\[ SP(n) = \min\{k : n | k^k\} \] is the case of \( f(k) = k^k \). We note that the Definitions 39 and 40 give the particular case of \( S_t \) for \( t = 2 \) and \( t = 3 \).

In our paper we have introduced also the following "dual" of \( F_j \). Let \( g : \mathbb{N}^* \rightarrow \mathbb{N}^* \) be a given arithmetical function, which satisfies the following assumption:

\((P_3)\) For each \( n \geq 1 \) there exists \( k \geq 1 \) such that \( g(k) | n \).

Let \( G_g : \mathbb{N}^* \rightarrow \mathbb{N}^* \) defined by

\[ G_g(n) = \max\{k \in \mathbb{N}^*: g(k) | n\}. \quad (2) \]

Since \( k^t | n, k!! | n, k^k | n, \frac{k(k + 1)}{2} | n \) all are verified for \( k = 1 \), property \((P_3)\) is satisfied, so we can define the following duals of the above considered functions:

\[ S^*_1(n) = \max\{k : k^t | n\}; \]
\[ SDF^*(n) = \max\{k : k!! | n\}; \]
\[ SP^*(n) = \max\{k : k^k | n\}; \]
\[ Z^*(n) = \max\left\{k : \frac{k(k + 1)}{2} | n\right\}. \]

These functions are particular cases of (2), and they could deserve a further study, as well.

**References**


SMARANDACHE STAR (STIRLING) DERIVED SEQUENCES
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Let \( b_1, b_2, b_3, \ldots \) be a sequence say \( S_b \) the base sequence. Then the Smarandache star derived sequence \( S_d \) using the following star triangle (ref. [1]) is defined

\[
\begin{array}{cccc}
1 \\
1 & 1 \\
1 & 3 & 1 \\
1 & 7 & 6 & 1 \\
1 & 15 & 25 & 10 & 1 \\
\ldots \\
\end{array}
\]

as follows
\[
d_1 = b_1 \\
d_2 = b_1 + b_2 \\
d_3 = b_1 + 3b_2 + b_3 \\
d_4 = b_1 + 7b_2 + 6b_3 + b_4 \\
\ldots \\

d_{n+1} = \sum_{k=0}^{n} a_{(m,r)} \cdot b_{k+1}
\]

where \( a_{(m,r)} \) is given by
\[
a_{(m,r)} = \left(\frac{1}{r!}\right) \sum_{t=0}^{r} (-1)^{r-t} \cdot C_1 \cdot t^m, \text{ Ref. [1]}
\]

e.g. (1) If the base sequence \( S_b \) is 1, 1, 1, \ldots then the derived sequence \( S_d \) is 1, 2, 5, 15, 52, \ldots , i.e. the sequence of Bell numbers. \( T_n = B_n \)

(2) \( S_b \rightarrow 1, 2, 3, 4, \ldots \) then \( S_d \rightarrow 1, 3, 10, 37, \ldots \), we have \( T_n = B_{n+1} - B_n \). Ref [1]

The Significance of the above transformation will be clear when we consider the inverse transformation. It is evident that the star triangle is nothing but the Stirling Numbers of the Second kind (Ref. [2]). Consider the inverse Transformation: Given the Smarandache Star Derived Sequence \( S_d \), to retrieve the original base sequence \( S_b \). We get \( b_k \) for \( k = 1, 2, 3, 4 \) etc. as follows:
\[
b_1 = d_1 \\
b_2 = -d_1 + d_2 \\
b_3 = 2d_1 - 3d_2 + d_3 \\
b_4 = -6d_1 + 11d_2 - 6d_3 + d_4 \\
b_5 = 24d_1 - 50d_2 + 35d_3 - 10d_4 + d_5 \\
\ldots \ldots \ldots \\
\]

we notice that the triangle of coefficients is

\[
\begin{array}{cccc}
1 \\
-1 & 1 \\
\end{array}
\]
Which are nothing but the Stirling numbers of the first kind.

Some of the properties are

1. The first column numbers are \((-1)^{r-1}(r-1)!\), where \(r\) is the row number.
2. Sum of the numbers of each row is zero.
3. Sum of the absolute values of the terms in the \(r\)th row = \(r!\).

More properties can be found in Ref. [2].

This provides us with a relationship between the Stirling numbers of the first kind and that of the second kind, which can be better expressed in the form of a matrix.

Let \([b_{1,k}]_{1x1}\) be the row matrix of the base sequence.

\([d_{1,k}]_{1x1}\) be the row matrix of the derived sequence.

\([S_{j,k}]_{n\times n}\) be a square matrix of order \(n\) in which \(s_{j,k}\) is the \(k\)th number in the \(j\)th row of the star triangle (array of the Stirling numbers of the second kind, Ref. [2]).

Then we have

\([T_{j,k}]_{n\times n}\) be a square matrix of order \(n\) in which \(t_{j,k}\) is the \(k\)th number in the \(j\)th row of the array of the Stirling numbers of the first kind, Ref. [2].

Then we have

\([b_{1,k}]_{1x1}\ast[S_{j,k}]_{n\times n}=[d_{1,k}]_{1x1}\)

\([d_{1,k}]_{1x1}\ast[T_{j,k}]_{n\times n}=[b_{1,k}]_{1x1}\)

Which suggests that \([T_{j,k}]_{n\times n}\) is the transpose of the inverse of the transpose of the Matrix \([S_{j,k}]_{n\times n}\).

The proof of the above proposition is inherent in theorem 10.1 of ref. [3].

Readers can try proofs by a combinatorial approach or otherwise.

REFERENCES: