P_2| = R_{\text{proton}}$ and centers-of-mass located at $O_1, O_2 \in X$ that are equidistant from $O$.

In accordance to [14], the first protium’s proton has a hadronic horizon at $P_1$ with the three RGB-TL locations at $r_1, g_1, b_1 \in P_1$, while the second protium’s proton has a hadronic horizon at $P_2$ with the three RGB-TL locations at $r_2, g_2, b_2 \in P_2$. 
Fig. 3: The topology of the two protium atoms: the two valence iso-electrons pair to form the iso-electronium quasi-particle with the overlapping constraint for the oo-shaped orbital and the Santilli-Shillady strong valence bond [8, 9, 10, 11, 12, 13]. The first protium’s iso-electron is located at $e_1 \in E_1$ and the second protium’s iso-electron is located at $e_2 \in E_2$.

Fig. 4: The two protium atoms form a singlet planar coupling on $X$ because they acquire parallel-but-opposite magnetic polarities (with a null value at sufficient distance) to permit the characteristic diamagnetic alignment under an external strong magnetic field [8, 9, 10, 11, 12, 13].
try breaking to emphasize this initiated establishment in Santilli’s iso-electronium model [8, 9, 10, 11, 12, 13]. Thus, at \( r_1, g_1, b_1 \in P_1 \) and \( r_2, g_2, b_2 \in P_2 \) we assign the “Cooper paired” strongly conserved order parameters \( \psi_J \) (“total angular momentum” for spin-orbit coupling), \( \psi_I \) (“iso-spin”), and \( \psi_C \) (“color charge”) from eq. (26) of [14] and, in accordance to eqs. (32–36) of [14] to define the (complex-valued) full wavefunction states for \( P_1, P_2 \in X \) of the two protium atoms as

\[
\begin{align*}
\Psi_{\text{total}}(r_1, g_1, b_1) &= \Psi(r_1) \times \Psi(g_1) \times \Psi(b_1) \\
\Psi_{\text{total}}(r_2, g_2, b_2) &= \Psi(r_2) \times \Psi(g_2) \times \Psi(b_2),
\end{align*}
\]

where the comprising RGB-TL wavefunction states for \( P_1 \) are

\[
\begin{align*}
\Psi(r_1) &= \psi_J(r_1) \times \psi_I(r_1) \times \psi_C(r_1) \times r_1 \\
\Psi(g_1) &= \psi_J(g_1) \times \psi_I(g_1) \times \psi_C(g_1) \times g_1 \\
\Psi(b_1) &= \psi_J(b_1) \times \psi_I(b_1) \times \psi_C(b_1) \times b_1
\end{align*}
\]

and the comprising RGB-TL wavefunction states for \( P_2 \) are

\[
\begin{align*}
\Psi(r_2) &= \psi_J(r_2) \times \psi_I(r_2) \times \psi_C(r_2) \times r_2 \\
\Psi(g_2) &= \psi_J(g_2) \times \psi_I(g_2) \times \psi_C(g_2) \times g_2 \\
\Psi(b_2) &= \psi_J(b_2) \times \psi_I(b_2) \times \psi_C(b_2) \times b_2.
\end{align*}
\]

