A SUM CONCERNING SEQUENCES

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Abstract. Let $A=\{a(n)\}_{n=1}^{\infty}$ be a sequence of positive integers. In this paper we prove that if the trailing digit of $a(n)$ is not zero for any $n$, then sum of $a(n)/\text{Rev}(a(n))$ is divergent.

Key words. decimal number, reverse, sequence of positive integers.

Let $a=a_{m}\ldots a_{2}a_{1}$ be a decimal number. Then the decimal number $a_{1}a_{2}\ldots a_{m}$ is called the reverse of $a$ and denote by $\text{Rev}(a)$. For example, if $a=123$, then $\text{Rev}(a)=321$. Let $S=\{s(n)\}_{n=1}^{\infty}$ be a certain Smarandache sequence such that $s(n)>0$ for any positive integer $n$. In [1], Russo that proposed to study the limit

$$L(s)=\lim_{N \to \infty} \sum_{n=1}^{N} \frac{s(n)}{\text{Rev}(s(n))}.$$ (1)

In this paper we prove a general result as follows.

Theorem. Let $A=\{a(n)\}_{n=1}^{\infty}$ be a sequence of positive integers if the trailing digit of $a(n)$ is not zero for any $n$, then the sum of $a(n)/\text{Rev}(a(n))$ is divergent.

Proof. Let $a(n)=a_{m}\ldots a_{2}a_{1}$, where $a_{1} \neq 0$. Then we have

$$\text{Rev}(a(n))=a_{1}a_{2}\ldots a_{m}. \quad (2)$$

We see from (2) that

$$\frac{a(n)}{\text{Rev}(a(n))} > \frac{1}{10}. \quad (3)$$

Thus, by (3), the sum of $a(n)/\text{Rev}(a(n))$ is divergent. The
theorem is proved.

References


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