ON THE SMARANDACHE UNIFORM SEQUENCES

Maohua Le

Abstract. Let \( t \) be a positive integer with \( t > 1 \). In this paper we give a necessary and sufficient condition for \( t \) to have the Smarandache uniform sequence.

Key words. Smarandache uniform sequence, decimal notation.

Let \( t \) be a positive integer with \( t > 1 \). If a sequence contains all multiples of \( t \) written with same digit in base 10, then it is called the Smarandache uniform sequence of \( t \). In [2], Smith showed that such sequence may be empty for some \( t \).

In this paper we give a necessary and sufficient condition for \( t \) to have the Smarandache uniform sequence. Clearly, the positive integer \( t \) can be expressed as

\[
t = 2^a 5^b c,
\]

where \( a, b \) are nonnegative integers, \( c \) is a positive integer satisfying \( \gcd(10, c) = 1 \). We prove the following result.

Theorem. \( t \) has the Smarandache uniform sequence if and only if

\[
(a, b) = (0, 0), (1, 0), (2, 0), (3, 0), (0, 1).
\]

Proof. Clearly, \( t \) has the Smarandache uniform sequence if and only if there exists a multiple \( m \) of \( t \) such that

\[
m = d d \ldots d, \ 1 \leq d \leq 9.
\]
By (1) and (3), we get

\[ ts = 2^q b \frac{10^r - 1}{10 - 1}, \]

where \( r, s \) are positive integers. From (4), we obtain

\[ 2^q b c s = d (10^r - 1). \]

Since \( \gcd (2^q b, 10^r - 1) = 1 \), we see from (5) that \( d \) is a multiple of \( 2^q b \). Therefore, since \( 1 \leq d \leq 9 \), we obtain the condition (2).

On the other hand, since \( \gcd (10, q) = 1 \), by Fermat-Euler theorem (see [1, Theorem 72]), there exists a positive integer \( r \) such that \( 10^r - 1 \) is a multiple of \( q \). Thus, if (2) holds, then \( t \) has Smarandache uniform sequence. The theorem is proved.

References


Department of Mathematics
Zhanjiang Normal College
Zhanjiang, Guangdong
P.R. CHINA