Again, we re-iterate that in this iso-electronium case the eqs. (1–3) do not apply to “quarks” as in [14], but instead serve as generalized RGB-TLs for encoding the hadronic horizons of \( P_1, P_2 \in X \) that are surrounded by the toroidal polarizations of [8, 9, 10, 11, 12, 13] in the 3D space \( Y \). Hence, the hadronic horizons of \( P_1, P_2 \in X \) circulate at the speed-of-light and simultaneously acquire geometric phases while the \( \psi_J, \psi_C, \) and \( \psi_I \) order parameters spontaneously generate effective mass for the simultaneous breaking of multiple gauge symmetries and rotate freely in 2D and/or 3D space and exhibit long range order in the shape of superfluidic toroidal helices, such that the Leggett superfluid B phase [14, 18] (the azimuthal-phase specific to the RGB-TL locations) between them remains constant and serves as an additional wavepacket wavefunction constraint [14]. For eqs. (1–3), the amplitude-radius variations in the underlying order parameters correspond to scalar amplitude-excitations, while the azimuthal-phase variations correspond to pseudo-scalar phase-excitations [14]. The spinon, holon, and orbiton excitations of eqs. (1–3) are spontaneously generated and confined to \( P_1 \) and \( P_2 \), which both simultaneously acquire geometric phases, where such perturbations are propagated across the micro space branes (the superluminal and non-local zones that are “inside” of \( P_1 \) and \( P_2 \), and correspond to Santilli’s interior dynamical systems) and the macro space brane (the non-superluminal and local zone that is “outside” of both \( P_1 \) and \( P_2 \), and corresponds to Santilli’s exterior dynamical system) in topological accordance to [14, 15, 16, 17]—see Figure 5.
Fig. 5: The hadronic horizons of $P_1, P_2 \in X$ circulate at the speed-of-light and simultaneously acquire geometric phases while the $\psi_I, \psi_C$, and $\psi_I$ order parameters spontaneously generate effective mass for the simultaneous breaking of multiple gauge symmetries and rotate freely in 2D and/or 3D space and exhibit long range order in the shape of superfluidic toroidal helices, such that Leggett’s superfluid B phase [14, 18] between the RGB-TL’s remains constant and serves as an additional wavepacket wavefunction constraint [14]. This important mechanism is directly connected to scalar amplitude-excitations and pseudo-scalar phase-excitations at the hadronic horizon of baryons [14]. Note: for the sake of illustration simplicity, only $g_1 \in P_1$ is shown.
5. Fifth, upon recalling the work of [14, 15, 17], the iso-electrons of the two protium atoms in the singlet planar coupling are equipped with topological deformation order parameters of fractional statistics to encode the iso-electronium’s toroidal distribution of the orbital [8, 9, 10, 11, 12, 13], which is due to Santilli’s iso-uncertainty principle of hadronic mechanics [6, 7, 19]. So, similarly to the protium’s proton case of eqs. (1–3), we hypothesize that the full iso-electron wavefunction states for the iso-electronium of the two protium atoms may be defined as

\[
\Psi_{total}(e_1) \equiv \Psi(e_1) = \psi_J(e_1) \times e_1
\]

\[
\Psi_{total}(e_2) \equiv \Psi(e_2) = \psi_J(e_2) \times e_2,
\]

where only the \( \psi_J \) for spin-orbit coupling applies to the iso-electronium because the comprising iso-electrons have neither iso-spin (\( \psi_I \)) nor color charge (\( \psi_C \))—see Figure 6. Therefore (similarly to \( P_1 \) and \( P_2 \) of the two RGB-TLs), the iso-electronium’s \( E_1 \) and \( E_2 \) are assigned spinon, holon, and orbiton excitations of fractional statistics for eq. (4) and simultaneously acquire geometric phases for the oo-orbital, where such perturbations are propagated across both the micro and macro space branes of [14, 15, 16, 17] and are inferred from the states of \( P_1 \) and \( P_2 \) because they are dual. Hence, as the massless iso-electron point-particles of the iso-electronium circulate along the oo-orbit of \( E_1 \) and \( E_2 \) at the speed-of-light, the \( \psi_J \)-type order parameters of eq. (4) vary to spontaneously generate scalar amplitude-excitations and pseudo-scalar phase-excitations as in [14] to form Santilli’s iso-electronium’s toroidal polarization distribution [8, 9, 10, 11, 12, 13]—see Figure 6.

