ON SMARANDACHE SIMPLE CONTINUED FRACTIONS

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Abstract. Let \( A = \{a_n\}_{n=1}^{\infty} \) be a Smarandache type sequence. In this paper we show that if \( A \) is a positive integer sequence, then the simple continued fraction \([a_1, a_2, \ldots]\) is convergent.

Let \( A = \{a_n\}_{n=1}^{\infty} \) be a Smarandache type sequence. Then

The simple continued fraction

\[
\frac{1}{a_1 + \frac{1}{a_2 + \frac{1}{a_3 + \cdots}}}
\]

(1)

is called the Smarandache simple continued fraction associated \( A \) (See [1]). By the usually symbol (see [2, Notion 10.1]), the continued fraction (1) can be written as \([a_1, a_2, a_3, \ldots]\).

Recently, Castillo [1] posed the following question:

Question. Is the continued fraction (1) convergent? In particular, is the continued fraction \([1, 12, 123, \ldots]\) convergent?

In this paper we give a positive answer as follows.

Theorem. If \( A \) is a positive integer sequence, then the

¹Editor's Note (M.L.Perez): This article has been done by each of the above authors independently.
continued fraction \( (1) \) is convergent.

Proof. If \( A \) is a positive integer sequence, then \( (1) \) is a usually simple continued fraction and its quotient are positive integers. Therefore, by [2,Theorem165], it is convergent. The Theorem is proved.

On applying [2, Theorems 165 and 176], we get a further result immediately.

Theorem 2. If \( A \) is an infinite positive integer sequence, then \( (1) \) is equal to an irrational number \( \alpha \). Further, if \( A \) is not periodic, then \( \alpha \) is not an algebraic number of degree two.

References