At this point, we’ve completed the initial step for our topological deformation order parameter upgrade of the iso-electronium comprised of dual protium atoms that are interlocked with the singlet planar coupling, which is consistent with our hypothesis and the limited scope of this paper. Consequently, we will use these preliminary outcomes as a platform to launch subsequent papers that aim to further revise, scrutinize, develop, and improve this emerging model. For example, it will be important to consider the possibility of adding one or more constraints to eq. (4) that further quantify the iso-electronium’s characteristic diamagnetic alignment and oo-orbit of the iso-electron \( e_1 \in E_1 \) and \( e_2 \in E_2 \) positions and/or \( \psi_J \)-type order parameters in the singlet planar coupling.

3 Conclusion and discussion

In this work, we deployed some pertinent aspects of the Inopin-Schmidt baryon IHCR topology and RGB-TL order parameter configuration [14] as tools to forge a hypothetical upgrade to Santilli’s iso-electronium model [8, 9, 10, 11, 12, 13]. The preliminary theoretical, conceptual, and mathematical outcomes of the presented procedure exhibit a promising degree of support for the said conjecture. For this, we presented initial results that support the idea that the oo-shaped iso-electron orbit of the iso-electronium [8, 9, 10, 11, 12, 13] can be equipped with topological deformation order parameters of fractional statistics for the quasi-particle, which become excited (or de-excited) as the
Fig. 6: The circular iso-electron orbits of the iso-electronium are equipped with order parameters of fractional statistics to encode the topological deformations for the toroidal polarization and the wavepacket wavefunction. Similarly to $P_1$ and $P_2$, the iso-electronium’s iso-electrons at $e_1 \in E_1$ and $e_2 \in E_2$ are massless point-particles that circulate at the speed-of-light along the oo-orbit while spontaneously generating effective mass as $E_1$ and $E_2$ simultaneously acquire geometric phases. This important mechanism is directly connected to scalar amplitude-excitations and pseudo-scalar phase-excitations for the iso-electronium, like in the proton’s case [14].
orbit iteratively acquires a geometric phase [14]. More specifically, we illustrated that the toroidal polarization of the iso-electronium can be further encoded with topological deformation order parameters for spontaneous superfluidic gauge symmetry breaking, where the parameters are correlated with Leggett’s superfluid B phase [14, 18] and generate correlated helices with long range order [14]; it is these spontaneously generated correlated helices that may effectively represent the states and transitions of the iso-electronium’s toroidal polarization [8, 9, 10, 11, 12, 13]. Moreover, we utilized the order parameters to construct a preliminary set of iso-electronium wavepacket wavefunctions, where amplitude-radius and azimuthal-phase variations in the (complex-valued) order parameters are connected to scalar amplitude-excitations and pseudo-scalar phase-excitations, respectively [14]. Thus, in our opinion, the hypothetical merger and consolidation of these constructions is compelling.

In upcoming future research, this hypothesis must be subjected to additional rigorous scrutiny via the scientific method, which is surely necessary in order to compel the general advancement of these models and systems. In this paper, we considered the simple base case of the iso-electronium molecule with the dual protium atoms interlocked in the singlet planar coupling [8, 9, 10, 11, 12, 13], so in near future work we plan to apply this hypothetical order parameter upgrade to Santilli’s magneecules [20, 19, 8, 21, 22, 23, 9, 24, 25] to initiate further consideration of sustainable, over-unity, clean energy systems such as Intermediate Controlled Nuclear Fusion [26, 27, 28, 29] which does not emit waste with harmful toxins and/or dangerous levels of radiation. As scientists, it is absolutely critical that all such avenues are explored to unlock such capabilities by actualizing and maximizing the technological and industrial applications of the said disciplines. Therefore, it is the opinion of the authors that any work in this realm is significant.

4 Acknowledgment

We wish to thank the R.M. Santilli Foundation for funding our trip to the 2013 ICNAAM in Rhodes, Greece and Professor R.M. Santilli for his discussions on iso-electronium, magneecules, MagneGas Fuel, and Intermediate Controlled Nuclear Fusion.
References
[19] R. M. Santilli. Physics of new clean energies and fuels according to hadronic me-


